Canonical Approach to High-Energy Scattering in Massive Quantum Electrodynamics

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By the canonical Hamiltonian approach to field theory, we have studied the very high-energy limit of massive quantum electrodynamics. We introduce a series of canonical transformations which bring the usual Hamiltonian of a neutral-vector-meson interaction with a conserved current to the effective Hamiltonian that operates at infinite energy. The result we obtain for the S-matrix operator is that it factorizes at very high energies:

$$S=S_0\left[U^\dagger T\left(\exp\left(-i\int_{-\infty}^{\infty} dt\, H_s(t)\right)\right)U\right],$$

where S_0 is the S matrix for the hard-meson "core," first given by Cheng and Wu and by Chang and Ma:

$$S_0=\exp\left(-i\frac{g^2}{(2\pi)^2}\int d^2k\frac{j_R(k_\perp)j_L(-k_L)}{k_\perp{}^2+\mu^2}\right),$$

and the second operator describes the contribution to scattering due to pionization effects. U is the dressing operator which clothes the hard-meson "core" in a coherent state of virtual soft mesons, and the T product is obviously the time-evolution operator for soft-meson dynamics.

I. INTRODUCTION

Physics at very high energies is believed to be simple. This belief, until recently, was based more on physical intuition than on a complete theoretical framework such as that of field theory. However, within the last two years, very much stimulated by the ideas of Feynman[1] and Yang,[2] Cheng and Wu[3] and others[3,4] have carried out an extensive investigation of field theory, summing various classes of Feynman diagrams and arriving at the infinite-energy limit. The results of their investigations, on the whole, lend support to the parton ideas of Feynman[1] and to the coherent-droplet ideas of Yang.[2]

In this paper we study the field theory of massive quantum electrodynamics in the limit of very high energies from a Hamiltonian approach, rather than in terms of summing Feynman diagrams. Our analysis thus is very close in spirit to the talk that Feynman gave at Stony Brook in 1969,[1] except that we commit ourselves to a specific model field theory. The language we use is the noncovariant one, viz., with respect to the center-of-mass frame, and the picture we describe in this language is similar and, we believe, adds to the one given by Cheng and Wu.

The picture we have for massive quantum electrodynamics at very high energies is this: An incident hadron, of momentum W along the z direction, is a fully dressed state with a "hard"-meson "core" as well as a "soft"-meson cloud. These are the "partons" in Feynman's language.[1] The hard-meson "core" is due to the virtual dissociation of a hadron of momentum $p_z=W$ to a hard meson $[p_z=xW,\ 1>x>0]$ and a hadron $[p_z=(1-x)W]$ and these correspond to the "wriggles" of Cheng and Wu.[5] The soft-meson cloud is due to the virtual bremsstrahlung of a soft meson (momentum $\vec{k}$, $k_z\ll W$) by the hadron of momentum W. The soft-meson cloud is in a coherent state,[6] and is attached to the "core," i.e., in classical physics terms, it is a persistent field and not a radiation field of the moving particle.

One could either follow the motion of the core or that of the cloud. Let us first follow the motion of the cores. The cores moving right (R) and left (L), if there were no soft-meson clouds attached to them, would simply not interact to leading order in W, since the energy cost is too great for a hard meson moving right to (in Feynman's words) jump the wagon.[1] But there are soft-meson clouds which overlap when the R and L cores meet and the resultant interaction Hamiltonian between the two cores is an effective current × current interaction; in particular, it is the current for the right-moving system interacting with the current for the left-moving system as well as the current for soft (pionization) particles, if they are present.

Now the current operator can connect between the initial dressed-hadron state and any final physical multiparticle state. The momentum transfer carried by this current is soft, so that the final "fragmented" state carries the same large overall momentum W as the initial hadron. The effective Hamiltonian of interaction between the two cores may thus be called the fragmentation Hamiltonian. Were it not for pionization, the resulting S-matrix element would simply be

$$S_0=\exp\left(\frac{-ig^2}{(2\pi)^2}\int d^2k\,\frac{j_R(k_\perp)j_L(-k_\perp)}{k_\perp^2+\mu^2}\right), \tag{1.1}$$

a result first obtained by Cheng and Wu,[3] Chang and Ma,[3] and others.[4] This S_0 corresponds to "black dots" in the impact picture of Cheng and Wu.[5] Matrix elements of j_R, j_L are their impact factors.[5] This form of the S matrix is consistent with the coherent-droplet model of Yang.[2] $j_R(\vec{k})$ and $j_L(\vec{k})$ are the Fourier transforms of the current operators. The effect of pionization is to append to S_0 the time-evolution operator for the soft-current interaction with the right and left currents [see (5.5)].

To understand better the physical process involved in pionization, we should next follow the motion of the clouds. As we mentioned earlier, for the class of experiments where we do not or are not able to measure the fragmentation products, we can regard the initial hadrons as sources of the coherent state of soft virtual mesons. These soft virtual mesons will, of course, interact according to the time-evolution operator appropriate to soft-meson dynamics,

$$T\left(\exp\left(-i\int_{-\infty}^{\infty}dt\;H_s(t)\right)\right). \tag{1.2}$$

Thus the full S matrix can be given by reading off from (1.2) in the coherent-state basis[6] rather than, as is usual, in the Fock space of definite numbers of mesons. More precisely, if U is the dressing operator for the soft-meson cloud [see Eq. (4.5)], then

$$S=S_0U^\dagger T\left(\exp\left(-i\int_{-\infty}^{\infty}dt\;H_s(t)\right)\right)U. \tag{1.3}$$

The approach we are taking in this paper bears a great similarity to those used in studies on scalar-field theories,[7] and also in the classic investigations on quantum electrodynamics (QED).[8] The basic Hamiltonian we start with is simple in structure, but leads to fairly complicated processes. Of these, only a certain class is expected to be important at high energies, and we can perform a canonical transformation on the original Hamiltonian so that this class is explicitly singled out. The analogy with QED is now clear. There, the initial Hamiltonian contains terms describing absorption and emission of mesons by free nucleons. These processes are forbidden if we impose energy conservation, and may be transformed away.[9] The way to do this has already been pointed out by Schwinger and Tomonaga, and our canonical transformation is constructed to parallel theirs.

The plan of the paper is as follows: In Sec. II we set up our notation, and exhibit the relevant kinematics at high energies in the center-of-mass frame. In Sec. II we also introduce the concept of order of processes that occur at high energies in our model. With respect to such an order, we may expect certain processes to be suppressed relative to others. This is analogous to the case in QED where certain processes do not occur because of energy conservation. The concept of suppression by order is of course absent there, although the basic mechanisms involved are the same. We then construct the appropriate canonical transformations (Secs. III and IV) to exhibit these suppressions manifestly in the Hamiltonian. The procedure is reminiscent of that used by Foldy and Wouthuysen[10] to exhibit the relevant nonrelativistic suppression factors for the motion of electrons in an external field. In Sec. V we obtain our final expression for the S-matrix operator, and conclude with some remarks on the process of pionization.

Our main conclusion in this paper is that high-energy scattering phenomena may be viewed in terms of a simple picture. Each step in this picture may be realized by means of an appropriate canonical transformation on the basic Hamiltonian. The final result of such transformations is to change the basis vectors into coherent vectors with respect to which semiclassical arguments describing the scattering phenomena may be phrased.

II. KINEMATICS AND MODEL HAMILTONIAN

In this section we introduce more precisely the notation as well as the language that we will find to be convenient for our later discussion. This language is not a Lorentz-covariant language, but will refer exclusively to the center-of-mass frame. In this frame we picture the two incident hadrons as having momentum W in the positive- and negative-z directions, respectively, W being large. We will refer to particles moving in the c.m. frame as right, left, or "soft" in accordance with their p_z ranges[11]:

$$R:\ \infty > p_z > x_0 W,$$
$$s:\ x_0 W > p_z > -x_0 W, \tag{2.1}$$
$$L:\ -x_0 W > p_z > -\infty .$$

In (2.1) x_0 is a small parameter, but not "wee" in the sense of Feynman,[1] i.e., not of order $1/W$.

As a matter of fact, we will build this distinction into our field theory directly through the field operators. Thus, we write for the fermion field

$$\psi(\vec{x}) = \psi_R(\vec{x}) + \psi_s(\vec{x}) + \psi_L(\vec{x}), \tag{2.2}$$

where

$$\psi_R(\vec{x}) = \frac{1}{[(2\pi)^3]^{1/2}} \int_R \frac{d^3p}{p_0} \sqrt{m} \sum_s [u(\vec{p}, s)a(\vec{p}, s)e^{i\vec{p}\cdot\vec{x}} + v(\vec{p}, s)b^\dagger(\vec{p}, s)e^{-i\vec{p}\cdot\vec{x}}] \tag{2.3}$$

and similarly for $\psi_L(\vec{x})$ and $\psi_s(\vec{x})$. In (2.3) we have used the invariant normalization

$$\{a(\vec{p}, s), a^\dagger(\vec{p}\,', s')\} = \delta_{ss'} p_0 \delta(\vec{p} - \vec{p}\,') . \tag{2.4}$$

The advantage of (2.4) is that with respect to R (right) and L (left) particles, the normalization is independent of W.

Next we introduce the notion of an order with respect to W of an operator such as the free Hamiltonian. Consider the free Hamiltonian for the fermions,

$$H_0 = \int d^3p \sum_s [a^\dagger(\vec{p}, s)a(\vec{p}, s) + b^\dagger(\vec{p}, s)b(\vec{p}, s)] . \tag{2.5}$$

The matrix elements of H_0 with respect to infinite-energy states, normalized in accordance with (2.4), are of order W as $W \to \infty$. We refer to the operator H_0 as being of order W.

In general, we determine the order of any operator by expressing it in terms of particle operators associated with invariantly normalized R, L, and s states and to scale the R, L ranges in terms of the Feynman x parameters,

$$p_z = \pm x W, \quad x_0 < x < \infty . \tag{2.6}$$

If the operator is scale-invariant with respect to W, then it is of order unity, while if the operator has a scale factor W remaining then it is of order W, etc. An operator which involves only s states will be defined to be of order unity.

By this prescription, the usual mass-renormalization term

$$\int d^3x\, \delta m \bar{\psi}(\vec{x})\psi(\vec{x})$$
$$= \delta m \int d^3x [\bar{\psi}_s(\vec{x})\psi_s(\vec{x}) + \bar{\psi}_R(\vec{x})\psi_R(\vec{x}) + \bar{\psi}_L(\vec{x})\psi_L(\vec{x})] \tag{2.7}$$

can be seen to be of order 1, $1/W$, $1/W$, respectively. With respect to infinite-energy states, the mass-renormalization term is of order $1/W$.

Next we write down the corresponding operator decomposition for the neutral-vector-meson field, $A_\mu(x)$:

$$A_\mu(\vec{x}) = A_\mu^R(\vec{x}) + A_\mu^s(\vec{x}) + A_\mu^L(\vec{x}), \tag{2.8}$$

where

$$A_\mu^R(\vec{x}) = \frac{1}{[2(2\pi)^3]^{1/2}} \int_R \frac{d^3k}{\omega} [A_\mu(\vec{k})e^{i\vec{k}\cdot\vec{x}} + A_\mu^\star(\vec{k})e^{-i\vec{k}\cdot\vec{x}}], \tag{2.9}$$

and similarly for A_μ^s and A_μ^L fields $[A_\mu^\star = (\vec{A}^\dagger, -A_4^\dagger)]$.

The quantization condition reads

$$[A_\mu(\vec{k}), A_\nu^\star(\vec{k}')] = \delta_{\mu\nu}\omega\,\delta(\vec{k} - \vec{k}') . \tag{2.10}$$

The spin content of $A_\mu(\vec{k})$ can be expressed by the relation

$$A_\mu(\vec{k}) = \sum_{\lambda=-1}^{1} \epsilon_\mu(\vec{k}, \lambda)a(\vec{k}, \lambda) + \frac{k_\mu}{m_\rho}\mathfrak{a}(\vec{k}), \tag{2.11}$$

where

$$\sum_{\lambda=-1}^{1} \epsilon_\mu(\vec{k}, \lambda)\epsilon_\nu^\star(\vec{k}, \lambda) = \left(\delta_{\mu\nu} + \frac{k_\mu k_\nu}{m_\rho^2}\right) \tag{2.12}$$

and $a(\vec{k}, \lambda)$ and $\mathfrak{a}(\vec{k})$ are the annihilation operators for the spin-1 and spin-0 particles described by $A_\mu(\vec{k})$. Notice that $\mathfrak{a}(\vec{k})$ is not a physical particle operator since from (2.10),

$$[\mathfrak{a}(\vec{k}), \mathfrak{a}^\dagger(\vec{k}')] = -\omega\,\delta(\vec{k} - \vec{k}'), \tag{2.13}$$

so that the spin-0 particle in A_μ is a ghost state.

If we had used a pure spin-1 vector field, the resulting interaction Hamiltonian, as is well known, would have had a normal dependent contact term.[5] For a conserved current interacting with the A_μ field, the two formulations are equivalent. For the sake of convenience as well as clarity, we will focus our attention on the Stückelberg formulation.

The usual vector-meson interaction is thus

$$ig \int d^3x\, \bar{\psi}(\vec{x})\gamma_\mu\psi(\vec{x})A_\mu(\vec{x}), \tag{2.14}$$

which, in our language, can be split into various configurations for the virtual emission and absorption of neutral vector mesons, viz.,

$$H_I = H_b + H_d + H_s + H', \tag{2.15}$$

where

$$H_b = \int d^3x : [ig\bar{\psi}_R(\vec{x})\gamma_\mu\psi_R(\vec{x}) + ig\bar{\psi}_L(\vec{x})\gamma_\mu\psi_L(\vec{x})] : A^s_\mu(\vec{x}), \tag{2.16}$$

$$H_d = ig\int d^3x : [\bar{\psi}_R(\vec{x})\gamma_\mu\psi_R(\vec{x})A^R_\mu(\vec{x}) + \bar{\psi}_L(\vec{x})\gamma_\mu\psi_L(\vec{x})A^L_\mu(\vec{x})] : , \tag{2.17}$$

$$H_s = ig\int d^3x : \bar{\psi}_s(\vec{x})\gamma_\mu\psi_s(\vec{x})A^s_\mu(\vec{x}) : , \tag{2.18}$$

$$\begin{aligned} H' = ig\int d^3x : \{&\bar{\psi}_R(\vec{x})\gamma_\mu\psi_R(\vec{x})A^L_\mu(\vec{x}) + \bar{\psi}_L(\vec{x})\gamma_\mu\psi_L(\vec{x})A^R_\mu(\vec{x}) \\ &+ [\bar{\psi}_R(\vec{x})\gamma_\mu\psi_L(\vec{x})A_\mu(\vec{x}) + \bar{\psi}_R(\vec{x})\gamma_\mu\psi_s(\vec{x})A^R_\mu(\vec{x}) + \bar{\psi}_L(\vec{x})\gamma_\mu\psi_s(\vec{x})A^L_\mu(\vec{x}) + \text{H.c.}]\} : . \end{aligned} \tag{2.19}$$

Now we observe that the different interaction pieces of the Hamiltonian can be classified in terms of their order with respect to W, in the sense that we have noted above. Thus, we find that the bremsstrahlung interaction for right-moving particles is of order unity, since

$$\begin{aligned} ig\int d^3x : \bar{\psi}_R(\vec{x})\gamma_\mu\psi_R(\vec{x}) : A^s_\mu(\vec{x}) \xrightarrow[W\to\infty]{} \frac{1}{[2(2\pi)^3]^{1/2}} \int_s \frac{d^3k}{\omega}\int d^2p_\perp \int_{x_0}^\infty \frac{dx}{x} \{&[a^\dagger(p_\perp + k_\perp, xW + k_z)a(p_\perp, xW) \\ &- b^\dagger(p_\perp + k_\perp, xW + k_z)b(p_\perp, xW)]A_-(\vec{k}) + \text{H.c.}\} \end{aligned} \tag{2.20}$$

and matrix elements of (2.20) with respect to infinite-energy states (normalized invariantly) clearly will no longer depend explicitly on W as $W \to \infty$. In (2.20) we have introduced the notation

$$A_\pm(\vec{k}) \equiv A_3(\vec{k}) \mp iA_4(\vec{k}) . \tag{2.21}$$

Similarly, we find that the dissociation of a right-moving hadron into a hard vector meson and a hadron is of order $1/W$, viz.,

$$\begin{aligned} H_d = \frac{1}{W}\frac{g}{[2(2\pi)^3]^{1/2}}\int d^2k\, d^2p \Bigg\{ &\int_{2x_0}^\infty dx \int_{x_0}^{x-x_0} dy \frac{1}{xy(x-y)} a^\dagger((x-y)W, p_\perp - k_\perp) \\ &\times \left[\frac{\sigma_3\sigma_a}{2}\left(\left(\frac{x-y}{x}\right)^{1/2} - \left(\frac{x}{x-y}\right)^{1/2}\right) + \left(\frac{x}{x-y}\right)^{1/2}\frac{\sigma_\perp\cdot(p-k)_\perp\sigma_a}{2m}\right. \\ &\left. + \left(\frac{x-y}{x}\right)^{1/2}\frac{\sigma_a\sigma_\perp\cdot p_\perp}{2m} - \frac{k_a}{m}\frac{[x(x-y)]^{1/2}}{y}\right] a(xW, p_\perp)A_a^*(yW, k_\perp) + \text{H.c.} \\ &+ \int_{2x_0}^\infty dy \int_{x_0}^{y-x_0} dx \frac{1}{xy(y-x)} a^\dagger((y-x)W, k_\perp - p_\perp) \\ &\times \left[-\frac{y}{2[x(y-x)]^{1/2}}\sigma_a + \left(\frac{x}{y-x}\right)^{1/2}\frac{\sigma_\perp\cdot(p-k)_\perp\sigma_a\sigma_3}{2m}\right. \\ &\left. + \left(\frac{y-x}{x}\right)^{1/2}\frac{\sigma_\perp\cdot p_\perp\sigma_a\sigma_3}{2m} + \frac{k_a}{m}\frac{[x(y-x)]^{1/2}}{y}\right] b^\dagger(xW, p_\perp)A_a(yW, k_\perp) + \text{H.c.}\Bigg\} \\ + \frac{1}{W}\frac{g}{[2(2\pi)^3]^{1/2}}\int d^2k\, d^2p \Bigg\{ &\int_{2x_0}^\infty dx \int_{x_0}^{x-x_0} dy \frac{1}{y[x(x-y)]^{1/2}} a^\dagger((x-y)W, p_\perp - k_\perp)a(xW, p_\perp)\alpha^\dagger(yW, k_\perp) \\ &\times \left[\frac{p_\perp^2 + m^2}{2x} - \frac{k_\perp^2 + \mu^2}{2y} - \frac{(p-k)_\perp^2 + m^2}{2(x-y)}\right]\Big/ m\mu + \text{H.c.} \\ &+ \int_{2x_0}^\infty dy \int_{x_0}^{y-x_0} dx \frac{1}{y[x(y-x)]^{1/2}} a^\dagger((y-x)W, k_\perp - p_\perp)\, b^\dagger(xW, p_\perp)\alpha(yW, k_\perp) \\ &\times \left[\frac{k_\perp^2 + \mu^2}{2y} - \frac{p_\perp^2 + m^2}{2x} - \frac{(k-p)_\perp^2 + m^2}{2(y-x)}\right]\Big/ \mu m + \text{H.c.}\Bigg\} - (a \leftrightarrow b) . \end{aligned} \tag{2.22}$$

In (2.22), A_a^* is the particle operator for the pure spin-1 field defined as

$$A_a^* = \sum_\lambda \epsilon_a(\vec{k}, \lambda) a^\dagger(\vec{k}, \lambda), \quad a = 1, 2\,. \tag{2.23}$$

In this way, we can construct a figure (Fig. 1) for the different pieces of the interaction. At the same time we shall note for each piece of the interaction the order with respect to W of the energy difference between the initial state and the final state after the interaction has acted.

In constructing Fig. 1, we have obviously omitted those interaction vertices that can be obtained by a mirror reflection. The role of the energy differences will be clear when we think in terms of the old-fashioned perturbation theory. For the present we shall simply note that if the energy difference across the interaction vertex is of order unity or smaller, we shall, following Feynman, refer to it as an energy-matching interaction vertex, while if the energy difference across the vertex is of order W we refer to it as an energy-nonmatching interaction vertex.

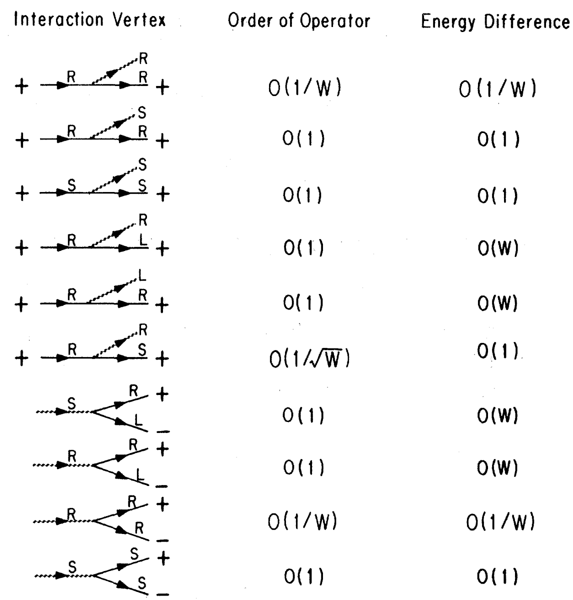

FIG. 1. A wavy line represents the neutral vector meson, a solid line the proton, say, while the ± charge will serve to distinguish between particle and antiparticle.

III. CANONICAL TRANSFORMATIONS: HARD MESONS

To prepare the ground work for our discussion of field theory in the infinite-energy limit, we present here a brief summary of the time-independent Lippmann-Schwinger formalism for the description of scattering.[12] Let

$$H = H_0 + V \tag{3.1}$$

and introduce eigenfunctions φ_α, $\psi_\alpha^\pm$ such that

$$\begin{aligned} H_0\varphi_\alpha &= \mathcal{E}_\alpha \varphi_\alpha\,, \\ H\psi_\alpha^\pm &= \mathcal{E}_\alpha \psi_\alpha^\pm\,. \end{aligned} \tag{3.2}$$

Then the scattering states $\psi_\alpha^\pm$ are related to φ_α by

$$\psi_\alpha^\pm = \varphi_\alpha + \frac{1}{\mathcal{E}_\alpha - H \pm i\epsilon} V\varphi_\alpha \equiv \Omega_\pm \varphi_\alpha \tag{3.3}$$

and the scattering-matrix element will be given by

$$S_{\beta\alpha} = \delta_{\beta\alpha} - 2\pi i\, \delta(\mathcal{E}_\beta - \mathcal{E}_\alpha)(\psi_\beta^-, V\varphi_\alpha) \tag{3.4}$$

$$\equiv \delta_{\beta\alpha} - 2\pi i\, \delta(\mathcal{E}_\beta - \mathcal{E}_\alpha)(\varphi_\beta, T(\mathcal{E}_\beta)\varphi_\alpha)\,, \tag{3.5}$$

where

$$T(E) = V + V\frac{1}{E - H_0 + i\epsilon}V + V\frac{1}{E - H_0 + i\epsilon}V\frac{1}{E - H_0 + i\epsilon}V + \cdots. \tag{3.6}$$

Equivalently, the S-matrix operator may be expressed in terms of the $\Omega_\pm$ operators introduced in (3.3), viz.,

$$S = \Omega_-^\dagger \Omega_+\,. \tag{3.7}$$

Equation (3.6) is the so-called old-fashioned perturbation theory which is not manifestly covariant. The usefulness of this theory for infinite-momentum analysis was first pointed out by Weinberg.[13] However, this was still for the case of total c.m. energy being finite, whereas we shall be interested in the limit as energy becomes large.

In the context of the old-fashioned perturbation theory, the relevance of the notions of the order of the operator and the energy matching across vertices becomes clear. Forgetting about the integration over intermediate states, the contribution to the T-matrix element of an interaction vertex that, though of order unity, does not match the energy across the vertex will be suppressed by a factor of W relative to the contribution of a vertex that is of order unity and matches the energy. Energy nonmatching, however, is not the only consideration, for even where energy matching occurs, the relative size of the contribution to the T-matrix element depends on the relative order (i.e., strength) of the interaction vertices.

But what about the integration over intermediate states? There are well-known ultraviolet divergences that come from the $|\vec{p}| \to \infty$ end of the integration of the momenta of all *on-shell* intermediate particles. For a renormalizable theory, the divergences can be handled through mass and vertex renormalization in a systematic way. For our purposes, we simply note that where the divergences occur in the dx integrals after W has been scaled out, they occur in diagrams on which the renormalization procedure is to be carried out, and this procedure that makes the dx integrals well defined does not depend on W. It is important here that x_0, the lower limit of the dx integral, be small, but not "wee" in the sense of Feynman,[1] i.e., x_0 should not be of order $1/W$, since otherwise the remaining integral over dx will have a dependence on W which will spoil our notions of the relative size of contributions of various interaction vertices.

In this context, we should make a remark here about the soft-range integration. We have heretofore defined operators such as H_s to be of order unity since it acted only on particles moving "slowly" in the c.m. frame. However, the range of p_z extends to x_0W, x_0 being small. There can be a dependence on W through the upper end of the "soft" integration, but this dependence by a usual power count can only be logarithmic: Our nomenclature of order unity is thus to be taken as order unity up to logarithms.[5]

To proceed with the old-fashioned perturbation theory, we could write down the systematic rules for the time-ordered graphs and classify the graphs in terms of their limiting behavior as $W\to\infty$.[14] While this procedure may be well defined mathematically, a better physical insight is gained by examining the Hamiltonian itself.

Our terminology has been invented to guide us as to which piece of the interaction is effective for high energies. Our aim therefore should be to exhibit the effective Hamiltonian that operates at high energies. From the figure, we see that H_s, H_d, and H_b are all interactions that could contribute to the T-matrix elements as $W\to\infty$. In H', however, there are vertices that survive (i.e., of order unity) which do not match energy and vertices that match energy, but whose strengths are relatively weak. By our previous discussion, we expect that H' will make a relatively small contribution to the T-matrix element as $W\to\infty$, although at the level of the Hamiltonian itself we cannot say that H' is negligible compared with H_b, H_d, and H_s. (In fact, H' is of order unity as $W\to\infty$.) To exhibit the suppression, we shall perform a canonical transformation which maps H' into an operator that is at most of order $1/\sqrt{W}$ and which does not match energy across the vertex.[8]

Define the canonical transformation

$$U'\equiv\exp\left(-i\int_{-\infty}^{0}dt'e^{\alpha t'}H'(t')\right),\quad \alpha\to 0_+ \tag{3.8}$$

$$\equiv e^{iF'}, \tag{3.9}$$

where

$$H'(t)\equiv e^{iH_0t}H'e^{-iH_0t}. \tag{3.10}$$

H' is the interaction Hamiltonian involving, from Fig. 1, energy-mismatched vertices as well as vertices that match energy but are themselves weak as $W\to\infty$. We note that F' is an operator that is at most of order $1/\sqrt{W}$ as $W\to\infty$.

Now the transformed Hamiltonian is

$$\begin{aligned}U'^{\dagger}HU'=&H_0+H_b+H_d+H_s+H'\\&-i[F',H_0]-\frac{1}{2!}[F',[F',H_0]]+\cdots\\&-i[F',H_b+H_d+H_s+H']\\&-\frac{1}{2!}[F'[F',H_b+H_d+H_s+H']]+\cdots,\end{aligned} \tag{3.11}$$

but

$$[F',H_0]=-iH'. \tag{3.12}$$

Therefore,

$$\begin{aligned}U'^{\dagger}HU'=&H_0+H_b+H_d+H_s\\&-i[F',H_b+H_d+H_s+\tfrac{1}{2}H']\\&-\tfrac{1}{2}[F',[F',H_b+H_d+H_s+\tfrac{2}{3}H']]+\cdots\end{aligned} \tag{3.13}$$

$$=H_0+H_b+H_d+H_s+O(1/\sqrt{W}). \tag{3.14}$$

The last statement follows from the fact that F' is of order $1/\sqrt{W}$ and H_b, H_s are of order unity (H_d is of order $1/W$). The commutator involves the basic commutation relations (2.4) and (2.10), which are invariant under scaling in W, and thus the commutators $[F',H_b]$, etc., will not introduce additional powers of W. To be sure, however, that the operator $O(1/\sqrt{W})$ in (3.14) will not contribute to the transition-matrix element, we have checked that the energy difference across the effective vertex $O(1/\sqrt{W})$ in (3.14) cannot be of order $1/W$, so that the energy denominator cannot overcome the weak strength of the effective vertex.

IV. CANONICAL TRANSFORMATIONS: SOFT MESONS

The Hamiltonian that concerns us thus at infinite energy then is

$$H=H_0+H_d+H_b+H_s+O(1/\sqrt{W}). \tag{4.1}$$

The physical significance of each piece of the Hamiltonian is clear. Notice, in particular, the H_0+H_d can be rewritten as H_R+H_L, where

$$\begin{aligned}H_R=&\int_{p_z>0}d^3p[a^{\dagger}(\vec{p},s)a(\vec{p},s)+b^{\dagger}(\vec{p},s)b(\vec{p},s)]\\&+\int_{k_z>0}d^3kA_{\mu}^{*}(\vec{k})A_{\mu}(\vec{k})\\&+ig\int d^3x:\bar{\psi}_R(\vec{x})\gamma_{\mu}\psi_R(\vec{x}):A_{\mu}^{R}(\vec{x})\end{aligned} \tag{4.2}$$

and H_L is similarly defined for the left-moving system. Since

$$[H_R, H_L] = 0, \tag{4.3}$$

the eigenstates of H_R and H_L are not confused with each other, so that the eigenstates of $H_R + H_L$ are direct products of R and L states. Physically, H_R and H_L generate the time evolution of right- and left-moving hadrons *minus* their virtual-soft-meson cloud.

The structure of the virtual-soft-meson cloud can of course be deduced from the full Hamiltonian (4.1); it is, however, easier to exhibit the virtual-soft-meson cloud by analogy with quantum electrodynamics[8,15] and perform the following canonical transformation.

Let

$$U \equiv e^{iF} \tag{4.4}$$

$$= \exp\left(-i\int_{-\infty}^{0} H_b(t)e^{\alpha t}dt\right), \quad \alpha \to 0_+ \tag{4.5}$$

where

$$H_b(t) \equiv e^{i(H_R+H_L)t}H_b e^{-i(H_R+H_L)t}. \tag{4.6}$$

Then

$$U^\dagger U'^\dagger HU'U = H_R + H_L - \tfrac{1}{2}i[F, H_b] - \tfrac{1}{3}[F, [F, H_b]] + \cdots + H_s - i[F, H_s] - \tfrac{1}{2}[F, [F, H_s]] + \cdots. \tag{4.7}$$

At this point we make use of the vectorial nature of the basic interaction to observe that, as $W \to \infty$,

$$H_b = g\int d^3x : \bar\psi_R(\vec{x})\gamma_+\psi_R(\vec{x}): A_-^s(\vec{x}) - g\int d^3x : \bar\psi_L(\vec{x})\gamma_-\psi_L(\vec{x}): A_+^s(\vec{x}) + O(1/W), \tag{4.8}$$

$$\gamma_\pm \equiv \tfrac{1}{2}(\gamma_4 \pm i\gamma_3), \tag{4.9}$$

$$A_\pm^s(\vec{x}) \equiv A_3^s(\vec{x}) \mp iA_4^s(\vec{x}), \tag{4.10}$$

so that

$$F = -\int_{-\infty}^{0} dt \int d^3x\, g[: \bar\psi_R(\vec{x}, t)\gamma_+\psi_R(\vec{x}, t): A_-^s(\vec{x}, t) - : \bar\psi_L(\vec{x}, t)\gamma_-\psi_L(\vec{x}, t): A_+^s(\vec{x}, t)] + O(1/W). \tag{4.11}$$

A very important property of the $A_\pm$ fields is the fact that

$$[A_\pm^s(\vec{x}, t), A_\pm^{s\dagger}(\vec{y}, 0)] = 0, \tag{4.12}$$

while

$$[A_\pm^s(\vec{x}, t), A_\mp^{s\dagger}(\vec{y}, t')] = 2\Delta^s(\vec{x}-\vec{y}, t-t'), \tag{4.13}$$

$$\Delta^s(\vec{x}, t) = \frac{1}{2(2\pi)^3}\int_s \frac{d^3k}{\omega}(e^{ik\cdot x} - e^{-ik\cdot x}). \tag{4.14}$$

This property follows directly from the quantization rule (2.11).

Another important fact to observe is that involving the commutators of currents. For this it is convenient to work with the Fourier transforms

$$\bar\psi_R(\vec{x})\gamma_+\psi_R(\vec{x}) = \int_s \frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot x}j_R(\vec{k}), \tag{4.15}$$

$$\bar\psi_L(\vec{x})\gamma_-\psi_L(\vec{x}) = \int_s \frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}j_L(\vec{k}); \tag{4.16}$$

then to order $1/W$, F may be given in terms of j_R, j_L as follows:

$$F = \frac{ig}{[2(2\pi)^3]^{1/2}}\int_s \frac{d^3k}{\omega}\left[\frac{j_R(\vec{k})A_-(\vec{k})}{k_z-\omega} + \frac{j_L(\vec{k})A_+(\vec{k})}{k_z+\omega} - \text{H.c.}\right]. \tag{4.17}$$

Equation (4.17) follows from the commutation rule

$$[H_R, j_R(\vec{k})] = k_z j_R(\vec{k}) + O(1/W). \tag{4.18}$$

In terms of the operators $j_R(\vec{k})$, a very important property that emerges is the commutator

$$[j_R(\vec{k}), j_R(\vec{k}')] = O(1/W), \quad \vec{k}, \vec{k}' \in s. \tag{4.19}$$

Because of (4.19) the transformed Hamiltonian (4.7) terminates if we are only interested in $O(1)$ operators.

From (4.17), (4.19), and (4.13), it is easy to see that the resulting Hamiltonian is

$$U^\dagger U'^\dagger HU'U = H_R + H_L + \frac{2g^2}{(2\pi)^3}\int_s d^3k\,\frac{j_R(\vec{k})j_L(-\vec{k})}{k_\perp^2+\mu^2} + H_s + \frac{2g^2}{(2\pi)^3}\int_s d^3k\,\frac{j_R(\vec{k})j_-^s(-\vec{k})}{k_\perp^2+\mu^2} + \frac{2g^2}{(2\pi)^3}\int_s d^3k\,\frac{j_+^s(\vec{k})j_L(-\vec{k})}{k_\perp^2+\mu^2} + O\left(\frac{1}{\sqrt{W}}\right). \tag{4.20}$$

V. SCATTERING OPERATOR

We have finally an effective Hamiltonian that for infinite-energy states has the form

$$\mathcal{H} = H_R + H_L + V + O(1/\sqrt{W}), \tag{5.1}$$

where H_R and H_L describe the dynamical evolution of the right- and left-moving hadron systems *minus* their virtual-soft-meson clouds. Or in more picturesque language, it would be as if H_R and H_L describe the motion of the hard-meson "core" and that in this picture the soft-meson cloud always follows the "core." V is the effective Hamiltonian that gives the interaction between the hard-meson "cores." Of course, this interaction comes about through the overlap of the virtual-soft-meson clouds.

Now a good basis for the application of perturbation theory is to use as H_0 the full $H_R + H_L$. This would be the basis where we recognize that the appropriate incoming states are not bare particles, but rather, in the language of field theory, dressed particles where the dressing is governed by $H_R + H_L$.

To obtain the S-matrix operator from this final form of H as given in (5.1), it is easiest now to work in the interaction picture. In this picture

$$V(t) = e^{i(H_R+H_L)t} V e^{-i(H_R+H_L)t} \tag{5.2}$$

$$= \frac{2g^2}{(2\pi)^3} \int_s d^3k e^{2ik_zt} \frac{j_R(\vec{k}) j_L(-\vec{k})}{k_\perp^2+\mu^2} + H_s(t) + \frac{2g^2}{(2\pi)^3} \int_s d^3k e^{ik_zt} \frac{j_R(\vec{k}) j_-^s(-\vec{k},t) + j_+^s(\vec{k},t) j_L(-\vec{k})}{k_\perp^2+\mu^2} + O\left(\frac{1}{\sqrt{W}}\right). \tag{5.3}$$

The S-matrix operator that follows from this is

$$S = T\left(\exp\left(-i\int_{-\infty}^{\infty} dt\, V(t)\right)\right) \tag{5.4}$$

$$= \exp\left(-i \frac{g^2}{(2\pi)^2} \int d^2k \frac{j_R(k_\perp) j_L(-k_\perp)}{k_\perp^2+\mu^2}\right)$$

$$\times T\left(\exp\left(-i\int_{-\infty}^{\infty} H_s(t)dt\right) \exp\left(-i\frac{2g^2}{(2\pi)^3}\int_{-\infty}^{\infty} dt \int_s d^3k\, e^{ik_zt} \frac{j_R(\vec{k}) j_-^s(-\vec{k},t) + j_+^s(\vec{k},t) j_L(-\vec{k})}{k_\perp^2+\mu^2}\right)\right). \tag{5.5}$$

If soft-current interactions were absent, the result (5.5) would correspond to the infinite-energy limit first obtained by Cheng and Wu in their summation of ladder diagrams for massive quantum electrodynamics.[3-5] The simple exponential nature of S_0 for the elastic-scattering matrix element translates into a simple eikonal form.[16] There are, however, differences.

Cheng and Wu,[3] and others,[4] have considered the ladder diagrams with point γ_μ vertices and found an exponentiation neglecting the H_s interaction. In our case we have the operators $j_R(\vec{k})$ and $j_L(\vec{k})$ which can connect between different eigenstates of $H_R + H_L$ and thus, in principle, include the possibility of fragmentation of initial particles. In the language of Cheng and Wu,[3,5] our S_0 includes the dissociation of hadrons into their component states.

S_0 also coincides with the coherent-droplet model proposed by Yang and co-workers,[2] when expressed in the operator formulation as given by Lee.[4]

Our major result however is to express how pionization affects the basic S_0 at infinite energy. By pionization we mean the production of soft mesons in the c.m. frame.

An important feature that we note at this point is the fact that the effective Hamiltonian at infinite energy does not allow for physical bremsstrahlung of soft mesons. This is easy to understand physically since at high energies the interaction Hamiltonian is finite and the momentum W of the incident hadron is changed only by a finite amount. The hadron thus maintains its initial straight-line path in the limit as $W \to \infty$ and, to leading order, the hadron will not radiate physical soft mesons. This is easy to see also from the Feynman diagram for bremsstrahlung, as was already noted by Cheng and Wu.[17]

Pionization, however, is a different matter. The virtual soft mesons in a coherent state[6] act as a semiclassical field which, however, can produce soft mesons or pions.

An alternative physical understanding of the pionization result is through the following: The infinite-energy hadron carries with it, as we already know, a virtual-soft-meson cloud in a coherent state.[6] Instead of describing the scattering by following the motion of the hard-meson "core," we can alternatively think in terms of following the motion of the virtual-soft-meson clouds.

For this picture we may think of the incident hadrons as being the classical source for soft mesons. The soft mesons interact in accordance with the Hamiltonian H_s and the corresponding S matrix would be simply

$$T\left(\exp\left(-i\int_{-\infty}^{\infty} dt\, H_s(t)\right)\right). \tag{5.6}$$

Now our picture would have it that instead of reading off matrix elements for initial states with a definite number of soft mesons going into final states with another *definite* number of mesons, we would read it off for initial and final states that are coherent states.[6,18]

In terms of the operator U, defined in (4.5), we have then

$$S = U^\dagger S_0 T\left(\exp\left(-i\int_{-\infty}^{\infty} dt\, H_s(t)\right)\right) U. \tag{5.7}$$

Since S_0 commutes with U to leading order on W, the resulting S-matrix operator is

$$S = S_0 U^\dagger T\left(\exp\left(-i\int_{-\infty}^{\infty} dt\, H_s(t)\right)\right)U\,. \qquad (5.8)$$

This form of the S-matrix operator agrees with the form (5.5) obtained earlier.

A detailed study of the amplitude when one evaluates the S-matrix operator between coherent states is now under way, and we will come back to the pionization process elsewhere.

We have thus obtained the S-matrix operator by utilizing the transformation properties of the Hamiltonian of the theory. It should perhaps be recognized that our method is, in principle, applicable to other field-theoretic models like those that possess chiral symmetry. In practice, however, internal-symmetry operators tend to make the manipulations more involved. The method, in any case, provides a good complementary approach in studying high-energy limits to those that exist in the literature.

Finally, we make a comment about the nonunique choice of x_0. Our implicit and intuitive assumption is that physical results do not depend sensitively on x_0, provided that x_0 is small enough. The reason for this observation is that if we set $x_0 = 1$, or 0.5, or even 0.1, and then proceed to transform away the "braking" vertex $\bar{\psi}_R\gamma_\mu\psi_s A_\mu^R$, we would be missing a large piece of the dissociation vertex which by a mere quirk of definition has been shifted into the braking vertex. But as we decrease x_0 and make it smaller and smaller, we would be including more and more of the dissociation vertex. Transforming away then the braking vertex will not mean dropping a large part of the dissociation vertex. Experience with the Cheng-Wu calculations indicates that, order by order, the dependence on x_0 is a weak logarithmic one. However, this does not mean that our total scattering phenomena do not receive contributions from vertices including particles with x_0 small, for these are now included in H_s. Such vertices are responsible for pionization, and their net contributions, as we have indicated earlier, are of order unity. Thus, contributions due to H_s may not be dropped.

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APPENDIX A

In this appendix we give a simple derivation of S_0 using old-fashioned perturbation theory.[12] For this derivation we ignore the possibility of H_s and deal directly with a Hamiltonian

$$H = H_0 + H_b + H_d \qquad (A1)$$

$$\equiv H_R + H_L + H_b\,. \qquad (A2)$$

Calling $H_R + H_L$ the free Hamiltonian, the problem is to find the S-matrix operator for (A2) as $W\to\infty$. Perform the canonical transformation U, as defined in (4.5), and obtain

$$U^\dagger H U = H_0 + H_d + \frac{2g^2}{(2\pi)^3}\int_s d^3k\,\frac{j_R(\vec{k})j_l(-\vec{k})}{k_\perp^2+\mu^2} \qquad (A3)$$

$$= H_R + H_L + V\,. \qquad (A4)$$

If we define

$$V \equiv \int d^3Q\, V(\vec{Q})\,, \qquad (A5)$$

an important property is that, to leading order of W,

$$[V(\vec{Q}),\, V(\vec{Q}')] = O(1/W) \qquad (A6)$$

and also

$$[H_R + H_L,\, V(\vec{Q})] = 2Q_z V(\vec{Q}) + O(1/W)\,. \qquad (A7)$$

Because of (A7), the Møller operator defined through

$$\psi_\alpha^\dagger \equiv \Omega_+\varphi_\alpha = \varphi_\alpha + \frac{1}{\mathcal{E}_\alpha - H_R - H_L + i\epsilon}V\psi_\alpha^\dagger \qquad (A8)$$

can be solved by direct iteration or by verification to be

$$\Omega_+ = \exp\left(\int \frac{d^3Q}{2Q_z + i\epsilon}V(\vec{Q})\right). \qquad (A9)$$

Similarly,

$$\Omega_- = \exp\left(\int \frac{d^3Q}{2Q_z - i\epsilon}V(\vec{Q})\right), \qquad (A10)$$

so that

$$S_0 = \Omega_-^\dagger\Omega_+ = \exp\left(-i\pi\int d^2Q\, V(Q_\perp)\right)$$
$$= \exp\left(-i\,\frac{g^2}{(2\pi)^2}\int d^2k\,\frac{j_R(k_\perp)j_L(-k_\perp)}{k_\perp^2+\mu^2}\right). \qquad (A11)$$

APPENDIX B

The physical significance of U is clearly exhibited when we suppress H_s. It is the dressing operator for a one-particle state in the absence of soft-meson dynamics. Turning on H_s would, of course, cause further "dressing" of the coherent state.

Consider the one-particle eigenstate of H as given in (A1):

$$H|\vec{p}\rangle_d = \left[W + \frac{p_\perp^2 + m^2}{2W} + O\left(\frac{1}{W^2}\right)\right]|\vec{p}\rangle_d . \qquad \text{(B1)}$$

To exhibit the soft-meson cloud, we do a canonical transformation

$$|\vec{p}\rangle_d = U|\vec{p}) . \qquad \text{(B2)}$$

If U is chosen to be (4.5), then the vector $|\vec{p})$ will satisfy a new equation

$$(H_0 + H_d + V)|\vec{p}) = \left(W + \frac{p_\perp^2 + m^2}{2W}\right)|\vec{p}) . \qquad \text{(B3)}$$

But V acting on $|\vec{p})$ is zero, since if $\vec{p}$ is right moving, it will have no overlap with a left-moving current and thus V annihilates $|\vec{p})$. Therefore, we have

$$(H_0 + H_d)|\vec{p}) = \left(W + \frac{p_\perp^2 + m^2}{2W}\right)|\vec{p}),$$

which shows that the structure in $|\vec{p})$ comes entirely from the virtual-dissociation process, and all the soft-meson structure has been obtained by the U transformation.

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