Study of Photon-Photon Interactions via Electron-Electron and Electron-Positron Colliding Beams*

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Some techniques are presented for studying the virtual photon-photon interaction available through electron-electron and electron-positron colliding beams. The two-photon processes are analyzed with the helicity methods presented before by Muzinich, Wang, and Wang, and with invariant amplitudes in the special case of forward elastic photon-photon scattering. A discussion of the high-energy behavior of some particular hadron production amplitudes as determined by Regge theory is given. In particular, we emphasize the possibility of measuring the real photon-photon hadron production cross sections. If only the final lepton momenta are measured and one attempts to isolate the total real photon-photon cross section into hadrons, there are electromagnetic background effects that may make this measurement difficult.

I. INTRODUCTION

A topic of considerable interest now is the role of the two-photon annihilation in electron-electron and electron-positron colliding-beam experiments. One striking feature immediate from the outset is the possibility of obtaining information about photon-photon interactions, in particular, hadron production. In fact, it now seems not only possible but perhaps unavoidable. Brodsky, Kinoshita, and Terazawa,[1] Balakin, Budnev, and Ginzburg,[2] and Arteaga-Romero, Jaccarini, and Kessler[3] have made the interesting observation that the two-photon annihilation processes can be dominant in the high-energy limit.

Theoretical interest in this topic is not new. Some time ago, Low[4] studied pseudoscalar (e.g., π^0 and η) production, and later, Calogero and Zemach[5] suggested studying lepton and pion pair production in terms of the two-photon annihilation mode. Because of the higher energies and improved luminosities that we will see at CEA, SLAC, and DESY, and because of the aforementioned dominance of this mode, photon-photon interactions may at last be of primary experimental interest.

The theoretical analyses seen in the literature have, for the most part, employed the Weizsäcker-Williams equivalent-photon approach.[6] Furthermore, specific hadron-channel estimates have been confined to perturbative phenomenological point couplings. Such analyses exhibit very nicely the enhancement due to the propagators for photons with small spacelike mass. However, we feel that a more complete treatment of the relevant Feynman diagrams will ultimately be useful as the technology of experiments improves.

Specifically we consider the possibility that the energies and directions of both final electrons can be measured. Therefore we apply a helicity and group-theoretical formalism[7] for extracting the dependence of the cross section upon lepton energy and angle. A similar helicity analysis is also contained in Arteaga-Romero *et al.*[3]; we became aware of their reports after starting to communicate our results. We also present and use a set of invariant amplitudes to analyze the total photon-photon cross section if only the final lepton momenta are measured. The total cross-section measurement has been suggested in Ref. 2.

We begin in Sec. II with a discussion of the specific production process in the envisioned colliding-beam experiment. The notation and Feynman amplitudes are given there. Next, the helicity formalism is contained in Sec. III. Section IV is concerned with Regge predictions for our photon amplitudes and some numerical estimates. We present a short description of competing processes in Sec. V. If only the final electrons are observed and one attempts to isolate the total real photon-photon cross section into hadrons, some background processes *appear* to be important. For example, at high energies the process $\gamma\gamma \to e^+e^-e^+e^-$, double electron pair production, dominates the hadron production process for real photons. However, effects due to finite virtual photon masses are crucial with regard to the size of this background.

An appendix is included which contains a kinematic-zero-free and kinematic-singularity-free amplitude expansion for the forward virtual photon-photon scattering amplitude.

II. FEYNMAN AMPLITUDE AND NOTATION

We concentrate here on the colliding-beam reaction

$$e(p_1)+e(p_2)\to e(p_1')+e(p_2')+\text{“anything”}. \qquad \text{(i)}$$

Here, $e(p_i)$ corresponds to either an electron or positron with four-momentum p_i and “anything” consists of even charge-conjugation ($C=+1$) states (which also have zero charge, baryon number, and strangeness). That is, we deal with the virtual intermediate reaction

$$\gamma(q_1)+\gamma(q_2)\to \text{“anything”}, \qquad \text{(ii)}$$

where $\gamma(q_i)$ refers to virtual photons with spacelike four-momentum q_i.

We assume that the final leptons $e(p_1')$ and $e(p_2')$ are detected (in coincidence). If only the leptons are detected, there is a background problem from competing processes. This background is discussed in Sec. V.

If information about reaction (ii) can be obtained from experiment, we have a new and very rich test of theoretical ideas. As examples, Regge behavior, production of $C=+1$ resonances, and current-algebra predictions could all be investigated.

In detail, the amplitude for reaction (ii) corresponds to the Feynman diagrams in Fig. 1. We need consider both Figs. 1(a) and 1(b) for the antisymmetric electron-electron amplitude but only Fig. 1(a) in electron-positron collisions. In order to be definite, let us restrict ourselves to the electron-electron case; later, we will work in a kinematic region where (a) dominates over (b) and hence our results will be applicable to the electron-positron case as well.

We define an assortment of notation[8] in terms of Fig. 1:

$$t_1=q_1{}^2=(p_1-p_1')^2,\quad t_2=q_2{}^2=(p_2-p_2')^2,$$
$$u_1=r_1{}^2=(p_1-p_2')^2,\quad u_2=r_2{}^2=(p_2-p_1')^2,$$
$$s=(p_1+p_2)^2,\quad s'=(q_1+q_2)^2=P_N{}^2,$$

and

$$p_i{}^2=E_i{}^2-\vec{p}_i{}^2=m^2,\quad p_i'^2=E_i'^2-\vec{p}_i'^2=m^2. \qquad (2.1)$$

The S-matrix element can now be written as

$$S_{fi}=-i\frac{1}{(2\pi)^2}\left(\frac{m}{E_1}\frac{m}{E_2}\frac{m}{E_1'}\frac{m}{E_2'}\right)^{1/2}$$
$$\times\,\delta^4(p_1'+p_2'+P_N-p_1-p_2)M_{fi}, \qquad (2.2)$$

where the Feynman amplitude is given by

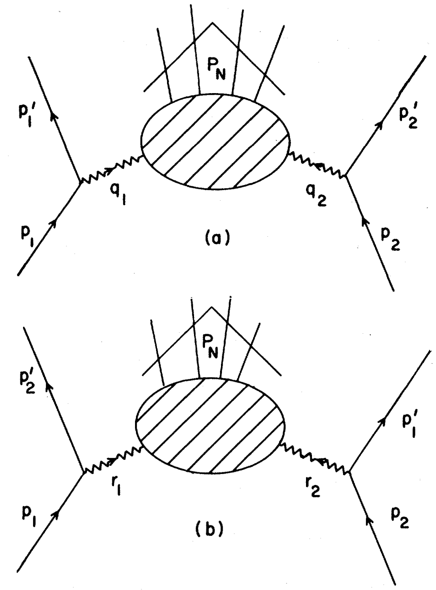

FIG. 1. Hadron production in electron-electron collisions via two-photon annihilation. For electron-positron processes, (b) is disregarded.

$$M_{fi}=-e^4\left[\frac{1}{t_1t_2}\bar{u}(p_1')\gamma_\sigma u(p_1)\bar{u}(p_2')\gamma_\beta u(p_2)T^{\sigma\beta}(q_1,q_2;N)\right.$$
$$\left.-\frac{1}{u_1u_2}\bar{u}(p_2')\gamma_\sigma u(p_1)\bar{u}(p_1')\gamma_\beta u(p_2)T^{\sigma\beta}(r_1,r_2;N)\right]. \qquad (2.3)$$

The quantity $T^{\sigma\beta}(q_1,q_2;N)$ in the expression (2.3) is the covariant amplitude for the reaction (ii) with σ (β) the polarization index of the virtual photon q_1 (q_2). (Also, N represents collectively all of the labels necessary to specify the $C=+1$ hadron state.) This amplitude is defined in terms of the covariant time-ordered product of two electromagnetic current operators:

$$T^{\sigma\beta}(q_1,q_2;N)=i\int d^4x\,e^{-iq_1x}$$
$$\times\langle N|T^*(J^\sigma(x)J^\beta(0))|0\rangle. \qquad (2.4)$$

Here, in the definition of the T^* product, all necessary Schwinger terms are understood to be subtracted. Thus $T^{\sigma\beta}$ is truly a second-rank Lorentz tensor and

$$q_1^\sigma T_{\sigma\beta}(q_1,q_2;N)=q_2^\beta T_{\sigma\beta}(q_1,q_2;N)=0. \qquad (2.5)$$

We wish now to choose more specific momentum coordinates for the electrons. For colliding beams, the laboratory is considered to be the c.m. reference frame. Then

$$p_1=(E,|\vec{p}|\hat{z}),\quad p_2=(E,-|\vec{p}|\hat{z}),$$
$$p_1'=(E_1',|\vec{p}_1'|\sin\theta_1\cos\phi_1,|\vec{p}_1'|\sin\theta_1\sin\phi_1,|\vec{p}_1'|\cos\theta_1), \qquad (2.6)$$
$$p_2'=(E_2',|\vec{p}_2'|\sin\theta_2\cos\phi_2,|\vec{p}_2'|\sin\theta_2\sin\phi_2,-|\vec{p}_2'|\cos\theta_2),$$

so that θ_1, ϕ_1, and $\pi-\theta_2$, ϕ_2 are the polar and azimuthal angles for the final electrons p_1' and p_2', respectively. As a consequence,

$$t_1=2(m^2-EE_1'+|\vec{p}|\,|\vec{p}_1'|\cos\theta_1)\cong -4EE_1'\sin^2(\tfrac{1}{2}\theta_1),$$
$$t_2=2(m^2-EE_2'+|\vec{p}|\,|\vec{p}_2'|\cos\theta_2)\cong -4EE_2'\sin^2(\tfrac{1}{2}\theta_2),$$
$$u_1=2(m^2-EE_2'-|\vec{p}|\,|\vec{p}_2'|\cos\theta_2)\cong -4EE_2'\cos^2(\tfrac{1}{2}\theta_2),$$
$$u_2=2(m^2-EE_1'-|\vec{p}|\,|\vec{p}_1'|\cos\theta_1)\cong -4EE_1'\cos^2(\tfrac{1}{2}\theta_1). \quad (2.7)$$

At this point, we make the important remark to the effect that we restrict ourselves to small θ_1 and θ_2 (and, of course, all energies much larger than m). Since

$$t_i(\theta_i=0)\approx -m^2\frac{(E-E_i')^2}{EE_i'},$$
$$u_1(\theta_2=0)\approx -4EE_2',\quad u_2(\theta_1=0)\approx -4EE_1', \quad (2.8)$$

the crossed final-state contribution corresponding to Fig. 1(b) and the $T(r_1, r_2; N)$ term in Eq. (2.3) can be neglected. [The dominance of the $(t_1t_2)^{-1}$ term produces the enhancement discussed in Sec. I.] Therefore, we can ignore this Pauli-principle effect in our work and the electron-electron and electron-positron cases can be treated simultaneously.

The differential cross section for reaction (i) and a particular final state N is

$$\frac{d\sigma}{d\Phi_N}=\frac{1}{(2\pi)^2}\frac{m^4}{2E^2}\frac{d^3p_1'}{E_1'}\frac{d^3p_2'}{E_2'} \times\delta^4(p_1'+p_2'+P_N-p_1-p_2)\tfrac{1}{4}\sum_{\text{spins}}|M_{fi}|^2, \quad (2.9)$$

in which $d\Phi_N$ denotes the phase-space volume for the hadronic state. A final remark before discussing Eq. (2.9) in terms of the helicity formalism is that we could neglect m in many of our expressions but often find it just as convenient to retain the m dependence.

III. THE HELICITY AND O(2,1) FORMALISM AND INVARIANT AMPLITUDES

We now relate the differential cross section in Eq. (2.9) directly to the $T^{\sigma\beta}$ amplitude in Eq. (2.4) through the density-matrix formalism of Ref. 7. The upshot of the analysis in that reference is that the lepton vertex can be handled in a simple manner using its invariance under Lorentz boosts along the direction of the virtual photons (say $\vec{q}_i$). Namely, we go to the frame where a given q_i is of the form $q_i=(-t_i)^{1/2}(0,\hat{q}_i)$; in such a (brick-wall) frame, the lepton pair distribution takes the form of a finite-dimensional nonunitary representation of the O(2, 1) group in the helicity basis.

Specifically, after squaring the amplitude and using the techniques discussed in Ref. 7, the colliding-beam differential cross section becomes

$$\frac{d\sigma}{dE_1'd\Omega_1'dE_2'd\Omega_2'}=\frac{\alpha^4}{8\pi^2}\frac{E_1'E_2'}{E^2}\sum_{\text{all } m_i, m_i', n_i}\Big[\frac{1}{t_1^2t_2^2}l_{m_1m_1'}(\xi_1,\psi_1)l_{m_2m_2'}(\xi_2,\psi_2)\rho_{m_1m_2m_1'm_2'}(q_1,q_2;N)$$
$$-\frac{1}{t_1u_1t_2u_2}2\,\mathrm{Re}[\bar{l}_{m_1m_2n_1n_2}(\xi_1,\psi_1;\xi_2,\psi_2;\bar{\xi}_1,\bar{\psi}_1;\bar{\xi}_2,\bar{\psi}_2)\bar{\rho}_{m_1m_2n_1n_2}(q_1,q_2,r_1,r_2;N)]$$
$$+\frac{1}{u_1^2u_2^2}l_{m_1m_1'}(\bar{\xi}_1,\bar{\psi}_1)l_{m_2m_2'}(\bar{\xi}_2,\bar{\psi}_2)\rho_{m_1m_2m_1'm_2'}(r_1,r_2;N)\Big]. \quad (3.1)$$

The newly introduced quantities here are discussed below; we note first that the helicity index m_i takes on the values $m_i=\pm1, 0$.

The density matrices for the lepton pairs ("meeting" at each electromagnetic vertex) in Eq. (3.1), $l_{m_1m_1'}$, etc., have been derived in Ref. 7. For example, we have[9]

$$l_{m_1m_1'}(\xi_1,\psi_1)=-t_1[\mathfrak{D}^{1*}_{m_11}(\xi_1,\psi_1)\mathfrak{D}^{1}_{m_1'1}(\xi_1,\psi_1)+\mathfrak{D}^{1*}_{m_1-1}(\xi_1,\psi_1)\mathfrak{D}^{1}_{m_1'-1}(\xi_1,\psi_1)],$$
$$l_{m_1m_1'}(\bar{\xi}_1,\bar{\psi}_1)=-u_1[\mathfrak{D}^{1*}_{m_11}(\bar{\xi}_1,\bar{\psi}_1)\mathfrak{D}^{1}_{m_1'1}(\bar{\xi}_1,\bar{\psi}_1)+\mathfrak{D}^{1*}_{m_1-1}(\bar{\xi}_1,\bar{\psi}_1)\mathfrak{D}^{1}_{m_1'-1}(\bar{\xi}_1,\bar{\psi}_1)], \quad (3.2)$$

if the beams are unpolarized and the final electron spins are unspecified. Also,

$$\bar{l}_{m_1m_2n_1n_2}(\xi_1,\psi_1;\xi_2,\psi_2;\bar{\xi}_1,\bar{\psi}_1;\bar{\xi}_2,\bar{\psi}_2)$$
$$=(t_1t_2u_1u_2)^{1/2}[\mathfrak{D}^{1*}_{m_11}(\xi_1,\psi_1)\mathfrak{D}^{1*}_{m_21}(\xi_2,\psi_2)\mathfrak{D}^{1}_{n_11}(\bar{\xi}_1,\bar{\psi}_1)\mathfrak{D}^{1}_{n_21}(\bar{\xi}_2,\bar{\psi}_2)+\mathfrak{D}^{1*}_{m_1-1}(\xi_1,\psi_1)\mathfrak{D}^{1*}_{m_2-1}(\xi_2,\psi_2)\mathfrak{D}^{1}_{n_1-1}(\bar{\xi}_1,\bar{\psi}_1)\mathfrak{D}^{1}_{n_2-1}(\bar{\xi}_2,\bar{\psi}_2)]. \quad (3.3)$$

In both Eqs. (3.2) and (3.3), electron mass terms have been neglected in our high-energy approximation [notice that these quantities appear in the *numerators* of Eq. (3.1)]. The complete formulas correct for nonzero electron mass can be found in Appendix B of Ref. 7.

In the preceding equations, $\mathfrak{D}^1_{mn}(\xi, \psi)$ is a finite-dimensional representation of O(2, 1) and is given by

$$\mathfrak{D}^1_{mn}(\xi, \psi) = e^{im\psi} d^1_{mn}(i\xi) i^{|m|-|n|}, \tag{3.4}$$

where d^J_{mn} is the reduced rotation matrix as defined by Rose.[10] The variables ξ_i, $\bar{\xi}_i$ parametrize the boosts transverse to the direction of the momentum transfer $\vec{q}_i$, $\vec{r}_i$, respectively, and are related to the laboratory quantities by[11]

$$\begin{aligned} &\sin\theta_i |\vec{p}|\, |\vec{p}_i'| = (m^2 - t_i/4)^{1/2} |\vec{q}_i| \sinh\xi_i, \quad i = 1, 2 \\ &\sin\theta_j |\vec{p}|\, |\vec{p}_j'| = (m^2 - u_i/4)^{1/2} |\vec{r}_i| \sinh\bar{\xi}_i, \quad i = 1,\ j = 2 \text{ or } i = 2,\ j = 1. \end{aligned} \tag{3.5}$$

The angles between the normals to the lepton planes and a fixed y axis have been called ψ_i and $\tilde{\psi}_i$. That is,

$$\begin{aligned} &(\vec{p}_i \times \vec{p}_i') \cdot \hat{y} = |\vec{p}_i \times \vec{p}_i'| \cos\psi_i, \quad i = 1, 2 \\ &(\vec{p}_i \times \vec{p}_j') \cdot \hat{y} = |\vec{p}_i \times \vec{p}_j'| \cos\tilde{\psi}_i, \quad i = 1,\ j = 2 \text{ or } i = 2,\ j = 1. \end{aligned} \tag{3.6}$$

Hence we use

$$\psi_1 = \phi_1, \quad \psi_2 = \phi_2 - \pi, \quad \tilde{\psi}_1 = \phi_2, \quad \tilde{\psi}_2 = \phi_1 - \pi, \tag{3.7}$$

in terms of the azimuthal angles introduced in Eq. (2.6). The O(2, 1) transformation specified, for example, by ξ_1 and ψ_1 is simply the Lorentz transformation that brings us to a frame where the pair of electron momenta, $\vec{p}_1$ and $\vec{p}_1'$, are collinear with the direction of the "current" $\vec{q}_1$.

The density matrices in Eq. (3.1) corresponding to the current correlation functions $T^{\sigma\beta}$ defined in Eq. (2.4) are denoted by ρ and take the form

$$\begin{aligned} \rho_{m_1 m_2 m_1' m_2'}(q_1, q_2; N) = {}& (2\pi)^4 \delta^4(P_N - q_1 - q_2) d\Phi_N (-)^{m_1 + m_1' + m_2 + m_2'} \\ & \times \epsilon^\sigma_{m_1}(q_1) \epsilon^{\lambda *}_{m_1'}(q_1) \epsilon^\beta_{m_2}(q_2) \epsilon^{\alpha *}_{m_2'}(q_2) T_{\sigma\beta}(q_1, q_2; N) T^*_{\lambda\alpha}(q_1, q_2; N) \end{aligned} \tag{3.8}$$

and

$$\begin{aligned} \bar{\rho}_{m_1 m_2 n_1 n_2}(q_1, q_2, r_1, r_2; N) = {}& (2\pi)^4 \delta^4(P_N - q_1 - q_2) d\Phi_N (-)^{m_1 + n_1 + m_2 + n_2} \\ & \times \epsilon^\sigma_{m_1}(q_1) \epsilon^{\lambda *}_{n_1}(r_1) \epsilon^\beta_{m_2}(q_2) \epsilon^{\alpha *}_{n_2}(r_2) T_{\sigma\beta}(q_1, q_2; N) T^*_{\lambda\alpha}(r_1, r_2; N). \end{aligned} \tag{3.9}$$

An expression analogous to Eq. (3.8) holds for $\rho_{m_1 m_1' m_2 m_2'}(r_1, r_2; N)$. We have introduced a standard set of polarization vectors $\epsilon^\mu_m(q)$ in projecting out the helicity matrix elements in Eqs. (3.1)–(3.3), (3.8), and (3.9). These have the usual properties[12]

$$\epsilon_m(q) \cdot q = 0, \quad \epsilon^*_m(q) \cdot \epsilon_{m'}(q) = (-)^m \delta_{mm'}, \quad \sum_m (-)^m \epsilon^\mu_m(q) \epsilon^{\nu *}_m(q) = g^{\mu\nu} - q^\mu q^\nu / q^2 \tag{3.10}$$

for spacelike $q^2 \leqslant 0$.

This completes our presentation of the general formula for the differential cross section. Any particular distribution function can be derived from Eq. (3.1); the formulation with the helicity basis and the O(2, 1) group has a big advantage over the usual tensor basis since all physical quantities are immediately accessible. Restricting ourselves, as we said, to small angles θ_i allows us to neglect the last two terms in Eq. (3.1), simplifying matters considerably. However, we have included the u_i-term formulas here for completeness.

A few remarks are in order here. We see[9] that there are six linearly independent elements implied by $l_{mm'}$ for the lepton pair. Of course, the number of linearly independent elements of ρ depends upon the nature of N but its detailed symmetry properties and relevant counting formulas have been given in Ref. 7. The relation between the O(2, 1) variables and laboratory variables exhibited in Eq. (3.5) seems to dictate that the helicity density-matrix elements are to be evaluated in the laboratory frame. But the density-matrix form of Eq. (3.1) is Lorentz-invariant. We can thus generalize our results readily to

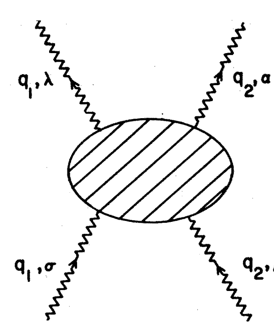

FIG. 2. The connected part of the forward virtual photon-photon scattering amplitude, $T_{\lambda\alpha\sigma\beta}$.

any frame provided the relation between the O(2, 1) variables and the momentum variables of the particular frame, Eq. (3.5), are changed accordingly.

We conclude this section with some discussion concerning an interesting relation to real photon-photon scattering and the total photon-photon cross-section measurement. If we stay in the region of small angles,[13] the longitudinal ($m=0$) components of ρ as well as the crossed terms can be neglected in Eq. (3.1).

If we sum over all final states N and drop the crossed term in Eq. (3.1), we obtain the following result in terms of the absorptive part of elastic forward virtual photon-photon scattering (Fig. 2):

$$\frac{d\sigma}{dE_1'd\Omega_1dE_2'd\Omega_2}=\frac{\alpha^4}{8\pi^2}\frac{E_1'E_2'}{E^2}\frac{1}{t_1^2t_2^2}\sum_{\text{all } m_i, m_i'} l_{m_1m_1'}(\xi_1,\psi_1)l_{m_2m_2'}(\xi_2,\psi_2)a_{m_1m_2m_1'm_2'}\,. \tag{3.11}$$

In Eq. (3.11), $a_{m_1m_2m_1'm_2'}(q_1, q_2)$ is given by

$$a_{m_1m_2m_1'm_2'}=(-1)^{m_1+m_1'+m_2+m_2'}\epsilon^{\sigma}_{m_1}(q_1)\epsilon^{\beta}_{m_2}(q_2)\epsilon^{\lambda *}_{m_1'}(q_1)\epsilon^{\alpha *}_{m_2'}(q_2)A_{\lambda\alpha\sigma\beta}(q_1,q_2)\,, \tag{3.12}$$

where $A_{\lambda\alpha\sigma\beta}(q_1, q_2)$ is the absorptive part of photon-photon scattering (Fig. 2) and use of completeness in Eq. (3.8) gives the expression

$$A_{\lambda\alpha\sigma\beta}(q_1,q_2)=\int d^4x d^4y d^4z\, e^{-iq_1x+iq_1y+iq_2z}\langle 0|T^*(J_\lambda(y)J_\alpha(z))^\dagger T^*(J_\sigma(x)J_\beta(0))|0\rangle\,. \tag{3.12$'$}$$

For some theoretical models it is desirable to have an invariant-amplitude description of the tensor $A_{\lambda\alpha\sigma\beta}$. In general there are eight independent covariants I_i corresponding to the invariants A_i and helicity projections $a_{m_1m_2m_1'm_2'}$ in the expansion

$$A_{\lambda\alpha\sigma\beta}(q_1,q_2)=\sum_{i=1}^{8}(I_i)_{\lambda\alpha\sigma\beta}A_i(w,t_1,t_2)\,, \tag{3.13}$$

where $q_1\cdot q_2=w$. The actual form of the covariant functions of q_1 and q_2 and the analysis of their kinematic constraints is lengthy (see the Appendix) and we quote only the essential results here. Use of the standard trace techniques yields the following exact expression for the differential cross section:

$$\begin{aligned}\frac{d\sigma}{dE_1'd\Omega_1'dE_2'd\Omega_2'}=\frac{\alpha^4}{8\pi^2}\frac{E_1'E_2'}{E^2}\frac{1}{t_1^2t_2^2}\big(&4t_1^2t_2^2A_1+\{(sr-u_1u_2)^2+\tfrac{1}{2}t_1t_2[(s-r)^2+2t_1t_2+(u_1-u_2)^2]\}(A_2+A_3)\\&+t_1t_2^2[(s+u_1)^2+(r+u_2)^2+2t_1t_2]A_4+t_1^2t_2[(s+u_2)^2+(r+u_1)^2+2t_1t_2]A_5\\&+t_1t_2[s^2+u_1^2+u_2^2+r^2+4t_1t_2](A_7+A_8)\\&+t_1t_2[(s+r)(sr-u_1u_2+t_1t_2)+(u_1+u_2)(u_1u_2-sr+t_1t_2)]A_6\big)\,.\end{aligned} \tag{3.13$'$}$$

Parity conservation at the lepton vertex limits the number of independent combinations of amplitudes that appear in Eq. (3.13′) to six of the eight in Eq. (3.13). The invariants s, u_i, and t_i are given in terms of laboratory quantities by Eq. (2.7) and $r\equiv 2p_1'\cdot p_2'=2w-s-u_1-u_2$. In addition there is an explicit factor of t_1t_2 in the expression in heavy parentheses in Eq. (3.13′) if one makes use of the relation

$$(sr-u_1u_2)^2=t_1t_2[(t_1t_2)^{1/2}-(u_1u_2)^{1/2}2\cos(\phi_1-\phi_2)]^2$$

in the coefficient of (A_2+A_3).

If we retain the leading terms at small t_i in Eq. (3.13′) or (3.11), we obtain the following approximate formula in terms of laboratory quantities:

$$\frac{d\sigma}{dE_1'd\Omega_1'dE_2'd\Omega_2'}\underset{\text{small } t_i}{\cong}\frac{\alpha^4}{4\pi^2}\frac{E_1'E_2'}{E^2}\frac{1}{t_1t_2}\left\{\frac{(E^2+E_1'^2)(E^2+E_2'^2)}{(E-E_1')^2(E-E_2')^2}[a_{1111}+a_{1-11-1}]+\frac{E^2E_1'E_2'}{(E-E_1')^2(E-E_2')^2}\cos 2(\phi_1-\phi_2)[a_{11-1-1}]\right\}. \tag{3.14}$$

The amplitude combinations in the square brackets of Eq. (3.14) are easily expressed in terms of the invariant amplitudes of Eq. (3.13′) by [see Eq. (A14)]

$$\begin{aligned}a_{1111}+a_{1-11-1}&=w^2(A_2+A_3+2A_7+2A_8+2t_2A_4+2t_1A_5)+t_1t_2(2A_1+A_2+A_8)\,,\\a_{11-1-1}&=w^2(A_2+A_3)+2wt_1t_2A_6+t_1t_2(A_7+A_8)\,.\end{aligned} \tag{3.14$'$}$$

In the region of large E, small t_i, and small E_i', we have the following limiting distribution which follows from Eqs. (3.13′) and (3.14):

$$\frac{d\sigma}{dE_1'd\Omega_1'dE_2'd\Omega_2'} \underset{E\to\infty;\,\text{small }t_i}{\cong} \frac{\alpha^4}{4\pi^2}\frac{E_1'E_2'}{E^2}\frac{1}{t_1t_2}\frac{E^2+E_1'^2}{(E-E_1')^2}\frac{E^2+E_2'^2}{(E-E_2')^2}(a_{1111}+a_{1-11-1}). \tag{3.15}$$

Also we obtain the following crude but effective differential cross-section formula from Eq. (3.14):

$$\frac{d\sigma}{dE_1'd\cos\theta_1 dE_2'd\cos\theta_2} \underset{\text{small }t_i}{\cong} \left(\frac{\alpha}{\pi}\right)^2\frac{E^2+(E-\omega_1)^2}{2E^2}\frac{E^2+(E-\omega_2)^2}{2E^2}\frac{1}{\sin^2(\frac{1}{2}\theta_1)\sin^2(\frac{1}{2}\theta_2)}\frac{\sigma_T(s')}{s'}, \tag{3.16}$$

where $\omega_i = E - E_i'$ are the equivalent-photon energies from each lepton pair. Here $s' \cong 4\omega_1\omega_2$ for small t_i and

$$\sigma_T(s') = e^4\frac{(a_{1111}^{\gamma\gamma}+a_{1-11-1}^{\gamma\gamma})}{16\omega_1\omega_2} \tag{3.17}$$

is the total *real* photon-photon hadron production cross section. Equation (3.16) was derived by ignoring the azimuthal angular dependence in the $a^{\gamma\gamma}$'s and considering t_i small but with $-t_i \gg m^2$. This azimuthal approximation should be reasonable since s' is weakly dependent upon $\phi_1 - \phi_2$ at small angles. We will use Eq. (3.16) for order-of-magnitude estimates later in Sec. IV.

It is interesting to note that we recover the Weizsäcker-Williams (WW) equivalent-photon formula[1,4]

$$\frac{d\sigma}{d\omega_1 d\omega_2} = \left(\frac{2\alpha}{\pi}\right)^2\left(\ln\frac{E}{m}\right)^2\frac{E^2+(E-\omega_1)^2}{2E^2\omega_1}\frac{E^2+(E-\omega_2)^2}{2E^2\omega_2}\sigma_T(s') \tag{3.18}$$

upon integrating Eq. (3.16) over θ_1 and θ_2. This entails reverting back to the $(t_1t_2)^{-1}$ form and also neglecting the θ_i dependence of s'. Furthermore, we have kept only the leading logarithm term and have understood $s' = 4\omega_1\omega_2$.

Equation (3.18) is applicable to the e^+e^- case generally, but it can be compared to the e^-e^- case only if the final electrons are limited to the forward cone – because of the Pauli principle. With this remark in mind, the total colliding-beam cross section for the two-photon annihilation into hadrons is[1,4]

$$\sigma_{ee}(s) = \int_{s_{\text{th}}}^{s} ds'\frac{d\sigma(s,s')}{ds'}, \tag{3.19}$$

where $s_{\text{th}} = m_\pi^2$ for π^0 production, $4m_\pi^2$ for pion pair production, etc. We have introduced

$$\frac{d\sigma(s,s')}{ds'} \cong 2\left(\frac{\alpha}{\pi}\right)^2\left(\ln\frac{E}{m}\right)^2\frac{\sigma_T(s')}{s'}\left[\frac{1}{2}\left(2+\frac{s'}{s}\right)^2\ln\left(\frac{s}{s'}\right)-\left(1-\frac{s'}{s}\right)\left(3+\frac{s'}{s}\right)\right], \tag{3.20}$$

with

$$E = \tfrac{1}{2}(s)^{1/2}.$$

IV. REGGE PHENOMENOLOGY AND LARGE s' ESTIMATES

In this section, we apply some techniques available from Regge or J-plane phenomenology in discussing the high-energy behavior of the various photon-photon amplitudes that arise in the cross sections, Eqs. (3.1) and (3.11). Although the production of high-energy hadronic systems is inhibited,[14] some general statements are in order in view of the foreseeable progress in colliding-beam technology. Also, some numerical estimates are included.

We proceed by making the following remarks:

(1) Production of final states N that can proceed through the exchange of the Pomeranchuk trajectory will ultimately dominate at high values of the invariant mass $s' = P_N^2$ (the c.m. energy squared of the two virtual photons). For example, production of four pions (and other systems which can be produced diffractively from two virtual photons) should dominate over the production of a pion pair, kaon pair, or a nucleon-antinucleon pair. For the latter, the production takes place according to the exchange of the leading trajectory with the quantum numbers of the $\gamma\pi$, γK, or γN system, respectively.

(2) Since the density-matrix elements occurring in Eqs. (3.1) and (3.11) involve second-order (in perturbation theory) electromagnetic amplitudes, there are fixed poles at nonsense correct-signature J values which contribute fixed-power s' growth. Therefore, a simple and general Regge-behavior statement cannot be made.

(3) At lower values of s', there is no reason that lower-lying trajectory exchange will be unimportant. Also, resonances (perhaps long-lived) with even charge-conjugation parity could be produced, and it is clearly interesting to search for them.[15]

(4) In the situation where only the electrons are

detected (where all the hadronic states N are summed) and we obtain a closed form for the cross section, Eq. (3.14), the Regge picture is again simple. The cross section involves the absorptive part of the reaction $\gamma(q_1)+\gamma(q_2)\to\gamma(q_1)+\gamma(q_2)$, depicted in Fig. 2. This reaction entertains only even-C trajectory exchange, there are no fixed powers in the absorptive part, and factorization yields

$$a_{lasb}(q_1, q_2) \xrightarrow[s'\to\infty]{} \sum_n b^n_{ls} b^n_{ab} s'^{\alpha_n}. \tag{4.1}$$

The index n ranges over the leading even-C trajectories with the quantum numbers of two photons. Since we are dealing with the forward absorptive part and since there are apparently only $M=0$ Lorentz-pole trajectories in the real world, we obtain the more restrictive no-helicity-flip form

$$a_{lasb} \xrightarrow[s'\to\infty]{} \sum_n b^n_l b^n_a s'^{\alpha_n} \delta_{ls}\delta_{ab}. \tag{4.2}$$

Thus the last term in Eq. (3.14), a_{1-11-1}, would vanish in this limit.

The problem concerning the size of these hadron-production cross sections follows on the heels of these remarks. One can give a simple numerical estimate[2] for the total hadron production cross section σ_T by factorization. We have

$$\sigma_T(s') \xrightarrow[s'\to\infty]{} \frac{[\sigma_{\gamma p}(\infty)]^2}{\sigma_{pp}(\infty)} \approx 0.25\ \mu\text{b}, \tag{4.3}$$

where $\sigma_{\gamma p}(\infty)\approx 100\ \mu$b is the total photoabsorption cross section off a proton and $\sigma_{pp}(\infty)\approx 40$ mb is the total proton-proton cross section in the infinite-energy limit. As a result, we can look back at Eq. (3.16) and say something about the size of the differential cross section.

Consider the following situation: $E=3$ ($s=36$) and $E_1'=E_2'=\omega_1=\omega_2=1.5$ ($s'=9$) in units of GeV (GeV^2). If $\theta_1=\theta_2=\frac{1}{2}^\circ$, we obtain about 10^{-29} cm^2/($\cos\theta_1 \cos\theta_2$ GeV^2) for (3.16). If $\theta_1=\theta_2=5^\circ$, we get 10^{-33} cm^2/($\cos\theta_1 \cos\theta_2$ GeV^2). If this sort of signal is satisfactory to the experimentalists when reasonable acceptances are taken into account and if the electromagnetic background can be separated out, one has a handle on the size of $\sigma_T(s')$ at high s' via the double inelastic experiment.

V. BACKGROUND PROBLEMS

In the event that only specific hadron states are detected (with the electrons integrated out), the major background of the α^4 photon-photon mechanism is the odd-C single-photon annihilation present in order α^2 for electron-positron collisions and in order α^4 for electron-electron collisions. (See Fig. 3 for the related Feynman diagrams.) The remarkable comment made most cogently by Brodsky *et al.* is appropriate here, namely, the α^2 e^+e^- competition is damped by the photon propagator which decreases as s^{-1} whereas the α^4 diagram in Fig. 1(a) has no such damping. In fact, our process of interest has an enhancement at small angles not present in the α^2 competition and, moreover, present in at least one less power of t_i^{-1} in the α^4 competition of Fig. 3(b). The latter suffers from damping due to the timelike photon propagator and there is no enhancement due to the electron propagator that competes with the additional t^{-1} of the process in Fig. 1(a). The two-timelike-photon production of even-C hadron states not pictured in Fig. 3 has s^{-1} damping.

In spite of the aforementioned possibility of overcoming the α^2 competitor, recent evidence from Frascati[16] indicates anomalously large multihadron production (at low enough energies that we probably have to describe the α^2 single-photon mode as the perpetrator) in e^+e^- collisions. As a consequence of the above remarks the electron coincidence detection scheme that we have been discussing may be just as interesting to the extent that it may be effective in measuring the *total* photon-photon hadron production.

There is, however, a background problem present in the situation where we attempt to measure photon-photon total cross sections into hadrons. Summing over N corresponds to detecting only the two electrons for an experimental comparison with Eq. (3.14). Thus we have to face up to new competitors, the escape of which requires coincidence detection of the final electrons at the very least.

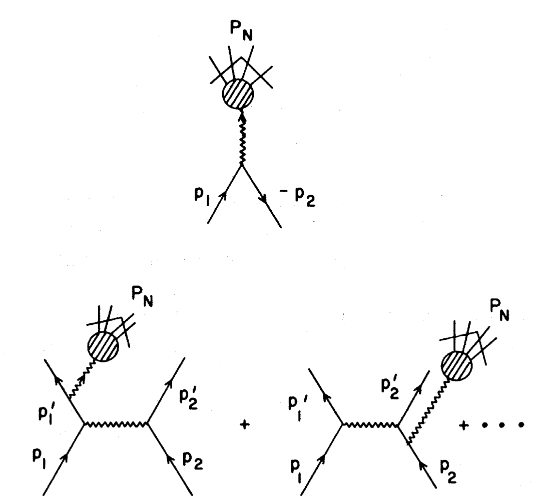

FIG. 3. (a) The $\alpha^2 C=-1$ competitor in e^+e^- collisions. (b) The $\alpha^4 C=-1$ competitor in both e^+e^- and e^-e^- experiments. The remaining graphs left understood by the three dots include the antisymmetric $p_1'\leftrightarrow p_2'$ ones in the e^-e^- case. We have not shown the two-timelike-photon $C=+1$ competitor in e^+e^- which is less favored.

This coincidence scheme furnishes the "missing mass" $(s')^{1/2}$ of the ensemble of particles produced if we know the energy and momentum of both final electrons. Simply knowing that the collision involved a final e^- with energy significantly less than E removes the elastic-channel [Fig. 4(a)] possibility. Knowledge that $s' \neq 0$ eliminates single bremsstrahlung [Fig. 4(b)]. Unfortunately, single and double pair production and double bremsstrahlung offer more resistance [Figs. 4(c) and 4(d)].

Inasmuch as the electron propagators offer enhancement for certain angles in Fig. 4(c) and inasmuch as there are three indistinguishable e^-'s in Fig. 4(d), we feel these competitors will require careful estimation. It is to be noted that double pair production via the real photon process, $\gamma+\gamma \to e^+ + e^- + e^+ + e^-$, has been calculated to be quite large.[17] In fact, at high s', $\sigma(\text{double pair}) \approx \frac{3}{2}\alpha^4/m^2 = 6\ \mu\text{b}$ for real photons, compared to the total hadron production cross section of 0.25 μb. This large cross section arises from the small Feynman mass in the electron propagators and the only parameter to set the scale for the asymptotically

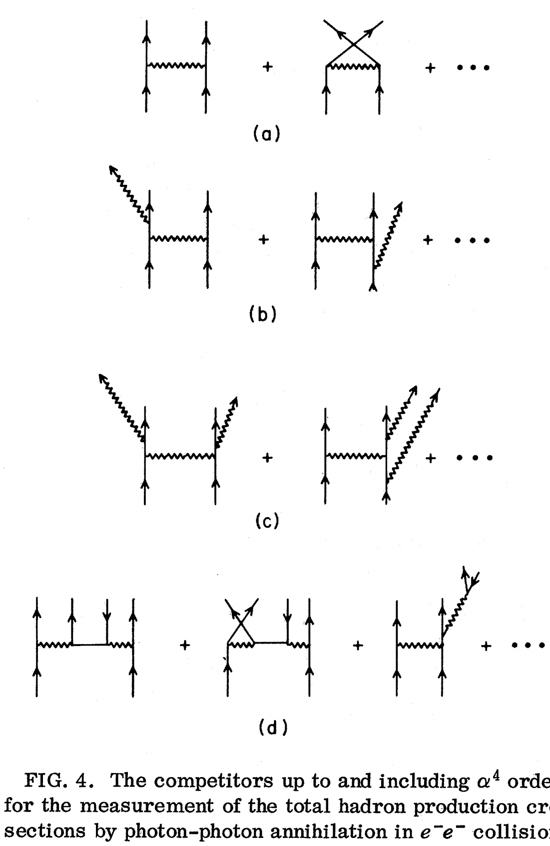

FIG. 4. The competitors up to and including α^4 order for the measurement of the total hadron production cross sections by photon-photon annihilation in e^-e^- collisions: (a) elastic, (b) single bremsstrahlung, (c) double bremsstrahlung, and (d) e^+e^- pair production. The three dots imply higher order in α for (a) and (b), and other permutations and antisymmetric contributions for (c) and (d).

constant cross section is the electron mass. It is quite probable that at small but finite photon mass t_1 and t_2, the scale is set by t_i^{-1} instead of the electron's mass. There is support for this behavior in other model investigations.[18] While it appears that double lepton pair production will swamp the hadron production total cross section for real photons, we feel that the question of how the scale changes for small but finite photon mass is an interesting question.

Note added in proof. Further investigation has shown that for finite values of photon mass, $-t_i \gg m^2$, the scale of σ(double pair) is given roughly by $(t_1 t_2)^{-1/2}$. Details will be forthcoming in a later publication.

Getting back to lepton pair production, we can estimate such a competitor by way of Eq. (3.16). [The odd-C pairs are damped for the same reasons given in the hadron discussion and the crossed even-C pairs seen in Fig. 4(d) lack some of the photon propagator enhancement.] From Bjorken and Drell,[19] we see that $\gamma+\gamma\to e^+ + e^-$ has a total cross section for real photons

$$\sigma_{\text{pair}}(s') = 4\pi\alpha^2 (s')^{-1}[\ln(s'/m^2) - 1] \qquad (5.1)$$

for $\omega_1 = \omega_2$ in the photon's c.m. frame and $s' = 4\omega_1^2 \gg m^2$. Hence this *dies out* as $s'^{-1}\ln s'$, in contrast to $\sigma_T(s')$, and for our earlier example ($s' = 9$ GeV^2), $\sigma_{\text{pair}} \lesssim 0.48\ \mu$b. (*Note added in proof.* The scale in the logarithm remains m^2 for $s' \gg -t_i, m^2$. Details will be published elsewhere.) So we are led to seriously consider *double inelastic scattering* at small angles where $s' \to s \to$ large (determined by detecting both electrons), since σ_{pair} decreases with respect to σ_T by a whole power of s'. The next competitor of interest is double bremsstrahlung.

The demand that s' be large implies that neither photon in double bremsstrahlung [see Fig. 4(c)] can be soft since each is massless. Double bremsstrahlung can, in principle, be separated from the process of interest, Fig. 1(a), since the external lepton pairs do not attach locally and the distribution in the electron's energy and angle is different. Preliminary indications from the photon spectrum analysis of Ref. 20 are that double bremsstrahlung will not dominate the process of interest at high s'.

In conclusion, the specific background problems mentioned above may not be disastrous and can be calculated explicitly and subtracted from the data. However, there are background problems of higher-order α, e.g., triple bremsstrahlung which may very well give rise to large radiative corrections.

VI. DISCUSSION

One of the most interesting results to come out of the recent inelastic electron-proton scattering experiments performed at SLAC was the precise measurement of the total photoabsorption cross sections at high energies. Consideration of the reaction $e+p\to e+$"anything" at small momentum transfers allowed by extrapolation the estimation[21] of the total cross sections for $\gamma+p\to$ "anything". We have discussed the analogous possibility that by double inelastic electron-electron scattering, $e+e\to e+e+$"anything", the total cross sections for $\gamma+\gamma\to$ "anything" could be measured.

Even though we have gone so far as to ask that the final electrons be detected in coincidence, serious background still persists for the total-cross-section measurement. This includes multiple bremsstrahlung, pair production, etc.

Both the total hadron production cross section and specific hadron production have been discussed in terms of Regge notions. This was done by considering "double inelastic" electron-electron scattering so that $s'\to\infty$. The possible confrontation of the enigmatic $C=+1$ photon-photon channel with Regge phenomenology certainly ranks as an important one when information about this channel is finally known.

In discussing the forward photon-photon scattering matrix element, we constructed a set of invariant amplitudes according to the outline in the Appendix and with an eye towards future dispersion-relation applications.

Recently, some other related work has come to our attention. Stodolsky[22] has proposed a method utilizing high-energy collisions with nuclei for measuring the γ-γ total cross section.

ACKNOWLEDGMENT

We have used Veltman's CDC 6600 algebraic computer program in dealing with the tensor analysis described in the Appendix.

APPENDIX

We present here an invariant-amplitude expansion of the rank-four forward amplitude $T_{\lambda\alpha\sigma\beta}$ for virtual photon-photon scattering [good, of course, for $A_{\lambda\alpha\sigma\beta}$ of (3.12) as well]. The steps in the derivation of this expansion are sketched below, in particular, those taken to reach kinematic-zero-free and kinematic-singularity-free invariant amplitudes. Also, we give the relations between these invariants and the helicity amplitudes in the c.m. system of the process $\gamma(k_1)+\gamma(k_2)\to\gamma(k_1)+\gamma(k_2)$.

There are 43 independent rank-four tensors which can be constructed out of the available tensors $g^{\mu\nu}$, q_1^μ, and q_2^μ. Omitting $\epsilon^{\mu\nu\sigma\rho}$ corresponds to assuming invariance under parity transformations, all of which agrees with helicity counting for the four virtual photon helicities and forward scattering. Further agreement with such counting is the fact that time-reversal invariance (or, equivalently, charge-conjugation invariance) limits us to 27 possible tensor combinations. Finally, gauge invariance connects the scalar and longitudinal photon spin states, leaving eight independent tensors. But the last requires care insofar as we wish to avoid forcing zeros or poles into the invariant amplitudes.[23]

Specifically, since the electromagnetic current is conserved,

$$q_1^\lambda T_{\lambda\alpha\sigma\beta}=q_2^\alpha T_{\lambda\alpha\sigma\beta}=q_1^\sigma T_{\lambda\alpha\sigma\beta}=q_2^\beta T_{\lambda\alpha\sigma\beta}=0\,, \tag{A1}$$

and we obtain 19 relations between the 27 invariant-amplitude coefficients of the 27 independent tensors. Let us use the three scalars, $t_1\equiv q_1^{\,2}$, $t_2\equiv q_2^{\,2}$, and $w\equiv q_1\cdot q_2=\frac{1}{2}(s'-t_1-t_2)$ as the independent variables for the invariant amplitudes. The first step then involves searching for those invariant amplitudes appearing in the 19 relations without a t_1, t_2, or w coefficient. These are then eliminated. Note that in a relation like $t_1A+t_1B=0$, where A and B represent invariant amplitudes, we say $A=-B$. (We assume that there are no δ-function ambiguities at $t_1=0$ by appealing to analyticity.) We then have remaining relations similar to $t_1A+wB=0$. These imply that $A=A'w$ and $B=B't$, and that finally $A'=-B'$, where A' and B' are the invariant amplitudes we would eventually use.

Needless to say, this is an exceedingly tedious task and it was made palatable by judicious use of an algebraic computer program. By listing the relations on a program and sequential substitution, a remarkable amount of effort was avoided and a minimization of errors-by-hand was achieved.

Finally, we write what we believe to be a kinematic-singularity-free and kinematic-zero-free complete basis for $T^{\lambda\alpha\sigma\beta}$ (but see our crossing discussion later),

$$T^{\lambda\alpha\sigma\beta}=\sum_{i=1}^{8} I_i^{\lambda\alpha\sigma\beta}A_i(w,t_1,t_2)\,, \tag{A2}$$

where the A_i are invariant amplitudes. The independent tensors are

$$I_1^{\lambda\alpha\sigma\beta}=(t_1g^{\lambda\sigma}-q_1^\lambda q_1^\sigma)(t_2g^{\alpha\beta}-q_2^\alpha q_2^\beta)\,, \tag{A3}$$

$$I_2^{\lambda\alpha\sigma\beta}=(wg^{\lambda\alpha}-q_2^\lambda q_1^\alpha)(wg^{\sigma\beta}-q_2^\sigma q_1^\beta)\,, \tag{A4}$$

$$I_3^{\lambda\alpha\sigma\beta}=(wg^{\lambda\beta}-q_2^\lambda q_1^\beta)(wg^{\alpha\sigma}-q_1^\alpha q_2^\sigma)\,, \tag{A5}$$

$$I_4^{\lambda\alpha\sigma\beta}=[t_1q_2^\lambda q_2^\sigma-w(q_1^\lambda q_2^\sigma+q_2^\lambda q_1^\sigma)+w^2g^{\lambda\sigma}]\times(t_2g^{\alpha\beta}-q_2^\alpha q_2^{\,\beta})\,, \tag{A6}$$

$$I_5^{\lambda\alpha\sigma\beta}=(t_1g^{\lambda\sigma}-q_1^\lambda q_1^\sigma)\times[t_2q_1^\alpha q_1^\beta-w(q_2^\alpha q_1^\beta+q_1^\alpha q_2^\beta)+w^2g^{\alpha\beta}]\,, \tag{A7}$$

and the remaining unsimplified forms

$$
\begin{aligned}
I_6^{\lambda\alpha\sigma\beta} = {} & 2wt_1t_2 g^{\lambda\beta}g^{\alpha\sigma} \\
& - wt_2(g^{\lambda\beta}q_1^\alpha q_1^\sigma + q_1^\lambda q_1^\beta g^{\alpha\sigma}) \\
& - t_1t_2(g^{\lambda\beta}q_1^\alpha q_2^\sigma + q_2^\lambda q_1^\beta g^{\alpha\sigma}) \\
& + w^2(g^{\lambda\beta}q_2^\alpha q_1^\sigma + q_1^\lambda q_2^\beta g^{\alpha\sigma}) \\
& - wt_1(g^{\lambda\beta}q_2^\alpha q_2^\sigma + q_2^\lambda q_2^\beta g^{\alpha\sigma}) \\
& + t_2(q_1^\lambda q_2^\sigma + q_2^\lambda q_1^\sigma)q_1^\alpha q_1^\beta \\
& + t_1 q_2^\lambda q_2^\sigma(q_1^\alpha q_2^\beta + q_2^\alpha q_1^\beta) \\
& - w(q_1^\lambda q_1^\alpha q_2^\sigma q_2^\beta + q_2^\lambda q_2^\alpha q_1^\sigma q_1^\beta),
\end{aligned} \tag{A8}
$$

$$
\begin{aligned}
I_7^{\lambda\alpha\sigma\beta} = {} & w^2 g^{\lambda\sigma}g^{\alpha\beta} + t_1t_2 g^{\lambda\beta}g^{\alpha\sigma} \\
& - t_2(g^{\lambda\beta}q_1^\alpha q_1^\sigma + q_1^\lambda q_1^\beta g^{\alpha\sigma}) \\
& + w(g^{\lambda\beta}q_2^\alpha q_1^\sigma + q_1^\lambda q_2^\beta g^{\alpha\sigma}) \\
& - t_1(g^{\lambda\beta}q_2^\alpha q_2^\sigma + q_2^\lambda q_2^\beta g^{\alpha\sigma}) \\
& + t_2 g^{\lambda\sigma}q_1^\alpha q_1^\beta + t_1 q_2^\lambda q_2^\sigma g^{\alpha\beta} \\
& - wg^{\lambda\sigma}(q_1^\alpha q_2^\beta + q_2^\alpha q_1^\beta) \\
& - w(q_1^\lambda q_2^\sigma + q_2^\lambda q_1^\sigma)g^{\alpha\beta} \\
& + q_1^\lambda q_2^\alpha q_2^\sigma q_1^\beta + q_2^\lambda q_1^\alpha q_1^\sigma q_2^\beta,
\end{aligned} \tag{A9}
$$

$$
\begin{aligned}
I_8^{\lambda\alpha\sigma\beta} = {} & w^2 g^{\lambda\sigma}g^{\alpha\beta} + t_1t_2 g^{\lambda\alpha}g^{\sigma\beta} \\
& + t_2 g^{\lambda\sigma}q_1^\alpha q_1^\beta + t_1 q_2^\lambda q_2^\sigma g^{\alpha\beta} \\
& - wg^{\lambda\sigma}(q_1^\alpha q_2^\beta + q_2^\alpha q_1^\beta) \\
& - w(q_1^\lambda q_2^\sigma + q_2^\lambda q_1^\sigma)g^{\alpha\beta} \\
& - t_2(g^{\lambda\alpha}q_1^\sigma q_1^\beta + q_1^\lambda q_1^\alpha g^{\sigma\beta}) \\
& - t_1(g^{\lambda\alpha}q_2^\sigma q_2^\beta + q_2^\lambda q_2^\alpha g^{\sigma\beta}) \\
& + w(g^{\lambda\alpha}q_1^\sigma q_2^\beta + q_1^\lambda q_2^\alpha g^{\sigma\beta}) \\
& + q_1^\lambda q_1^\alpha q_2^\sigma q_2^\beta + q_2^\lambda q_2^\alpha q_1^\sigma q_1^\beta .
\end{aligned} \tag{A10}
$$

It should be noted that $I_2 - I_3 \propto w$ (that is, $I_2 - I_3$ has an over-all factor of w). But we do not choose to include $(1/w)(I_2 - I_3)$ as part of our "minimal polynomial" set since the general nonforward photon-photon amplitude should be $O(q_1q_2q_3q_4)$. In order to have a smooth extrapolation to the forward set, we ask that our expansion be fourth order in q_i. The reason for the second-order possibility is that we only have two independent q_i in the forward direction.

In order to explicitly implement crossing symmetry like $q_1 \to -q_1$, $\lambda \leftrightarrow \sigma$ or $q_2 \to -q_2$, $\alpha \leftrightarrow \beta$ we should form the even combinations

$$I_1, \quad I_2 + I_3, \quad I_4, \quad I_5, \quad I_7 + I_8 \tag{A11}$$

and the odd combinations

$$I_2 - I_3, \quad I_6', \quad I_7 - I_8, \tag{A12}$$

where

$$I_6' = 2I_6 - 3wI_7 - wI_8 + (t_1t_2/w)(I_2 - I_3). \tag{A13}$$

The invariant coefficients (amplitudes) of the odd tensors (A13) would have explicit w factors according to crossing symmetry. [Note that crossing here means $A_i(w, t_1, t_2) \to A_i(-w, t_1, t_2)$.]

Because we have restricted the discussion to the forward case (where q_1 and q_2 are both initial *and* final momenta), we cannot uncover restrictions on the invariant amplitudes for the other crossing (Bose) symmetries, e.g., q_1 (initial) $\leftrightarrow q_2$ (initial) and $\sigma \leftrightarrow \beta$.

We give the relations between the eight invariant absorptive amplitudes A_i and the eight helicity amplitudes of Eq. (3.11). These are found most easily by going to the c.m. frame of the two virtual photons using the polarization representation seen in Ref. 7. We have

$$
\begin{aligned}
a_{1111} &= t_1t_2(A_1 + A_8) + w^2(A_2 + A_7 + A_8) + w^2(t_2A_4 + t_1A_5), \\
a_{1100} &= w(t_1t_2)^{1/2}(A_2 + A_8), \\
a_{11-1-1} &= w^2(A_2 + A_3) + 2wt_1t_2A_6 + t_1t_2(A_7 + A_8), \\
a_{0000} &= t_1t_2(A_1 + A_2 + A_3 + 2A_7 + 2A_8) \\
&\quad + t_1t_2(t_2A_4 + t_1A_5 + 2wA_6), \\
a_{0101} &= -t_1t_2(A_1 + A_7 + A_8) - t_1t_2^2A_4 - w^2t_1A_5, \\
a_{01-10} &= -w(t_1t_2)^{1/2}(A_3 + A_7) - (t_1t_2)^{1/2}(w^2 + t_1t_2)A_6, \\
a_{1010} &= -t_1t_2(A_1 + A_7 + A_8) - w^2t_2A_4 - t_1^2t_2A_5, \\
a_{1-11-1} &= t_1t_2(A_1 + A_7) + w^2(A_3 + A_7 + A_8) \\
&\quad + w^2(t_2A_4 + t_1A_5) + 2wt_1t_2A_6 .
\end{aligned} \tag{A14}
$$

At threshold and at pseudothreshold defined by the condition $w^2 = t_1t_2$ the number of independent amplitudes drops from eight to three; the reduction in amplitudes takes place because of conservation of total spin, $\vec{s} = \vec{s}_1 + \vec{s}_2$ ($s_1 = 1$, $s_2 = 1$). We have checked that the relations between the helicity amplitudes of Eq. (A14) on the manifold defined by $w^2 = t_1t_2$ due to conservation of total spin (i.e., orbital angular momentum zero) are indeed satisfied.

For real photons, $t_1 = t_2 = 0$, the nonvanishing helicity absorptive amplitudes are

$$
\begin{aligned}
a_{1111} &= w^2(A_2 + A_7 + A_8), \\
a_{11-1-1} &= w^2(A_2 + A_3), \\
a_{1-11-1} &= w^2(A_3 + A_7 + A_8),
\end{aligned} \tag{A15}
$$

as expected.

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[9]From Ref. 7, $l_{mn}(\xi,\psi)=-q^2e^{i(m-n)\psi}\tau_{mn}(\xi)$, where $\tau_{11}=\tau_{-1-1}=\frac{1}{2}(1+\cosh^2\xi)$, $\tau_{00}=\sinh^2\xi$, $\tau_{10}=\tau_{01}=-\tau_{-10}=-\tau_{0-1}=2^{-3/2}\sinh 2\xi$, and $\tau_{1-1}=\tau_{-11}=-\frac{1}{2}\sinh^2\xi$.

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[12]See Ref. 7 for explicit polarization representations. However, the factor $(-)^{m+1}$ in Eq. (2.10) of that reference should read $(-)^{(m+1)/2}$. Further, ϵ_m^λ should be replaced by ϵ_m^ν in Eq. (2.9) there.

[13]Notice that the first, second, and third terms vary as $(t_1t_2)^{-1}$, $(t_1u_1t_2u_2)^{-1/2}$, and $(u_1u_2)^{-1}$ due to the implicit t_i and u_i dependence in l and $\bar{l}$. See Eqs. (3.2) and (3.3).

[14]According to Eq. (3.20) the coefficient of $\sigma_T(s')$ favors low values of s'.

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