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Peculiar Properties of Tachyon Signals

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The gedanken experiment as introduced recently by Pirani to demonstrate the noncausal behavior of classical tachyons is reconsidered and analyzed in Minkowski space. Upon adhering to the principles used by Sudarshan and Feinberg with respect to the possible existence of tachyons, it is shown that the noncausal behavior of classical tachyons as deduced from this experiment is inconclusive.

I. INTRODUCTION

Recently Pirani[1] has proposed a gedanken experiment, which he claims to be different from those that have been discussed by Bilaniuk, Deshpande, and Sudarshan[2] and Feinberg,[3] and many others,[4] to demonstrate the noncausal behavior of classical tachyons. Feinberg's original article was concerned in part with the possibility of the existence of causal anomalies with tachyon signals. In order to be precise we wish to quote Feinberg's remarkable conclusion: "Therefore, while it does appear possible to construct kinematic closed cycles using tachyons in which signals are sent back to the past, a careful examine [examen] of the methods of detection, with due regard to the interpretation of absorption of negative-energy tachyons as emission of positive-energy tachyons, leads to the conclusion that such closed cycles will not be interpreted as reciprocal signaling, but rather as uncorrelated spontaneous emission. It therefore does not appear that causal anomalies can be used as an argument against the existence of tachyons." On the other hand, Pirani has pointed out that the experiments discussed by Feinberg cannot lead to violations of causality because they involved only one space dimension; whereas he has discovered that it is possible to construct thought experiments in two or more space dimensions so that the difficulties of interpreting the behavior of tachyon receivers which may become spontaneous emitters can be avoided and yet still lead to causality violations. In other words, it is possible to arrange observers in two space dimensions so that each observer definitely agrees that his apparatus has *received* and *emitted* a tachyon. However, upon a careful analysis, we find that his results on causality violations as deduced from these thought experiments are inconclusive.

The plan of the present article runs as follows: We first summarize the essential ingredients of Pirani's thought experiment and then we translate his anticipated observations, pieced together by four different observers into Minkowski space as viewed by a single observer. Such a representation, in our opinion, clarifies the question on the causal behavior of tachyons, and it further enables one to generate all other equivalent descriptions (consistent with the principle of the special theory of relativity) by means of Lorentz transformations. In particular, for a curve in a spacelike region, one observer may interpret the evolution of the space-time arrow as proceeding from past to future, but for another observer the world line will exhibit a discontinuity in the space-time arrow,[5] hence giving rise to the lack of Lorentz invariance of the number of tachyons for all observers. Such a phenomenon provides the noncausal correlation associated with the tachyon signals received and emitted by the observer, hence leaving Pirani's thought experiment subject to question.

II. GEDANKEN EXPERIMENT IN TWO SPACE DIMENSIONS

According to Pirani, four observers A, B, C, and D are moving with prescribed velocities relative to an observer S. Each of these observers is instructed in advance to emit a tachyon as soon as one is received from another observer. From the point of view of each observer, the energy of the tachyons interacting with his apparatus is always measured to be positive. The over-all view of the experiment is represented in Fig. 1. The oblique arrows with respect to the coordinate directions represent the velocities of each observer relative to the observer S. The observer and tachyon velocities are chosen in such a way that according to S all the tachyon arrows are pointing in opposite directions to those specified in Fig. 1. It is important to note that the figure below does not represent the actual description by one observer, but the tachyon arrows show the sense of motion as seen by each observer for those tachyons which interact with his apparatus. In fact, Pirani's experiment, if described in words, would proceed as follows:

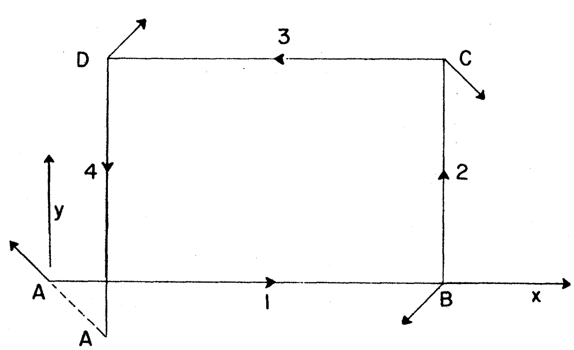

FIG. 1. Pirani's thought experiment as pieced together by the four observers A, B, C, and D. The oblique arrows denote the observer three-velocities and the numbered lines denote the tachyon paths.

TABLE I. Kinematic table: kinematic quantities as observed by A which are obtained under Lorentz transformations from the quantities specified in S from Pirani's thought experiment. $F_\pm = 1 \pm (1-2\omega^2/c^2)^{1/2}$, $I_\pm = (1-H)v\omega/c^2 \pm 1$, $J_\pm = 1 \pm v\omega/c^2$, $G = 1/(1+2\omega^2/c^2)^{1/2}$, $H = (1-2\omega^2/c^2)^{1/2}$.

Observers	Kinematic quantity	A	B	C	D	1	2	3	4
S	Relative velocity components (x, y)	$(-\omega, \omega)$	$(-\omega, -\omega)$	$(\omega, -\omega)$	(ω, ω)	$(-v, 0)$	$(0, -v)$	$(v, 0)$	$(0, v)$
	Tachyon path lengths					$(-l_1, 0)$	$(0, -l_2)$	$(l_3, 0)$	$(0, l_4)$
	Energy of tachyons					E	E	E	E
A	Relative velocity components (x, y)	$(0, 0)$	$(\omega F_-, -\omega F_+)$	$(2\omega G, -2\omega G)$	$(\omega F_+, -\omega F_-)$	$\left(\frac{-vH-\omega I_-}{J_-}, \frac{\omega I_-}{J_-}\right)$	$\left(\frac{\omega I_+}{J_+}, \frac{-vH-\omega I_+}{J_+}\right)$	$\left(\frac{vH+\omega I_+}{J_+}, \frac{-\omega I_+}{J_+}\right)$	$\left(\frac{-\omega I_-}{J_-}, \frac{vH+\omega I_-}{J_-}\right)$
	Time interval of tachyon signals with condition $v\omega > c^2$					$t_{A1} = \frac{l_1 J_-}{vH}$	$t_{A2} = \frac{l_2 J_+}{vH}$	$t_{A3} = \frac{l_3 J_+}{vH}$	$t_{A4} = \frac{l_4 J_-}{vH}$
	Energy of tachyon signals with condition $v\omega > c^2$					$E_{A1} = \frac{EJ_-}{vH}$	$E_{A2} = \frac{EJ_+}{vH}$	$E_{A3} = \frac{EJ_+}{vH}$	$E_{A4} = \frac{EJ_-}{vH}$

(1) Observer A initiates the experiment by emitting tachyon 1 to B.

(2) Upon reception B immediately emits tachyon 2 to C.

(3) Upon reception C immediately emits tachyon 3 to D.

(4) Upon reception D immediately sends tachyon 4 back to A with the result that A receives tachyon 4 before the initiation of tachyon 1. Upon reception of tachyon 4, A destroys his emitter and thereby prevents the emission of tachyon 1 which initiated the complete tachyon loop, hence the causality violation.

In order to infer any conclusion about the causality violation from this experiment, it is necessary to have a complete description of the over-all experiment by one observer who could correlate the tachyons emitted and absorbed by his apparatus. In other words, we are looking for the space-time description of the four exchanged tachyons by the observer A, which can be obtained from S by means of a Lorentz transformation. In our opinion, the most convenient way of representation is to translate these kinematic properties into Minkowski space. The primary reason for doing this is to generate all the Lorentz equivalent descriptions from this one. Since the role of each observer is to provide an immediate exchange of a tachyon, then from a dynamical point of view, this is equivalent to the scattering of the tachyon by an external force field (for example, an atom which is capable of tachyonic emission and absorption). Therefore, we can represent A's observations as multiple scattering events in Minkowski space as shown in Fig. 2, where the world-point vertices B, C, D represent the interactions of the exchanged tachyons with the external field. The direction of the space-time arrows indicates the direction of propagation of positive-energy tachyons. The kinematic properties of the four tachyons as viewed by the observers S and A are summarized in Table I.

From Fig. 2 we see that tachyon 4 is absorbed before the emission of tachyon 1 which is in contradiction with the prescribed instructions. Since tachyon 1 was supposed to initiate the absorption of tachyon 4, then apparently one may infer the noncausal behavior of tachyons, which is the conclusion drawn by Pirani in a slightly different way. However, according to the principle of relativity, once a given sequence of events has been observed by any Lorentz observer, then each of the other Lorentz observers is entitled to his own interpretation which, of course, is related to the original description by a Lorentz transformation; consequently, we shall investigate how observer A would interpret the experiment.

According to A, the two most remarkable features concerning the sequence of events are the discontinuity of the space-time arrows and the time ordering of a given sequence of events as compared to observer S. The physical interpretation of such a discontinuity corresponds to the emission of two tachyons at D and the absorption of two tachyons at B. These processes, in which the number of tachyons is relative, are necessary in order to keep the energy of all the tachyons positive as measured by A. Furthermore, it is evident from the space-time diagram that the initial and the final states for A are different than for S. Both of these phenomena have been clarified in elaborate

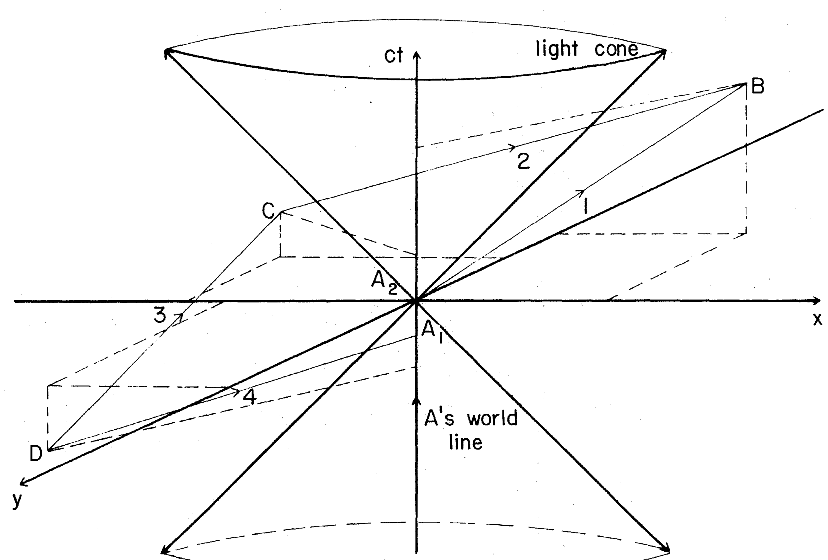

FIG. 2. Minkowskian representation of A's observations of the multiple scattering events of Pirani's thought experiment. A_1 and A_2 represent the absorption and emission of tachyons 4 and 1, respectively. D represents the emission of tachyons 4 and 3 in the initial state, with a subsequent scattering of tachyon 3 into 2, and the absorption of tachyons 1 and 2 at B in the final state.

detail by Sudarshan and Feinberg.

In view of the processes just discussed, observer A would describe the sequence of events[6] as follows:

(1) At the world point D, a pair of tachyons (i.e., tachyons 3 and 4) are emitted by the external field. Tachyon 4 proceeds from D to be absorbed at A_1, and tachyon 3 proceeds from D to be scattered at C.

(2) At the world point A_2, A emits tachyon 1 to be absorbed at B simultaneously with tachyon 3, which has been scattered at C.

(3) According to observer A, the most remarkable features involved in this sequence of events are the initial pair of tachyons at D and the final pair of tachyons at B.

(4) The above deduction allows us to conclude that according to observer A there is no causal correlation between tachyon 1 and tachyon 4, hence the instructions which Pirani has introduced do not comply with the observations of A. In other words, according to A, tachyon 1 is not the initiation of the absorption of tachyon 4, and hence the effect does not precede the cause.

Now with regard to the destruction of A's emitter at world point A_1, we return to observer S who interprets a normal series of events. According to S if A destroys his emitter after emitting tachyon 4 to D, then the continuity of space-time arrows will result in a tachyon being emitted by B and proceeding out to infinity, past observer A. Now suppose we transform S's description of the events to observer A and represent A's description in Minkowski space, then A will interpret the events in the following way:

(1) Due to a remote tachyon source at infinity,[7] which is the initial state in this situation, A observes an absorption of tachyon 4 from world point D at A_1, and tachyon 1 passes him at A_2.

(2) The description of the remaining events is the same as before with B remaining as the final state for observer A, and hence we have a perfectly normal situation.

III. CONCLUSIONS

Although Pirani's method of making *each* observer agree that his apparatus has definitely received and emitted a tachyon is quite ingenious, it does not necessarily imply that each observer interprets the others that way. In fact, we can classify all the possible descriptions of the tachyon signals as described by all Lorentz observers into two classes. The first class of descriptions corresponds to an emission at A_1 and then an absorption at A_2; we call this class of descriptions the *normal* class. A member in the normal class consists of continuous space-time arrows along the interconnected series of spacelike world lines. The second class corresponds to an absorption at A_1 and then an emission at A_2; we may call this class the *abnormal* class. Observer A's description belongs to this class. Each member of this class has the impression of a causal anomaly. However, we find that the space-time arrows of each member of this class are discontinuous and thus lead to an uncorrelated tachyon signal. Just as the concepts of tachyon emission and tachyon absorption were found to be relative, likewise, we find that the concept of a correlated signal for tachyons is not a Lorentz-invariant concept. In other words, a correlated tachyon signal (continuous space-time arrows) for one observer may be uncorrelated (discontinuous space-time arrows) for other observers. Because of the existence of this rather peculiar property of tachyon signals[8] the difficulties of causal anomalies can be avoided.

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[8]The word *signal* has the connotation of conveying reliable information. However, we have just argued that tachyons do not possess such a property; therefore it is used figuratively here. We would like to thank Professor Feinberg for pointing this out to us.