The π^+-π^0 Mass Difference Using Gravity-Modified Hadron Electrodynamics

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Computations of gravity-modified hadron electrodynamics are performed using nonpolynomial Lagrangian field-theory techniques. In particular, it is shown that the infinity encountered in a standard Lagrangian approach to the electromagnetic mass difference of pions is removed. The numerical value obtained for the mass shift is a little high, but consideration of the effects of tensor-meson dominance of gravity leads to a closer agreement with experiment.

I. INTRODUCTION

The idea has recently been revived[1-3] that the universal and nonlinear coupling of gravitation to matter may provide a natural mechanism for the damping of ultraviolet infinities in field theory. The reality of the built-in cutoff was demonstrated in a calculation of the electron's self-mass and self-charge,[1,3] in which the traditional logarithmic infinities of quantum electrodynamics are regularized with an effective cutoff mass equal to κ_g^{-1} (κ_g^2 is 8π times G, the Newtonian gravitational constant). Conventional electrodynamics, of course, treats these infinite quantities as unmeasurable, so their appearance causes no inconsistencies. However, in hadron physics, electromagnetic mass shifts are observable through the breaking of the internal SU(2) symmetry. For example, the value of the π^+-π^0 mass difference is experimentally well established. We present in this paper, then, the first prediction of this gravity-modified theory which can be compared directly with experiment. A simple model shows how the inclusion of gravity removes the infinity encountered in the calculation of the electromagnetic mass difference of pions from a physically reasonable Lagrangian. This model is also used to investigate the implications of "strong gravity" as proposed in Ref. 4. Here, hadrons do not couple directly to gravitons but only through massive spin-2 mesons, in analogy with the vector-dominance model of hadron electrodynamics. The role of Einstein's gravity in strong interactions is then naturally played by tensor mesons, which provide a built-in cutoff at κ_m^{-1}, much earlier than κ_g^{-1}. (Here κ_m is the universal coupling constant of the strongly interacting tensor meson to hadronic matter and, to within an order of magnitude, is equal to the inverse of its mass.)

The plan of the paper is as follows. Section II briefly reviews the standard prescription for introducing gravity into a Lagrangian theory. The Lagrangian to be modified for our calculation is the chiral Lagrangian used by Lee and Nieh,[5] described in Sec. III. Whereas for soft pions they obtain a result close to that observed experimentally, for massive pions the electromagnetic mass difference becomes logarithmically divergent. The same result was obtained independently by Wick and Zumino.[6] In Sec. IV a calculation is carried out with a simplified version of the gravity-modified chiral Lagrangian. Finally, Sec. V gives the numerical results. Pure gravity modifications yield a mass difference of 6.9 MeV for nonzero-mass pions compared with the experimentally determined value of 4.6 MeV. Extrapolation of κ to values typical of the tensor-meson model leads to a significantly lower value (between 4 and 6 MeV).

II. INCLUSION OF GRAVITY INTO A LAGRANGIAN THEORY

The standard way of introducing gravity into any theory is by the requirement that the equations of the theory be invariant under general coordinate transformations. In Lagrangian language this means that the action integral $\int d^4x\, \mathcal{L}(x)$ must be invariant under this group, implying that the Lagrange function must be constructed so that it transforms as a scalar density with weight -1. Such a Lagrangian can be generated from any Lorentz-invariant one by the following rules. Firstly, replace the Minkowski metric, $\eta^{\mu\nu} = \text{diag}(1, -1, -1, -1,)$, wherever it appears in the Lagrangian, by the Einstein metric, $g^{\mu\nu}(x)$. Secondly, replace the ordinary derivatives of the fields by the standard covariant derivatives of general relativity. Finally, adjust the total weight of each term in the Lagrangian to -1 by adjoining to each a factor $|\det g_{\mu\nu}|^{-w/2}$ which transforms as a Lorentz scalar with weight w. The crucial point about this last step is that it renders the new Lagrangian automatically nonpolynomial. To the Lagrangian generated by means of these rules it is of course necessary to add a purely gravitational term. The graviton field $\phi^{\mu\nu}(x)$ is defined by

$$\kappa\phi^{\mu\nu}(x)=g^{\mu\nu}(x)-\eta^{\mu\nu} \qquad (1)$$

so that $g^{\mu\nu}$ reduces to the flat-space metric, $\eta^{\mu\nu}$, as $\phi^{\mu\nu}\to 0$. It has been established[3] that the scalar gravity replacement

$$\phi^{\mu\nu}\to\eta^{\mu\nu}\phi\,,$$

and so

$$|\det g_{\mu\nu}|\to(1+\kappa\phi)^{-4}\,, \qquad (2)$$

gives numerical results essentially the same as tensor gravity, and use of the scalar field $\phi(x)$ is a very convenient simplification. Denoting the fields in the original Lagrangian by $A_\alpha(x)$, the over-all modification now reads

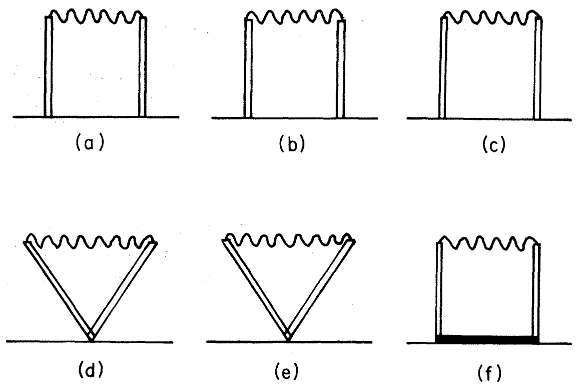

FIG. 1. Feynman graphs from the chiral Lagrangian. The wavy line, single line, double line, and thick line represent the photon, pion, ρ, and A_1, respectively. (a), (b), and (c) differ by the powers of momenta at the vertices; similarly for (d) and (e).

$$\mathfrak{L}(A_\alpha,\partial_\mu A_\beta,\eta^{\nu\sigma})\to\mathfrak{L}(A_\alpha(1+\kappa\phi)^{-2w_\alpha},\,D_\mu A_\beta(1+\kappa\phi)^{-2w_\beta},\eta^{\nu\sigma}(1+\kappa\phi)^{-2})+\mathfrak{L}(\text{gravity})\,. \qquad (3)$$

w_α is the weight of the field A_α, and D_μ is the appropriate covariant derivative.

III. THE LAGRANGIAN OF LEE AND NIEH

Lee and Nieh[5] construct a phenomenological Lagrangian appropriate to the group SU(2)× SU(2) which includes pions, ρ mesons, and axial-vector mesons (A_1 mesons). The π^+-π^0 mass difference is then calculated to order e^2 by considering all the tree diagrams for the process $\pi^++\rho^0\to\pi^++\rho^0$ and closing the ρ^0-γ-ρ^0 loop as shown in Fig. 1. The relevant vertices are given by

$$\mathfrak{L}=-g\vec\rho_\mu\cdot(\vec\pi\times\partial_\mu\vec\pi)+\tfrac{1}{2}g(\sqrt{2}\,m_\rho)^{-2}(\partial_\mu\vec\rho_\nu-\partial_\nu\vec\rho_\mu)\cdot\partial_\mu\vec\pi\times\partial_\nu\vec\pi+\tfrac{1}{2}g^2(\vec\rho_\mu\times\vec\pi)^2-\tfrac{1}{2}g(\sqrt{2}\,m_\rho)^{-1}(\partial_\mu\vec a^1_\nu-\partial_\nu\vec a^1_\mu)\cdot(\partial_\mu\vec\rho_\nu-\partial_\nu\vec\rho_\mu)\times\vec\pi$$
$$-\tfrac{1}{2}g(\sqrt{2}\,m_\rho)^{-1}(\partial_\mu\vec\rho_\nu-\partial_\nu\vec\rho_\mu)\cdot(\vec a^1_\mu\times\partial_\nu\vec\pi-\vec a^1_\nu\times\partial_\nu\vec\pi)-\tfrac{1}{4}g^2(\sqrt{2}\,m_\rho)^{-2}[(\partial_\mu\vec\rho_\nu-\partial_\nu\vec\rho_\mu)\times\vec\pi]^2+(e/g)m_\rho^2\rho^0_\mu A_\mu\,, \qquad (4)$$

where A_μ, $\vec\pi$, $\vec\rho_\mu$, and $\vec a^1_\mu$ denote the photon, pion, ρ-meson, and A_1 fields respectively; m_ρ is the mass of the ρ meson.

For zero-mass pions ($\mu^2\to 0$), the answer is finite and in reasonable agreement with experiment ($\alpha=e^2/4\pi$):

$$\delta\mu^2=(3\alpha/4\pi)m_\rho^2\times 2\ln 2\,, \qquad (5)$$

which gives the mass difference $\delta\mu=5.0$ MeV, the experimentally determined value being 4.6 MeV. When the pions are massive, however, ($\mu^2\neq 0$) the calculated mass shift is logarithmically divergent. To order $(\mu/m_\rho)^2$,

$$\delta\mu^2=\frac{3\alpha}{4\pi}m_\rho^2\left[2\ln 2+\frac{\mu^2}{m_\rho^2}\left(\ln\frac{m_\rho^2}{\mu^2}+\frac{19}{4}\ln 2-\frac{5}{2}+\frac{1}{8}\ln\frac{\Lambda^2}{m_\rho^2}\right)\right], \qquad (6)$$

where Λ is the ultraviolet cutoff momentum. This relation may be expressed in the form

$$\delta\mu=6.0\text{ MeV}+\frac{3\alpha}{4\pi}\,\frac{\mu}{8}\ln\frac{\Lambda}{m_\rho}\,. \qquad (7)$$

It is this logarithmic infinity which we will seek to remove by the introduction of gravity.

IV. GRAVITATIONAL MODIFICATIONS

We modify the Lagrangian of Lee and Nieh using the techniques of Sec. II. The most convenient coordinates for the calculation are those in which the weights of all the fields are equal to zero. The Lagrangian of Eq. (4) now becomes

$$\mathcal{L} = -g\vec{\rho}_\mu \cdot (\vec{\pi}\times\partial_\mu\vec{\pi})(1+\kappa\phi)^{-1} + \tfrac{1}{2}g(\sqrt{2}\,m_\rho)^{-2}(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)\cdot\partial_\mu\vec{\pi}\times\partial_\nu\vec{\pi} + \tfrac{1}{2}g^2(\vec{\rho}_\mu\times\vec{\pi})^2(1+\kappa\phi)^{-1}$$
$$- \tfrac{1}{2}g(\sqrt{2}\,m_\rho)^{-1}(\partial_\mu\vec{a}^1_\nu - \partial_\nu\vec{a}^1_\mu)\cdot(\partial_\nu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)\times\vec{\pi} - \tfrac{1}{2}g(\sqrt{2}\,m_\rho)^{-1}(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)\cdot(\vec{a}^1_\mu\times\partial_\nu\vec{\pi} - \vec{a}^1_\nu\times\partial_\mu\vec{\pi})$$
$$- \tfrac{1}{4}g^2(\sqrt{2}\,m_\rho)^{-2}[(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)\times\vec{\pi}]^2 + (e/g)m_\rho^{\,2}\rho^0_\mu A_\mu(1+\kappa\phi)^{-1}\,. \tag{8}$$

Covariant derivatives D_μ (which would involve couplings to $\partial_\mu\phi$) do not appear in the above since[7]

$$D_\mu\vec{V}_\nu - D_\nu\vec{V}_\mu = \partial_\mu\vec{V}_\nu - \partial_\nu\vec{V}_\mu \tag{9}$$

for $\vec{V}$ a vector (or axial-vector) field, and

$$D_\mu\vec{\pi} = \partial_\mu\vec{\pi} \tag{10}$$

for $\vec{\pi}$ a scalar field. Note that only the ρ-photon and some of the ρ-pion vertices are changed. Those involving A_1 mesons remain the same. These modifications correspond diagrammatically to the inclusion of superpropagators between some of the vertices in the diagrams of Fig. 1 as shown in Fig. 2. However, the explicit calculation of diagrams involving more than one superpropagator is at the present time an unsolved technical problem, and we make the approximation of including only one superpropagator in each graph as shown in Fig. 3. Since only one superpropagator per diagram is sufficient to make the theory finite, this approximation still retains the main features of the gravitational regularization. The originally divergent diagrams have only one superpropagator anyway [Figs. 2(c), (e), and (f)], and diagrams in which superpropagators are neglected gave finite contributions. The inclusion of the other superpropagators would serve only to modify slightly this already finite answer.

The calculation is performed by the standard Volkov-Salam-Strathdee momentum-space method for nonpolynomial Lagrangians. We refer the reader to Refs. 8 and 9 for detailed discussions of the procedure. The superpropagator in configuration space is given by

$$G(x) = \langle T(V[\phi(x)]\;V[\phi(0)])\rangle_0\,, \tag{11}$$

where

$$V[\phi] = \frac{1}{1+\kappa\phi} = \sum_{n=0}^{\infty}(-\kappa\phi)^n\,,$$

which gives in momentum space the (massless) superpropagator

$$\tilde{G}(p^2) = \tfrac{1}{2}\pi(4\pi)^2\int_{a-i\infty}^{a+i\infty} dz\left(\frac{\kappa}{4\pi}\right)^{2z}\frac{1}{\tan\pi z\,\sin\pi z}(-p^2)^{z-2}\,\frac{\Gamma(z+1)}{\Gamma(z)\Gamma(z-1)}\,, \tag{12}$$

where $-1 < a < 0$. The pion mass difference may now be computed from the diagrams of Fig. 3,

$$\delta\mu^2 = \frac{e^2}{i}\int\frac{d^4k}{(2\pi)^4}\,\frac{d^4q}{(2\pi)^4}\,M_{\mu\nu}(k^2)D_{\mu\nu}(q^2,\lambda)\tilde{G}[(k-q)^2]\,, \tag{13}$$

where (with $p^2 = \mu^2$ and the A_1 mass $m_a = \sqrt{2}\,m_\rho$),

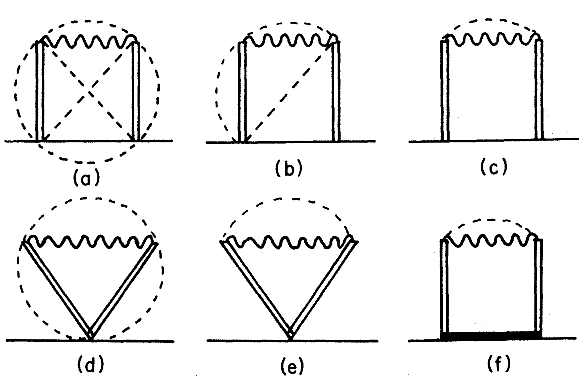

FIG. 2. Full gravity-modified graphs. The dashed line represents the multigraviton propagator.

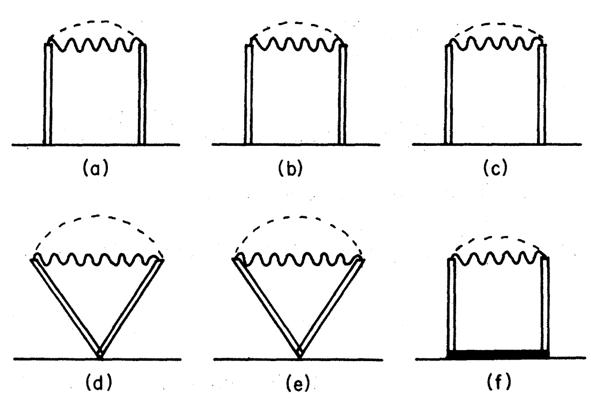

FIG. 3. Single-superpropagator graphs evaluated.

$$M_{\mu\nu}(k^2)=\left(\frac{1}{k^2-m_\rho{}^2}\right)^2\left(\eta_{\mu\tau}-\frac{k_\mu k_\tau}{m_\rho{}^2}\right)\Bigg[\left(-\frac{1}{(p-k)^2-\mu^2}\right)[(p-k)_\alpha p_\beta+p_\alpha(p-k)_\beta+(p-k)_\alpha(p-k)_\beta+p_\alpha p_\beta]\,[\eta_{\tau\alpha}\eta_{\beta\delta}$$
$$-(m_\rho\sqrt{2})^{-2}\eta_{\tau\alpha}(\eta_{\beta\delta}k^2-k_\beta k_\delta)+(\tfrac{1}{2})^2(m_\rho\sqrt{2})^{-4}(\eta_{\tau\alpha}k^2-k_\tau k_\alpha)(\eta_{\beta\delta}k^2-k_\tau k_\alpha)]$$
$$+\eta_{\tau\delta}-(m_\rho\sqrt{2})^{-2}(\eta_{\tau\delta}k^2-k_\tau k_\delta)-\left(\frac{1}{(p-k)^2-m_a{}^2}\right)(m_\rho\sqrt{2})^{-2}(\eta_{\tau\alpha}k^2-k_\tau k_\alpha)$$
$$\times\left(\eta_{\alpha\beta}-\frac{(p-k)_\alpha(p-k)_\beta}{m_a{}^2}\right)(\eta_{\beta\delta}k^2-k_\beta k_\delta)\Bigg]\left(\eta_{\delta\nu}-\frac{k_\delta k_\nu}{m_\rho{}^2}\right) \tag{14}$$

and $D_{\mu\nu}(q^2,\lambda)$ is the photon propagator. It will be instructive to work in an arbitrary covariant gauge parametrized by λ:

$$D_{\mu\nu}(k^2,\lambda)=\left(\eta_{\mu\nu}-\lambda\frac{k_\mu k_\nu}{k^2}\right)\frac{1}{k^2}\,. \tag{15}$$

In Eq. (13) the internal integral around the photon-graviton loop may be carried out first and the result written in terms of a "modified photon propagator" $D'_{\mu\nu}(k^2)$,

$$\delta\mu^2=\frac{e^2}{i}\int\frac{d^4k}{(2\pi)^4}M_{\mu\nu}(k^2)D'_{\mu\nu}(k^2)\,, \tag{16}$$

where

$$D'_{\mu\nu}(k^2)=\int\frac{d^4q}{(2\pi)^4}D_{\mu\nu}(q^2)\tilde{G}((k-q)^2)\,. \tag{17}$$

Explicitly, using Eqs. (12) and (15),

$$D'_{\mu\nu}(k^2)=\tfrac{1}{2}\pi(4\pi)^2\int_{a-i\infty}^{a+i\infty}dz\left(\frac{\kappa}{4\pi}\right)^{2z}\frac{1}{\tan\pi z\,\sin\pi z}\,\frac{\Gamma(z+1)}{\Gamma(z)\Gamma(z-1)}\int\frac{d^4q}{(2\pi)^4}\left(\eta_{\mu\nu}-\frac{\lambda q_\mu q_\nu}{q^2}\right)\frac{1}{q^2}[-(k-q)^2]^{z-2}\,. \tag{18}$$

Here we have changed the order of integration, the contour integral will always be performed last in accordance with usual nonpolynomial techniques. Evaluating the q integral,

$$D'_{\mu\nu}(k^2)=\frac{\pi}{2i}\int_{a-i\infty}^{a+i\infty}dz\,\frac{(\kappa^2/16\pi^2)^z}{\sin\pi z\,\tan\pi z\,\Gamma(z)}(-k^2)^{z-1}\left[\eta_{\mu\nu}\left(1-\frac{\lambda z}{2(z+1)}\right)-\lambda\frac{k_\mu k_\nu}{k^2}\left(1-\frac{2z}{z+1}\right)\right]. \tag{19}$$

Substituting this expression into Eq. (16) and performing the momentum integration, we obtain to order $(\mu^2/m_\rho{}^2)$

$$\delta\mu^2=\left(\frac{3\alpha}{4\pi}\right)\frac{i}{2\pi^2}\int_{a-i\infty}^{a+i\infty}\frac{dz(m_\rho{}^2\kappa^2/16\pi^2)^z}{\sin^2\pi z\,\tan\pi z\,\Gamma(z)}$$
$$\times\left\{2m^2(2^z-1)+\mu^2\left[(\tfrac{1}{2}z^2-2\tfrac{1}{2}z+4\tfrac{3}{4})2^z-(\tfrac{3}{2}z+3\tfrac{7}{8})-\frac{2}{\sqrt{\pi}}\,\frac{\Gamma(z+\frac{1}{2})}{\Gamma(z+3)}\left(4\frac{\mu^2}{m_\rho{}^2}\right)^z\right]\right\}\left(1-\frac{\lambda z}{2(z+1)}\right), \tag{20}$$

which is linear in λ. The contour along which the integral in Eq. (20) is taken stands to the left of $z=0$, where the integrand has a double pole. Collapsing the contour around the positive real axis also picks up the singularities at $z=1,2,3,\ldots$ which are tripoles. For $\lambda=0$,

$$\delta\mu^2=(\delta\mu^2)_{z=0}+\sum_{n=1}^{\infty}C_n(\kappa)(\kappa^2m_\rho{}^2/16\pi^2)^n\,, \tag{21}$$

where

$$(\delta\mu^2)_{z=0}=\frac{3\alpha}{4\pi}m_\rho{}^2\left\{2\ln2+\frac{\mu^2}{m_\rho{}^2}\left[\ln\frac{m_\rho{}^2}{\mu^2}+\frac{19}{4}\ln2-\frac{5}{2}+\frac{1}{8}\left(\ln\frac{16\pi^2}{\kappa^2m_\rho{}^2}+\Psi(1)\right)\right]\right\}, \tag{22}$$

and the power series coefficients are

$$C_n(\kappa)=\frac{1}{2}\frac{3\alpha}{4\pi}m_\rho{}^2\frac{1}{(n-1)!}\Big([2(2^n-1)+4\Psi(n)\ln a-2\Phi(n)\ln^2a-2^{n+2}\Psi(n)\ln2a+2^{n+1}\Phi(n)\ln^2(2a)]$$
$$+\frac{\mu^2}{m_\rho{}^2}\{(\tfrac{1}{2}n^2-\tfrac{5}{2}n+\tfrac{19}{4})2^n-(\tfrac{3}{2}n+3\tfrac{7}{8})-2[2^n(n-\tfrac{5}{2})-\tfrac{3}{2}]\Psi(n)+2^n\Phi(n)+[2(\tfrac{3}{2}n+3\tfrac{7}{8})\Psi(n)-3\Phi(n)]\ln a$$
$$-(\tfrac{3}{2}n+3\tfrac{7}{8})\Phi(n)\ln^2a+[(2n-5)\Phi(n)-2(\tfrac{1}{2}n^2-\tfrac{5}{2}n-4\tfrac{3}{4})\Psi(n)]2^n\ln(2a)+(\tfrac{1}{2}n^2-\tfrac{5}{2}n+4\tfrac{3}{4})2^n\Phi(n)\ln^2(2a)\}\Big)\,, \tag{23}$$

where

$$a = m_\rho^2 \kappa^2/16\pi^2 ,$$

$$\Phi(n) = \Psi^2(n) - \Psi'(n) ,$$

$$\Psi(n) = \frac{\partial}{\partial z} \ln\Gamma(z)\bigg|_{z=n} ,$$

$$\Psi'(n) = \frac{\partial}{\partial z} \Psi(z)\bigg|_{z=n} .$$

It has been pointed out in Ref. 3 that computation with only one superpropagator is not a gauge-invariant procedure. In this calculation, the gauge dependence of $\delta\mu$ is made manifest by the explicit appearance of the gauge parameter λ in Eq. (19). As a multiplier of $k_\mu k_\nu/k^2$ it is harmless; after integrating over k this part will vanish by symmetric integration. As a multiplier of $z\eta_{\mu\nu}/2(z+1)$, however, it exhibits the nongauge invariance of our result, since, as can be seen from Eq. (20), λ still remains in the expression for $\delta\mu^2$ even after the momentum integrations have been carried out. However, on evaluating Eq. (20), the coefficient of λ is only 0.005 MeV when $\kappa = \kappa_g$ of graviton theory ($\sim 2\times 10^{-22} m_e^{-1}$) and still only 0.01 MeV when $\kappa = \kappa_m$ of tensor meson theory (~ 1 BeV^{-1}). The smallness of such gauge-dependent effects increases our optimism that the problem of gauge invariance is not as serious as one might first expect. All numerical results in Sec. V are quoted with $\lambda = 0$.

V. RESULTS AND DISCUSSION

Graviton theory. The power-series contribution of the tripoles [Eq. (23)] is negligible (2×10^{-35} MeV) when κ is put equal to κ_g of graviton theory ($\kappa_g = 4.3 \times 10^{-22}$ MeV^{-1}). The contribution of the leading dipole [Eq. (22)] clearly reproduces the ordinary zero-gravity result of Lee and Nieh [Eq. (6)] except that there remains a dependence on κ in the form of an effective cutoff

$$\Lambda \sim 4\pi/\kappa . \qquad (24)$$

The important point is that the ultraviolet infinity in the old theory has disappeared via the mechanism of the induced cutoff. The ultraviolet infinity still leaves its mark as a singularity in the κ plane and reappears if the limit $\kappa \to 0$ is taken. Equation (22) gives

$$\delta\mu = 6.9 \text{ MeV} \qquad (25)$$

for massive pions. Thus gravity-modified hadron electrodynamics produces a finite pion mass difference not much greater than that observed.

Tensor meson theory. A rigorous calculation of the effects of tensor-meson dominance of gravity is not possible at the present time since the analytic form of the massive superpropagator is unknown and also the effects of our approximations in neglecting superpropagators become more serious. However, if one assumes that the behavior of the zero-mass propagator provides a good approximation to the massive case,[9] the effects of strong gravity may be estimated by extrapolating to large values of κ. For values of the order of one (BeV)$^{-1}$ the contribution of the tripoles is no longer negligible, and the whole of Eq. (21) must be taken.

Taking as a typical tensor-meson mass that of the $f(1260)$, one has for $10^{-1} < \kappa m_f < 10$,

$$5.7 > \delta\mu > 4.1 \text{ MeV} .$$

This indicates that a full tensor-meson theory would give a more physically reasonable prediction.

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