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Δ_δ Regge Trajectory and Nondegenerate Parity Partners*

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The hypothesis that the $D(1670)$ is the MacDowell partner of the $\Delta(1234)$ is presented. This idea predicts a G_{37} πN resonance with mass 2530 MeV, and implies that for fixed, nonzero u, the sign of the $\pi^- p$ polarization should change with increasing s.

I. INTRODUCTION

The empirical observation that fermion Regge trajectories are approximately linear functions of $W^2 \equiv u$ has been frequently made.[1] The observation is based on Chew-Frautschi plots, such as that for the Δ trajectory shown in Fig. 1. In spite of the fact that the spin assignment of the highest two states is only tentative, the five resonances on this plot provide the best evidence for the idea that fermion Regge trajectories are even functions of W. Still, it is important to remember that the empirical approximation

$$\mathrm{Re}\,\alpha_\Delta(W) \cong 0.15 + 0.9W^2 \qquad (1.1)$$

is only fitted to information obtained in the resonance region, $W > M + \mu$, and it is not obvious that it should be taken seriously in extrapolating, for example, to W on the imaginary axis, the scattering region. There is really very little evidence that (1.1) remains valid. The usual clue about the behavior of a trajectory function in the scattering region, a possible wrong-signature nonsense dip, is absent in $\pi^- p$ backward scattering. Errors involved in determining an effective trajectory from fitting the shrinkage of the differential cross section are too large to provide any support for (1.1).

A related problem is the continuation of (1.1) to large W on the negative real axis. The importance of the extrapolation of the trajectory function to negative W curs because of MacDowell symmetry.[2] If (1.1) is approximately valid for negative as well as positive W, MacDowell symmetry predicts the existence of an unobserved $J^P = \frac{3}{2}^-$ state degenerate with the Δ unless one of two things happens. The first alternative is that the residue function associated with the trajectory can develop a dynamical zero precisely at the point where the real part of α passes through $J = \frac{3}{2}$.[3] The other is that the residue contains a branch cut which removes the unwanted portion of the trajectory function from the physical region of the J plane.[4]

A third, simple explanation of the absence of the Δ's degenerate partner is just that the trajectory is not an even function of W. There is, after all, a relatively well-established $J^P = \frac{3}{2}^-$ Δ resonance, the $D_{33}(1670)$, available.[5] What is the chance that this state is the MacDowell parity partner of the $P_{33}(1234)$? Surprisingly enough, this possibility does not necessarily conflict with the empirical approximation (1.1) applied to the positive-parity resonance region $W > 1.0$ GeV. The possibility definitely warrants more serious consideration than it has been heretofore allotted.

This paper discusses the analytic structure of fermion Regge trajectories implied by unitarity using a method which has been found useful in the study of boson trajectories.[6,7] Within this framework, it is found that the condition that the approximation (1.1) be valid *above the elastic threshold* is not the same as the condition that the trajectory function be even in W. An explicit parametrization of the Δ trajectory which includes the $D_{33}(1670)$ and which fits the masses and total widths of the Δ recurrences is given. If the assignment of the D_{33} as the negative-parity MacDowell partner of the P_{33} is accepted, the reduced residue function of the trajectory is found to obey a particularly simple parametrization. The mass, total width, and elastic width of the possible G_{37} recurrence of the D_{33} are, of course, then found to be closely constrained. The asymptotic form of the scattering

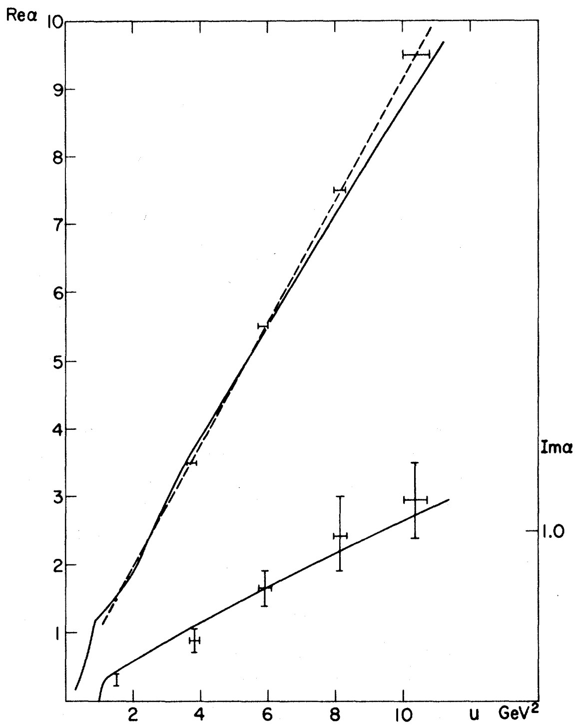

FIG. 1. Chew-Frautschi plot for the Δ_δ trajectory. The dashed line is the approximate form $\mathrm{Re}\,\alpha = 0.15 + 0.90u$ which ignores the analytic structure of the trajectory function, but which passes cleanly through the experimental values of $(\mathrm{mass})^2$ for the resonances. The solid pair of lines gives the trajectory function $\alpha = (0.13 + 0.95u)[1 - u/(M+\mu)^2]^{-0.035}$ which has both a real and imaginary part above the elastic threshold. The idea that fermion Regge trajectories are even functions of $W = u^{1/2}$ is supported by quasilinear fits to Chew-Frautschi resonance plots, but a trajectory can have a significant odd part and still be consistent with the $(\mathrm{mass})^2$ of the observed resonance.

differential cross section and polarization are also predicted by this assumption. For a fixed nonzero value of u, the sign of the polarization is predicted to change sign as s is increased. For example, at $u = -0.8$ GeV^2, the period of the changeover is such that the polarization changes sign when s is tripled. In contrast to the case of conventional Regge parametrizations of $\pi^- p$ backward scattering, this polarization prediction does not depend crucially on the amount of absorption present.[8]

II. PHASE REPRESENTATION OF A NONDEGENERATE FERMION TRAJECTORY

Consider the $\pi^- p$ scattering process with the kinematic variables defined as depicted in Fig. 2. The invariant matrix element is given in terms of the spinor amplitudes[9]

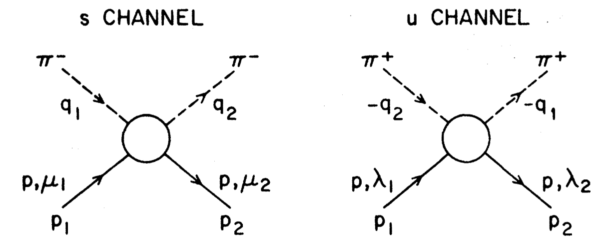

FIG. 2. Kinematic variables in $\pi^- p$ backward scattering. The λ_i are u-channel helicity indices. $s = (p_1 + q_1)^2$, $t = (q_1 - q_2)^2$, $u = (p_1 - q_2)^2$, $W = u^{1/2}$, $E_u = (u + M^2 - \mu^2)/2W$, $q_u^2 = [u - (M+\mu)^2][u - (M-\mu)^2]/4u$, $z_u = 1 + t/2q_u^2 = \cos\theta_u$.

$$T(\lambda_1 p_1 q_1, \lambda_2 p_2 q_2) = \bar{u}_{\lambda_2}(p_2)[A(s,u) - \tfrac{1}{2}(q_1+q_2)\cdot\gamma B(s,u)]u_{\lambda_1}(p_1)\,, \tag{2.1}$$

where λ_1 and λ_2 are u-channel helicity indices. For each physical value of J there are two u-channel partial-wave amplitudes of definite parity, $P = (-1)^{L+1}$, given by

$$f^{L=J\pm 1/2}(J,W) = \frac{E+M}{8\pi W}[A(J\pm\tfrac{1}{2},u) + (W-M)B(J\pm\tfrac{1}{2},u)] + \frac{E-M}{8\pi W}[-A(J\mp\tfrac{1}{2},u) + (W+M)B(J\mp\tfrac{1}{2},u)]\,, \tag{2.2}$$

where $W = u^{1/2}$ and

$$A(L,u) = \tfrac{1}{2}\int_{-1}^{+1} dz\, P_L(z) A(s(z),u)\,, \tag{2.3a}$$

$$B(L,u) = \tfrac{1}{2}\int_{-1}^{+1} dz\, P_L(z) B(s(z),u)\,. \tag{2.3b}$$

The MacDowell symmetry relation,[2]

$$f^{J+1/2}(J,W) = -f^{J-1/2}(J,-W)\,, \tag{2.4}$$

then follows immediately from (2.2) and the fact that (2.3) implies $A(L,u)$ and $B(L,u)$ are even functions of W. Since $A(L,u)$ and $B(L,u)$ have signatured Froissart-Gribov continuations just like spinless amplitudes, the Regge singularities occurring in $f^{J+1/2}$ and $f^{J-1/2}$ are related through (2.4). Assume that for some $W > 0$, the leading J-plane singularity surface in the positive-signature continuation of $f^{J-1/2}(J,W)$ is a nondegenerate pole trajectory. That is, in the limit $J \to \alpha_1(W)$,

$$f^{J-1/2}(J,W) \sim \beta_1(W)/[J - \alpha_1(W)]\,, \tag{2.5}$$

where the signature index is suppressed. Equation (2.4) then implies

$$f^{J+1/2}(J,W) \sim \beta_2(W)/[J-\alpha_2(W)]\,, \tag{2.6}$$

where

$$\alpha_2(W) = \alpha_1(-W)\,, \tag{2.7a}$$

$$\beta_2(W) = -\beta_1(-W)\,. \tag{2.7b}$$

If $\alpha_1(W)$ contains the physically observed states of positive parity such as the P_{33} resonance, $\Delta(1234)$, and its recurrences, $\alpha_2(W)$ will generate physical states of negative parity unless $\beta_1(W)$ contains a branch cut which prevents the continuation implied by (2.7) being valid for physical W.[5] Even in the absence of other J-plane singularities, certain states predicted by (2.7) can always be removed by dynamical zeros in the residue function. The fact remains that by investigating the continuation of $\alpha_1(W)$ and $\beta_1(W)$ to negative, unphysical W, we obtain information about the trajectory in the opposite-parity amplitude. Conversely, once the existence of this continuation is assumed, information about resonances on the opposite-parity trajectory can be used to investigate the behavior of $\alpha_1(W)$ and $\beta_1(W)$ in the entire W plane.

In order to discuss the continuation of $\alpha_1(W)$ to negative W, we have to consider its analytic structure. The first thing is to recall that $\alpha_1(W)$ does not inherit the singularities at negative u (imaginary W) of $A(L,u)$ and $B(L,u)$.[10] It is also convenient to assume that no singularities are generated on the imaginary axis from a collision with other singularity surfaces.[11,12] In this case, $\alpha_1(W)$ is analytic in the region shown in Fig. 3 and, using (2.9), can be parametrized in terms of the dispersion relation,[13]

$$\alpha_1(W) = \frac{1}{\pi}\int_{M+\mu}^{\infty} \frac{\mathrm{Im}\,\alpha_1(x)}{x-W}\,dx + \frac{1}{\pi}\int_{M+\mu}^{\infty} \frac{\mathrm{Im}\,\alpha_2(x)}{x+W}\,dx\,, \tag{2.8}$$

where the subtractions necessary to make the integrals converge have not been explicitly shown.

Of some interest in interpreting (2.8) are the threshold expansions for $\alpha_{1,2}(W)$ implied by u-channel elastic unitarity[14]

$$\mathrm{Im}\,\alpha_1(W) \sim C_1[W-(M+\mu)]^{\mathrm{Re}\,\alpha_1(M+\mu)}\,, \tag{2.9a}$$

$$\mathrm{Im}\,\alpha_2(W) \sim C_2[W-(M+\mu)]^{\mathrm{Re}\,\alpha_2(M+\mu)+1}\,, \tag{2.9b}$$

where C_1 and C_2 are both positive. The expressions (2.9) are important because, when inserted into (2.8), they show that $\alpha_1(W)$ cannot be a strictly even function of W. Even if, in some strict narrow-resonance limit, the trajectory functions α_1 and α_2 were degenerate, the constraints (2.9) imply that elastic unitarity would break this degeneracy. Of course, mere threshold expansions do not indicate how big the breaking of the degeneracy must be, and this is an important question. In the case of the Δ trajectory, for example, can the splitting between α_1 and α_2 be big enough so that $\alpha_2(W)$ passes through $J=\frac{3}{2}$ at the position of the $D_{33}(1670)$? One way to answer this question is to confront a specific parametrization of the trajectory function $\alpha_1(W)$ with the observed resonance spectrum. A particularly simple parametrization of $\alpha_1(W)$ is implied by recasting the dispersion relation (2.8) in the form[6,15]

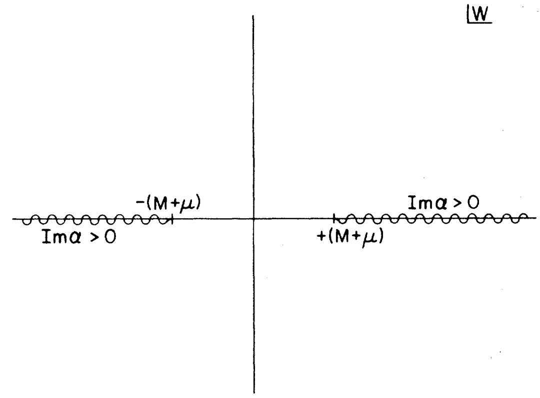

FIG. 3. The analytic structure of a fermion Regge trajectory which does not collide with other singularity surfaces. Unitarity requires that the imaginary part of α be positive above the right-hand and below the left-hand cut. The Schwartz reflection principle requires $\alpha(W^*) = \alpha^*(W)$.

$$\alpha_1(W) = (a_0 + a_1 W + a_2 W^2) \times \exp\left[\frac{W}{\pi}\int_{M+\mu}^{\infty} \frac{\delta_1(x)\,dx}{x(x-W)}\right] \exp\left[\frac{-W}{\pi}\int_{M+\mu}^{\infty} \frac{\delta_2(x)\,dx}{x(x+W)}\right], \tag{2.10}$$

where

$$\delta_{1,2}(W) = \arctan\left[\frac{\mathrm{Im}\,\alpha_{1,2}(W+i\epsilon)}{\mathrm{Re}\,\alpha_{1,2}(W+i\epsilon)}\right]. \tag{2.11}$$

This parametrization is simple because the number of subtractions in the integrals is determined by the unitarity requirement

$$\delta_{1,2}(W) \in (0,\pi), \quad W > M+\mu\,. \tag{2.12}$$

Also, following an argument developed by Childers[6] for boson trajectories, it can be shown that the polynomial in (2.10) must be of second order.[16] Empirically, it seems as if the phases defined by (2.11) are approximately constant.[7,17,18] This suggests that we can approximately take into account the constraints of unitarity by writing

$$\alpha_1(W) \cong (a_0 + a_1 W + a_2 W^2) \times [1 - W/(M+\mu)]^{-\epsilon_1} [1 + W/(M+\mu)]^{-\epsilon_2} . \tag{2.13}$$

If the D_{33} resonance is accommodated on the Δ trajectory, the parameters

$$a_0 = -0.26, \quad \epsilon_1 = 0.036, \quad a_1 = 0.46 \text{ GeV}^{-1}, \quad \epsilon_2 = 0.045, \quad a_2 = 0.89 \text{ GeV}^{-2}, \tag{2.14}$$

provide an excellent fit to the observed masses *and* total widths of the resonances. The real part of this trajectory is shown in Fig. 4 as is a simple second-order polynomial of the type suggested by Desai,[19]

$$\text{Re}\alpha_1(W) \cong \tfrac{3}{2} + 0.8(W - 1.23)(W + 1.67). \tag{2.15}$$

Given the errors on the masses of the high-spin recurrences,[5] both of these forms give a good approximation to the observed spectrum.

The point is, even if you take very seriously the spin assignments projected for the Δ recurrences, the five resonances on the Chew-Frautschi plot do not rule out a substantial deviation from the approximation (1.1) at negative W. Observe that putting the $D_{33}(1670)$ on a trajectory function which is a linear function of W^2 of the canonical slope gives

$$\text{Re}\alpha_{\text{"}D\text{"}}(W) \cong -1.01 + 0.9W^2 , \tag{2.16}$$

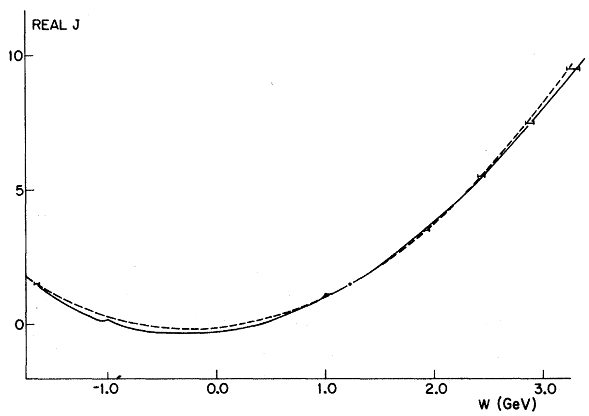

FIG. 4. $\text{Re}\alpha_\Delta$ vs W for two trajectories which contain the $D_{33}(1670)$ as the MacDowell partner of the $\Delta(1234)$. The dashed line is $\text{Re}\alpha = 1.5 + 0.8(W - 1.23)(W + 1.67)$ which ignores the analytic structure of the trajectory function. The solid line is $\alpha(W) = (-0.26 + 0.46W + 0.89W^2) \times [1 - W/(M+\mu)]^{-0.036} [1 + W/(M+\mu)]^{-0.045}$ which fits both the masses and total widths of the resonances. The latter function actually looks straighter in a Chew-Frautschi plot, $\text{Re}\alpha$ vs W^2, than does the even function of W, $\alpha = (0.13 + 0.95u)[1 - u/(M+\mu)^2]^{-0.035}$, plotted in Fig. 1.

so a large apparent deviation from (1.1) is possible without distorting badly its validity for positive W.

III. THE REDUCED RESIDUE FUNCTION AND NEGATIVE-PARITY RECURRENCES

If the hypothesis about the connection between the $\Delta(1234)$ and the $D(1670)$ is correct, quite a bit of information about the reduced residue function can be extracted. Following Berger and Fox,[3,8] it is convenient to normalize the reduced residue function to the elastic widths of the positive-parity Δ resonances

$$\Gamma^+_{J,\,\text{elastic}} = (E_J + M)(4q_J{}^2)^{J-1/2} \frac{\Gamma(J+\frac{3}{2})}{\Gamma(2J+2)} \frac{q_J}{4\pi M_J{}^2} \gamma_1(M_J) . \tag{3.1a}$$

Then the elastic widths of the negative-parity states are given by

$$\Gamma^-_{J,\,\text{elastic}} = -(E_J - M)(4q_J{}^2)^{J-1/2} \frac{\Gamma(J+\frac{3}{2})}{\Gamma(2J+2)} \frac{q_J}{4\pi M_J{}^2} \gamma_1(-M_J) , \tag{3.1b}$$

so the $\gamma_1(W)$ must change sign between $W = M_\Delta$ and $W = -M_D$.[19] In this paper, it is assumed that $\gamma_1(W)$ is analytic at $W = 0$. Figure 5 shows a plot of the reduced residue evaluated at the position of the resonances using (3.1a) and (3.1b). The value of the residue function at $W = 0$ is also strongly constrained by the scattering data for $d\sigma/du$ in the backward direction, and this constraint is also shown in Fig. 5. As can be seen, the experimental elastic widths and the scattering data are consistent with a linear polynomial in W,

$$\gamma_1(W) \cong g_0 + g_1 W, \tag{3.2}$$

where g_0 is small, $g_0 < 0.05$, and $g_1 \cong 0.25$ GeV^{-1}. Also shown is a parametrization where the residue is forced to be zero at the points where $\alpha_1 = \frac{1}{2}$, as suggested by exchange-degeneracy arguments connecting the Δ with the $N\gamma$ trajectory.[20]

If the trajectory function is assumed to be an even function of W, the absence of a negative-parity state degenerate with the $\Delta(1234)$ forces a completely different parametrization of the reduced residue function. Either a zero must appear at $W = -1.23$, or the residue function must contain a branch cut. The fact that either of the possibilities is difficult to reconcile with the magnitude of $\gamma_1(0)$, implied by the scattering data and the value obtained at the resonance positions, has been discussed by Berger and Fox.[3,8]

Once both the trajectory function and the residue

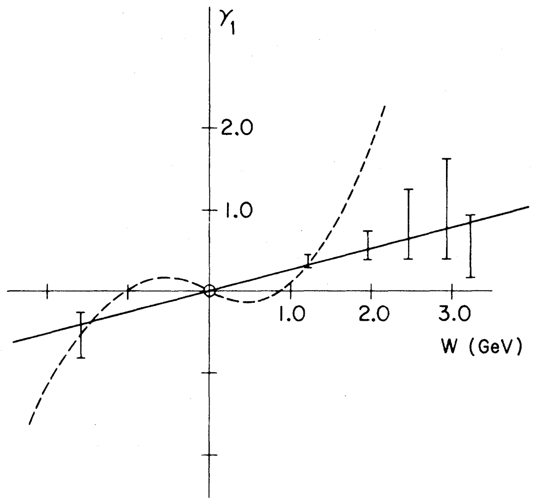

FIG. 5. Plot of the reduced residue function, $\gamma_1(W)$, determined from the experimental elastic widths and (3.1a) and (3.1b). The circle at $W=0$ gives the range of values consistent with the backward $\pi^- p$ scattering data. The linear fit $\gamma_1 \cong g_0 + g_1 W$ passes through all the experimental values if $|g_0| < 0.05$ and $g_1 = 0.26$. The dashed line includes two extra zeros at $W = 0.82$ and $W = -1.05$, where $\alpha = \frac{1}{2}$. The concept that γ_1 might be approximated by a low-order polynomial of W is discussed by Berger and Fox, Ref. 3.

function have been parametrized, we can consider the consequences. In the next section, we will discuss the asymptotic behavior of $d\sigma/du$ and the backward polarization implied by (2.13), (2.14), and (3.2). However, a very simple consequence of the hypothesis is the prediction of the properties of a G_{37} resonance, the $J^P = \frac{7}{2}^-$ state on the trajectory

$$\begin{aligned} &M_G \cong 2.53 \text{ GeV}, \\ &\Gamma_{\text{tot}} \cong 0.3\text{–}0.4 \text{ GeV}, \\ &\Gamma_{\text{elastic}} \cong 30 \text{ MeV}. \end{aligned} \tag{3.3}$$

Compare these values to those implied by other alternatives. If the trajectory function is even in W, but the residue contains a dynamic zero to remove the parity partner of the $\Delta(1234)$, the G_{37} should be degenerate with the F_{37}. Its total width and elastic width are unknown since they could depend crucially on the details of the mechanism which removed the $D_{33}(1234)$

$$\begin{aligned} &M_G \cong 1.98 \text{ GeV}, \\ &\Gamma_{\text{tot}} = ?\,, \\ &\Gamma_{\text{el}} = ?\,. \end{aligned} \tag{3.4}$$

If the Carlitz and Kislinger mechanism has anything to do with the $I = \frac{3}{2}$ particle spectrum, then the negative-parity states have no connection with the positive-parity ones. Taking the extrapolation (2.16) for the trajectory containing the D_{33} gives a possible G_{37} state at

$$\begin{aligned} &M_G \cong 2.24 \text{ GeV}, \\ &\Gamma_{\text{tot}} = ?\,, \\ &\Gamma_{\text{el}} = ?\,. \end{aligned} \tag{3.5}$$

Again, the total width and elastic width are not predicted.

The next round of phase shifts could produce evidence for a G_{37} resonance. Its properties would aid in deciding between these three alternatives for fermion Reggeization.

IV. ASYMPTOTIC BEHAVIOR OF $d\sigma/du$ AND P

If the assumption that the Δ Regge trajectory interpolates between the $\Delta(1234)$ and the $D_{33}(1670)$ is correct, there are also severe constraints on the form of the trajectory and residue functions in the scattering region (ImW). These constraints can be used to predict the asymptotic form of the differential cross section and the polarization.

For W imaginary, the two trajectories $\alpha_1(W)$ and $\alpha_2(W)$ are complex conjugates as can be seen from (2.7). For small ImW, the imaginary part of the two trajectories can be estimated from the representation (2.10),

$$\begin{aligned} \left.\frac{d\alpha_1(W)}{dW}\right|_{W=0} &\cong \frac{\alpha_1(W)}{W} \\ &= a_1 + \frac{a_0}{\pi}\int_{M+\mu}^{\infty} \frac{\delta_1(x) - \delta_2(x)}{x^2}\,dx\,, \end{aligned} \tag{4.1}$$

or, using the specific forms (2.13) and (2.14),

$$\begin{aligned} \left.\frac{d\alpha_1(W)}{dW}\right|_{W=0} &= a_1 + a_0(\epsilon_2 - \epsilon_1)/(M+\mu) \\ &\cong 0.46 \text{ GeV}^{-1}. \end{aligned} \tag{4.2}$$

The real and imaginary parts are plotted in Fig. 6. From the discussion in Sec. III, it is also evident that a reasonable approximation to the reduced residue function, $\gamma_1(W)$, is given by (3.2). In terms of the reduced residues of the invariant amplitudes A and B,[8]

$$\gamma_1(W) = \gamma_A^1(W) + (W - M)\gamma_B^1(W)\,. \tag{4.3}$$

To get the even and odd parts of γ_A^1 and γ_B^1, we can write

$$\gamma_A^1 = a_R(u) + W a_I(u)\,, \tag{4.4a}$$

$$\gamma_B^1 = b_R(u) + W b_I(u)\,. \tag{4.4b}$$

The parametrization (3.2) then gives

$$a_R - Mb_R + W^2 b_I = g_0 , \tag{4.5a}$$

$$a_I - Mb_I + b_R = g_1 . \tag{4.5b}$$

The residues of the complex-conjugate trajectory are given by

$$\gamma_A^2 = \gamma_A^{1*} = a_R(u) - Wa_I(u) , \tag{4.6a}$$

$$\gamma_B^2 = \gamma_B^{1*} = b_R(u) - Wb_I(u) . \tag{4.6b}$$

The asymptotic behavior of the differential cross section, assuming the dominance of the MacDowell pair of trajectories $\alpha_1(W)$ and $\alpha_2(W)$ and the residue function (3.2), is

$$\frac{d\sigma}{du}(s,u) \sim \frac{1}{16\pi s}|A(s,u)+MB(s,u)|^2 , \tag{4.7}$$

$$\frac{d\sigma}{du}(s,u) \sim G^2(u) s^{2\alpha_R(u)-1} \times [1 + e^{-2\pi\alpha_I(u)} - 2e^{-2\pi\alpha_I(u)} \sin\pi\alpha_R(u)] , \tag{4.8}$$

where the last factor, written explicitly, is the modification that a pair of complex-conjugate trajectories make to the usual fermion Regge signature factor (τ denotes signature)

$$R(\tau, u) = 2 - 2\tau \sin\pi\alpha(u) . \tag{4.9}$$

Since (4.9) has a double zero when α passes through a wrong-signature value, the differential cross section of an amplitude dominated by a single, real trajectory is usually expected to have a dip at such a point unless other factors in the residue specifically remove this dip.[21] For $\pi^- p$ backward scattering, the first wrong-signature nonsense point is at $\mathrm{Re}\alpha = -\frac{3}{2}$. For the parametrization (2.13), (2.14), this occurs at $u = -1.55\ \mathrm{GeV}^2$, and for the real trajectory (1.1) at $u = -1.8\ \mathrm{GeV}^2$. No dip is seen in the differential cross section in this neighborhood.[22]

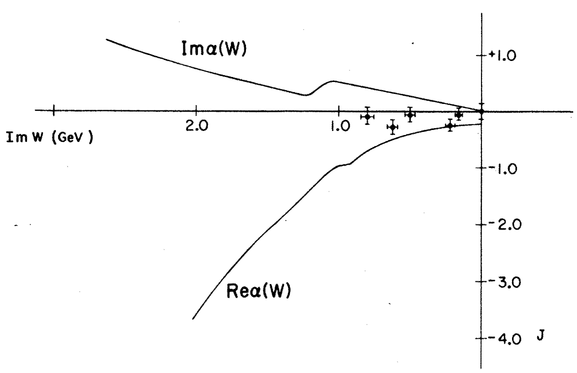

FIG. 6. The real (lower curve) and imaginary (upper curve) parts of the complex-conjugate trajectories $\alpha_1(W) = \mathrm{Re}\alpha(W) + i\,\mathrm{Im}\alpha(W)$ and $\alpha_2(W) = \mathrm{Re}\alpha(W) - i\,\mathrm{Im}\alpha(W)$. The wrong-signature point $\mathrm{Re}\alpha = -\frac{3}{2}$ occurs at $W = +1\ 23i$ or $u = -1.55$. At this point $\mathrm{Im}\alpha \cong 0.29$.

The absence of such a dip is sometimes attributed to the fact that at energies at which a search for it has been made, t-channel exchanges can be important at this value of u. Also, absorptive cuts could

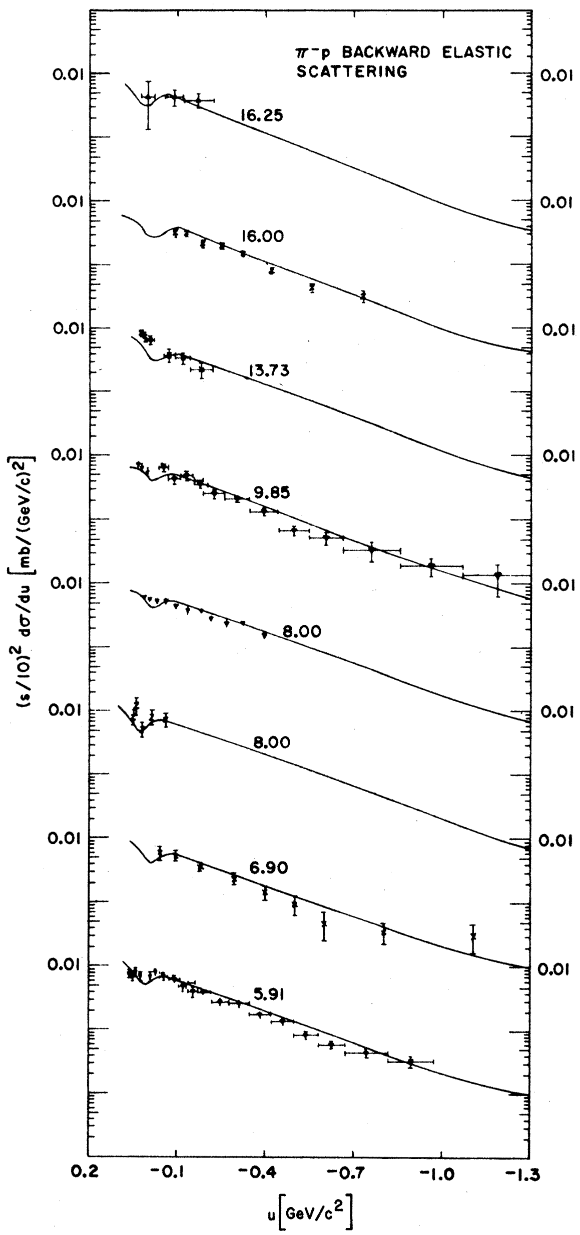

FIG. 7. Fit to the backward $\pi^- p$ differential cross section using (4.8) and (4.10). The data are multiplied by scale factors, as discussed by Berger and Fox, Ref. 8. The factors determined are: Kormanyos *et al.*, 1.1; Baker *et al.*, 1.1; Anderson *et al.*, 0.7. The absence of a wrong-signature dip in (4.8) allows a better chance of a fit than Regge parametrizations using a single real Δ trajectory. The dip at $u = 0$, shown in the plot, assumes that $\gamma_1(0) = 0$.

fill in the dip to some extent.[23] For the parametrization, in terms of a pair of complex-conjugate MacDowell-partner trajectories, no such dip is predicted unless α_I is very small, smaller than is consistent with (2.13) and (2.14). As can be seen in Fig. 6, the imaginary part of the complex-conjugate trajectories, α_I, is about equal to 0.3 in the region of the wrong-signature value of α_R.

If the value of the reduced residue function vanishes exactly at $W=0$, $g_0=0$ in (3.2),[24] then the coefficient $G^2(u)$ in (4.8) will be proportional to u,

$$G^2(u)\propto|u|\ . \tag{4.10a}$$

In this case, the data should show a dip at this point. Figure 7 shows a fit to backward data using the parameters implied by our hypothesis. The data are consistent with (4.10) and the absence of the wrong-signature dip.

The relative phases of the two amplitudes as determined by this parametrization can be seen by (4.5) and (4.6). The prediction for the polarization was first given by Gribov[13]:

$$P=\left[1-\frac{\sin^2\pi\alpha_R(u)}{\cosh^2\pi\alpha_I(u)}\right]^{1/2}\sin[2\pi\alpha_I\ln(s/s_0)+2\varphi+\theta]\ , \tag{4.10b}$$

where $s_0=1$ GeV,

$$\tan\varphi(u)=(-u)^{1/2}\,a_I(u)/a_R(u)\ , \tag{4.11a}$$

$$\tan\theta(u)=\frac{\sinh\pi\alpha_I(u)}{\cos\pi\alpha_R(u)}\ . \tag{4.11b}$$

The important thing to notice about (4.10) is that, for fixed u, as one goes to higher s, the sign of the polarization changes. The condition that the sine function goes through a half period between $s_{\max}$ and $s_{\min}$ is

$$\ln(s_{\max}/s_{\min})=\frac{1}{2\alpha_I(u)}\ . \tag{4.12}$$

For the parametrization (2.13) and (2.14), $\alpha_I\cong 0.5$ for $u\cong-1.0$ GeV^2 so the sign of the polarization in this region of u should change every time s is tripled, say from 5 to 15 GeV^2.

The prediction about the behavior of the polarization with increasing s is strikingly different from that of the models discussed by Berger and Fox,[8] where there is little or no energy dependence. The prediction (4.10) is not sensitive to absorptive corrections in the sense that a small, energy-independent change in the phase of the spin-flip or non-spin-flip amplitudes would not change things drastically. The relative phase of the amplitudes should still change with energy in order to produce a polarization which changes sign.

V. CONCLUSIONS AND SUMMARY

There is insufficient evidence that the parity-doublet trajectories implied by MacDowell symmetry are closely degenerate. In particular, the possibility exists that the $D_{33}(1670)$ is the parity partner of the $\Delta(1234)$. The testable consequences of this trajectory assignment include a prediction for the mass, total width, and elastic width of a $G_{37}(2530)$ recurrence. Assuming the dominance of the complex-conjugate MacDowell-partner trajectories, the asymptotic behavior of π^-p backward scattering is predicted without making absorptive corrections. The outstanding feature of the backward polarization, the change in sign of the polarization at finite u as s is increased, is not sensitive to absorptive effects and can be used to approximate the size of the odd portion of the trajectory function.

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[1]See, for example, V. Barger and D. Cline, *Phenomenological Theories of High-Energy Scattering* (Benjamin, New York, 1969) and references therein.

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[16]To see that the polynomial must be of second order, write a phase representation for the function

$$\alpha_1(W)-j_N=P(N,W)\exp\left[\frac{W}{\pi}\int_{M+\mu}^{\infty}\frac{\delta_1(N,x)}{x(x-W)}dx\right]$$

$$\times\exp\left[\frac{-W}{\pi}\int_{M+\mu}^{\infty}\frac{\delta_2(N,x)}{x(x+W)}dx\right],$$

where $j_N=2N+\frac{3}{2}$ is a right-signature point for the Δ trajectory. It follows that $\alpha_1(W)-j_N$ can have no zero on the physical sheet of the cut W plane since these would correspond to poles on the physical sheet of $f^{L=j_N\pm 1/2}(j_N,W)$ and would violate unitarity. The phases in the integrals are given by

$$\delta_{1,2}(N,W)=\arctan\left[\frac{\mathrm{Im}\alpha_{1,2}(W)}{\mathrm{Re}\alpha_{1,2}(W)-j_N}\right].$$

The exponentials have no zeros and can be shown to be well behaved everywhere in the W plane except at $\pm(M+\mu)$ where they behave as

$$\exp\left[\frac{W}{\pi}\int_{M+\mu}^{\infty}\frac{\delta_1(N,x)}{x(x-W)}dx\right]\sim C_1[(M+\mu)-W]^{-\delta_1(N,M+\mu)/\pi},$$

$$\exp\left[\frac{W}{\pi}\int_{M+\mu}^{\infty}\frac{\delta_2(N,x)}{x(x+W)}dx\right]\sim C_2[(M+\mu)+W]^{-\delta_2(N,M+\mu)/\pi}.$$

When $j_N>\max[\alpha_1(M+\mu),\alpha_2(M+\mu)]$, the threshold expansions (2.9) imply

$$\delta_{1,2}(N,M+\mu)=\lim_{x\to(M+\mu)+}\delta_{1,2}(N,x)=\pi.$$

If we assume that $\alpha_1(W)$ is bound at $W=\pm(M+\mu)$, the polynomial $P(N,W)$ must contain simple zeros in order to cancel the threshold behavior of the exponentials. Since $\alpha_1(W)-j_N$ can have no first-sheet zeros, the polynomial can have no other zeros and must be exactly of second order. For further details, see the complete discussion in Ref. 6.

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