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Tests of Light-Cone Commutators: Fixed-Mass Sum Rules*

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By use of light-cone commutators, corrected versions of six infinite-momentum sum rules are derived. The light-cone commutators are given by bilocal operators whose matrix elements are shown to be Fourier transforms of deep-inelastic-scattering structure functions. A modification of the Bég sum rule is found, and a sum rule for the axial-vector coupling constant in terms of a nucleon (spin-flip) structure function is presented. The latter is related to an old result of Bjorken. We deal with the full spin-dependent forward Compton amplitude and discuss in detail the form-factor decomposition of the bilocal operators which give the fermion-quark-model light-cone commutators. The model dependence of the present results is discussed, and boson-model commutators are given for comparison.

I. INTRODUCTION

It has now become possible to give a plausible model for the commutator of currents restricted to a lightlike surface.[1,2] This model is obtained by quantizing a quantum theory on such a surface,[3] in complete analogy with the conventional derivation of equal-time commutators. This scheme provides an elegant formulation of Bjorken scaling in deep-inelastic electron scattering,[4] and summarizes in a compact fashion the various deep-inelastic sum rules.[5]

In order to provide alternate tests of the hypothesis, we examine in this paper the consequences that the present ideas have for the fixed-mass sum rules of conventional current algebra – the most famous of these being due to Dashen, Fubini, and Gell-Mann.[6] The usual derivation of such sum rules makes use of the $p\to\infty$ technique, which involves an interchange of limit with integral.[7] This operation is not in general justified, and results have been obtained which are not verified in free-field theory. A viewpoint which is frequently proposed is that according to Regge lore the integrals which occur in the bad relations diverge, and therefore, one should not be surprised that nonsense emerges from manipulations of infinities.

We feel however that this is not an adequate resolution of the problem. Three reasons may be advanced. (1) A divergent sum rule should not be inconsistent. It should be a statement of the fact that the matrix element of some commutator diverges, as a consequence of a physically sensible growth of cross sections. The vacuum Schwinger-term sum rule, which involves an integral over the total lepton annihilation cross section, is an example of a sensible, divergent sum rule.[8] (2) The Regge model, which indicates divergences, is not the only model, while the sum rules should be largely model-independent. In particular the free-field model has convergent integrals, yet the sum rules fail. (3) Even if one accepts the Regge model, there exists a sum rule due to Bég[9] which should converge; yet it fails in free-field theory. Moreover, numerical evaluation of that relation indicates that it is not satisfied in nature.[10]

Consequently we feel that the proper criticism of the bad sum rules is not that they diverge, but that they are *improperly derived*. On the other hand, it has been known for some time that the light cone is relevant to the fixed-mass sum rules.[11] Since we now have a model for light-cone behavior of commutators, we are in a position to derive the relevant sum rules without invoking the $p\to\infty$ technique. To the extent that the present results differ from the $p\to\infty$ results, we see that the light-cone methods *are not* equivalent to $p\to\infty$ methods.

This paper is organized as follows. In Sec. II, we set up our conventions and define the kinematics. Section III is devoted to a summary of the $p\to\infty$ sum rules; while in Sec. IV we rederive the results with our light-cone commutators. Corrections are found to many relations. The corrections involve matrix elements of bilocal operators, and can be expressed in terms of deep-inelastic scaling functions, which in principle are measurable. Specifically, we present a modified version of the Bég sum rule,[9] with contributions from deep-inelastic cross sections. In Sec. V we discuss how all the bilocal operators occurring in light-cone commutators can be measured. We show that a certain bilocal operator has matrix elements which coincide in the local limit with the matrix elements of the vector current. We also derive a sum rule for g_A in terms of the deep-inelastic cross sections. Concluding remarks, which stress the relevance of our results to the question of subtractions in dispersion relations, comprise Sec. VI. Also, we discuss the model dependence of our light-cone commutators and of our results.

In Appendix A we investigate the constraints that conventional Regge ideas impose on the high-energy behavior of the various functions which we discuss. In Appendix B we demonstrate explicitly, with several examples, how the infinite-momentum frame fails to provide the proper formula for the light-cone commutator. In Appendix C, we present an alternative scheme of light-cone current commutators, based on a boson theory, rather than on the fermion theory which is used in the text. Finally, in Appendix D *(added in proof)* we discuss the reliability of our final results. We find that the present techniques improve upon the $p\to\infty$ methods in that fixed poles with polynomial residues are now properly handled. However, if there are poles with nonpolynomial residues, then the light-cone method seems to fail; as of course, so does the $p\to\infty$ technique.

II. PRELIMINARIES

The object with which we shall concern ourselves is the diagonal matrix element of the commutator of two vector currents between fermion states with momentum p:

$$C^{\mu\nu}_{ab}(p,q)\equiv\int d^4x\, e^{iq\cdot x}\langle p|[V^\mu_a(x),\ V^\nu_b(0)]|p\rangle. \tag{2.1}$$

The vector current V^μ_a is conserved, since we shall restrict our considerations to electromagnetic and isospin currents. The state is characterized by a spin vector $s^\alpha\equiv\bar{u}(p)i\gamma^\alpha\gamma^5u(p)$. This vector is orthogonal to p and transforms as an axial vector. Its form is $s^\mu=(s^0,\vec{s})$:

$$s^0=\vec{p}\cdot\hat{n}\,,\qquad \vec{s}=m\hat{n}+\frac{\vec{p}\cdot\hat{n}}{E+m}\vec{p}. \tag{2.2}$$

Here $\hat{n}$ is an arbitrary unit vector specifying the rest-frame spin direction $\hat{n}=\langle\vec{\sigma}\rangle$, $p^2=-s^2=m^2$.

With the three available vectors p, q, and s, the following seven parity-conserving tensor amplitudes, transverse to q, can be constructed ($\nu=p\cdot q$):

$$A^{\mu\nu}_L=-g^{\mu\nu}+q^\mu q^\nu/q^2, \tag{2.3a}$$

$$A^{\mu\nu}_2=p^\mu p^\nu-(\nu/q^2)(p^\mu q^\nu+p^\nu q^\mu)+g^{\mu\nu}\nu^2/q^2, \tag{2.3b}$$

$$A^{\mu\nu}_3=\epsilon^{\mu\nu\alpha\beta}s_\alpha q_\beta, \tag{2.3c}$$

$$A^{\mu\nu}_4=q\cdot s\,\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta, \tag{2.3d}$$

$$A^{\mu\nu}_5=q^2\epsilon^{\mu\nu\alpha\beta}s_\alpha p_\beta+q^\mu\epsilon^{\nu\rho\alpha\beta}q_\rho s_\alpha p_\beta-q^\nu\epsilon^{\mu\rho\alpha\beta}q_\rho s_\alpha p_\beta, \tag{2.3e}$$

$$A^{\mu\nu}_6=(p^\mu q^2-\nu q^\mu)\epsilon^{\nu\rho\alpha\beta}q_\rho s_\alpha p_\beta-(p^\nu q^2-\nu q^\nu)\epsilon^{\mu\rho\alpha\beta}q_\rho s_\alpha p_\beta, \tag{2.3f}$$

$$A^{\mu\nu}_7=(p^\mu q^2-\nu q^\mu)\epsilon^{\nu\rho\alpha\beta}q_\rho s_\alpha p_\beta+(p^\nu q^2-\nu q^\nu)\epsilon^{\mu\rho\alpha\beta}q_\rho s_\alpha p_\beta. \tag{2.3g}$$

Time-inversion invariance eliminates $A^{\mu\nu}_7$, while $A^{\mu\nu}_3$ through $A^{\mu\nu}_6$ are not independent:

$$-\nu A^{\mu\nu}_3+A^{\mu\nu}_4+A^{\mu\nu}_5=0, \tag{2.4a}$$

$$(\nu^2-m^2q^2)A^{\mu\nu}_3-\nu A^{\mu\nu}_4+A^{\mu\nu}_6=0. \tag{2.4b}$$

For our purposes it is convenient to eliminate $A^{\mu\nu}_5$ and $A^{\mu\nu}_6$. Therefore, we are left with

$$C^{\mu\nu}_{ab}(p,q)=A^{\mu\nu}_LW^{ab}_L+A^{\mu\nu}_2W^{ab}_2+iA^{\mu\nu}_3W^{ab}_3+iA^{\mu\nu}_4W^{ab}_4. \tag{2.5}$$

The invariants are functions of q^2 and ν. We shall decompose them into parts symmetric in ab, denoted by (ab), and parts antisymmetric in ab, denoted by $[ab]$:

$$W^{ab}_i=W^{(ab)}_i+iW^{[ab]}_i,\quad i=L,2,3,4. \tag{2.6}$$

Crossing now implies that

$$W^{(ab)}_i(q^2,\nu)=-W^{(ab)}_i(q^2,-\nu),\quad i=L,2,3 \tag{2.7a}$$

$$W^{(ab)}_4(q^2,\nu)=+W^{(ab)}_4(q^2,-\nu), \tag{2.7b}$$

and the opposite symmetry obtains for the invariants which are antisymmetric in a and b. It can be verified that the invariants occurring on the right-hand side of (2.6) are real.

Since the current has dimension three, and our states are covariantly normalized, the following functions are dimensionless: W_L^{ab}, νW_2^{ab}, νW_3^{ab}, and $\nu^2 W_4^{ab}$. Hence, these objects approach a limit, as $\nu\to\infty$ with fixed $-q^2/2\nu\equiv\omega$. In the present context this is *not* a hypothesis, but a consequence of our use of the light-cone commutators. In the pre-asymptotic region, we shall frequently consider the above functions to depend on ω and q^2. In that case W_L^{ab} will be denoted by $-(1/2\omega)\tilde{F}_L^{ab}(\omega, q^2)$, and the others by $\tilde{F}_i^{ab}(\omega, q^2)$. In the scaling limit, the $\tilde{F}_i(\omega, q^2)$ become $F_i(\omega)$, $i=L, 2, 3, 4$. It is easy to see that the F_i vanish for $|\omega|>1$.

In a Regge model one expects the following leading large-ν behavior at fixed q^2:

$$W_L^{(ab)}\sim \nu^{\alpha_V}, \qquad W_L^{[ab]}\sim\nu^{\alpha_\rho}, \tag{2.8a}$$

$$W_2^{(ab)}\sim \nu^{\alpha_V-2}, \qquad W_2^{[ab]}\sim\nu^{\alpha_\rho-2}, \tag{2.8b}$$

$$W_3^{(ab)}\sim \nu^{\tilde\alpha_V-1}, \qquad W_3^{[ab]}\sim\nu^{\tilde\alpha_\rho-1}, \tag{2.8c}$$

$$W_4^{(ab)}\sim \nu^{\tilde\alpha_V-2}, \qquad W_4^{[ab]}\sim\nu^{\tilde\alpha_\rho-2}, \tag{2.8d}$$

where α_V and α_ρ are the Pomeranchon and ρ-meson trajectories and $\tilde\alpha_V$ and $\tilde\alpha_\rho$ are, respectively, the leading odd- and even-signature trajectories of proper G parity. The absence of candidates for $\tilde\alpha_V$ and $\tilde\alpha_\rho$ will indicate convergence of three of the sum rules investigated below. The absence of a leading ρ contribution to $W_3^{[ab]}+\nu W_4^{[ab]}$ is the familiar reason for the convergence of the Bég sum rule.[7,9] The Regge content of $C_{ab}^{\mu\nu}$ and its relation to the s- and t-channel helicity amplitudes is discussed in Appendix A.

The Compton amplitude, whose imaginary part is proportional to (2.1), is the following:

$$T_{ab}^{\mu\nu}(p,q)=i\int d^4x\, e^{iq\cdot x}\langle p|T^*(V_a^\mu(x)V_b^\nu(0))|p\rangle$$

$$=A_L^{\mu\nu}T_L^{ab}+A_2^{\mu\nu}T_2^{ab}+iA_3^{\mu\nu}T_3^{ab}+iA_4^{\mu\nu}T_4^{ab}+if_{abc}(1/q^2)\Gamma_c(g^{\mu\nu}\nu-p^\mu q^\nu-p^\nu q^\mu), \tag{2.9a}$$

$$\langle p|V_c^\nu(0)|p\rangle\equiv p^\nu\Gamma_c. \tag{2.9b}$$

The invariant functions T_i^{ab} may be represented by a dispersion relation of the form

$$T_i^{ab}(q^2,\nu)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\nu'\,\frac{W_i^{ab}(q^2,\nu')}{\nu'-\nu}, \qquad q^2\leqslant 0 \tag{2.10}$$

where there may, of course, be subtractions.

The one-nucleon (Born) contribution to W_i^{ab} is the following:

$$W_L^{ab}=-\tfrac14\pi q^2 d_{abc}\lambda_c[2F_EF_M+F_M{}^2(1+q^2/4m^2)][\delta(q^2+2\nu)-\delta(q^2-2\nu)]$$

$$-\tfrac14 i\pi q^2 f_{abc}\lambda_c[2F_EF_M+F_M{}^2(1+q^2/4m^2)][\delta(q^2+2\nu)+\delta(q^2-2\nu)], \tag{2.11a}$$

$$W_2^{ab}=\pi d_{abc}\lambda_c[F_E{}^2-(q^2/4m^2)F_M{}^2][\delta(q^2+2\nu)-\delta(q^2-2\nu)]+i\pi f_{abc}\lambda_c[F_E{}^2-(q^2/4m^2)F_M{}^2][\delta(q^2+2\nu)+\delta(q^2-2\nu)], \tag{2.11b}$$

$$W_3^{ab}=\tfrac12\pi d_{abc}\lambda_c[F_E{}^2+F_EF_M(1+q^2/4m^2)+F_M{}^2(q^2/4m^2)][\delta(q^2+2\nu)-\delta(q^2-2\nu)]$$

$$+\tfrac12 i\pi f_{abc}\lambda_c[F_E{}^2+F_EF_M(1+q^2/4m^2)+F_M{}^2(q^2/4m^2)][\delta(q^2+2\nu)+\delta(q^2-2\nu)], \tag{2.11c}$$

$$W_4^{ab}=\tfrac14\pi d_{abc}\lambda_c(1/m^2)(F_EF_M+F_M{}^2)[\delta(q^2+2\nu)+\delta(q^2-2\nu)]+\tfrac14 i\pi f_{abc}\lambda_c(1/m^2)(F_EF_M+F_M{}^2)[\delta(q^2+2\nu)-\delta(q^2-2\nu)]. \tag{2.11d}$$

Here F_E and F_M are the electric and magnetic form factors and are functions of q^2 such that $F_E(0)=1$ and $F_M(0)=$ anomalous magnetic moment; i.e., the photon-fermion vertex is

$$\gamma^\mu F_E(q^2)+(i/2m)\sigma^{\mu\nu}q_\nu F_M(q^2). \tag{2.12}$$

The free-field values for W_i^{ab} are obtained from the above by setting $F_E=1$ and $F_M=0$.

We conclude this section by recording various cross sections. The cross sections for inelastic lepton-nucleon scattering are given in terms of the invariant functions W_i by

$$\frac{d^2\sigma^{(\pm)}}{d\Omega dE'}=\frac{\alpha^2}{4\pi}\frac{E'}{mE}\frac{1}{q^4}[-2q^2W_L(q^2,\nu)+(2E^2+2E'^2+q^2)m^2W_2(q^2,\nu)]$$

$$\mp\frac{\alpha^2}{2\pi}\frac{E'}{E}\frac{1}{q^2}[(E+E'\cos\theta)W_3(q^2,\nu)+m(E+E')(E-E'\cos\theta)W_4(q^2,\nu)]. \tag{2.13}$$

E and E' are, respectively, the initial and final lepton energies, as viewed in the laboratory frame, and θ is the lepton scattering angle. $d\sigma^{(\pm)}$ is the cross section when the spins of the lepton and nucleon are parallel (+) or antiparallel (−) and along the direction of the incident lepton. The lepton mass is neglected.

III. INFINITE-MOMENTUM DERIVATION OF SUM RULES

We summarize the $p\to\infty$ sum rules which emerge from the 0μ components of (2.1). Some of these results are well-known; others presumably have been recorded in the literature. We collect them here for easy refere.ıce. Equation (2.1) and the usual current algebra imply

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} dq^0 C_{ab}^{0\alpha}(p,q) = if_{abc}p^\alpha\Gamma_c\,. \tag{3.1a}$$

We have assumed that the Schwinger term is a c number. Throughout this section we set $\vec{p}\cdot\vec{q}=0$. A change of variable $q^0=\nu/p^0$ is now performed:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\nu}{p^0}C_{ab}^{0\alpha}(p,q)=if_{abc}p^\alpha\Gamma_c. \tag{3.1b}$$

In (3.1b) we first set $\alpha=0$, and obtain from (2.3) and (2.5)

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}d\nu\left[\frac{\vec{q}^2}{q^2{p_0}^2}W_L^{[ab]}(q^2,\nu)+\left(1-\frac{\nu^2}{q^2{p_0}^2}\right)W_2^{[ab]}(q^2,\nu)\right]=f_{abc}\Gamma_c,\qquad q^2=\frac{\nu^2}{{p_0}^2}-\vec{q}^2. \tag{3.2a}$$

Because of (2.7), only the antisymmetric part in ab contributes in (3.2a). Letting $p\to\infty$, we are left with the Dashen–Fubini–Gell-Mann result[6]

$$\frac{1}{\pi}\int_0^\infty d\nu\,W_2^{[ab]}(q^2,\nu)=f_{abc}\Gamma_c,\qquad q^2\le 0. \tag{3.2b}$$

Differentiating with respect to q^2 and setting $q^2=0$ reduces this to the Cabibbo-Radicati sum rule.[12]
Next we take $\alpha=i$ in (3.1b):

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\nu}{p^0}C_{ab}^{0i}(p,q)=if_{abc}p^i\Gamma_c. \tag{3.3}$$

The integrand is given by

$$C_{ab}^{0i}(p,q)=q^0q^iW_L^{ab}+\left[p^0p^i-\frac{\nu}{q^2}(p^0q^i+q^0p^i)\right]W_2^{ab}+i\epsilon^{ijk}s_jq_kW_3^{ab}+i\epsilon^{ijk}p_jq_kq\cdot sW_4^{ab}. \tag{3.4}$$

Evidently the terms involving s must integrate to zero. Moreover, since $\epsilon^{ijk}s_jq_k$ and $\epsilon^{ijk}p_jq_k$ are independent, we have

$$\int_{-\infty}^{\infty}d\nu\,W_3^{ab}(q^2,\nu)=0,\qquad q^2=\frac{\nu^2}{{p_0}^2}-\vec{q}^2 \tag{3.5}$$

$$\int_{-\infty}^{\infty}d\nu\left(\nu\frac{s^0}{p^0}-\vec{q}\cdot\vec{s}\right)W_4^{ab}(q^2,\nu)=0,\qquad q^2=\frac{\nu^2}{{p_0}^2}-\vec{q}^2. \tag{3.6}$$

Letting $p\to\infty$ in (3.5), we obtain from (2.7) the nontrivial result

$$\int_0^\infty d\nu\,W_3^{[ab]}(q^2,\nu)=0,\qquad q^2\le 0. \tag{3.7}$$

Now we extract information from (3.6). According to (2.2),

$$\int_{-\infty}^{\infty}d\nu\left(\nu\frac{\vec{p}\cdot\hat{n}}{p^0}-m\vec{q}\cdot\hat{n}\right)W_4^{ab}(q^2,\nu)=0,\qquad q^2=\frac{\nu^2}{{p_0}^2}-\vec{q}^2. \tag{3.8}$$

In the $p_0\to\infty$ limit, we are left with[13]

$$\int_0^\infty d\nu\,\nu W_4^{[ab]}(q^2,\nu)=0,\qquad q^2\le 0 \tag{3.9}$$

$$\int_0^\infty d\nu\,W_4^{(ab)}(q^2,\nu)=0,\qquad q^2\le 0. \tag{3.10}$$

Finally we return to (3.3) and (3.4) and discuss the s-independent terms. Equating coefficients of q^i and p^i yields

$$\int_{-\infty}^{\infty} d\nu \left[\frac{\nu}{p_0^{\,2}} W_L^{ab}(q^2, \nu) - \frac{\nu}{q^2} W_2^{ab}(q^2, \nu)\right] = 0, \quad q^2 = \frac{\nu^2}{p_0^{\,2}} - \vec{q}^{\,2} \tag{3.11}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu \left(1 - \frac{\nu^2}{q^2 p_0^{\,2}}\right) W_2^{ab}(q^2, \nu) = i f_{abc}\Gamma_c, \quad q^2 = \frac{\nu^2}{p_0^{\,2}} - \vec{q}^{\,2}. \tag{3.12}$$

Because of the symmetry properties in ν, (3.11) is equivalent, in the $p\to\infty$ limit, to

$$\int_0^{\infty} d\nu\, \nu W_2^{(ab)}(q^2, \nu) = 0, \quad q^2 \leq 0. \tag{3.13}$$

To extract the consequences of (3.12), we substitute that relation for the right-hand side of (3.2a). Since only $W_2^{[ab]}$ is present in (3.12), this operation yields, after some rearrangement,

$$\int_0^{\infty} d\nu \frac{\vec{q}^{\,2}}{q^2} W_L^{[ab]}(q^2, \nu) = 0, \quad q^2 = \frac{\nu^2}{p_0^{\,2}} - \vec{q}^{\,2}. \tag{3.14a}$$

As $p\to\infty$ we find

$$\int_0^{\infty} d\nu\, W_L^{[ab]}(q^2, \nu) = 0, \quad q^2 \leq 0. \tag{3.14b}$$

In Table I the sum rules are summarized. In the second column, they are rewritten in terms of the functions $\vec{F}_i(\omega, q^2)$ which possess a finite limit as $-q^2\to\infty$. In the third column, we comment on the convergence of these results in the Regge model. Finally, in the last column, their validity in free-field theory is indicated. The entry $0=0$ indicates that in free-field theory, the invariant function vanishes.

Numerical evaluation is consistent for sum rule I, i.e., the Cabibbo-Radicati[12] relation appears to be valid.[10] The Bég sum rule,[9] which is the sum of II and III at $q^2=0$, is not verified.[10] We may also see that V cannot be valid, since the scaling version of it, in the limit $-q^2\to\infty$, implies

$$\int_0^{\infty} \frac{d\omega}{\omega^2} F_2^{(ab)}(\omega) = 0.$$

This is in contradiction with the observed MIT-SLAC experiments.[14] (In a Regge model the left-hand side diverges.)

IV. LIGHT-CONE DERIVATION OF SUM RULES

To give the light-cone analysis of the fixed-mass sum rules, we first introduce the following notation for coordinates (and all other tensor quantities): $x^+ = (\frac{1}{2})^{1/2}(x^0 + x^3)$, $x^- = (\frac{1}{2})^{1/2}(x^0 - x^3)$. The "perpendicular" direction $\vec{x}_\perp$ will sometimes be denoted by x^i, $i=1, 2$. The metric tensor is $g^{+-}=g^{-+}=1$, $g^{++}=g^{--}=0$. The antisymmetric tensor is $\epsilon^{+-ij}=\epsilon^{ij}$, $\epsilon^{12}=1$. To obtain a fixed-mass sum rule, we set q^+ to zero in the $\mu=+$ component of (2.1), and integrate over q^-:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dq^- C_{ab}^{+\alpha}(p, q)\bigg|_{q^+=0} = \int d^2x_\perp e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\left\langle p\left|\left[\int dx^- V_a^+(x), V_b^\alpha(0)\right]\right|p\right\rangle_{\mathrm{LC}} \tag{4.1}$$

The subscript LC indicates the light-cone commutator, i.e., $x^+=0$ in the right-hand side of (4.1).
We now assume that the light-cone commutator of $\int dx^- V_a^+(x)$ with $V_b^\alpha(0)$ can be computed from that of $V_a^+(x)$ with $V_b^\alpha(0)$. This point will be further discussed in Appendix D. Consequently (4.1) becomes

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dq^- C_{ab}^{+\alpha}(p, q)\bigg|_{q^+=0} = \int d^2x_\perp dx^- e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\langle p|[V_a^+(x), V_b^\alpha(0)]|p\rangle_{\mathrm{LC}}. \tag{4.2}$$

Choosing $\alpha=+$, the left-hand side of (4.1) becomes

$$\begin{aligned}\frac{1}{2\pi}\int_{-\infty}^{\infty} dq^- C_{ab}^{++}(p, q)\bigg|_{q^+=0} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} dq^- p^+p^+ W_2^{ab}(-\vec{q}_\perp^{\,2}, p^+q^- - \vec{p}_\perp\cdot\vec{q}_\perp)\\ &= \frac{p^+}{2\pi}\int_{-\infty}^{\infty} d\nu\, W_2^{ab}(-\vec{q}_\perp^{\,2}, \nu) = \frac{ip^+}{\pi}\int_0^{\infty} d\nu\, W_2^{[ab]}(-\vec{q}_\perp^{\,2}, \nu).\end{aligned} \tag{4.3}$$

Therefore, without ever using the $p\to\infty$ frame, *we have arrived at the integral relevant* to the Dashen–

TABLE I. Fixed-mass sum rules as derived by the $p\to\infty$ method.

	Sum rule	Scaling version	Regge-model convergence	Free-field validity
I	$\int_0^\infty d\nu\, W_2^{[ab]}(q^2,\nu)=\pi f_{abc}\Gamma_c$	$\int_0^1 \frac{d\omega}{\omega}\tilde F_2^{[ab]}(\omega,q^2)=\pi f_{abc}\Gamma_c=\int_0^1\frac{d\omega}{\omega}F_2^{[ab]}(\omega)$	Yes	Yes
II	$\int_0^\infty d\nu\, W_3^{[ab]}(q^2,\nu)=0$	$\int_0^1 \frac{d\omega}{\omega}\tilde F_3^{[ab]}(\omega,q^2)=0=\int_0^1\frac{d\omega}{\omega}F_3^{[ab]}(\omega)$	Yes	No
III	$\int_0^\infty d\nu\,\nu W_4^{[ab]}(q^2,\nu)=0$	$\int_0^1 \frac{d\omega}{\omega}\tilde F_4^{[ab]}(\omega,q^2)=0=\int_0^1\frac{d\omega}{\omega}F_4^{[ab]}(\omega)$	Yes	Yes $(0=0)$
IV	$\int_0^\infty d\nu\, W_4^{(ab)}(q^2,\nu)=0$	$\int_0^1 d\omega\tilde F_4^{(ab)}(\omega,q^2)=0=\int_0^1 d\omega F_4^{(ab)}(\omega)$	Yes	Yes $(0=0)$
V	$\int_0^\infty d\nu\,\nu W_2^{(ab)}(q^2,\nu)=0$	$\int_0^1 \frac{d\omega}{\omega^2}\tilde F_2^{(ab)}(\omega,q^2)=0=\int_0^1\frac{d\omega}{\omega^2}F_2^{(ab)'}(\omega)$	No	No
VI	$\int_0^\infty d\nu\, W_L^{[ab]}(q^2,\nu)=0$	$\int_0^1 \frac{d\omega}{\omega^3}\tilde F_L^{[ab]}(\omega,q^2)=0=\int_0^1\frac{d\omega}{\omega^3}F_L^{[ab]}(\omega)$	No	Yes $(0=0)$

Fubini–Gell-Mann[6] sum rule. However, this object is *not* equal to an equal-time commutator, but rather to a light-cone commutator.[11] The p-infinity technique can now be understood as the attempt to convert the light-cone commutator, occurring on the right-hand side of (4.1), to an equal-time commutator. This point will be elaborated in Appendix B. Here we need not engage in this dubious procedure, if we have a model for the light-cone commutator.

Before presenting the model, we take $\alpha=-$ in (4.1). The left-hand side becomes

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}dq^- C_{ab}^{+-}(p,q)\Big|_{q^+=0}=\frac{1}{2\pi}\int_{-\infty}^{\infty}dq^-\Big[-W_L^{ab}(-\vec q_\perp^{\,2},\nu)+\Big(p^+p^-+\frac{\nu}{\vec q_\perp^{\,2}}\vec p_\perp\cdot\vec q_\perp\Big)W_2^{ab}(-\vec q_\perp^{\,2},\nu)$$
$$+i\epsilon^{ij}s_iq_jW_3^{ab}(-\vec q_\perp^{\,2},\nu)+i\epsilon^{ij}p_iq_j(q^-s^+-\vec q_\perp\cdot\vec s_\perp)W_4^{ab}(-\vec q_\perp^{\,2},\nu)\Big]$$
$$=\frac{i}{\pi p^+}\int_0^\infty d\nu[-W_L^{[ab]}(-\vec q_\perp^{\,2},\nu)+p^+p^-W_2^{[ab]}(-\vec q_\perp^{\,2},\nu)]+\frac{\vec p_\perp\cdot\vec q_\perp}{\pi\vec q_\perp^{\,2}p^+}\int_0^\infty d\nu\,\nu W_2^{(ab)}(-\vec q_\perp^{\,2},\nu)$$
$$-\frac{\epsilon^{ij}q_j}{\pi p^+}\int_0^\infty d\nu\Big[s_iW_3^{[ab]}(-\vec q_\perp^{\,2},\nu)+\frac{s^+p_i\nu}{p^+}W_4^{[ab]}(-\vec q_\perp^{\,2},\nu)\Big]$$
$$+\frac{i\epsilon^{ij}p_iq_j}{\pi p^+}\Big(\vec p_\perp\cdot\vec q_\perp\frac{s^+}{p^+}-\vec q_\perp\cdot\vec s_\perp\Big)\int_0^\infty d\nu\,W_4^{(ab)}(-\vec q_\perp^{\,2},\nu). \qquad (4.4)$$

We have used the symmetry properties of the W_i^{ab} to decompose the final expression in (4.4). It is seen that all the integrals occurring in Table I have reoccurred in (4.3) and (4.4).

To complete the derivation of the sum rules, we need to specify the light-cone commutator $[V_a^+, V_b^\alpha]$. What can we say about it in a model-independent way? First, it is easy to show that for *conserved currents* the charge Q_a, conventionally given by an integral on a timelike surface $Q_a=\int d^3x\,V_a^0(x)$, is identically equal to an integral over a lightlike surface: $Q_a=\int d^2x_\perp dx^-\,V_a^+(x)$. From the assumed transformation properties of V_b^α, we therefore deduce that

$$[V_a^+(x),V_b^\alpha(y)]_{\rm LC}=if_{abc}V_c^\alpha(x)\delta(x^--y^-)\delta^2(\vec x_\perp-\vec y_\perp)+\partial_-^xS_{ab}^\alpha(x|y)+\partial_i^xS_{ab}^{i\alpha}(x|y). \qquad (4.5)$$

The additional terms, which vanish upon integration over x^- and $\vec x_\perp$, are rather like the Schwinger terms encountered in conventional equal-time calculations. Indeed, by taking vacuum expectation values, one can prove that they *must* be present.[1] They differ, however, in one important fashion from the structures encountered previously: They need not be local in $x^- - y^-$, though of course they are local in $\vec x_\perp-\vec y_\perp$. The reason for this is that $(x-y)^2=2(x-y)^+(x-y)^- - (\vec x-\vec y)_\perp^{\,2}$ does not depend on $x^- - y^-$ when $x^+=y^+$, and causality is insured merely by locality in the transverse components, with no restriction on the "minus" component. These objects are called *bilocal operators*.

Let us remark here that for the Dashen–Fubini–Gell-Mann[6] sum rule to be valid, there must be no transverse gradients in $[V_a^+(x), V_b^+(y)]_{\rm LC}$.[11] Such gradients would, according to (4.1) and (4.3), produce $\vec{q}_\perp$-dependent corrections. Similarly it has been shown that $F_L(\omega)$, as measured in the MIT-SLAC electroproduction experiments,[13] vanishes if and only if there are no "minus" gradients in this commutator.[1] Of course in question here is only the q-number portion of such objects. Also, since we are dealing with a diagonal matrix element, no information about off-diagonal terms is present.

To specify the bilocal operators, we turn to a model field theory. By quantizing it on a null surface,[3] we can deduce light-cone commutators canonically.[1] Two alternative theories may be considered: a fermion theory or a boson theory. In both, the Dashen–Fubini–Gell-Mann[6] sum rule is valid, but in the latter $F_L(\omega) \neq 0$. Hence we accept the former. In the Conclusion, Sec. VI, we discuss some questions concerning the model dependence of these commutators. In Appendix C we present the boson theory.

The current commutators which emerge in the fermion (= quark) theory are the following[1]:

$$[V_a^+(x), V_b^+(y)]_{\rm LC} = if_{abc}V_c^+(x)\delta(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp) - \tfrac{1}{4}i\partial_-^x\partial_-^y[S_{ab}(x|y)\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)], \tag{4.6a}$$

$$\begin{aligned}[V_a^+(x), V_b^-(y)]_{\rm LC} = {} & if_{abc}V_c^-(x)\delta(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp) \\ & - \tfrac{1}{2}if_{abc}\{\partial_-^x[\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)\mathcal{V}_c^-(x|y)] \\ & \qquad + \tfrac{1}{2}\partial_i^x[\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)\mathcal{V}_c^i(x|y)] - \tfrac{1}{2}\partial_i^x\epsilon^{ij}[\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)\overline{\mathcal{A}}_{jc}(x|y)]\} \\ & - \tfrac{1}{2}id_{abc}\{\partial_-^x[\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)\overline{\mathcal{V}}_c^-(x|y)] \\ & \qquad + \tfrac{1}{2}\partial_i^x[\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)\overline{\mathcal{V}}_c^i(x|y)] + \tfrac{1}{2}\partial_i^x\epsilon^{ij}[\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)\mathcal{A}_{jc}(x|y)]\} \\ & + i\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)M_{ab}(x|y) - \tfrac{1}{8}iS_{ab}(x|y)\epsilon(x^- - y^-)\partial_i\partial^i\delta^2(\vec{x}_\perp - \vec{y}_\perp). \end{aligned} \tag{4.6b}$$

These results were obtained in a quark model with a vector-gluon interaction and a symmetry-breaking mass term.[1] It is not hard to verify that they are also true when the gluons are spinless. In (4.6) all terms except $S_{ab}(x|y)$ emerge with canonical manipulations; however, $S_{ab}(x|y)$ can be shown to have nonzero vacuum expectation value. The inability to compute it canonically is a reflection of the fact that the ordinary Schwinger term is not evaluated canonically, and is related to the nonoccurrence of dimension-two fermion operators in the theory. We shall assume that S_{ab} is a c number; $S_{ab}(x|y) = \delta_{ab}S$. This assumption, which is equivalent to setting $F_L(\omega) = 0$, is not true in perturbation theory.[15]

The term involving $M_{ab}(x|y)$ is present in (4.6b) only when the currents are not conserved, and need not concern us any further.[1] The remaining bilocal operators are defined as follows:

$$V_a^\mu(x|y) = \tfrac{1}{2}\bar{\psi}(x)\gamma^\mu\lambda_a\psi(y), \tag{4.7a}$$

$$A_a^\mu(x|y) = \tfrac{1}{2}i\bar{\psi}(x)\gamma^\mu\gamma^5\lambda_a\psi(y). \tag{4.7b}$$

These are bilocal non-Hermitian generalizations of the vector and axial-vector currents. We now extract the Hermitian and anti-Hermitian parts, as these are the objects which occur in (4.6b):

$$\mathcal{V}_a^\mu(x|y) \equiv \tfrac{1}{2}V_a^\mu(x|y) + \tfrac{1}{2}V_a^\mu(y|x), \tag{4.8a}$$

$$\overline{\mathcal{V}}_a^\mu(x|y) \equiv (1/2i)V_a^\mu(x|y) - (1/2i)V_a^\mu(y|x), \tag{4.8b}$$

$$\mathcal{A}_a^\mu(x|y) \equiv \tfrac{1}{2}A_a^\mu(x|y) + \tfrac{1}{2}A_a^\mu(y|x), \tag{4.8c}$$

$$\overline{\mathcal{A}}_a^\mu(x|y) \equiv (1/2i)A_a^\mu(x|y) - (1/2i)A_a^\mu(y|x). \tag{4.8d}$$

It is clear that these enter (4.6b) with $x^+ = y^+$, $\vec{x}_\perp = \vec{y}_\perp$, $x^- \neq y^-$.

The occurrence of the bilocal generalizations of the vector and axial-vector currents in the right-hand side of (4.6b) may also be understood as follows. As $x^- \to y^-$, the tip of the light cone is attained, and the light-cone commutators approach equal-time commutators.[16] It is not hard to see that the requirement that (4.6b) reproduce, in that limit, the appropriate equal-time commutator, requires that $\mathcal{V}_c^-(x|x) = V_c^-(x)$. The relevant equal-time commutator involves the space components, hence the bilocal operators of (4.6b) are of the given form only in the quark model. Different model realizations of SU(3)×SU(3) would lead to different bilocal operators. Furthermore, since the space-component equal-time algebra is not maintained in perturbation theory,[17] we must expect that the light-cone commutators will have different bilocal structures, when perturbative calculations are performed. [Similar considerations about the light-cone commutator of vector and axial-vector currents, show that $\mathcal{A}_a(x|x)$ is the local axial-vector current.]

The sum rules may now be derived. From (4.1), (4.3), and (4.6a), we readily get

$$\int_0^\infty d\nu\, W_2^{[ab]}(-\vec{q}_\perp^{\,2}, \nu) = \pi f_{abc}\Gamma_c. \tag{4.9}$$

Thus the Dashen–Fubini–Gell-Mann[6] result has no corrections.

Similarly from (4.1), (4.4), and (4.6b), two relations are derived, by equating terms symmetric or antisymmetric in ab:

$$\frac{\vec{p}_\perp\cdot\vec{q}_\perp}{\pi\vec{q}_\perp^{\,2}p^+}\int_0^\infty d\nu\,\nu W_2^{(ab)}(-\vec{q}_\perp^{\,2},\nu)+\frac{i\epsilon^{ij}p_iq_j}{\pi p^+}\left(\vec{p}_\perp\cdot\vec{q}_\perp\frac{s^+}{p^+}-\vec{q}_\perp\cdot\vec{s}_\perp\right)\int_0^\infty d\nu\,W_4^{(ab)}(-\vec{q}_\perp^{\,2},\nu)$$
$$=-\tfrac{1}{4}d_{abc}q_i\int_{-\infty}^{\infty}dx^-\epsilon(x^-)\langle p|\bar{\mathcal{V}}_c^i(x|0)|p\rangle\Big|_{x^+=0;\ \vec{x}_\perp=0}-\tfrac{1}{4}d_{abc}q_i\epsilon^{ij}\int_{-\infty}^{\infty}dx^-\epsilon(x^-)\langle p|\mathcal{A}_{jc}(x|0)|p\rangle\Big|_{x^+=0;\ \vec{x}_\perp=0}, \tag{4.10a}$$

$$\frac{i}{\pi p^+}\int_0^\infty d\nu\left[-W_1^{[ab]}(-\vec{q}_\perp^{\,2},\nu)+\left(p^+p^-+\frac{\nu^2}{\vec{q}_\perp^{\,2}}\right)W_2^{[ab]}(-\vec{q}_\perp^{\,2},\nu)\right]-\frac{\epsilon^{ij}q_j}{\pi p^+}\int_0^\infty d\nu\left[s_iW_3^{[ab]}(-\vec{q}_\perp^{\,2},\nu)+p_i\frac{\nu s^+}{p^+}W_4^{[ab]}(-\vec{q}_\perp^{\,2},\nu)\right]$$
$$=if_{abc}\Gamma_cp^--\tfrac{1}{4}f_{abc}q_i\int_{-\infty}^{\infty}dx^-\epsilon(x^-)\langle p|\mathcal{V}_c^i(x|0)|p\rangle\Big|_{x^+=0;\ \vec{x}_\perp=0}+\tfrac{1}{4}f_{abc}q_i\epsilon^{ij}\int_{-\infty}^{\infty}dx^-\epsilon(x^-)\langle p|\bar{\mathcal{A}}_{jc}(x|0)|p\rangle\Big|_{x^+=0;\ \vec{x}_\perp=0}. \tag{4.10b}$$

The expressions (4.10a) and (4.10b) simplify. We note that $\langle p|\mathcal{V}_c^\mu(x|0)|p\rangle$ and $\langle p|\mathcal{A}_c^\mu(x|0)|p\rangle$ are even in x, while the same matrix elements of $\bar{\mathcal{V}}_c^\mu(x|0)$ and $\bar{\mathcal{A}}_c^\mu(x|0)$ are odd. This is seen by translation invariance and the definitions (4.8). Also, (4.9) may be used in (4.10b). Therefore, instead of (4.10a) and (4.10b), we have upon equating spin-dependent and spin-independent terms the following equalities:

$$\frac{\vec{p}_\perp\cdot\vec{q}_\perp}{\pi\vec{q}_\perp^{\,2}p^+}\int_0^\infty d\nu\,\nu W_2^{(ab)}(-\vec{q}_\perp^{\,2},\nu)=-\tfrac{1}{2}d_{abc}q_i\int_0^\infty dx^-\langle p|\bar{\mathcal{V}}_c^i(x|0)|p\rangle\Big|_{s=0;\ x^+=0;\ \vec{x}_\perp=0}, \tag{4.11}$$

$$\frac{i}{\pi p^+}\int_0^\infty d\nu\,W_L^{[ab]}(-\vec{q}_\perp^{\,2},\nu)=-\tfrac{1}{2}f_{abc}q_i\epsilon^{ij}\int_0^\infty dx^-\langle p|\bar{\mathcal{A}}_{jc}(x|0)|p\rangle\Big|_{s=0;\ x^+=0;\ \vec{x}_\perp=0}, \tag{4.12}$$

$$\frac{i\epsilon^{ij}p_iq_j}{\pi p^+}\left(\vec{p}_\perp\cdot\vec{q}_\perp\frac{s^+}{p^+}-\vec{q}_\perp\cdot\vec{s}_\perp\right)\int_0^\infty d\nu\,W_4^{(ab)}(-\vec{q}_\perp^{\,2},\nu)=\tfrac{1}{2}d_{abc}q_i\int_0^\infty dx^-\langle p|\bar{\mathcal{V}}_c^i(x|0)|p\rangle\Big|_{s\neq0;\ x^+=0;\ \vec{x}_\perp=0}, \tag{4.13}$$

$$-\frac{\epsilon^{ij}q_j}{\pi p^+}\int_0^\infty d\nu\left[s_iW_3^{[ab]}(-\vec{q}_\perp^{\,2},\nu)-\frac{p_is^+}{p^+}\nu W_4^{[ab]}(-\vec{q}_\perp^{\,2},\nu)\right]=\tfrac{1}{2}f_{abc}q_i\epsilon^{ij}\int_0^\infty dx^-\langle p|\bar{\mathcal{A}}_{jc}(x|0)|p\rangle\Big|_{s\neq0;\ x^+=0;\ \vec{x}_\perp=0}. \tag{4.14}$$

Here the subscripts $s=0$ or $s\neq0$ denote, respectively, the spin-independent or the spin-dependent parts. Finally we define the following real form factors:

$$\langle p|\bar{\mathcal{V}}_c^\mu(x|0)|p\rangle=p^\mu\bar{V}_c^1(x^2,x\cdot p)+x^\mu\bar{V}_c^2(x^2,x\cdot p), \tag{4.15a}$$

$$\langle p|\bar{\mathcal{A}}_c^\mu(x|0)|p\rangle=s^\mu\bar{A}_c^1(x^2,x\cdot p)+p^\mu x\cdot s\bar{A}_c^2(x^2,x\cdot p)+x^\mu x\cdot s\bar{A}_c^3(x^2,x\cdot p). \tag{4.15b}$$

T inversion eliminates a possible structure of the form $\epsilon^{\mu\alpha\beta\rho}x_\alpha p_\beta s_\rho$ in (4.15a). The sum rules now become (for $q^2\leq0$)

$$\int_0^\infty d\nu\frac{\nu}{-q^2}W_2^{(ab)}(q^2,\nu)=\tfrac{1}{2}\pi d_{abc}\int_0^\infty d\alpha\,\bar{V}_c^1(0,\alpha), \tag{4.16}$$

$$\int_0^\infty d\nu\,W_4^{(ab)}(q^2,\nu)=0, \tag{4.17}$$

$$\int_0^\infty d\nu\,W_L^{[ab]}(q^2,\nu)=0, \tag{4.18}$$

$$\int_0^\infty d\nu\,\nu W_4^{[ab]}(q^2,\nu)=\tfrac{1}{2}\pi f_{abc}\int_0^\infty d\alpha\,\alpha\bar{A}_c^2(0,\alpha), \tag{4.19}$$

$$\int_0^\infty d\nu\,W_3^{[ab]}(q^2,\nu)=\tfrac{1}{2}\pi f_{abc}\int_0^\infty d\alpha\bar{A}_c^1(0,\alpha). \tag{4.20}$$

It is seen that three sum rules possess corrections. The corrections are expressible in terms of integrals over matrix elements of the bilocal operators. In the next section we shall show that these matrix elements are measurable, even when an integration is not performed. Here we merely demonstrate how (4.16), (4.19), and (4.20) may be exploited.

Observe that the right-hand sides of (4.16), (4.19), and (4.20) are independent of q^2. Let us rewrite the left-hand sides in terms of the scaling functions (for $q^2 \leqslant 0$) (see Table I):

$$\int_0^\infty d\nu \frac{\nu}{-q^2} W_2^{(ab)}(q^2, \nu) = \int_0^1 \frac{d\omega}{2\omega^2} \tilde{F}_2^{(ab)}(\omega, q^2), \tag{4.21}$$

$$\int_0^\infty d\nu\, W_3^{[ab]}(q^2, \nu) = \int_0^1 \frac{d\omega}{\omega} \tilde{F}_3^{[ab]}(\omega, q^2), \tag{4.22}$$

$$\int_0^\infty d\nu\, \nu W_4^{[ab]}(q^2, \nu) = \int_0^\infty \frac{d\omega}{\omega} \tilde{F}_4^{[ab]}(\omega, q^2). \tag{4.23}$$

According to the sum rules, these integrals are q^2-independent and may be evaluated by letting $-q^2 \to \infty$. Hence we conclude (for $q^2 \leqslant 0$)

$$\int_0^\infty d\nu \frac{\nu}{-q^2} W_2^{(ab)}(q^2, \nu) = \int_0^1 \frac{d\omega}{2\omega^2} F_2^{(ab)}(\omega), \tag{4.24}$$

$$\int_0^\infty d\nu\, W_3^{[ab]}(q^2, \nu) = \int_0^1 \frac{d\omega}{\omega} F_3^{[ab]}(\omega), \tag{4.25}$$

$$\int_0^\infty d\nu\, \nu W_4^{[ab]}(q^2, \nu) = \int_0^1 \frac{d\omega}{\omega} F_4^{[ab]}(\omega). \tag{4.26}$$

These then are the corrected versions of the previously improperly derived relations. We summarize them in Table II.

The sum rule (4.24) has already been derived by an entirely different method, involving dispersion relations, by Cornwall, Corrigan, and Norton.[18] The sum of (4.25) and (4.26) represents our modification of the Bég sum rule.[9] The modifications (4.24), (4.25), and (4.26) ensure that all the sum rules are now valid in the free-field model.

V. MEASURING BILOCAL OPERATORS

We demonstrate that the bilocal operators encountered in light-cone commutators are measured by the deep-inelastic limits of the W_i^{ab}. The present results are an obvious generalization of the investigation by Cornwall and one of us (R. J.),[1] where it was shown that the deep-inelastic structure functions measured in the electroproduction experiments similarly determine the relevant bilocal operators. Two methods are available. One may write a representation of $\int d^4q\, e^{-iq\cdot x} C_{ab}^{\mu\nu}(p, q)$ in position space, parametrized by the

TABLE II. Fixed-mass sum rules as derived from light-cone commutators.

I	$\int_0^\infty d\nu\, W_2^{[ab]}(q^2, \nu)$	$= \pi f_{abc} \Gamma_c$	$= \int_0^1 \frac{d\omega}{\omega} \tilde{F}_2^{[ab]}(\omega, q^2)$	$= \int_0^1 \frac{d\omega}{\omega} F_2^{[ab]}(\omega)$
II	$\int_0^\infty d\nu\, W_3^{[ab]}(q^2, \nu)$	$= \frac{1}{2}\pi f_{abc} \int_0^\infty d\alpha\, \overline{A}_c^1(0, \alpha)$	$= \int_0^1 \frac{d\omega}{\omega} \tilde{F}_3^{[ab]}(\omega, q^2)$	$= \int_0^1 \frac{d\omega}{\omega} F_3^{[ab]}(\omega)$
III	$\int_0^\infty d\nu\, \nu W_4^{[ab]}(q^2, \nu)$	$= \frac{1}{2}\pi f_{abc} \int_0^\infty d\alpha\, \alpha \overline{A}_c^2(0, \alpha)$	$= \int_0^1 \frac{d\omega}{\omega} \tilde{F}_4^{[ab]}(\omega, q^2)$	$= \int_0^1 \frac{d\omega}{\omega} F_4^{[ab]}(\omega)$
IV	$\int_0^\infty d\nu\, W_4^{(ab)}(q^2, \nu)$	$= 0$	$= \int_0^1 d\omega\, \tilde{F}_4^{(ab)}(\omega, q^2)$	$= \int_0^1 d\omega\, F_4^{(ab)}(\omega)$
V	$\int_0^\infty d\nu \frac{\nu}{-q^2} W_2^{(ab)}(q^2, \nu)$	$= \frac{1}{2}\pi d_{abc} \int_0^\infty d\alpha\, \overline{V}_c^1(0, \alpha)$	$= \int_0^1 \frac{d\omega}{2\omega^2} \tilde{F}_2^{(ab)}(\omega, q^2)$	$= \int_0^1 \frac{d\omega}{2\omega^2} F_2^{(ab)}(\omega)$
VI	$\int_0^\infty d\nu\, W_L^{[ab]}(q^2, \nu)$	$= 0$	$= \int_0^1 \frac{d\omega}{\omega^3} \tilde{F}_L^{[ab]}(\omega, q^2)$	$= \int_0^1 \frac{d\omega}{\omega^3} F_L^{[ab]}(\omega)$

$F_i^{ab}(\omega)$, and then pass to $x^+ \to 0$. Alternatively, the light-cone version of the BJL limit, which was discussed in Ref. 1, may be used. Since the former method was previously employed in Ref. 1, we now, for variety, use the latter. The technique is more tedious than the position-space approach, though it is instructive.

Consider the Compton amplitude (2.9a). It has been shown that[19]

$$T_{ab}^{\mu\nu}(p,q) \xrightarrow[q^- \to \infty]{} \text{polynomials} - \frac{1}{q^-}\int dx^- d^2x_\perp e^{iq^+x^-} e^{-i\vec{q}_\perp\cdot\vec{x}_\perp} \langle p|[V_a^\mu(x), V_b^\nu(0)]|p\rangle_{\rm LC} \tag{5.1}$$

From (2.3) and (2.9) we have

$$T_{ab}^{++}(p,q) \xrightarrow[q^- \to \infty]{} -\frac{if_{abc}\Gamma_c p^+}{q^-} + \frac{q^+}{2q^-} T_L^{ab} + O(1/q^-) T_2^{ab}, \tag{5.2a}$$

$$T_{ab}^{+-}(p,q) \xrightarrow[q^- \to \infty]{} \left(-\frac{1}{2} + \frac{1}{4}\frac{\vec{q}_\perp^{\,2}}{q^+q^-}\right) T_L^{ab} + \left(p^+p^- - \frac{p^+}{2q^+}\vec{p}_\perp\cdot\vec{q}_\perp\right) T_2^{ab} + \epsilon^{ij} s_i q_j T_3^{ab} + \epsilon^{ij} p_i q_j q^- s^+ T_4^{ab} - if_{abc}\Gamma_c \frac{\vec{p}_\perp\cdot\vec{q}_\perp}{2q^+q^-}. \tag{5.2b}$$

The large-q^- behavior of the T_i^{ab} is deduced from the dispersive representation (2.10), which may be written in terms of the scaling variables:

$$T_L^{ab} = -\frac{\omega}{4\pi}\int_{-1}^{1} \frac{d\omega'}{\omega'^2} \frac{\tilde{F}_L^{ab}(\omega', q^2)}{\omega - \omega'}, \tag{5.3a}$$

$$T_2^{ab} = -\frac{\omega}{\pi q^2}\int_{-1}^{1} d\omega' \frac{\tilde{F}_2^{ab}(\omega', q^2)}{\omega - \omega'}, \tag{5.3b}$$

$$T_3^{ab} = -\frac{\omega}{\pi q^2}\int_{-1}^{1} d\omega' \frac{\tilde{F}_3^{ab}(\omega', q^2)}{\omega - \omega'}, \tag{5.3c}$$

$$T_4^{ab} = \frac{2\omega}{\pi q^4}\int_{-1}^{1} d\omega' \omega' \frac{\tilde{F}_4^{ab}(\omega', q^2)}{\omega - \omega'}. \tag{5.3d}$$

Here we have continued to use an unsubtracted form, which, in general, is incorrect. However, for present purposes the question of subtractions is irrelevant. The reason for this is that the large-q^- limit of the T_i^{ab} is unaffected by subtractions, since in that limit ω is fixed, $\omega = -q^+/p^+$. Specifically, for (5.3a) we have

$$T_L^{ab} \xrightarrow[q^- \to \infty]{} \frac{\omega}{4\pi}\int_{-1}^{1} \frac{d\omega'}{\omega'^2} \frac{F_L^{ab}(\omega')}{\omega' - \omega}.$$

If a subtracted in ω dispersion relation is used, then

$$T_L^{ab} \xrightarrow[q^- \to \infty]{} C + \frac{1}{2\pi}\int_{-1}^{1} \frac{d\omega'}{\omega'} \frac{F_L^{ab}(\omega')}{\omega' - \omega}.$$

At the end of the calculation, we shall take the discontinuity in ω [see (5.5) and (5.8) below], and therefore the final result is the same, independent of subtractions. Consequently we remain with (5.3) and find

$$T_L^{ab} \to \frac{-\omega}{4\pi}\int_{-1}^{1} \frac{d\omega'}{\omega'^2} \frac{F_L^{ab}(\omega')}{\omega - \omega'} - \frac{\omega}{8\pi q^+ q^-}\int_{-1}^{1} \frac{d\omega'}{\omega'^2} \frac{G_L^{ab}(\omega')}{\omega - \omega'}, \tag{5.4a}$$

$$T_2^{ab} \to \frac{-\omega}{2\pi q^+ q^-}\int_{-1}^{1} d\omega' \frac{F_2^{ab}(\omega')}{\omega - \omega'}, \tag{5.4b}$$

$$T_3^{ab} \to \frac{-\omega}{2\pi q^+ q^-}\int_{-1}^{1} d\omega' \frac{F_3^{ab}(\omega')}{\omega - \omega'}, \tag{5.4c}$$

$$T_4^{ab} \to \frac{\omega}{2\pi (q^+ q^-)^2}\int_{-1}^{1} d\omega' \omega' \frac{F_4^{ab}(\omega')}{\omega - \omega'}. \tag{5.4d}$$

Here $\omega = -q^+/p^+$ and the $F_i(\omega)$ are the scaling limits of the $\tilde{F}_i$. In (5.4a) we have taken

$$\tilde{F}_L^{ab}(\omega, q^2) \xrightarrow[q^2 \to \infty]{} F_L^{ab}(\omega) + (1/q^2) G_L^{ab}(\omega)$$

because the next-to-leading terms contribute to (5.2b).

Substituting (5.4) into (5.2a) gives with the help of (4.6a) and (5.1)

$$\int_{-1}^{1}\frac{d\omega'}{\omega'^2}\frac{F_L^{ab}(\omega')}{\omega-\omega'}=0, \tag{5.5a}$$

since we take the light-cone Schwinger term to be a c number. We find, therefore, the result previously encountered in the Abelian case:

$$F_L^{ab}(\omega)=0. \tag{5.5b}$$

Next we insert (5.4) into (5.2b) and use (5.5b). After some rearrangement, we find

$$\begin{aligned}\frac{\omega}{16\pi q^+}\int_{-1}^{1}\frac{d\omega'}{\omega'^2}\frac{G_L^{ab}(\omega')}{\omega-\omega'}+\left(p^- -\frac{\vec{p}_\perp\cdot\vec{q}_\perp}{2q^+}\right)\frac{1}{2\pi}\int_{-1}^{1}d\omega'\frac{F_2^{ab}(\omega')}{\omega-\omega'}+i\epsilon^{ij}s_iq_j\frac{\omega}{2\pi q^+}\int_{-1}^{1}d\omega'\frac{F_3^{ab}(\omega')}{\omega-\omega'}+if_{abc}\Gamma_c\frac{\vec{p}_\perp\cdot\vec{q}_\perp}{2q^+}\\ -i\epsilon^{ij}p_iq_j\frac{s^+}{(q^+)^2}\frac{\omega}{2\pi}\int_{-1}^{1}d\omega'\omega'\frac{F_4^{ab}(\omega')}{\omega-\omega'}=\int dx^-d^2x_\perp e^{iq^+x^-}e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\langle p\,\|[V_a^+(x),V_b^-(0)]\,|p\rangle.\end{aligned} \tag{5.6a}$$

The commutator is evaluated from (4.6b). Therefore, the right-hand side of (5.6a) becomes

$$\begin{aligned}&if_{abc}p^-\Gamma_c-\tfrac{1}{2}f_{abc}\Big[q^+\int_{-\infty}^{\infty}dx^-e^{iq^+x^-}\epsilon(x^-)\langle p|\mathcal{V}_c^-(x|0)|p\rangle\\&\qquad+\tfrac{1}{2}q_i\int_{-\infty}^{\infty}dx^-e^{iq^+x^-}\epsilon(x^-)\langle p|\mathcal{V}_c^i(x|0)|p\rangle-\tfrac{1}{2}q_i\epsilon^{ij}\int_{-\infty}^{\infty}dx^-e^{iq^+x^-}\epsilon(x^-)\langle p|\overline{\mathcal{A}}_{jc}(x|0)|p\rangle\Big]\\&\quad-\tfrac{1}{2}d_{abc}\Big[q^+\int_{-\infty}^{\infty}dx^-e^{iq^+x^-}\epsilon(x^-)\langle p|\overline{\mathcal{V}}_c^-(x|0)|p\rangle\\&\qquad+\tfrac{1}{2}q_i\int_{-\infty}^{\infty}dx^-e^{iq^+x^-}\epsilon(x^-)\langle p|\overline{\mathcal{V}}_c^i(x|0)|p\rangle+\tfrac{1}{2}q_i\epsilon^{ij}\int_{-\infty}^{\infty}dx^-e^{iq^+x^-}\epsilon(x^-)\langle p|\mathcal{A}_{jc}(x|0)|p\rangle\Big].\end{aligned} \tag{5.6b}$$

Finally, we introduce the tensor decomposition for matrix elements of $\mathcal{V}_c^\mu$ and $\mathcal{A}_c^\mu$, analogous to (4.14), and equate appropriate terms in the two formulas (5.6a) and (5.6b). We find

$$2f_{abc}\Gamma_c+\frac{1}{\pi}\int_{-1}^{1}d\omega'\frac{F_2^{ab}(\omega')}{\omega-\omega'}=-\omega\int_{-\infty}^{\infty}d\alpha\,e^{-i\omega\alpha}\epsilon(\alpha)[d_{abc}\overline{V}_c^1(0,\alpha)+f_{abc}V_c^1(0,\alpha)], \tag{5.7a}$$

$$\frac{1}{\pi}\int_{-1}^{1}d\omega'\frac{F_3^{ab}(\omega')}{\omega-\omega'}=\tfrac{1}{2}i\int_{-\infty}^{\infty}d\alpha\,e^{-i\omega\alpha}\epsilon(\alpha)[d_{abc}A_c^1(0,\alpha)-f_{abc}\overline{A}_c^1(0,\alpha)], \tag{5.7b}$$

$$\frac{1}{\pi}\int_{-1}^{1}d\omega'\omega'\frac{F_4^{ab}(\omega')}{\omega-\omega'}=\tfrac{1}{2}i\omega\int_{-\infty}^{\infty}d\alpha\,e^{-i\omega\alpha}\epsilon(\alpha)\alpha[d_{abc}A_c^2(0,\alpha)-f_{abc}\overline{A}_c^2(0,\alpha)], \tag{5.7c}$$

$$\frac{1}{\pi}\int_{-1}^{1}\frac{d\omega'}{\omega'^2}\frac{G_L^{ab}(\omega')}{\omega-\omega'}=8\omega\int_{-\infty}^{\infty}d\alpha\,e^{-i\omega\alpha}\epsilon(\alpha)\alpha[d_{abc}\overline{V}_c^2(0,\alpha)+f_{abc}V_c^2(0,\alpha)]. \tag{5.7d}$$

All the terms on the right-hand side of (5.7) are of the form

$$\int_{-\infty}^{\infty}d\alpha\,e^{-i\omega\alpha}\epsilon(\alpha)f(\alpha).$$

By the convolution theorem this is equivalent to

$$-\frac{i}{\pi}\int_{-\infty}^{\infty}\frac{d\omega'}{\omega-\omega'}\int_{-\infty}^{\infty}d\alpha\,e^{-i\omega'\alpha}f(\alpha).$$

[If there are subtractions in the integrals occurring on the left-hand side of (5.7), the convolution integral must be similarly subtracted.] Consequently, we conclude that

$$F_2^{ab}(\omega)=i\omega\int_{-\infty}^{\infty}d\alpha\,e^{-i\omega\alpha}[d_{abc}\overline{V}_c^1(0,\alpha)+f_{abc}V_c^1(0,\alpha)], \tag{5.8a}$$

$$F_3^{ab}(\omega)=\tfrac{1}{2}\int_{-\infty}^{\infty}d\alpha\, e^{-i\omega\alpha}[d_{abc}A_c^1(0,\alpha)-f_{abc}\overline{A}_c^1(0,\alpha)], \tag{5.8b}$$

$$F_4^{ab}(\omega)=\tfrac{1}{2}\int_{-\infty}^{\infty}d\alpha\, e^{-i\omega\alpha}\alpha[d_{abc}A_c^2(0,\alpha)-f_{abc}\overline{A}_c^2(0,\alpha)], \tag{5.8c}$$

$$G_L^{ab}(\omega)=-8i\omega^3\int_{-\infty}^{\infty}d\alpha\, e^{-i\omega\alpha}\alpha[d_{abc}\overline{V}_c^2(0,\alpha)+f_{abc}V_c^2(0,\alpha)]. \tag{5.8d}$$

This shows that the proton matrix elements of the bilocal operators are measurable in terms of the deep-inelastic cross sections $F_i^{ab}(\omega)$. Explicitly, we have

$$d_{abc}\overline{V}_c^1(0,\alpha)+f_{abc}V_c^1(0,\alpha)=\frac{1}{2\pi i}\int_{-1}^{1}\frac{d\omega}{\omega}e^{i\omega\alpha}F_2^{ab}(\omega), \tag{5.9a}$$

$$d_{abc}A_c^1(0,\alpha)-f_{abc}\overline{A}_c^1(0,\alpha)=\frac{1}{\pi}\int_{-1}^{1}d\omega\, e^{i\omega\alpha}F_3^{ab}(\omega), \tag{5.9b}$$

$$d_{abc}\alpha A_c^2(0,\alpha)-f_{abc}\alpha\overline{A}_c^2(0,\alpha)=\frac{1}{\pi}\int_{-1}^{1}d\omega\, e^{i\omega\alpha}F_4^{ab}(\omega), \tag{5.9c}$$

$$d_{abc}\alpha\overline{V}_c^2(0,\alpha)+f_{abc}\alpha V_c^2(0,\alpha)=\frac{i}{16\pi}\int_{-1}^{1}\frac{d\omega}{\omega^3}e^{i\omega\alpha}G_L^{ab}(\omega). \tag{5.9d}$$

This formula can also be obtained by the position-space method of Ref. 1. We emphasize that, whereas the relations in (5.7) may in general be modified by subtractions, the results (5.8) and (5.9) do not suffer from this shortcoming.

Additional results now follow from (5.9). Setting $\alpha=0$ in (5.9a) we have

$$f_{abc}V_c^1(0,0)=\frac{1}{\pi}\int_0^1\frac{d\omega}{\omega}F_2^{[ab]}(\omega). \tag{5.10}$$

(The terms symmetric in ab vanish.) From the Dashen–Fubini–Gell-Mann sum rule,[6] evaluated in the scaling region, we can determine the right-hand side of (5.10) (see Table II). Hence (5.10) becomes

$$f_{abc}V_c^1(0,0)=f_{abc}\Gamma_c. \tag{5.11}$$

Let us now recall the definitions of $V_c^1(0,0)$ and Γ_c:

$$p^\mu V_c^1(0,0)=\langle p|\mathcal{V}_c^\mu(0|0)|p\rangle=\langle p|V_c^\mu(0|0)|p\rangle, \tag{5.12a}$$

$$p^\mu\Gamma_c=\langle p|V_c^\mu(0)|p\rangle. \tag{5.12b}$$

[We have assumed that $x^\mu V_c^2(0,x\cdot p)$ vanishes with x.] Thus (5.11) shows that the proton matrix element of the bilocal operator $V_c^\mu(x|y)$ reduces to that of the vector current as $x\to y$, as is obvious in our quark model where

$$V_c^\mu(x|y)=\overline{\psi}(x)\tfrac{1}{2}\lambda_c\gamma^\mu\psi(y).$$

This provides a satisfying consistency check on the theory. Moreover when we discuss the model dependence of our results, we shall show that (5.11) is an important model-independent constraint on the internal-symmetry structure of the bilocal operators.

We now assume that the same is true for the axial-vector bilocal operator, as is indicated by the operator formulas (4.7b) and (4.8c). We have from (5.9b)

$$\begin{aligned}d_{abc}A_c^1(0,0)s^\mu&=d_{abc}\langle p|A_c^\mu(0|0)|p\rangle\\&=d_{abc}\langle p|A_c^\mu(0)|p\rangle=d_{abc}\Gamma_c^A s^\mu=\frac{2s^\mu}{\pi}\int_0^1 d\omega\, F_3^{(ab)}(\omega).\end{aligned} \tag{5.13}$$

The sum rule which has emerged, relating Γ^A to the spin-odd, isospin-even, deep-inelastic cross section, is similar to a relation first obtained by Bjorken.[20,21]

Another result is obtained by letting $\alpha\to 0$ in (5.9d). We are already committed to the vanishing of the left-hand side in this limit, see (5.12). Consequently, we obtain

$$\int_{-1}^{1}\frac{d\omega}{\omega^3}G_L^{[ab]}(\omega)=0.$$

This also follows from sum rule VI, upon multiplication by q^2, and passage to $-q^2=\infty$.

The other relations in (5.9) do not seem to give anything new. Thus the integral with respect to α from 0 to ∞ of the symmetric part in ab of (5.9a), of the antisymmetric part of (5.9b), and of the antisymmetric part of (5.4c) yields, respectively, (4.21), (4.22), and (4.23). Setting α to zero in (5.9c) reproduces sum rule IV. Finally we see that sum rule VI in Table II is satisfied since $F_L(\omega)=0$.

Note that the Regge region, in terms of the scaling variables, corresponds to $\omega\to 0$. Consequently, the large-α limit of the left-hand side of (5.9) is relevant in this context. In other words, the bilocal operators, in the limit of large separation, probe Regge behavior.

VI. CONCLUSION

With this investigation we have answered an old question which plagued applications of current algebra: how to include the contribution of "Z graphs" to the fixed-mass sum rules. Our answer, contained in Table II, expresses them as integrals over deep-inelastic scaling functions. Of course there is no guarantee that *experimentally* these objects are nonzero; however, in principle they can be measured. In the free-field model, the scaling contributions to sum rules II and V are nonvanishing.

Some of the fixed-mass sum rules are equivalent to low-energy theorems and unsubtracted dispersion relations. Consequently when we find a modification, we are asserting that the dispersion relation *needs a subtraction*. In the case that the Regge model indicates that the absorptive part decreases sufficiently rapidly for the dispersive integral to converge in an unsubtracted form, as in sum rules II and III (the Bég result[9]), the subtraction evidently is necessitated by *growth of the real part*. This has bearing on the question of fixed poles.

A question which naturally arises is the model dependence of our results. We may inquire what the possible generalization might be beyond the quark model which we have employed. We require, however, that the commutators of currents possess a structure so that three conditions are satisfied.

(i) $F_2(\omega)\neq 0$,

(ii) $F_L^{ab}(\omega)=0$,

i.e., no q-number bilocal Schwinger terms in the ++ commutator;

(iii) $\displaystyle\int_0^\infty d\nu\, W_2^{[ab]}(q^2,\nu)=\pi f_{abc}\Gamma_c$,

i.e., validity of the Dashen–Fubini–Gell-Mann[6] sum rule. It seems to us that these constraints force the use of a fermion model, though not necessarily with the triplet-quark realization of SU(3). [The σ model violates (i).] Consequently we suspect that the space-time tensor structure of the light-cone commutators – it is this which is responsible for (i), (ii), and (iii) – must be of the form given in (4.6), though the SU(3) content of the bilocal operators might be different in nature. The specific SU(3) form of (4.6b) follows from the following property of the quark model:

$$\lambda_a\lambda_b=if_{abc}\lambda_c+d_{abc}\lambda_c\,.$$

If we imagine that the fermions transform according to other representations of SU(3), we would replace this relation by

$$\lambda_a\lambda_b=if_{abc}\lambda_c+d_{ab}\,,$$

where d_{ab} is an unknown, symmetric SU(3) matrix containing the (1), (8), (10), and (27) representations. Therefore, the commutator (4.6b) more generally should be of the given form, except that the bilocal operators $d_{abc}\mathcal{V}_c^\mu(x|y)$, etc., are to be replaced by $\mathcal{V}_{(ab)}^\mu(x|y)$, etc. A further generalization would be to replace the antisymmetric bilocal currents $f_{abc}\mathcal{V}_c^\mu(x|y)$ by $\mathcal{V}_{[ab]}^\mu(x|y)$.

With such generalizations, our results would be affected as follows. The relations of Sec. V describing measurement of bilocal operators, Eqs. (5.9), will be modified in that the form factors on the left-hand side of (5.9) may possess an SU(3) structure which is more complicated. Thus the left-hand side of (5.9a) would read

$$\overline{V}^1_{(ab)}(0,\alpha)+V^1_{[ab]}(0,\alpha)\,,$$

with similar changes in the remaining equations. Consequently the measurement of the SU(3) content of the $F_i^{ab}(\omega)$, a task more possible in principle than in practice, provides information about the SU(3) content of the bilocal operators, and will test the validity of the quark model. In spite of this generalization, and the concomitant loss of information, the requirement that the Dashen–Fubini–Gell-Mann[6] sum rule be valid imposes a condition on $V_{[ab]}^\mu(x|y)$. According to (5.10)–(5.12), it must be true that

$$V_{[ab]}^\mu(x|x)=f_{abc}V_c^\mu(x)\,.$$

We expect that an investigation of the vector–axial-vector commutator leads to a similar result:

$$A_{[ab]}^\mu(x|x)=f_{abc}A_c^\mu(x)\,.$$

On the other hand, the sum rule for the axial-vector current (5.13) can no longer be established. All we can say is that

$$\langle p|A^{\mu}_{(ab)}(0|0)|p\rangle = \frac{2}{\pi} s^{\mu} \int_0^1 d\omega\, F_3^{(ab)}(\omega)\,,$$

but there is no way to establish that

$$A^{\mu}_{(ab)}(x|x) = d_{abc} A^{\mu}_c(x)\,.$$

Thus (5.13) is a crucial test of the quark model.

The fixed-mass sum rules in Table II are much less model dependent, since they are determined only by the space-time tensor structure of the light-cone commutators. It should be realized that results II, III, and V, as well as those parts of I, IV, and VI, which equate an appropriate integral over ν of a W_i^{ab} to the relevant moment of F_i^{ab}, follow merely from scaling. The point is the following. According to (4.2), which to be sure involves the assumptions to be discussed in Appendix D, a fixed-mass sum rule is given by an integral over x^- and a Fourier transform with respect to $\vec{q}_\perp$ of a light-cone commutator. This commutator is local in $\vec{x}_\perp$; i.e., it is composed of a δ function and derivatives thereof. Consequently, in momentum space, the integral over ν of an invariant function $W_i^{ab}(-\vec{q}_\perp{}^2, \nu)$ must be a polynomial in $\vec{q}_\perp{}^2$. However, the degree of the polynomial is fixed by scaling. Specifically, for example, for the sum rule II we may conclude that

$$\int_0^\infty d\nu\, W_3^{[ab]}(-\vec{q}_\perp{}^2, \nu) = \int_0^1 \frac{d\omega}{\omega} \bar{F}_3^{[ab]}(\omega, -\vec{q}_\perp{}^2) = \sum_{n=0}^{M} (\vec{q}_\perp{}^2)^n C_n^{[ab]}\,. \tag{6.1}$$

Scaling requires that the limit as $\vec{q}_\perp{}^2 \to \infty$ of (6.1) exists. Hence we learn that

$$\int_0^\infty d\nu\, W_3^{[ab]}(-\vec{q}_\perp{}^2, \nu) = C_0^{[ab]} = \int_0^1 \frac{d\omega}{\omega} F_3^{[ab]}(\omega)\,. \tag{6.2}$$

In this connection see also the work of Georgelin, Stern, and Jersák.[22]

Further investigations along the lines of this paper, which can be pursued, are the following. One may evaluate canonically other light-cone commutators, including the ones involving scalar and tensor densities, and deduce additional sum rules. Also the effect of the additional terms in the commutators of *nonconserved* currents may be studied. Evidently appropriate sum rules will be sensitive to symmetry-breaking effects, and it will be most interesting to expose these. (Preliminary results indicate that nothing new emerges from the $[V_a^+, V_b^i]$ commutator. Also,

$$\int \frac{d\omega}{\omega^3} G_L^{[ab]}(\omega) \neq 0$$

when symmetry-breaking effects are included.)

Another important problem is to consider nondiagonal matrix elements, in which case many form factors of bilocal operators, that vanished in the present investigation, contribute.

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APPENDIX A

We first compute the $t=0$ s- and t-channel helicity amplitudes. Using $\bar{u}_s u_{s'} = 2m\delta_{ss'}$ and $\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3$, we have the following formula from (2.9a), where terms which vanish when contracted with conserved polarization vectors have been omitted.

$$2mT^{\nu\mu} = -g^{\nu\mu} T_1 + p^\nu p^\mu T_2 - m\epsilon^{\nu\mu\alpha\beta}\gamma_\alpha\gamma^5 q_\beta T_3 - m\epsilon^{\nu\mu\alpha\beta} p_\alpha q_\beta q_\rho \gamma^\rho \gamma^5 T_4. \tag{A1}$$

The s-channel center-of-mass amplitudes are given by

$$T^s_{\lambda_c,\lambda_a;\lambda_d,\lambda_b} = \epsilon_\nu^{a*}(\vec{q})\bar{u}_c(\vec{p}) T^{\nu\mu} u_d(\vec{p})\epsilon_\mu^b(\vec{q}), \tag{A2}$$

with

$$p = (E, 0, 0, |\vec{p}|), \quad q = (q^0, 0, 0, -|\vec{p}|),$$

$$\epsilon^0 = (1/\sqrt{q^2})(|\vec{p}|, 0, 0, -q^0), \quad \epsilon^{\pm 1} = (\tfrac{1}{2})^{1/2}(0, \pm 1, i, 0)\,.$$

There are four independent nonvanishing amplitudes. They may be taken to be

$$T^s_{\pm\frac{1}{2},1;\pm\frac{1}{2},1} = T_1 \mp [(Eq^0 + |\vec{p}|^2)T_3 + |\vec{p}|^2(E+q^0)^2 T_4]$$

$$\xrightarrow[\nu\to\infty]{} T_1 \mp \nu(T_3 + \nu T_4), \tag{A3a}$$

$$T^s_{\frac{1}{2},0;\frac{1}{2},0} = T_1 + \frac{|\vec{p}|^2(E+q^0)^2}{q^2}T_2$$

$$\xrightarrow[\nu\to\infty]{} T_1 + \frac{\nu^2}{q^2}T_2, \tag{A3b}$$

$$T^s_{\frac{1}{2},1;-\frac{1}{2},0} = -m\sqrt{2q^2}\,T_3. \tag{A3c}$$

The t-channel center-of-mass amplitudes are given by

$$T^t_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} = \epsilon^a_\nu(-\vec{q})\bar{u}_c(\vec{p})T^{\nu\mu}v_d(-\vec{p})\epsilon^b_\mu(\vec{q}), \tag{A4}$$

with

$$p = i(0,0,0,m), \quad q = i\sqrt{q^2}(0,\sin\theta,0,\cos\theta),$$

$$\epsilon_b^{\pm1} = (\tfrac{1}{2})^{1/2}(0,\pm\cos\theta,i,\mp\sin\theta), \quad \epsilon^0_{a,b} = (i,0,0,0),$$

$$\epsilon_a^{\pm1} = (\tfrac{1}{2})^{1/2}(0,\mp\cos\theta,i,\pm\sin\theta), \quad \cos\theta = -\nu/m\sqrt{q^2}.$$

The four independent amplitudes are

$$T^t_{\frac{1}{2},\frac{1}{2};1,1} = i(T_1 - \tfrac{1}{2}m^2\sin^2\theta\,T_2 - m\sqrt{q^2}\,T_3)$$

$$\xrightarrow[\nu\to\infty]{} i[T_1 - (\nu^2/2q^2)T_2 - m\sqrt{q^2}T_3], \tag{A5a}$$

$$T^t_{\frac{1}{2},\frac{1}{2};1,-1} = \tfrac{1}{2}im^2\sin^2\theta\,T_2$$

$$\xrightarrow[\nu\to\infty]{} -i(\nu^2/2q^2)T_2, \tag{A5b}$$

$$T^t_{\frac{1}{2},\frac{1}{2};0,0} = iT_1, \tag{A5c}$$

$$T^t_{\frac{1}{2},-\frac{1}{2};0,1} = (im/\sqrt{2})[\sqrt{q^2}\,T_3(1+\cos\theta) + q^2\sin^2\theta\,T_4]$$

$$\xrightarrow[\nu\to\infty]{} (-i\nu/\sqrt{2})(T_3 + \nu T_4). \tag{A5d}$$

Regge contributions to the t-channel helicity amplitudes are even, or odd, in ν according to the (even or odd) signature of the trajectory. The s-channel discontinuities of the t-channel helicity amplitudes have the opposite symmetry. Because of the symmetry of the T_i and W_i in ν, they receive contributions from a trajectory α of the form

$$\nu^\alpha \pm (-\nu)^\alpha + \nu^{\alpha-2} \pm (-\nu)^{\alpha-2} + \cdots.$$

We can then see the absence of a ρ contribution to $W_3^{[ab]}$ and $W_4^{[ab]}$ as follows: $W_3^{[ab]} + \nu W_4^{[ab]}$ must be even in ν, but, since it has odd signature, a leading ρ contribution to $T^t_{\frac{1}{2},-\frac{1}{2};0,1}$ would have to be odd in ν. $T_3 + \nu T_4$ then would be even from (A5d) and $W_3^{[ab]} + \nu W_4^{[ab]}$ odd, contradicting (2.7). $W_3^{[ab]}$ alone, from (A5a), could receive a ν^{α_ρ} if $T^t_{\frac{1}{2},\frac{1}{2};1,1}$, T_1, or $\nu^2 T_2$ could have a $\nu^{\alpha_\rho - 1}$. $T^t_{\frac{1}{2},\frac{1}{2};1,1}$ cannot have such a term because Eqs. (2.9) and (A11) of Gell-Mann, Goldberger, Low, Marx, and Zachariasen[23] show it is directly proportional to $P_{\alpha_\rho}(z) - P_{\alpha_\rho}(-z)$; T_1 and $\nu^2 T_2$ cannot have such a term by their symmetry.

The case of $q^2 = 0$, where there are only two independent amplitudes, has been discussed by Adler and Dashen[7] in terms of the $\sqrt{t}$ derivative of the amplitude $T^t_{\frac{1}{2},-\frac{1}{2};1,-1}$ which vanishes at $t=0$. For $q^2 \neq 0$ the two discussions are related by a derivative conspiracy condition.[24] The exclusion of the Pomeranchon from $W_3^{(ab)}$ and $W_4^{(ab)}$ follows similarly.

Finally we note for completeness that the optical theorem for helicity amplitudes is

$$\operatorname{Im} T = \frac{p\sqrt{s}}{m}\sigma^T = \left(\frac{\nu^2}{m^2} - q^2\right)^{1/2}\sigma^T. \tag{A6}$$

APPENDIX B

It has been known for some time that the $p\to\infty$ technique is *formally* equivalent to evaluating commutators on the light cone. Since our light-cone results differ from the $p\to\infty$ ones, we demonstrate here that the equivalence is in fact false. Consider, for example, the Dashen–Fubini–Gell-Mann[6] sum rule,

$$\frac{i}{\pi}\int_0^\infty d\nu\, W_2^{[ab]}(q^2,\nu) = \lim_{p\to\infty}\frac{1}{p^0}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\langle p|[V_a^0(x), V_b^0(0)]|p\rangle_{\text{ETC}}\,. \tag{B1}$$

By inserting a Lorentz boost in the direction of p (say the 3 direction), the state $|p\rangle$ may be brought to rest:

$$\frac{1}{p^0}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\langle p|[V_a^0(x), V_b^0(0)]|p\rangle_{\text{ETC}} = \frac{1}{p^0}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\langle p'|[\tilde{V}_a^0(x'), \tilde{V}_b^0(0)]|p'\rangle, \tag{B2}$$

where

$$|p'\rangle = U(\lambda)|p\rangle, \quad U(\lambda) = e^{i\lambda M^{03}}$$

and M^{03} is the Lorentz generator. The vector current transforms as

$$\tilde{V}_a^0(x') = U(\lambda)V_a^0(x)U^{-1}(\lambda) = (\tfrac{1}{2})^{1/2}V_a^+(x')e^{\lambda} + (\tfrac{1}{2})^{1/2}V_a^-(x')e^{-\lambda} \tag{B3}$$

and the transformed coordinate x' has the components

$$\begin{aligned}(x')^+ &= x^+e^{-\lambda} = (\tfrac{1}{2})^{1/2}x^3e^{-\lambda},\\ (x')^- &= x^-e^{\lambda} = (\tfrac{1}{2})^{1/2}x^3e^{\lambda},\\ (\vec{x}')_\perp &= \vec{x}_\perp.\end{aligned} \tag{B4}$$

We have used the fact that $x^0=0$ in (B2). If the following change of variables is performed, $z=-(\frac{1}{2})^{1/2}x^3e^{\lambda}$, (B2) becomes

$$\frac{1}{p^0}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\langle p|[V_a^0(x), V_b^0(0)]|p\rangle_{\text{ETC}} = \frac{\sqrt{2}\,e^{-\lambda}}{p^0}\int dz\, d^2x_\perp e^{izq^3\sqrt{2}\,e^{-\lambda}}e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\langle p'|[\tilde{V}_a^0(x'), \tilde{V}_b^0(0)]|p'\rangle, \tag{B5a}$$

$$(x')^+ = -e^{-2\lambda}z, \quad (x')^- = z, \quad (\vec{x}')_\perp = \vec{x}_\perp. \tag{B5b}$$

It is not hard to verify that as $p\to\infty$, if $e^{\lambda}=2|\vec{p}|/m$, the state $|p'\rangle$ is at rest. Hence we have, with this choice,

$$\begin{aligned}\lim_{p\to\infty}\frac{1}{p^0}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\langle p|[V_a^0(x), V_b^0(0)]|p\rangle_{\text{ETC}} &= \frac{2\sqrt{2}}{m}\int dz\, d^2x_\perp e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\lim_{p\to\infty}e^{-2\lambda}\langle p'|[\tilde{V}_a^0(x'), \tilde{V}_b^0(0)]|p'\rangle\\ &= \frac{\sqrt{2}}{m}\int dx^-d^2x_\perp e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\langle p'|[V_a^+(x), V_b^+(0)]|p'\rangle_{\text{LC}}.\end{aligned} \tag{B6}$$

(B3) was used, as well as the fact that, according to (B5b), $(x')^+\to 0$ as $p\to\infty$. Finally we may set $(p')^+ = m/\sqrt{2}$, and we get from (B1)

$$\frac{i}{\pi}\int_0^\infty d\nu\, W_2^{[ab]}(q^2,\nu) = \frac{1}{p^+}\int dx^-d^2x_\perp e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\langle p|[V_a^+(x), V_b^+(0)]|p\rangle_{\text{LC}}\,, \tag{B7}$$

where we have dropped the primes on p. Equation (B7) is the same expression as (4.1) and (4.3).

We now see that the validity of the $p\to\infty$ technique for 0, 0 components is equivalent to the validity of the formal statement, which seems to follow from (B3) and (B4):

$$\lim_{\lambda\to\infty}e^{-2\lambda}U(\lambda)[V_a^0(x), V_b^0(0)]_{\text{ETC}}U^{-1}(\lambda) = \tfrac{1}{2}[V_a^+(x), V_b^+(0)]_{\text{LC}}. \tag{B8}$$

On the left-hand side x^3 is changed to $-\sqrt{2}\,e^{-\lambda}z$, and on the right-hand side $x^-=z$, with z fixed in the limiting process. It is this λ-dependent change of x^3 which invalidates conclusions drawn from (B8); the equation is correct as written. Similar formal statements must be true for the validity of this technique for + − components. We demonstrate that (B8) is never valid, though a fortuitous turn of events assures the correctness of the Dashen–Fubini–Gell-Mann sum rule.

By direct evaluation of the left-hand side, we find

$$\begin{aligned}\lim_{\lambda\to\infty}e^{-2\lambda}U(\lambda)[V_a^0(x), V_b^0(0)]_{\text{ETC}}U^{-1}(\lambda) &= if_{abc}\lim_{\lambda\to\infty}e^{-2\lambda}U(\lambda)V_c^0(0)U^{-1}(\lambda)\delta^2(\vec{x}_\perp)\delta(-\sqrt{2}\,e^{-\lambda}z)\\ &= if_{abc}\lim_{\lambda\to\infty}e^{-2\lambda}[(\tfrac{1}{2})^{1/2}V_a^+(0)e^{\lambda}+(\tfrac{1}{2})^{1/2}V_a^-(0)e^{-\lambda}](e^{\lambda}/\sqrt{2})\delta^2(\vec{x}_\perp)\delta(z)\\ &= if_{abc}\tfrac{1}{2}V_a^+(0)\delta^2(\vec{x}_\perp)\delta(z).\end{aligned} \tag{B9a}$$

The right-hand side of (B8) is given by (4.6a):

$$\tfrac{1}{2}[V_a^+(x), V_b^+(0)]_{\text{LC}} = \tfrac{1}{2}if_{abc}V_c^+(0)\delta^2(\vec{x}_\perp)\delta(z) - \tfrac{1}{8}i\partial_-^x\partial_-^y[S_{ab}(x|y)\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)]_{y=0;\,x^-=z^-}. \tag{B9b}$$

Thus the $p\to\infty$ technique misses the bilocal Schwinger term. This, of course, is not serious when that object is a c number. Moreover, the Dashen–Fubini–Gell-Mann[6] sum rule involves an integration over x^-, hence, even a q-number Schwinger term of the form (B9b) does not contribute. However, in the general case, one may not rely on the validity of the formal manipulation.

Recently, Segrè[25] has given a $p\to\infty$ sum rule based on the 0, 0 commutator, which, however, does not involve simply an integral over x^-, but rather a Fourier transform. Thus his result is not true if there is a q-number bilocal Schwinger term, i.e., if $F_L(\omega)\neq 0$.[26]

For the sum rules from other components, the $p\to\infty$ technique requires

$$\lim_{\lambda\to\infty} U(\lambda)[V_a^0(x), V_b^0(0)-V_b^3(0)]_{\mathrm{ETC}}U^{-1}(\lambda)=[V_a^+(x), V_b^-(0)]_{\mathrm{LC}}. \tag{B10}$$

The validity of this relation appears to follow from (B3), (B4), and from the analogous transformation law for V_b^3

$$U(\lambda)V_b^3(0)U^{-1}(\lambda)=(\tfrac{1}{2})^{1/2}V_a^+(0)e^{\lambda}-(\tfrac{1}{2})^{1/2}V_a^-(0)e^{-\lambda}. \tag{B11}$$

However, direct evaluation of the left-hand side gives

$$\begin{aligned}\lim_{\lambda\to\infty} U(\lambda)\{[V_a^0(x), V_b^0(0)]_{\mathrm{ETC}}-[V_a^0(x), V_b^3(0)]_{\mathrm{ETC}}\}U^{-1}(\lambda)\\
&=\lim_{\lambda\to\infty} U(\lambda)\{if_{abc}V_c^0(0)\delta^2(\vec{x}_\perp)\delta(-\sqrt{2}e^{-\lambda}z)-if_{abc}V_b^3(0)\delta^2(\vec{x}_\perp)\delta(-\sqrt{2}e^{-\lambda}z)\}U^{-1}(\lambda)\\
&=\lim_{\lambda\to\infty} if_{abc}\sqrt{2}\,V_c^-(0)e^{-\lambda}(e^{\lambda}/\sqrt{2})\delta^2(\vec{x}_\perp)\delta(z)\\
&=if_{abc}V_c^-(0)\delta^2(\vec{x}_\perp)\delta(z).\end{aligned} \tag{B12}$$

Comparing this with the *correct* formula for the right-hand side of (B11), Eq. (4.6b), we see that *all* the bilocal operators have been missed. Since some of these survive even upon integration over x^-, the $p\to\infty$ method fails completely.

It is not hard to see what the problem is. Consider for simplicity a boson field, with the equal-time commutators

$$i[\phi^*(x), \phi(0)]_{\mathrm{ETC}}=0 \tag{B13}$$

and the bilocal light-cone commutator (see Appendix C)

$$i[\phi^*(x), \phi(0)]=\tfrac{1}{4}\epsilon(x^-)\delta^2(\vec{x}_\perp). \tag{B14}$$

The $p\to\infty$ technique would assert that

$$\lim_{\lambda\to\infty} U(\lambda)[\phi^*(x), \phi(0)]_{\mathrm{ETC}}U^{-1}(\lambda)=[\phi^*(x), \phi(0)]_{\mathrm{LC}}. \tag{B15}$$

However there is simply no way to transform the zero of the left-hand side, into a nonvanishing quantity. The point is that on the light cone there are operators which are not limits of expressions that exist outside the light cone. These are the bilocal operators which are lost by the $p\to\infty$ technique.

The ideas in this Appendix were developed through conversations with Professor G. Segrè. These are gratefully acknowledged.

APPENDIX C

We present here the light-cone commutators associated with a boson theory. These commutators lead to a nonvanishing F_L, and hence they appear to be physically unacceptable; nevertheless their structure is sufficiently interesting to warrant exposure. The Lagrangian is

$$\mathcal{L}=\partial_\mu\phi\partial^\mu\phi^*-m^2\phi\phi^*+\mathcal{L}_I(\phi, \phi^*), \tag{C1}$$

where $\mathcal{L}_I$ is a Hermitian function. The current is given by (we suppress internal symmetry)

$$J^\mu=i\phi^*\partial^\mu\phi-i\phi\partial^\mu\phi^* \tag{C2}$$

and the canonical light-cone commutator of the fields is

$$i[\phi^*(x), \phi(y)]_{\mathrm{LC}}=\tfrac{1}{4}\epsilon(x^--y^-)\delta^2(\vec{x}_\perp-\vec{y}_\perp). \tag{C3}$$

It is now straightforward to compute the light-cone commutator $[J^+, J^\alpha]$, $\alpha=+, i$. We find

$$[J^+(x), J^+(y)]_{\mathrm{LC}}=-\tfrac{1}{4}i\partial_-^x\partial_-^y[\epsilon(x^--y^-)\delta^2(\vec{x}_\perp-\vec{y}_\perp)S(x|y)], \tag{C4a}$$

$$[J^+(x), J^i(y)]_{\mathrm{LC}} = -\tfrac{1}{4}i\partial_-^x[\epsilon(x^- - y^-)\partial^i\delta^2(\vec{x}_\perp - \vec{y}_\perp)S(x|y)]$$
$$-\tfrac{1}{4}i\partial_-^x[\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)\partial_y^i S(x|y)] + iS(y|y)\delta(x^- - y^-)\partial^i\delta^2(\vec{x}_\perp - \vec{y}_\perp), \tag{C4b}$$

$$S(x|y) = \phi^*(x)\phi(y) + \phi^*(y)\phi(x). \tag{C5}$$

It is seen that everything is expressible in terms of $S(x|y)$, the bilocal generalization of the Schwinger term.

The $[J^+, J^-]$ commutator cannot be so simply expressed. The reason is that J^- involves $\partial_+\phi$, and the equations of motion must be used to express $\partial_+\phi$ in terms of ϕ. Unlike in the fermion case, the dependence on $\mathcal{L}_I$ does not seem to be removable.

It is instructive to rederive these commutators by Schwinger's method.[27] We consider the Lagrangian (C1) to be minimally coupled to an external vector field A^μ

$$\mathcal{L} = (\partial_\mu + ieA_\mu)\phi(\partial^\mu - ieA^\mu)\phi^* - m\phi\phi^* + \mathcal{L}_I(\phi, \phi^*). \tag{C6}$$

The current is now

$$J^\mu = -\frac{\delta\mathcal{L}}{\delta eA_\mu} = i\phi^*\partial^\mu\phi - i\phi\partial^\mu\phi^* - 2\phi\phi^*A^\mu. \tag{C7}$$

The commutator is given by

$$[J^+(x), J^\mu(y)] = -i\partial_\alpha\frac{\delta J^\alpha(x)}{\delta eA_\mu(y)}, \tag{C8}$$

where the functional variation is performed with fixed canonical variables: fields and conjugate momenta.

One may rewrite (C8) in the following fashion. By the reciprocity relation,[27]

$$\frac{\delta J^\alpha(x)}{\delta eA_\mu(y)} = \frac{\delta J^\mu(y)}{\delta eA_\alpha(x)}, \tag{C9}$$

Eq. (C9) becomes

$$[J^+(x), J^\mu(y)] = -i\partial_\alpha\frac{\delta J^\mu(y)}{\delta eA_\alpha(x)}. \tag{C10}$$

Since the variation is arbitrary, we may choose to perform it by setting $eA_\alpha(x) = \partial_\alpha\Lambda(x)$ and varying Λ. It is not hard to see that under these circumstances (C10) becomes

$$[J^+(x), J^\mu(y)] = i\frac{\delta J^\mu(y)}{\delta\Lambda(x)}. \tag{C11}$$

Hence all we need is to compute the dependence of $J^\mu(x)$ on Λ, when $eA_\alpha = \partial_\alpha\Lambda$.

A peculiar feature of the boson theory is that one cannot consider ϕ to be independent of Λ. The reason for this is that the canonical momentum depends on ϕ and Λ; i.e.,

$$\Pi = \frac{\delta\mathcal{L}}{\delta\partial_+\phi^*} = \partial_-\phi + i\partial_-\Lambda\phi. \tag{C12}$$

The canonical commutator now is

$$[\partial_-\phi(x) + i\partial_-\Lambda(x)\phi(x), \phi(y)]_{\mathrm{LC}} = -\tfrac{1}{2}i\delta(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp). \tag{C13}$$

Differentiating this with respect to Λ and setting $\Lambda = 0$, we find

$$\left[\partial_-\frac{\delta\phi(x)}{\delta\Lambda(z)}, \phi^*(y)\right]_{\mathrm{LC}} + \left[\partial_-\phi(x), \frac{\delta\phi^*(y)}{\delta\Lambda(z)}\right]_{\mathrm{LC}} = -i\partial_-\delta(x^- - z^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)[\phi(x), \phi^*(y)]_{\mathrm{LC}}$$
$$= \tfrac{1}{4}\partial_-\delta(x^- - z^-)\delta^2(\vec{x}_\perp - \vec{z}_\perp)\epsilon(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp), \tag{C14}$$

where we have used (C3), since $\Lambda = 0$. It is seen that the left-hand side cannot vanish. The correct formula for $\delta\phi(x)/\delta\Lambda(y)$ may be inferred from (C14):

$$\frac{\delta\phi(x)}{\delta\Lambda(y)} = \tfrac{1}{4}i\partial_-^y[\epsilon(x^- - y^-)\phi(y)\delta^2(\vec{x}_\perp - \vec{y}_\perp)]. \tag{C15}$$

To compute the commutators, we now have from (C7) and (C11), in the limit $eA_\mu = \partial_\mu\Lambda \to 0$

$$[J^+(x), J^+(y)] = -\frac{\delta\phi^*(y)}{\delta\Lambda(x)}\partial_-\phi(y) - \phi^*(y)\partial_-\frac{\delta\phi(y)}{\delta\Lambda(x)} + i\phi(y)\phi^*(y)\partial_-\delta(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp) - \mathrm{H.c.}, \tag{C16a}$$

$$[J^+(x), J^i(y)] = -\frac{\delta\phi^*(y)}{\delta\Lambda(x)}\partial^i\phi(y) - \phi^*(y)\partial^i\frac{\delta\phi(y)}{\delta\Lambda(x)} + i\phi(y)\phi^*(y)\delta(x^- - y^-)\partial^i\delta^2(\vec{x}_\perp - \vec{y}_\perp) - \text{H.c.} \tag{C16b}$$

Substituting (C15) into (C16) reproduces (C4). Again it is seen that in order to determine $[J^+(x), J^-(y)]$, one needs the equations of motion to compute the dependence of $\partial_+\phi$ on Λ.

We summarize the important differences between the boson and fermion models for the light-cone commutators. (1) In the boson theory there exists a dimension-two operator, hence the Schwinger term emerges canonically. Indeed everything is expressible in terms of the bilocal Schwinger term. In the fermion theory the basic bilocal operator is a generalization of the current, and no Schwinger terms are present. (2) The $[J^+(x), J^-(y)]$ commutator can be expressed in the fermion model in terms of bilocal operators without any reference to the interaction, provided there is no derivative coupling. In the boson theory this does not appear to be possible.

In conclusion, we deduce the implications that the present model has for electroproduction. In the usual fashion, we write[28]

$$\begin{aligned} i\langle p|[J^\mu(x), J^\nu(0)]|p\rangle = {} & [g^{\mu\nu}\Box - \partial^\mu\partial^\nu]\epsilon(x\cdot p)\left[\delta(x^2)\frac{1}{8\pi^2}\int_{-1}^{1} d\omega\frac{\cos\omega(x\cdot p)}{\omega^2}F_L(\omega) + \theta(x^2)f_L(x^2, x\cdot p)\right] \\ & + [p^\mu p^\nu\Box - p\cdot\partial(p^\mu\partial^\nu + p^\nu\partial^\mu) + g^{\mu\nu}(p\cdot\partial)^2]\epsilon(x\cdot p)\theta(x^2)\left[\frac{1}{8\pi^2}\int_{-1}^{1} d\omega\frac{\sin\omega(x\cdot p)}{\omega x\cdot p}F_2(\omega) + f_2(x^2, x\cdot p)\right], \\ & x^2 f_L(x^2, x\cdot p)\xrightarrow[x^2\to 0]{} 0, \quad f_2(x^2, x\cdot p)\xrightarrow[x^2\to 0]{} 0. \end{aligned} \tag{C17}$$

It now follows that

$$\langle p|[J^+(x), J^+(0)]|p\rangle_{\text{LC}} = i\partial_-\partial_-\left[\epsilon(x^-)\delta^2(\vec{x}_\perp)\frac{1}{16\pi}\int_{-1}^{1} d\omega\frac{\cos\omega(x^-p^+)}{\omega^2}F_L(\omega)\right], \tag{C18a}$$

$$\begin{aligned} \langle p|[J^+(x), J^i(0)]|p\rangle_{\text{LC}} = {} & i\partial_-\partial^i\left[\epsilon(x^-)\delta^2(\vec{x}_\perp)\frac{1}{16\pi}\int_{-1}^{1} d\omega\frac{\cos\omega(x^-p^+)}{\omega^2}F_L(\omega)\right] \\ & + i(p^+\partial^i - p^i\partial_-)\left[\epsilon(x^-)\delta^2(\vec{x}_\perp)\frac{1}{8\pi}\int_{-1}^{1} d\omega\frac{\sin\omega(x^-p^+)}{\omega}F_2(\omega)\right]. \end{aligned} \tag{C18b}$$

On the other hand, from (C17) we have

$$\langle p|[J^+(x), J^+(0)]|p\rangle = \tfrac{1}{4}i\partial_-\partial_-[\epsilon(x^-)\delta^2(\vec{x}_\perp)h(x^-p^+)], \tag{C19a}$$

$$\langle p|[J^+(x), J^i(0)]|p\rangle = i\delta^2(\vec{x}_\perp)[\delta(x^-)p^i h'(0) + \tfrac{1}{2}\epsilon(x^-)p^i p^+ h''(x^-p^+)] + i\partial^i\delta^2(\vec{x}_\perp)[\tfrac{1}{2}\delta(x^-)h(0) - \tfrac{1}{4}\epsilon(x^-)p^+h'(x^-p^+)], \tag{C19b}$$

where

$$h(x\cdot p) = \langle p|S(x|0)|p\rangle|_{x^2=0}.$$

Comparing (C18a) with (C19a) gives

$$h(x\cdot p) = \frac{1}{4\pi}\int_{-1}^{1} d\omega\frac{\cos\omega(x\cdot p)}{\omega^2}F_L(\omega), \tag{C20a}$$

while (C18b) and (C19b) imply

$$h'(0) = 0, \tag{C20b}$$

$$h(0) = \frac{1}{4\pi}\int_{-1}^{1}\frac{d\omega}{\omega^2}F_L(\omega), \tag{C20c}$$

$$h'(x\cdot p) = \frac{1}{4\pi}\int_{-1}^{1} d\omega\frac{\sin\omega(x\cdot p)}{\omega}[F_L(\omega) - 2F_2(\omega)], \tag{C20d}$$

$$h''(x\cdot p) = -\frac{1}{4\pi}\int_{-1}^{1} d\omega\cos\omega(x\cdot p)F_2(\omega). \tag{C20e}$$

All these relations may be summarized by

$$F_2(\omega) = F_L(\omega), \tag{C21a}$$

$$\langle p|S(x|0)|p\rangle_{x^2=0} = \frac{1}{4\pi}\int_{-1}^{1} d\omega \frac{\cos\omega(x\cdot p)}{\omega^3} F_2(\omega) \tag{C21b}$$

Thus the transverse deep-inelastic cross section, $F_2 - F_L$, vanishes in this model.

The discussion in this Appendix of Schwinger's method for calculating commutators was developed with help from Professor K. Johnson. We wish to thank him for his assistance.

APPENDIX D (added in proof): VALIDITY OF LIGHT-CONE FIXED-MASS SUM RULES

Though the $p\to\infty$ technique, which is known to fail in general, was not employed by us, we did perform various formal operations which can be unsound. We now examine the validity of these results more carefully. Our conclusion is that whereas the "Z graphs" seem to be properly included by the present methods, the graphs which have fixed-mass singularities in the external current cannot be handled by light-cone techniques unless certain superconvergence relations, discussed below, are true. (These are the "Class-2" graphs in the terminology of Adler and Dashen.[7]) Our statement that the Z graphs are correctly included is based merely on the observation that in the free-field quark theory the modified sum rules are valid. However, the free-field model does not have fixed-mass singularities, and further analysis is required.

Let us return to the defining equation (2.1). Suppressing all indices, this is of the following form:

$$W(q^2,\nu) = \int d^4x\, e^{iq\cdot x}\langle p|[V_1(x), V_2(0)]|p\rangle\,. \tag{D1}$$

To obtain a fixed-mass sum rule, one must *first* set $q^+=0$, *then* integrate over q^-. The rigorously true result is the analog of (4.1):

$$\frac{1}{2\pi}\int\frac{d\nu}{p^+}W(-\vec{q}_\perp^{\,2},\nu) = \int d^2x_\perp e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\left\langle p\left|\left[\int dx^- V_1(x), V_2(0)\right]\right|p\right\rangle_{\rm LC}\,. \tag{D2}$$

However, our model for light-cone commutators provides us with the commutators $[V_1(x), V_2(0)]_{\rm LC}$ and therefore $\int dx^-[V_1(x), V_2(0)]_{\rm LC}$, rather than the formula required in (D2). The two are identical only when the x^- integral is sufficiently well behaved so that the interchange with the limit $x^+\to 0$ is allowed. When the interchange is performed we arrive at the analog of (4.2),

$$\frac{1}{2\pi}\int\frac{d\nu}{p^+}W(-\vec{q}_\perp^{\,2},\nu) = \int d^2x_\perp dx^- e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\langle p|[V_1(x), V_2(0)]|p\rangle_{\rm LC}\,. \tag{D3}$$

To see what is involved, it is useful to consider the T product,

$$T(q^2,\nu) = i\int d^4x\, e^{iq\cdot x}\langle p|T^*V_1(x)V_2(0)|p\rangle\,. \tag{D4}$$

According to the light-cone BJL theorem,[1] the quantity of interest in (D2) is (apart from seagulls, which we ignore)

$$\lim_{q^-\to\infty}[q^-T]_{q^+=0}\,.$$

On the other hand, the interchanged order relevant to (D3) is

$$\left[\lim_{q^-\to\infty} q^-T\right]_{q^+=0}\,.$$

Since $q^2 = 2q^+q^- - \vec{q}_\perp^{\,2}$, this interchange clearly may be dangerous.

Suppose now that T has a contribution from a pole in the external current,

$$T(q^2,\nu)\sim\frac{1}{q^2-\mu^2}A(\nu)\,.$$

We have as the contribution to (D3)

$$\left[\lim_{q^-\to\infty} q^-T\right]_{q^+=0}\sim\left[\frac{1}{2q^+}A(\infty)\right]_{q^+=0} = 0\,. \tag{D5}$$

(It has been assumed that $|A(\infty)|<\infty$, i.e., that the commutator exists.) The reason that $1/q^+$ vanishes at $q^+=0$ is that the principal-value definition is appropriate here; i.e., in position space $\int_{-\infty}^{\infty}dx^-\epsilon(x^-) = 0$. The noninterchanged order, relevant to (D2), gives

$$\lim_{q^-\to\infty}[q^-T]_{q^+=0}\sim -\lim_{\nu\to\infty}\nu A(\nu)\frac{1}{p^+(q_\perp^{\,2}+\mu^2)}\,. \tag{D6}$$

Thus for the interchange to be valid, it must be true that

$$\lim_{\nu\to\infty}\nu A(\nu) = 0\,. \tag{D7}$$

The same result may be obtained by considering the dispersive derivation of the sum rules. Let us assume that the fixed-mass sum rule under consideration converges. The light-cone technique gives

$$\int_{-\infty}^{\infty}d\nu\, W(-\vec{q}_\perp^{\,2},\nu) = P(-\vec{q}_\perp^{\,2})\,, \tag{D8}$$

where $P(-\vec{q}_\perp^{\,2})$ is a polynomial in $\vec{q}_\perp^{\,2}$. The T product is given by a dispersive formula, which by hypothesis is convergent:

$$\overline{T}(-\vec{q}_\perp^{\,2},\nu) = \frac{1}{2\pi}\int_{-\infty}^{\infty}d\nu'\frac{W(-\vec{q}_\perp^{\,2},\nu')}{\nu'-\nu}\,. \tag{D9}$$

The bar on the T indicates that polynomials in ν have been dropped. From (D8) it follows that

$$\lim_{\nu\to\infty} \nu\overline{T}(-\vec{q}_\perp{}^2, \nu) = -P(-\vec{q}_\perp{}^2)\,. \qquad \text{(D10)}$$

If $\overline{T}(-\vec{q}_\perp{}^2, \nu)$ has a contribution of the form $(\vec{q}_\perp{}^2+\mu^2)^{-1}A(\nu)$, the only way that $\lim_{\nu\to\infty}\nu\overline{T}(-\vec{q}_\perp{}^2, \nu)$ can equal a polynomial in $\vec{q}_\perp{}^2$ is if (D7) is true.

The condition (D7), which assures the validity of the present results, is of course the same condition which was previously found to be necessary for the Class-2 graphs not to spoil the $p\to\infty$ technique. However, the present techniques are more general, since they can accommodate a polynomial in q^2 in (D10), while the $p\to\infty$ method requires the right-hand side of (D10) to be constant in the Dashen–Fubini–Gell-Mann[6] case, and zero otherwise. Phrased in another way, this means that the light-cone sum rules can accommodate fixed poles, provided the residues are polynomials in q^2. Presumably these fixed poles arise from the Z graphs, which were previously missed.

When the sum rule converges, there is hope that (D7) is true, since convergence implies $\lim_{\nu\to\infty}\mathrm{disc}\,\nu A(\nu)=0$. For the divergent sum rules, like V in Table II due to Cornwall, Corrigan, and Norton,[18] a truncation procedure has been developed by these authors, which possibly may make such relations well defined. The present considerations indicate that this truncation must remove all the fixed-mass singularities. It is doubtful that this is practicable procedure.

Note that if (D7) is not true, then $\int dx^- V_1(x)$ is not a local operator in $\vec{x}_\perp$. We mean that the commutator of this quantity with another local operator does not vanish for $x^+=0$, $\vec{x}_\perp\neq 0$. This is seen from (D6), where the commutator exhibits a nonpolynomial $\vec{q}_\perp{}^2$ dependence. The possible troubles with which we are here concerned arise from the lack of sufficiently uniform convergence of the integral $\int dx^- V_1(x)$; or more exactly, of the integral $\int dx^- V_a^+(x)$. It may be that physical considerations should be brought forward here. After all, we know that the charge $\int d^2x_\perp dx^- V_a^+(x)$ is a well-behaved, physically interesting operator, independent of x^+. Perhaps it is possible to use this fact to establish regularity for the partially integrated charge $\int dx^- V_a^+(x)$.

It is not difficult to exhibit models which possess some of the pathological features under discussion here. For example, in the vector-meson theory which gives rise to the algebra of fields,[29] the Dashen–Fubini–Gell-Mann[6] sum rule is violated in Born approximation,[30] if the field-current identity is made. This failure is traceable to the existence of fixed-mass singularities in the current, which arise from an "elementary" vector particle.[31] The integral $\int d\nu\, W_2^{[ab]}(q^2, \nu)$ comes out nonpolynomial in q^2, indicating that the light-cone technique fails. It appears that this model has several peculiarities on the light cone, which are under further study.[30]

One may avoid the entire problem of interchanges by not setting q^+ to zero. Then the interchange is legitimate, since one is dealing with Fourier integrals. Unfortunately the mass is no longer fixed in the sum rules, but rather $q^2=\alpha\nu+\beta$. Such sum rules have been considered by the $p\to\infty$ method,[25] and in general the results are false in free-field theory.[26] The light-cone versions of these sum rules have been studied,[22] and they provide the necessary corrections to the $p\to\infty$ results; see also Appendix B.

It should be emphasized that the purely deep-inelastic results of Sec. V are not affected by any of the problems discussed here. However, it must be remembered that the operation of dividing by ω in the scaling region may lead to additional δ functions of ω, as explained by Jackiw, Van Royen, and West.[28] It has been shown by Zee[32] that in perturbation theory for a quark-gluon model such functions are indeed present. Also it appears that in the algebra of fields these δ functions occur canonically.[30]

The considerations in this Appendix were developed in collaboration with Professor D. J. Gross. We are grateful for his comments.

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