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# Theory of $\eta$-$\pi$ Mixing with Applications to Meson Decays

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From a consideration of the intrinsic and radiative contributions to $\eta$-$\pi$ mixing, calculated in an effective-chiral-Lagrangian model, a satisfactory account is given of the process $\eta \to 3\pi$. The model lends itself to further application to the $G$-violating decay $\rho^\pm \to \eta\pi^\pm$, as well as to the weak processes $K^\pm \to \pi^\pm + \pi^0$ and $K^\pm \to \pi^\pm + \gamma + \gamma$. Also predicted are rates for $\varphi \to \eta + \gamma$ and $\omega \to \eta + \gamma$.

## I. INTRODUCTION

The interactions which give rise to electromagnetic mass differences within isospin multiplets lead naturally to a mixing of the neutral pion and the $\eta$ meson which provides a useful mechanism for explaining $G$-parity-violating decays like $\eta \to 3\pi$ and $\rho^\pm \to \eta\pi^\pm$. Furthermore, weak interactions which violate the nonleptonic $\Delta I = \frac{1}{2}$ rule, such as the well-known example of $K^\pm \to \pi^\pm + \pi^0$, can be understood in terms of an $\eta$-$\pi$ transition. Since an adequate description of the electromagnetic mass differences of hadrons appears to require both *intrinsic* and *radiative* contributions,[1-3] our model of $\eta$-$\pi$ mixing also includes these two kinds of interaction. The intrinsic, or tadpole-type, contribution is independent of the dynamics but the radiative contribution, calculated in an effective-chiral-Lagrangian model, turns out to be proportional to the square of the off-shell mass of the $\eta$-$\pi$ line and thus contributes differently to different processes.

After an evaluation of the $\eta$-$\pi$ transition in Sec. II, we apply it to calculate certain meson decay rates in Sec. III. We find a satisfactory rate for $\eta \to 3\pi$ and a prediction for $\rho^\pm \to \eta + \pi^\pm$ which, although considerably below the quoted experimental upper limit, is still experimentally accessible.

In addition, without assuming a specific nonleptonic weak-interaction Hamiltonian, we are able, within our model, to calculate the ratio of the decay rates of $K^\pm \to \pi^\pm + \gamma + \gamma$ and $K^\pm \to \pi^\pm + \pi^0$ to be $1.6 \times 10^{-3}$, which is larger than the results yielded by other models.[4] In our model, the effective mass distribution of the two $\gamma$ rays is very different from that usually assumed; for this reason, the quoted upper limit in a recent experimental analysis[5] is inapplicable, as discussed below.

In an attempt to obtain the absolute rate for the decay $K^+ \to \pi^+ + \pi^0$, using the pion reduction technique, we look at the transformation properties of the weak nonleptonic Hamiltonian which preserves $\Delta I = \frac{1}{2}$. The correct rate can be obtained providing the $\underline{27}$ representation is present in addition to the usually assumed octet.

## II. THE $\eta$-$\pi$ TRANSITION

We consider two types of contributions to the $G$-parity-violating transition of $\eta$ to $\pi$: (a) an intrinsic symmetry breaking in the energy density, which transforms like the third component of an $SU(3)$ octet, and which is realized in a Lagrangian context as a Coleman-Glashow tadpole[1]; (b) a radiative transition involving a photon loop, treated in the vector-dominance picture. While contribution (a) has the same value whether $\eta$ or $\pi$ is on its mass shell, contribution (b) turns out to be essentially proportional to the squared mass of the on-shell particle. Their relative magnitudes are such that in the applications to be considered, either (a) or (b) is dominant.

Contributions of types (a) and (b) are recognized as being important in the theory of hadron electromagnetic mass differences. The tadpole contribution appears to be essential to explain the mass ordering of members within the isospin multiplets, for both mesons and baryons. In our work we will adopt the value of the electromagnetic tadpole deduced from these considerations.

Using the Hamiltonian formulation with density

$$H = H_0 + \alpha_8 u_8 + \alpha_3 u_3 , \tag{1}$$

where $H_0$ is $SU(3)$-symmetric, one can use the results of Socolow[6,7] to deduce the ratio

$$\alpha_3/\alpha_8 = 0.016 , \tag{2}$$

where we have used his value for the electromagnetic tadpole which gives a best fit to the baryon mass differences. This value gives for the intrinsic symmetry-breaking contribution to the $\eta$-$\pi$ transition

$$\langle\eta|\pi\rangle_{\text{int}} = -2.37\times10^3 \text{ MeV}^2 . \tag{3}$$

To calculate the radiative $\eta$-$\pi$ transition (b) we make use of the chain

$$\eta \to V + \text{“}\gamma\text{”} \to \pi^0 ,$$

in which "$\gamma$" represents an effective photon propagator, such as the chain

$$\rho \to \gamma \to \omega(\varphi) ,$$

which changes the $G$ parity. Virtual particles of $J^P = 0^-$ or $1^+$ are forbidden by $G$-parity and isospin invariance of the strong couplings.

A detailed calculation of the radiative transition is presented in the Appendix, and we discuss here only its general features. The vertices are assumed to be of the Gell-Mann–Sharp–Wagner (GMSW) type[8] with octet-broken $SU(3)$ couplings[9,10] which are related to other observable physical processes such as $\eta\to\pi^+\pi^-\gamma$, $\eta\to2\gamma$, and $\pi^0\to2\gamma$. However, the diagram containing a unitary singlet intermediate vector meson requires a knowledge also of the rate of one of the decay modes $\varphi\to\eta\gamma$ or $\omega\to\eta\gamma$, for which only upper limits are known. In the absence of more definite knowledge, experimental comparison will depend on a parameter whose value is known only to lie within a prescribed range. If, for example, we fit the $\eta\to3\pi$ partial width, this parameter takes on a reasonable value near the middle of its allowed range. For this value we predict rates for $\varphi\to\eta\gamma$, $\omega\to\eta\gamma$, $\rho^\pm\to\eta\pi^\pm$, and $K^\pm\to\pi^\pm\pi^0$.

The fact that the radiative part of the $\eta$-$\pi$ transition is essentially proportional to the squared mass of the external particle is a consequence of the momentum dependence of the GMSW-type vertex, as shown in the Appendix.

The value of the $\eta$-$\pi$ radiative transition, to be obtained in Sec. III, is

$$\langle\eta|\pi\rangle_{\text{rad}} = -(4.3\pm1.0)(p^2/m_\eta{}^2)\times10^3\,\text{MeV}^2 . \tag{4}$$

This gives as predicted rates

$$\Gamma(\varphi\to\eta\gamma) = 10 \text{ keV} \quad (\text{experiment: } <270 \text{ keV}) , \tag{5}$$

$$\Gamma(\omega\to\eta\gamma) = 40 \text{ keV} \quad (\text{experiment: } <180 \text{ keV}) , \tag{6}$$

with an accuracy of about 25%.

## III. APPLICATION TO MESON DECAY

### A. $\eta\to3\pi$ Decay

Previous attempts to calculate the rate of $\eta\to3\pi$ decay have not been entirely successful, although the slope of the projection of the Dalitz plot on the $\pi^0$ energy axis is correctly given in current-algebra treatments.[11] For example, Sutherland[12] has shown that if the usual electromagnetic interaction is used to account for the $G$ violation in the process, the decay amplitude vanishes in the soft-pion limit. Oakes[13] has introduced an intrinsic breaking of the type $\alpha_3 u_3$, whose value is related to the Cabibbo angle and which turns out to be several times larger than the Coleman-Glashow tadpole. Based on this theory, his $\eta\to3\pi$ decay rate is approximately twice as large as the observed rate.

In the present work we use the pion-pole model and the effective-Lagrangian technique, in which the process is visualized as an $\eta$-$\pi$ transition, followed by off-mass-shell pion-pion scattering. The other possible diagram, that containing an $\eta$ pole, is neglected.[14] The decay amplitude is then given by

$$A(\eta\to\pi^+\pi^-\pi^0) = \langle\eta|\pi^0\rangle\,(m_\eta{}^2 - m_\pi{}^2)^{-1}B(\pi^0\to\pi^0\pi^+\pi^-) , \tag{7}$$

where the off-mass-shell amplitude $B$, obtained by extrapolating Weinberg's[15] on-shell result, is

$$B(\pi^0\to\pi^+\pi^-\pi^0) = m_\eta{}^2(1-2E_0/m_\eta)(2g_V/f_\pi)^2 , \tag{8}$$

where $E_0$ is the energy of the final $\pi^0$ and

$$(2g_V/f_\pi)^2 = 1.52\times10^{-4}\text{ MeV}^{-2} . \tag{9}$$

By comparing the amplitude $A$ with the experimental rate, i.e.,

$$\Gamma(\eta\to\pi^+\pi^-\pi^0) = 605\pm150 \text{ eV} , \tag{10}$$

we obtain

$$|\langle\eta|\pi\rangle|^2 = (6.7\times10^3 \text{ MeV}^2)^2 . \tag{11}$$

Using $\langle\eta|\pi\rangle_{\text{int}}$ of Sec. II, we obtain for $\langle\eta|\pi\rangle_{\text{rad}}$ the values $-4.3$ or $+9.1$ (in units of $10^3$ MeV$^2$). As discussed in the Appendix, the value $+9.1$ is inconsistent with the rates of other meson decays in our model and is, therefore, rejected.

On the other hand, adopting the value $-4.3$ leads to the predictions of Eqs. (5) and (6) and puts additional constraints on the symmetry-breaking parameters of the octet-broken $PVV$ interaction.[9] These constraints are entirely consistent with the meson decay rates where known, and do not violate the known experimental upper limits.

### B. $\rho^\pm\to\eta\pi^\pm$ Decay

Another test of the $G$-violating $\eta$-$\pi$ transition is furnished by the decay of the charged $\rho$ meson into $\eta$ plus pion. Because the value of the $\eta$-$\pi$ matrix element is the same for this process as in the $\eta\to3\pi$ decay, we expect this rate to be similarly enhanced. This decay can be realized through the chain

$$\rho^+ \to \pi^+ + \pi^0 \to \eta$$

and a simple calculation yields the partial decay width

$$\Gamma(\rho^\pm \to \pi^\pm + \eta) = 3.5\times 10^{-5}\Gamma_\rho \,, \tag{12}$$

where $\Gamma_\rho$ is the $\rho$-meson width. It is interesting that this prediction is an order of magnitude larger than an estimate based on $\alpha^2$ multiplied by the phase space. Although our prediction is several orders of magnitude smaller than the presently quoted experimental upper limit for this decay, its measurement should be feasible.

### C. Charged $K$-Meson Decays

In this section we consider two decay modes of the charged $K$ mesons, $K^\pm \to \pi^\pm + \pi^0$ and $K^\pm \to \pi^\pm + \gamma + \gamma$. Since the violation of the $\Delta I = \frac{1}{2}$ rule in other nonleptonic weak interactions is rather small, it is attractive to consider that $\Delta I = \frac{3}{2}$ contributions occur as "electromagnetic" corrections. An attractive model[4,16] for the two processes mentioned involves $K^+ \to \pi^+ + (\eta)$, where the virtual $(\eta)$ then decays into either a $\pi^0$ or into two $\gamma$ rays.

In this model, the decay rate for $K^+ \to \pi^+ + \pi^0$ is given by

$$\Gamma(K^+ \to \pi^+ + \pi^0) = |\langle K^+|H_w|\pi\eta\rangle|^2 |\langle \eta|\pi^0\rangle|^2 \times 16\pi m_K^2 \times (m_\eta^2 - m_\pi^2)^{-1}(m_K^2 - 4m_\pi^2)^{1/2} \,. \tag{13}$$

In this case, since $\pi^0$ is on its mass shell, the radiative contribution to $\langle \eta|\pi^0\rangle$ is negligible.

The decay rate for $K^+ \to \pi^+ + \gamma + \gamma$, in the same model, is

$$\Gamma(K^+ \to \pi^+ + \gamma + \gamma) = |\langle K^+|H_w|\pi\eta\rangle|^2 |F|^2 \times (8\pi)^{-3} 2.16 (m_\pi^4/m_K^3) \,, \tag{14}$$

where

$$|F|^2 = 64\pi \times 10^{-6}/m_\pi^2 \,,$$

and characterizes the decay $\eta \to 2\gamma$. (Details will be found in the Appendix.) In calculating the total rate, a possible momentum dependence of the $K^+ \to \pi^+ + \eta$ vertex was neglected. Under the same assumption, the ratio of the rates is

$$\frac{\Gamma(K^+ \to \pi^+ + \gamma + \gamma)}{\Gamma(K^+ \to \pi^+ + \pi^0)} = 1.6\times 10^{-3} \,. \tag{15}$$

Our value for this ratio is considerably larger than that obtained by Fäldt, Petersson, and Pilkuhn,[4] who also employ the $\eta$-pole model. The reason is that these authors use a large intrinsic $\eta$-$\pi$ transition rate, obtained by fitting the $\eta \to 3\pi$ rate without radiative contribution. Our predicted value of the ratio is also larger than the most recent quoted experimental upper limit[5] $2\times 10^{-4}$; however, the experimental analysis assumed a phase-space distribution of pion momentum and examined a range of pion momenta which is insensitive to the predictions of the $\eta$-pole model. The experiment is, therefore, inapplicable if the effective mass distribution is close to that predicted by the $\eta$-pole model:

$$P_\pi s^2 (m_\eta^2 - s)^{-2} \,, \tag{16}$$

where $s$ is the effective mass of the two photons and $P_\pi$ is the pion momentum. Our predicted ratio, it may be remarked, is at least an order of magnitude larger than that expected on the basis of the pion-pole model, which is an $\alpha$ correction to the $K^+ \to \pi^+ + \pi^0$ amplitude.

We are unable to calculate the absolute rate of the two charged $K$-meson decays considered, without a specific model for the vertex $K^+ \to \pi^+ + \eta$. It is interesting to observe that in the $SU(3)$-symmetric limit, using the Cabibbo Hamiltonian, the arguments regarding the forbiddenness of the $K_s \to 2\pi$ decays can be extended to apply equally to $K^+ \to \eta + \pi^+$. One can try to relate $K^+ \to \pi^+ + \eta$ to $K_s \to 2\pi$ by reducing one pion and using the Callan-Treiman technique,[17] assuming that, after the reduction, the Hamiltonian acts symmetrically on the two remaining mesons. There are thus two reduced matrix elements to be considered, corresponding to the $\underline{8}$ and $\underline{27}$ representations (keeping only $\Delta I = \frac{1}{2}$) of $SU(\overline{3})$.[18] Thus, if we write

$$\langle K_s|2\pi^0\rangle = a_8 + a_{27} \,, \tag{17}$$

we obtain

$$\langle K^+|\eta\pi^+\rangle = (\tfrac{1}{3})^{1/2} a_8 - 3\sqrt{3}\, a_{27} \,. \tag{18}$$

The observed experimental rate for $K^+ \to \pi^+ + \pi^0$ is obtained in our model by setting

$$\left|\frac{\langle K^+|\eta\pi^+\rangle}{\langle K_s|2\pi^0\rangle}\right|^2 = 27 \,, \tag{19}$$

which gives the two solutions

$$a_8/a_{27} = 0,\ -\tfrac{9}{4} \,. \tag{20}$$

Notice, however, that any sufficiently large admixture of $\underline{27}$ will give a reasonable rate.

In the case of the kaon decaying to three pions, the Dalitz-plot slopes agree with the $\Delta I = \frac{1}{2}$ rule predictions within 2 standard deviations. However, the ratio of the rates of neutral-kaon decays to charged-kaon decays shows a well-established deviation of about 20% from the $\Delta I = \frac{1}{2}$ ratios.[19] Our model suggests a mechanism for a reduction in the rate of $K_L^0$ decay arising from $\eta$-$\pi^0$ mixing in the pion-pole contribution to the amplitude. This effect is of the correct sign and rather strong, but we cannot estimate its magnitude without a detailed theory of the weak nonleptonic interaction. Because the $\eta$ is on the kaon mass shell, the amplitude is very sensitive to the $\eta$ pole if the momentum dependence of the $K$-$\pi$ transition is neglected.


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## APPENDIX

We calculate here the radiative contribution to $\eta$-$\pi$ mixing, using an effective Lagrangian composed of vector and pseudoscalar-meson fields. The vector Lagrangian is of the Yang-Mills variety, with the mass term providing field-current identities. Local gauge invariance of the pseudoscalar Lagrangian induces the usual couplings to the vector fields via the covariant derivative. An $SU(3)$-broken, but locally gauge-invariant, $PVV$ coupling of the GMSW type[8] is assumed.[20] The electromagnetic interaction, introduced by the minimal replacement in conjunction in the field-current identities, gives[21]

$$\mathfrak{L}_{\text{em}} = (em^2/g)[V_\mu^3 + (\tfrac{1}{3})^{1/2} V_\mu^8]A_\mu\,, \tag{A1}$$

where $A_\mu$ is the electromagnetic four-potential and $V_\mu^a$ is the vector meson field, described by $(a, b = 0, 1, \ldots, 8)$

$$\mathfrak{L}_V = \tfrac{1}{4}K^{ab}V_{\mu\nu}^a V_{\mu\nu}^b + \tfrac{1}{2}m^2 V_\mu^a V_\mu^a\,, \tag{A2}$$

where

$$V_{\mu\nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a - g f^{abc} V_\mu^b V_\nu^c. \tag{A3}$$

The matrix $K^{ab}$ is responsible in this model for mass splittings among the nine vector mesons and for the $\omega$-$\varphi$ mixing in the vector-mixing model[22]; it is diagonal except in the 0–8 sector, and gives the following connections between the $V_\mu^a$ and the physical meson fields:

$$\begin{aligned}
&V_\mu^{1,2,3} = \frac{m_\rho}{m}\rho_\mu^{1,2,3}, \quad V_\mu^{4,5,6,7} = \frac{m^*}{m}K_\mu^{*4,5,6,7},\\
&V_\mu^8 = -\frac{m_\omega}{m}\sin\theta\,\omega_\mu + \frac{m_\varphi}{m}\cos\theta\varphi_\mu\,,\\
&V_\mu^0 = \frac{m_\omega}{m}\cos\theta\,\omega_\mu + \frac{m_\varphi}{m}\sin\theta\,\varphi_\mu\,,\\
&\theta = 27.5^\circ, \quad m = 847\text{ MeV}.
\end{aligned} \tag{A4}$$

The $PVV$ coupling is

$$\mathfrak{L}_{PVV} = \epsilon^{\alpha\beta\mu\nu}(hD^{abc}V_{\alpha\beta}^a V_{\mu\nu}^b P^c + \lambda D^{ab} V_{\alpha\beta}^a V_{\mu\nu}^0 P^b)\,, \tag{A5}$$

with

$$D^{abc} = d^{abc} + \sqrt{3}\,\epsilon_1 d^{abk} d^{k8c} + \tfrac{1}{2}\sqrt{3}\,\epsilon_2(d^{ack}d^{k8b} + d^{bck}d^{k8a}) + (\tfrac{1}{3})^{1/2}\epsilon_3\delta^{ab}\delta^{c8}\,, \tag{A6}$$

$$D^{ab} = \delta^{ab} + \sqrt{3}\,\epsilon_4 d^{ab8}\,. \tag{A7}$$

Diagrams contributing to the $\eta$-$\pi$ transition have the structure

$$\eta \to V_a + V \to \gamma + V \to V_b + V \to \pi\,, \tag{A8}$$

where $V_a$ and $V_b$ have opposite $G$ parity. The summed contributions of such diagrams give the amplitude

$$M = \frac{2}{\sqrt{3}}h^2(1+\epsilon_1)^2\left(\frac{e}{g}\right)^2\frac{m^4}{(4\pi)^2}\frac{p^2}{m_\eta{}^2}S\,, \tag{A9}$$

with

$$S = 0.176S_1 + 0.208S_2 + 1.217S_3\,. \tag{A10}$$

Note that $M$ is proportional to $p^2$ as a consequence of the GMSW type $PVV$ coupling. The $S_i$ are

$$\begin{aligned}
S_1 &= \frac{1-\epsilon_1+\epsilon_2+\epsilon_3}{1+\epsilon_1}\,,\\
S_2 &= \frac{-1+\epsilon_1+\epsilon_2+\epsilon_3}{1+\epsilon_1}\,,\\
S_3 &= \frac{1-\epsilon_4}{1+\epsilon_1}\,.
\end{aligned} \tag{A11}$$

The parameters $\lambda$, $g$, and $h(1+\epsilon_1)$ of (A5) and (A9) are overdetermined by the experimental rates of $\pi^0 \to 2\gamma$, $\omega \to 3\pi$, $\varphi \to 3\pi$, and $\omega \to \pi\gamma$. $S_1$ and $S_2$ are determined by the $\eta \to 2\gamma$ and $\eta \to \pi^+\pi^-\gamma$ rates. Note that the dominant contribution $S_3$ contains $\epsilon_4$, which determines the rates of $\varphi \to \eta\gamma$ and $\omega \to \eta\gamma$. While the rates of the latter two processes have not been measured, their quoted experimental upper limits restricts the range of allowed values of $\epsilon_4$. If we choose the value of $\epsilon_4$ in the center of its allowed range, we find an $\eta \to 3\pi$ value in excellent agreement with experiment. Using the $\eta \to 3\pi$ value as input, we deduce $M = (-4.3 \pm 1.5) \times 10^3$ MeV$^2$, giving a value of $\epsilon_4 = 2.8 \pm 0.6$, leading to the predictions of Eqs. (5) and (6). The other solution for $M$ obtained from the $\eta \to 3\pi$ rate, $M = +9.1 \times 10^3$ MeV$^2$, gives $\epsilon_4 = -0.66$, which gives too large rates for $\omega \to \eta\gamma$ and $\varphi \to \eta\gamma$.

We remark that the $\eta \to 2\gamma$ vertex was derived from this same effective Lagrangian. The $\eta \to 2\gamma$ rate resulting from the Lagrangian is

$$\Gamma(\eta \to 2\gamma) = (m_\eta{}^3/64\pi)F^2\,, \tag{A12}$$

where

$$F = \frac{2}{\sqrt{3}}\frac{e^2}{g^2}h(1+\epsilon_1)(S_1 + \tfrac{1}{3}S_2)\,. \tag{A13}$$

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# Multiplicity Distributions in Regge-Pole-Dominated Inclusive Reactions*

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Multiplicity distributions, the dependence on $n$ of $\psi_n=\sigma_n/\sigma$, are discussed. Within the framework of the Amati-Fubini-Stanghellini model, a cluster expansion for the moments of $\psi_n$ is derived. This same expansion is then derived as a consequence of asymptotic dominance of inclusive reactions by an isolated, factorizable Regge pole. Such an expansion furnishes a systematic way of describing the shape of $\psi_n$. It is argued that a Poisson distribution for multiple particle production can not be expected to occur, even for very high energies.

## I. INTRODUCTION

The topic to be discussed in this paper is multiplicity distributions,[1] that is, the dependence on $n$ of $\sigma_n/\sigma=\psi_n$ for a fixed large energy. $\sigma_n$ is the production cross section for two particles to go into $n$ particles, while $\sigma$ is the total cross section. (Throughout this paper, only a single type of particle is considered. This is not necessary, but such an assumption simplifies the discussion.)

In Sec. II the model of Amati, Fubini, and Stanghellini[2] (AFS) will be used to derive a general expression [Eq.(2.11)] for the binomial moments in $n$ of $\psi_n$. The approach used in deriving this equation is not unlike that used to obtain the cluster expansion in statistical mechanics. Equation (2.11) is in fact a cluster expansion.

In Sec. III the basic result, Eq. (2.11), of this paper is derived anew, this time without using the AFS model. In this derivation the ingredients are of a more general character, although they may well not be correct in the physical world. In par-