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PHYSICAL REVIEW D VOLUME 4, NUMBER 1 1 JULY 1971

K_{l3} Decay, Elastic Unitarity, and the Symmetry-Breaking Parameter of the Vacuum States

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K_{l3} form factors are derived in the hard-pion approach without using vector- or axial-vector-meson dominance and with more general consideration of symmetry breaking. Elastic unitarity is employed for the vector form factor. The ratio $f_-(0)/f_+(0)$ is expressed in terms of the symmetry-breaking parameter, $b=\langle 0|u_8|0\rangle/\langle 0|u_0|0\rangle$. It is found that current algebra can provide a satisfactory explanation for the decay parameters.

I. INTRODUCTION

The current-algebra approach towards hadron physics has proved to be most fruitful. Applied to the problem of K_{l3} decay there have been numerous theoretical treatises since the work of Callan and Treiman.[1] However, a good deal remains to be said on the subject because of the ill-defined nature of symmetry breaking and the uncertainty in the experimental data involved. Recently, the hard-pion approach of Schnitzer, Weinberg, and Gerstein[2] has been used to derive an effective-range formula for the pion form factor with the application of the principles of unitarity.[3] Also, the symmetry-breaking argument of Gell-Mann, Oakes, and Renner[4] has been challenged and extended.[5] With this and the updated experimental numbers in mind, we intend to review and formulate the K_{l3} problem without using the vector-meson or axial-vector-meson dominance approximation.

This being an on-shell current-algebra calculation, much of the ambiguity involving soft-pion extrapolation can be eliminated. Unlike many previous results, our finding shows that the ratio of the form factors, $f_-(0)/f_+(0)$, is largely determined by the symmetry-breaking parameter of the vacuum state and favors the result given by the X2 Collaborations,[6] with λ_+ about 0.029 and $\xi = f_-(0)/f_+(0) = -0.65$. In Sec. II we introduce the commutation relations, the spectral representation of the two-point functions, and we define the relevant three-point functions. Section III deals with the solution of the Ward identities, which gives rise to expressions for the form factors, $f_+(s)$ and $f_-(s)$. In Sec. IV, the effective-range formula for $f_+(s)$ is proposed and the ratio $f_-(0)/f_+(0)$ is estimated to be in agreement with experiment.

II. TWO-POINT FUNCTIONS AND COMMUTATION RELATIONS

We consider the decay matrix element $\langle \pi^0(q)K^+(p)|V_c|0\rangle$ with the $SU(3)$ quantum numbers of π^0 and K^+ denoted by a and b, respectively, and $V_c = (V_4 + iV_5)/\sqrt{2}$. The following spectral representations of the two-point functions are relevant. For vector currents,

$$\int dx\, e^{-ikx}\langle T\{V_b^\nu(x), V_c^\lambda(0)\}\rangle_0 = \tfrac{1}{2}\left\{-i\Delta_{bc}^V(k)^{\nu\lambda} - ik^\nu k^\lambda \int \frac{dx}{x+k^2}\,\sigma_{bc}^S(x) + i\eta^\nu\eta^\lambda\left[C_{bc}^V + \int dx\,\sigma_{bc}^S(x)\right]\right\}, \tag{1}$$

$$\int dx\, e^{-ikx}\langle T\{\partial^\nu V_b^\nu(x), V_c^\lambda(0)\}\rangle_0 = -\tfrac{1}{2}k^\lambda \int \frac{dx\, x}{x+k^2}\,\sigma_{bc}^S(x), \tag{2}$$

while for axial-vector currents,

$$\int dy\, e^{-ipy}\langle T\{A_b^\nu(y), A_c^\lambda(0)\}\rangle_0 = \tfrac{1}{2}\left\{-i\,\Delta_{bc}^A(p)^{\nu\lambda} - iq^\nu q^\lambda \int \frac{dx}{x+p^2}\,\sigma_{bc}^P(x) + i\eta^\nu\eta^\lambda\left[C_{bc}^A + \int dx\,\sigma_{bc}^P(x)\right]\right\}, \tag{3}$$

$$\int dy\, e^{-ipy}\langle T\{\partial^\nu A_b^\nu(y), A_c^\lambda(0)\}\rangle_0 = -\tfrac{1}{2}p^\lambda \int \frac{dx\, x}{x+p^2}\,\sigma_{bc}^P(x), \tag{4}$$

where

$$\Delta_{bc}^{V,A}(k) = \int \frac{dx}{x+k^2}\,\rho_{bc}^{V,A}\left(\delta^{\nu\lambda} + \frac{k^\nu k^\lambda}{x}\right), \tag{5}$$

$$C_{bc}^{V,A} = \int \frac{dx}{x}\,\rho_{bc}^{V,A}, \tag{6}$$

ρ^V (ρ^A) and σ^S (σ^P) are the vector and scalar spectral functions, and $\eta^\nu = (0,0,0,1)$ is a unit timelike vector.

Now the three-point functions of currents, W_{abc}, are defined with the pion and kaon poles explicitly separated out.

$$W_{abc}^\lambda = \int dx dy\, e^{-iqx}e^{-ipy}\langle T\{\partial^\mu A_a^\mu(x), \partial^\nu A_b^\nu(y), V_c^\lambda(0)\}\rangle_0 = -\frac{f_\pi f_K m_\pi^2 m_K^2}{2(q^2+m_\pi^2)(p^2+m_K^2)}\,F_{abc}^\lambda, \tag{7}$$

$$W_{abc}^{\nu\lambda} = \int dx dy\, e^{-iqx}e^{-ipy}\langle T\{\partial^\mu A_a^\mu(x), A_b^\nu(y), V_c^\lambda(0)\}\rangle_0 = -i\frac{f_\pi m_\pi^2}{q^2+m_\pi^2}\left[F_{abc}^{\nu\lambda} + \frac{f_K}{2(p^2+m_K^2)}p^\nu F_{abc}^\lambda\right], \tag{8}$$

$$W_{abc}^{\mu\nu\lambda} = \int dx dy\, e^{-iqx}e^{-ipy}\langle T\{A_a^\mu(x), A_b^\nu(y), V_c^\lambda(0)\}\rangle_0 = F_{abc}^{\mu\nu\lambda} + \frac{f_K}{p^2+m_K^2}p^\nu F_{bac}^{\mu\lambda} + \frac{f_\pi}{q^2+m_\pi^2}q^\mu F_{abc}^{\nu\lambda} + \frac{f_\pi f_K}{2(q^2+m_\pi^2)(p^2+m_K^2)}q^\mu p^\nu F_{abc}^\lambda, \tag{9}$$

where f_π and f_K are the charged-pion and -kaon decay constants. The K_{l3} matrix element considered is just the vertex function F_{abc}^λ when the mesons are on their mass shell. Thus, on shell,

$$F_{abc}^\lambda \sim \tfrac{1}{2}[f_+(s)(q-p)^\lambda - f_-(s)k^\lambda] \tag{10}$$

and the form factors are

$$f_\pm(s) = f_\pm(0)\left(1 + \lambda_\pm \frac{s}{m_\pi^2}\right), \tag{11}$$

following the conventional parametrization and setting $s = -k^2 = -(p+q)^2$.

The commutation relations between the charges of the weak currents and the scalar and pseudoscalar densities belonging to the $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)\times SU(3)$ algebra have been given by Gell-Mann.[7]

$$[F_i, u_j] = if_{ijk}u_k, \tag{12}$$

$$[F_i^5, v_j] = id_{ijk}u_k. \tag{13}$$

Specifically for this work, the commutation relations

$$\delta(x^0-y^0)\{[A_a^0(x), \partial^\nu A_b^\nu(y)] + [A_b^0(y), \partial^\mu A_a^\mu(x)]\} = [-\tfrac{1}{2}\partial^\nu V_b^\nu(y) + i\beta_\pi u_b(y)]\,\delta(x-y) \tag{14}$$

are assumed, where β_π is defined in the partially conserved axial-vector current (PCAC) hypothesis as

$$\partial^\mu A_a^\mu(x) = \beta_\pi v_a(x). \tag{15}$$

In the model of Gell-Mann, Oakes, and Renner (with the symmetry-breaking hadronic Hamiltonian density, $\mathcal{H}' = -\epsilon_0 u_0 - \epsilon_8 u_8$)

$$\beta_\pi = (\tfrac{2}{3})^{1/2}\epsilon_0 + (\tfrac{1}{3})^{1/2}\epsilon_8. \tag{16}$$

The commutation relation (14) is derivable based on such Hamiltonian density. However, it remains true even if there should be an extra piece of $\mathcal{H}'$ belonging to the (1, 8) and (8, 1) representations. The possibility of such addition has been suggested. Thus we have in mind the more general symmetry-breaking Hamiltonian by assuming (14).

We further assume that the scalar and pseudoscalar densities are proportional to their respective fields, namely,

$$v_a = \chi_\pi \varphi_\pi, \tag{17}$$

$$u_b = \chi_K \varphi_K. \tag{18}$$

Then with these definitions for the π-meson and κ-meson fields, Eqs. (12) and (13) enable us to conclude that in the pole model,

$$\frac{\chi_K F_K}{\chi_\pi f_\pi} = \frac{3b}{i(2\sqrt{2}+2b)}, \tag{19}$$

where

$$b = \frac{\langle 0|u_8|0\rangle}{\langle 0|u_0|0\rangle} \tag{20}$$

is the symmetry-breaking parameter of the vacuum states.

III. WARD IDENTITIES

An independent set of the Ward identities for the three-point functions of currents follows from the standard current-algebra commutation relations and technique.

$$iq^\mu W_{abc}^{\mu\nu\lambda} = W_{abc}^{\nu\lambda} + if_{abd}\int dx\, e^{-ikx}\langle T\{V_d^\nu(x), V_c^\lambda(0)\}\rangle_0 + if_{ace}\int dy\, e^{-ipy}\langle T\{A_b^\nu(y), A_e^\lambda(0)\}\rangle_0, \tag{21}$$

$$ip^\nu W_{abc}^{\nu\lambda} = W_{abc}^{\lambda} + \int dxdy\, e^{-iqx}e^{-ipy}\delta(x^0-y^0)\langle T\{[A_b^0(y), \partial^\mu A_a^\mu(x)], V_c^\lambda(0)\}\rangle_0 + if_{bcg}\int dx\, e^{-iqx}\langle T\{\partial^\mu A_a^\mu(x), A_g^\lambda(0)\}\rangle_0, \tag{22}$$

$$iq^\mu W_{abc}^{\mu\lambda} = W_{abc}^{\lambda} + \int dxdy\, e^{-iqx}e^{-ipy}\delta(x^0-y^0)\langle T\{[A_a^0(x), \partial^\nu A_b^\nu(y)], V_c^\lambda(0)\}\rangle_0 + if_{acg}\int dy\, e^{-ipy}\langle T\{\partial^\nu A_b^\nu(y), A_g^\lambda(0)\}\rangle_0. \tag{23}$$

The result which emerges from the Ward identities is a solution for the vertex function F_{abc}^λ in terms of the two-point functions of currents and an unknown function $F_{abc}^{\mu\nu\lambda}$ related to the primitive function, as given by Gerstein and Schnitzer.[8] We give below the symmetrized result after the application of Eqs. (1)–(11) and Eq. (14).

$$\lim_{q^2\to -m_\pi^2;\; p^2\to -m_K^2} q^\mu p^\nu F_{abc}^{\mu\nu\lambda}$$

$$= \tfrac{1}{4}f_\pi f_K[f_+(s)(q-p)^\lambda - f_-(s)k^\lambda] + \tfrac{1}{8}(p-q)^\nu\Delta_{bc}^V(k)^{\nu\lambda} - \tfrac{1}{8}(p-q)^\lambda\left[C_{bc}^V + \int dx\,\sigma_{bc}^S(x)\right] + \tfrac{1}{4}(p^\lambda f_K{}^2 - q^\lambda f_\pi{}^2)$$

$$+ \tfrac{1}{8}(m_\pi{}^2 - m_K{}^2)k^\lambda\int\frac{dx\,\sigma_{bc}^S(x)}{x+k^2} + \tfrac{1}{8}k^\lambda\int\frac{dx\,x\sigma_{bc}^S(x)}{x+k^2} + \tfrac{1}{2}i\beta_\pi\int dx\, e^{-ikx}\langle T\{u_b(x), V_c^\lambda(0)\}\rangle_0. \tag{24}$$

This expression is exact given the assumed commutation relations, the standard PCAC, and Weinberg's first sum rule.[9]

IV. FORM FACTORS

To proceed further, the left-hand side of Eq. (24) must be expanded in linear powers of momenta. There is only one invariant scalar k^2 to consider in the expansion. Taking the primitive functions to be slowly varying functions of momenta, consistent with the Ward identities, we let

$$\lim q^\mu p^\nu F^{\mu\nu\lambda}_{abc} = \int \frac{dx\,\rho^V_{bc}(x)}{x-s}[\Gamma_1(p-q)^\lambda + \Gamma_2 k^\lambda] + \int \frac{dx\,\sigma^S_{bc}(x)}{x-s}\Gamma_3 k^\lambda, \tag{25}$$

where

$$\Gamma_1 = -\tfrac{1}{4}Gs, \tag{26}$$

$$\Gamma_3 = -\frac{m_\pi^2 + m_K^2 - s}{8}\,\frac{f_\pi^2 - f_K^2}{\int dx\,\sigma^S_{bc}(x)}. \tag{27}$$

It will soon be clear that Γ_2 is fixed by Γ_1, and Γ_3 has the effect of removing the otherwise linearly divergent term in the scalar form factor for the divergence of vector current. The last term in Eq. (24) can now be evaluated in the κ-pole model with the help of Eqs. (15)–(19). When the resulting expression together with Eqs. (25)–(27) is introduced into (24), we have, for the form factors of K_{l3} decay,

$$f_+(s) = \frac{1}{2f_\pi f_K}\left[f_\pi^2 + f_K^2 - \int dx\,\sigma^S_{bc}(x)\right] + \frac{s}{2f_\pi f_K}\int \frac{dx\,\rho^V_{bc}(x)}{x(x-s)}(1+2Gs), \tag{28}$$

$$f_-(s) = \frac{1}{2f_\pi f_K}\left\{ f_K^2 - f_\pi^2 - \int \frac{dx\,\rho^V_{bc}(x)}{x(x-s)}(8\Gamma_2 x + m_K^2 - m_\pi^2) \right.$$
$$\left. + \int \frac{dx\,\sigma^S_{bc}(x)}{x-s}\left[\frac{(f_K^2 - f_\pi^2)(s - m_K^2 - m_\pi^2)}{\int dx'\sigma^S_{bc}(x')} + x + m_\pi^2 - m_K^2\right] + \frac{3f_\pi^2 m_\pi^2}{\mu_K^2 - s}\left(\frac{b}{\sqrt{2}+b}\right)\right\}. \tag{29}$$

It is obvious from Eq. (28) that $f_+(s)$ satisfies a once-subtracted dispersion relation and

$$f_+(0) = \frac{1}{2f_\pi f_K}\left[f_\pi^2 + f_K^2 - \int dx\,\sigma^S_{bc}(x)\right]. \tag{30}$$

An effective-range formula for $f_+(s)$ can be derived if ρ^V_{bc} is taken as

$$\rho^V_{bc} = \frac{|f_+(s)|^2}{4\pi^2\sqrt{s}}P^3(s), \tag{31}$$

where

$$P(s) = \tfrac{1}{2}\left[\frac{s^2 - 2(m_K^2 + m_\pi^2)s + (m_K^2 - m_\pi^2)^2}{s}\right]^{1/2}. \tag{32}$$

This gives

$$\mathrm{Im} f_+(x) = \frac{1}{8\pi f_\pi f_K}\,\frac{|f_+(x)|^2}{\sqrt{x}}P^3(x)(1+2Gx). \tag{33}$$

The integral equation has the solution

$$f_+(s) = \frac{f_+(0)}{D(s)}, \tag{34}$$

with

$$D(s) = 1 + \frac{f_+(0)}{8\pi^2 f_\pi f_K}\left\{ sB + (1+2Gs)\left[\frac{C(s) - C(0) - sC'(0)}{s^2}\right] - \tfrac{1}{2}C''(0)\right\}, \quad \text{for } s < (m_K - m_\pi)^2 \tag{35}$$

where

$$C(s)=s^{3/2}P^3(s)\ln\left[\frac{m_K{}^2+m_\pi{}^2-s-2\sqrt{s}\,P(s)}{2m_Km_\pi}\right], \tag{36}$$

$$C(0)=\tfrac{1}{8}(m_K{}^2-m_\pi{}^2)^3\ln(m_\pi/m_K), \tag{37}$$

$$C'(0)=\tfrac{1}{8}(m_K{}^2-m_\pi{}^2)[(m_K{}^2-m_\pi{}^2)-3(m_K{}^2+m_\pi{}^2)\ln(m_\pi/m_K)], \tag{38}$$

$$C''(0)=\frac{3[(m_K{}^2-m_\pi{}^2)^2+(m_K{}^2+m_\pi{}^2)^2]}{8(m_K{}^2-m_\pi{}^2)}\ln(m_\pi/m_K)-\tfrac{5}{8}(m_K{}^2+m_\pi{}^2). \tag{39}$$

For $s>(m_K+m_\pi)^2$, $C(s)$ becomes $\overline{C}(s)$ where

$$\overline{C}(s)=s^{3/2}P^3(s)\left[\ln\frac{s+2\sqrt{s}\,P(s)-m_K{}^2-m_\pi{}^2}{2m_Km_\pi}-i\pi\right]. \tag{40}$$

In the effective-range approximation, B is determined by the condition

$$\mathrm{Re}D(m_{K^*}{}^2)=0, \tag{41}$$

which gives

$$B=-\frac{8\pi^2f_\pi f_K}{f_+(0)m_{K^*}{}^2}-\frac{(1+2Gs)}{m_{K^*}{}^6}[\mathrm{Re}\overline{C}(m_{K^*}{}^2)-C(0)-m_{K^*}{}^2C'(0)]+\frac{1}{2m_{K^*}{}^2}C''(0). \tag{42}$$

The K^* decay width, in turn, fixes the parameter G,[10] and

$$f_+'(0)=-\frac{f_+^2(0)}{8\pi^2f_\pi f_K}[B+GC''(0)+\tfrac{1}{6}C'''(0)]. \tag{43}$$

With $G=0.55$, Eqs. (42) and (43) give $f_+'(0)=1.17\,f_+(0)$, which predicts $\lambda_+=0.025$.

To find the ratio $f_-(0)/f_+(0)$, it is noted that the scalar form factor,

$$F(s)\equiv f_+(s)(m_K{}^2-m_\pi{}^2)+f_-(s)s, \tag{44}$$

being proportional to $\langle\pi^0K^+|\partial^\mu V^\mu|0\rangle$, receives no contribution from the spin-1 part of the spectral representation of vector currents. Indeed the terms involving ρ^V obtained by combining Eqs. (28) and (29) will be canceled if $\Gamma_2=\frac{1}{4}(m_K{}^2-m_\pi{}^2)G$. This was derived before with the assumption $q^\mu p^\nu k^\lambda\Gamma^{\mu\nu\lambda}=0$, where $\Gamma^{\mu\nu\lambda}$ is the relevant primitive function.[8]

Thus in the κ-pole model, with $\int dx\,\sigma_{bc}^S(x)=F_\kappa{}^2$, we have

$$F=(m_K{}^2-m_\pi{}^2)f_+(0)+\frac{s}{2f_\pi f_K(\mu_\kappa{}^2-s)}[(f_K{}^2-f_\pi{}^2)(\mu_\kappa{}^2-m_K{}^2-m_\pi{}^2)+F_\kappa{}^2(\mu_\kappa{}^2+m_\pi{}^2-m_K{}^2)]+\tfrac{3}{2}\left(\frac{f_\pi m_\pi{}^2s}{f_K(\mu_\kappa{}^2-s)}\right)\left(\frac{b}{\sqrt{2}+b}\right) \tag{45}$$

from which it follows that

$$\frac{f_-(0)}{f_+(0)}=-\lambda_+\left(\frac{m_K{}^2-m_\pi{}^2}{m_\pi{}^2}\right)+\frac{1}{2f_\pi f_Kf_+(0)\mu_\kappa{}^2}[F_\kappa{}^2(\mu_\kappa{}^2+m_\pi{}^2-m_K{}^2)+(f_K{}^2-f_\pi{}^2)(\mu_\kappa{}^2-m_K{}^2-m_\pi{}^2)]+\frac{3f_\pi m_\pi{}^2}{2f_+(0)f_K\mu_\kappa{}^2}\left(\frac{b}{\sqrt{2}+b}\right). \tag{46}$$

The result as expressed in (46) is clearly consistent with many previous current-algebra works including that of Dashen-Weinstein[11] if the last term can be neglected. In fact, the first term on the right-hand side gives about −0.28 and the second term has a contribution of about +0.21, determined largely by the condition $f_K/[f_\pi f_+(0)]=1.28$ and Eq. (30). The last term in Eq. (46), however, is sensitive to the parameter b. With $\mu_\kappa\cong1.05$ the ratio $f_-(0)/f_+(0)$ reaches about −0.6 for values of b lying between −1.32 and −1.33. Proposals with $b=c\equiv\epsilon_8/\epsilon_0$ have been suggested and considered.[12] We do not elaborate here.

In summary, we have formulated our problem without the use of the vector-dominance approximation. Elastic unitarity has been used for the vector form factor $f_+(s)$ and λ_+ is determined to be close to the K^*-dominance result. Our calculation is performed with mesons on the mass shell so that meson dominance should be good. The Callan-Treiman expressions for the sum and difference of the form factors follow in our approach at the proper off-shell points. Although we have used PCAC in the form of Eq. (15) for the pion, a similar form for the kaon has not been adopted in anticipation of correction to the model of Gell-Mann, Oakes, and Renner. The commutation relation (14) is the assumption and is true for extended models including the addition of the (1, 8) and (8, 1) representations to the symmetry-breaking Hamiltonian. In comparison with other current-algebra calculations, we agree with their results if the last term is neglected. This last term, however, is expected to be significant if $b = c = \epsilon_8/\epsilon_0$. By choosing b, we can achieve agreement with experiment.

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PHYSICAL REVIEW D VOLUME 4, NUMBER 1 1 JULY 1971

Predictions on $\pi^- + p \to \rho^0 + n$ and Some Related Processes Using Vector-Meson Dominance*†

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In the usual application of vector-meson dominance to analyze the process $\pi^- + p \to \rho^0 + n$, only the amplitudes for transversely polarized ρ are related to single-pion photoproduction. In this paper, it is shown that both the longitudinal and the transverse amplitudes for the process $\pi^- + p \to \rho^0 + n$ can be obtained from single-pion photoproduction amplitudes by assuming that the off-shell Ball amplitudes not only satisfy the constraints imposed by current conservation, but also are smooth in the vector-meson mass. The smoothness assumption is discussed in particle-exchange models in detail. We also extend our predictions to somewhat larger $|t|$ than in our previous work, and comparison is made with recent 15-GeV SLAC data on $\pi^- + p \to \rho^0 + n$. We also apply the same assumptions to analyze some related processes such as the electroproduction of a charged pion.

I. INTRODUCTION

Part of the objective of the vector-meson-dominance model (VMD)[1] is to relate processes involving ρ mesons to processes involving isovector photons. The idea of VMD is most easily understood in a theory in which the ρ meson and the isovector part of the photon are coupled to the same