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Derivation of the Blackbody Radiation Spectrum by Classical Statistical Mechanics

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It is assumed that the fluctuating radiation energy density in a blackbody cavity is the sum of two stochastically independent terms: a zero-point energy density ρ_0 with Lorentz-invariant spectrum which persists at the absolute zero of temperature, and a temperature-dependent energy density ρ_T which satisfies the laws of statistical mechanics. The mean-square fluctuation $\langle(\delta\rho_T)^2\rangle$ of ρ_T is calculated from classical electromagnetic theory and is shown to depend explicitly on $\langle\rho_0\rangle$. Classical statistical mechanics leads then uniquely from $\langle(\delta\rho_T)^2\rangle$ to $\langle\rho_T\rangle$, which turns out to satisfy Planck's formula.

I. INTRODUCTION

Some fascinating new ideas concerning the physical meaning of the quantum theory have been developed in a series of papers by Boyer[1-4] and a related paper by Nelson.[5] In Boyer's work the main new concept is the existence, at the absolute zero of temperature, of a classical, fluctuating, electromagnetic background radiation which is, in some unknown fashion, equivalent to the ground state of the radiation field in quantum electrodynamics. Boyer demonstrates that incorporating his radiation background into classical statistical physics makes possible a classical derivation of Planck's blackbody spectrum. He also suggests that the universal background radiation might be the source of the random perturbations, postulated by Nelson, which transform continuous classical particle motion into an equivalent random-walk process. Since Nelson is able to derive Schrödinger's equation for particles from this classical random-walk model, we may well witness the emergence of an exciting, new interpretation of the quantum theory.

The present paper makes a small contribution to Boyer's work by deriving some of his results in a simple, axiomatic fashion. This procedure would be quite unconvincing without Boyer's penetrating analysis of classical statistical mechanics. However, once the foundations of the new theory have been established, an axiomatic approach has the virtue of conciseness, and may help to make the new ideas more readily accessible to a large audience.

Boyer has presented two different classical derivations of Planck's blackbody spectrum. One is the Einstein-Hopf derivation[6] of the Rayleigh-Jeans radiation law which leads to Planck's law if the classical radiation background is taken into account. This approach promotes valuable insights into the processes which establish dynamical equilibrium between radiation and matter. Unfortunately, the method is formally very cumbersome[7] and subject to doubts as to its general validity. The situation is such as if one derives Maxwell's velocity distribution from Boltzmann's statistical analysis of binary collisions between rigid spheres, and wonders what would happen if a more realistic model of molecules was used. The simple and universal approach to Maxwell's distribution is statistical mechanics, and that is the second road to the radiation law adopted by Boyer.

Following Einstein's pioneering work on energy fluctuations in the electromagnetic field, Boyer emphasizes the concept of entropy and, in order to describe the effect of the radiation background, he distinguishes between a caloric and a probabilistic entropy. While this distinction contributes much to the understanding of the general theory, it is not the simplest approach to our problem, and an alternative method is presented in this paper which treats the electromagnetic fluctuations more explicitly than in Boyer's work. A qualitative discussion of the photoelectric effect is also included which supports the basic assumptions of Boyer's theory.

II. CLASSICAL STATISTICAL MECHANICS OF BLACKBODY RADIATION IN THE PRESENCE OF A UNIVERSAL RADIATION BACKGROUND

We shall obtain all results from a small number of basic assumptions which are either self-evident or equivalent to Boyer's postulates. The following axioms go beyond conventional statistical mechanics and establish the conceptual framework for the subsequent analysis.

(1) There is a classical, fluctuating, electromagnetic radiation at the absolute zero of temperature with a Lorentz-invariant spectral energy density function of the form

$$\langle\rho_0(\omega)\rangle = (\omega^2/\pi^2c^3)\tfrac{1}{2}\hbar\omega\,, \tag{1}$$

where $(\omega^2/\pi^2c^3)d\omega$ is the number of modes in the frequency interval $d\omega$, and $\frac{1}{2}\hbar\omega$ is the average energy per mode. $\hbar$ is an adjustable universal constant which turns out to be Planck's constant divided by 2π. Thus, the universal radiation background has formally the properties of the radiation ground state in quantum electrodynamics. Note, however, that this result has nothing to do with quantum theory, and is a unique consequence of the required Lorentz invariance of the spectrum. A derivation of Eq. (1) is given in Boyer's paper[1] and will not be reproduced here.

(2) The zero-point radiation fluctuates randomly just as if it was produced by a large number of incoherent sources.

(3) At a finite temperature T, a blackbody cavity contains the zero-point radiation and a temperature-dependent component which is the conventional blackbody radiation. Thus,

$$\rho(\omega) = \rho_0(\omega) + \rho_T(\omega)\,, \tag{2}$$

where the subscripts indicate the temperature. Both ρ_0 and ρ_T fluctuate randomly and are stochastically independent of each other.

(4) In line with a well-established tradition, classical statistical mechanics holds for the temperature-dependent density ρ_T and its fluctuations. In particular, the mean-square density fluctuation of ρ_T satisfies the general relation[8]

$$\langle(\delta_{\rho_T})^2\rangle \equiv \langle\rho_T{}^2\rangle - \langle\rho_T\rangle^2 = \frac{\partial\langle\rho_T\rangle}{\partial\alpha}\,, \tag{3}$$

where

$$\alpha = -\frac{1}{k_BT}\,, \tag{4}$$

and k_B is the Boltzmann constant.

In spite of the fact that statistical mechanics is only concerned with the properties of ρ_T, atoms and molecules in dynamical equilibrium with the blackbody radiation interact with the total radiation field, including the universal radiation background. The dynamics of this interaction is discussed by Boyer[1,2] and the nature of the fluctuations is investigated in Sec. III.

(5) The principle of detailed balancing is valid such that thermal equilibrium is established separately for each mode characterized by the wave vector $\vec{k}$, frequency ω_k, and polarization index p^l which distinguishes between clockwise (p^+) and counterclockwise (p^-) circular polarization.

III. RANDOM FLUCTUATIONS OF ELECTROMAGNETIC RADIATION

In view of the principle of detailed balancing we consider only one Fourier component $\vec{E}(\vec{k}, p^l)$ of the radiation field, and omit the labels $\vec{k}$ and p^l for simplicity. We assume that the zero-point radiation is produced by a large number N_0 of incoherent sources each of which produces at some point $\vec{r}$ the field $\vec{E}_{0s}$ with a random phase angle θ_s ($s = 1, 2, \ldots, N_0$). Since all the $\vec{E}_{0s}$ belong to the same $\vec{k}$ and p^l, vector notation may be dropped, and we get

$$E_0 = \sum_{s=1}^{N_0} E_{0s}e^{i\theta_s}\,. \tag{5}$$

The corresponding fluctuating energy density is

$$\begin{aligned}\rho_0 &= (4\pi)^{-1}|E_0|^2 \\ &= (4\pi)^{-1}\Big[\sum_s E_{0s}{}^2 + \sum_s \sum_{t\neq s} E_{0s}E_{0t}\cos(\theta_s - \theta_t)\Big]\,.\end{aligned} \tag{6}$$

The average density is then

$$\langle\rho_0\rangle = (4\pi)^{-1}\sum_s \langle E_{0s}{}^2\rangle = (4\pi)^{-2}N_0\epsilon_0\,, \tag{7}$$

where

$$(4\pi)^{-1}N_0\epsilon_0 = \tfrac{1}{2}\hbar\omega_k \tag{8}$$

is the average energy density associated with the Fourier component $\vec{E}(\vec{k}, p^l)$. The average of the double sum in Eq. (6) is zero because of the random phases.

The mean-square energy density is

$$\langle\rho_0{}^2\rangle=(4\pi)^{-2}\Big[\sum_s\sum_t\langle E_{0s}{}^2E_{0t}{}^2\rangle+2\sum_r\sum_s\sum_{t\neq s}\langle E_{0r}{}^2E_{0s}E_{0t}\cos(\theta_s-\theta_t)\rangle$$
$$+\sum_q\sum_{r\neq q}\sum_s\sum_{t\neq s}\langle E_{0q}E_{0r}E_{0s}E_{0t}\cos(\theta_q-\theta_r)\cos(\theta_s-\theta_t)\rangle\Big]. \tag{9}$$

The triple sum vanishes on taking the average, and the quadruple sum reduces to two double sums which satisfy the conditions $q=s$, $r=t$, or $q=t$, $r=s$. Since

$$\langle\cos^2(\theta_s-\theta_t)\rangle=\tfrac{1}{2}, \tag{10}$$

we get

$$\lim_{N_0\to\infty}\langle\rho_0{}^2\rangle=(4\pi)^{-2}N_0{}^2(\epsilon_0{}^2+2\epsilon_0{}^2/2)=2\langle\rho_0\rangle^2. \tag{11}$$

The difference between $\langle E_{0s}{}^4\rangle$ and $\langle E_{0s}{}^2\rangle^2$ has been neglected since the contribution from $\langle E_{0s}{}^4\rangle$ is negligible in the limit $N_0\to\infty$. Combining Eqs. (11) and (7),

$$\langle(\delta\rho_0)^2\rangle\equiv\langle\rho_0{}^2\rangle-\langle\rho_0\rangle^2=\langle\rho_0\rangle^2. \tag{12}$$

Equation (12) is typical for classical wave fields in which strong fluctuations can occur because of local constructive and destructive interference.

Next we add to the zero-point radiation a temperature-dependent radiation field which is produced by N_T random sources and the corresponding fields $E_{T\sigma}$. The total fluctuating field is then

$$E=E_0+E_T=\sum_{s=1}^{N_0}E_{0s}e^{i\theta_s}+\sum_{\sigma=1}^{N_T}E_{t\sigma}e^{i\theta_\sigma}, \tag{13}$$

and the total energy density is

$$\rho=\rho_0+\rho_T=(4\pi)^{-1}\left\{\left[\Big|\sum_s E_{0s}e^{i\theta_s}\Big|^2\right]+\left[\Big|\sum_\sigma E_{T\sigma}e^{i\theta_\sigma}\Big|^2+2\sum_s\sum_\sigma E_{0s}E_{T\sigma}\cos(\theta_s-\theta_\sigma)\right]\right\}, \tag{14}$$

where the first square bracket is ρ_0 and the second square bracket, containing all the temperature-dependent fields, is ρ_T.

Invoking the random phases as before, we get for the various averages

$$\langle\rho\rangle=\langle\rho_0\rangle+\langle\rho_T\rangle, \tag{15}$$

$$\langle\rho^2\rangle=2\langle\rho\rangle^2, \tag{16}$$

and

$$\langle(\delta\rho)^2\rangle=\langle\rho\rangle^2. \tag{17}$$

Next we shall prove that the mean-square fluctuations are additive. To this purpose we invoke Eq. (14) and present $\langle(\delta\rho)^2\rangle$ in the form

$$\begin{aligned}\langle(\delta\rho)^2\rangle&=\langle(\rho_0+\rho_T)^2\rangle-(\langle\rho_0\rangle+\langle\rho_T\rangle)^2\\&=\langle\rho_T{}^2\rangle-\langle\rho_T\rangle^2+\langle\rho_0{}^2\rangle-\langle\rho_0\rangle^2\\&=\langle(\delta\rho_0)^2\rangle+\langle(\delta\rho_T)^2\rangle,\end{aligned} \tag{18}$$

since

$$\langle\rho_0\rho_T\rangle=\langle\rho_0\rangle\langle\rho_T\rangle, \tag{19}$$

as may be readily deduced from Eq. (14). Combining Eqs. (18), (17), (15), and (12), we get finally the fundamental relation

$$\langle(\delta\rho_T)^2\rangle=\langle(\delta\rho)^2\rangle-\langle(\delta\rho_0)^2\rangle=\langle\rho_T\rangle^2+2\langle\rho_0\rangle\langle\rho_T\rangle. \tag{20}$$

If we form the fractional fluctuation we get a sum of two terms:

$$\frac{\langle(\delta\rho_T)^2\rangle}{\langle\rho_T\rangle^2}=1+\frac{2\langle\rho_0\rangle}{\langle\rho_T\rangle}. \tag{21}$$

The first term represents the familiar classical result for fluctuations in a radiation field without a zero-point background radiation. The second term is inversely proportional to $\langle\rho_T\rangle$ and, if taken separately, has the typical behavior of energy fluctuations in particle systems which are associated with fluctuations in the particle density. Thus, Eq. (21) represents the fundamental result which Einstein[9] used as an argument in favor of the photon hypothesis, and of the wave-particle duality of light.

IV. DERIVATION OF PLANCK'S FORMULA

Combining the Eqs. (3) and (20) gives the differential equation

$$\langle\rho_T\rangle^2+2\langle\rho_0\rangle\langle\rho_T\rangle=\frac{\partial\langle\rho_T\rangle}{\partial\alpha} \tag{22}$$

with solution

$$\int\frac{d\langle\rho_T\rangle}{\langle\rho_T\rangle^2+2\langle\rho_0\rangle\langle\rho_T\rangle}=\alpha+C. \tag{23}$$

Evaluating the integral and using Eq. (8) gives, with $C=0$, Planck's formula

$$\langle\rho_T\rangle=\frac{2\langle\rho_0\rangle}{e^{2\langle\rho_0\rangle/k_BT}-1}=\frac{\hbar\omega_k}{e^{\hbar\omega_k/k_BT}-1}. \tag{24}$$

V. PHYSICAL INTERPRETATION

According to the preceding analysis it is the zero-point radiation which fully accounts for the apparently nonclassical features of blackbody radiation. If that is more than a formal accident at least two questions should be answered:

(1) What is the origin of the zero-point radiation?

(2) Can the zero-point radiation explain other

particlelike properties of the radiation field, e.g., those exhibited in the low-intensity photoelectric effect?

At present, both questions can be answered only in terms of hypotheses which are still unconfirmed but, in my opinion, deserve serious consideration. Concerning the first question the following hypothesis comes naturally to one's mind:

The zero-point radiation is a self-consistent radiation field in dynamical equilibrium with all the electrically charged particles in the universe. These particles perform a complicated Brownian motion, in the spirit of Nelson's work, which is caused by random absorption and emission of the self-consistent zero-point radiation. And this radiation has such an energy density that there is no net, time-averaged energy exchange between matter and radiation at the absolute zero of temperature.

Concerning the low-intensity photoelectric effect, it was found[10] that photoelectrons are released immediately after the incident light beam hits the photocathode. This is surprising from a classical viewpoint since it takes time until a very weak radiation field with a continuous energy distribution can transfer an energy $\hbar\omega$ to an atom with a tiny cross section for electron excitation. Thus, the instantaneous release of photoelectrons is accepted by many as a proof for the existence of localized, particlelike photons.

However, the classical fluctuation theory presented in this paper can explain the instant response, at least in a qualitative manner, without invoking a nonclassical particle nature of light. To understand this explanation let us first consider the behavior of electrons in the photocathode before the light beam is switched on.[11] At low temperature all the electrons are in full valence bands and do not produce a current. However, these electrons are exposed to the fluctuating zero-point field, and in addition to the Brownian motion which is induced by this field, they are occasionally excited up into the conduction band. However, since energy is conserved in the time average, the electrons fall back to the ground state after a very short time which, presumably, satisfies Heisenberg's uncertainty principle. This model is quite similar to that used in quantum electrodynamics for Lamb-shift calculations,[12,13] with the main difference that our fluctuating-radiation ground state is real, while it is only virtual in the quantum theory.

Let us now switch on the incident beam of light. This beam will immediately affect the magnitude of the fluctuations in the radiation field and will make it possible that some of the "virtual" electrons in the conduction band can stay there for good without being forced back to the ground state by the exigencies of energy conservation. We see then that the incident beam does not by itself excite electrons into the conduction band; it only affects the statistics of fluctuating phenomena and determines the number of "virtual" electrons which are transformed into real, permanent conduction electrons. This statistical effect is not localized and operates instantaneously after switching on the beam.[14]

It still remains true that energy is exchanged between radiation and matter only in quanta of magnitude $\hbar\omega$. However, in our model that is a consequence of the quantization of matter and does not imply a particle nature of light. Furthermore, as pointed out in the introduction, it is possible that the quantization of matter has the same origin as the quantization of the electromagnetic field, namely, the self-consistent zero-point radiation. While it is the prospect of a universal classical theory which makes Boyer's work so interesting, the present paper does not require a commitment to this idea since its sole purpose is an analysis of the blackbody radiation spectrum.

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Collapsed Nuclei*

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We discuss the observational consistency, possible properties, and detection of collapsed nuclei C_A. These may be considered as elementary particles with mass number $A > 1$ and of much smaller radius than ordinary nuclei N_A. The existence of C_A of (perhaps much) lower energy than N_A is observationally consistent if N_A are very long-lived isomers against collapse because of a "saturation" barrier between C_A and N_A. Barrier-penetrability estimates show that sufficiently long lifetimes $\gtrsim 10^{31}$ sec are plausible for $A \gtrsim 16$–40. The properties of C_A are discussed using composite baryon and quark models; small charges and hypercharges and, especially, neutral C_A are possible. C_A can be effectively a source or sink of baryons. Some astrophysical implications are briefly discussed, in particular the possible large scale presence of C_A and the possibility that accelerated collapse in massive objects may be a source of energy comparable to the rest mass.

I. INTRODUCTION

We consider the possibility of collapsed nuclei and conjecture about their possible properties and about the observational consequences of their existence. By collapsed nuclei C_A we understand systems with baryon number $A > 1$ and with (presumably much) smaller radii than normal nuclei N_A. Collapsed nuclei are thus best regarded as elementary particles, in contrast to ordinary nuclei. C_A of (perhaps much) lower energy than N_A can be consistent with observation if N_A are extremely long-lived isomers. We show that this is possible for moderate A. States with $A = 2$ and nonzero strangeness have been previously considered[1] using SU_3. However, these states consist of baryons not bound or only loosely bound together.

Our conjectures arose on the one hand from quark models which give no obvious reason why tightly bound systems of quarks with $A > 1$ should not exist. Secondly, recent phenomenological nuclear forces with soft (or momentum-dependent) repulsive cores[2] do not necessarily satisfy the saturation conditions.[3]

The observational consequences of the existence of C_A are dependent on the properties conjectured for C_A. To obtain some indications about these we have considered two types of models for C_A, namely quark models with quarks as the constituents, and composite hadron models with the known hadrons as the constituents. Some possible astrophysical implications are also briefly discussed.

II. SATURATION CONDITIONS; "STABLE NUCLEI AS ISOMERS"

Since the A-baryon Hamiltonian H_A is not known for collapsed conditions which could be vastly different from normal ones, we use $H_A(C)$ and $H_A(N)$ for the appropriate forms of H_A. Collapsed and normal conditions, in particular states, are denoted by superscripts (C) and (N), respectively. Some speculations about $H_A(C)$ are given in Sec. IV in connection with the discussion of the properties of C_A. For the time being we merely assume that the radius R_C of C_A is much less than the radius $R_N = r_0 A^{1/3}$ of N_A and we consider two options for R_C: (1) R_C is roughly constant, about the nuclear-force range ($\lesssim 0.5$ F) with C_A resembling the usual collapsed state with all nucleons (generally hadrons) within R_C; (2) $R_C \approx r_C A^{1/3}$ with $r_C \lesssim 0.4$ F, corresponding to saturation at very high densities, perhaps appropriate to a saturating quark model. The binding energy calculated with a trial function is barred. The exact eigenvalue is unbarred; e.g., for $H_A(N)$ the binding energies $\bar{B}_A^{(C)}(N)$ and $\bar{B}_A^{(N)}(N)$ are for collapsed and normal trial functions, re-