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Phenomenology of $K\to 2\pi$ Decays

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The theoretical expressions for experimentally determined quantities in $K\to 2\pi$ decays and leptonic kaon decays are used to determine solutions for various theoretical parameters. Two solutions are found: one for which the $|\Delta I|=\frac{1}{2}$ dominance rule is valid, and another for which it is not. Both solutions yield $\epsilon\cong\eta_{+-}\cong\eta_{00}$. Even precise measurements of ϕ_{00} may not allow one to distinguish between the two solutions.

I. INTRODUCTION

With the recent measurement[1] of the phase angle of the ratio of $K_L\to\pi^0\pi^0$ to $K_S\to\pi^0\pi^0$ decay, ϕ_{00}, it has become of interest to recalculate the phenomenological parameters of $K\to 2\pi$ decays previously calculated by Roper.[2] In the calculation of Ref. 2 no approximations were made concerning the relative sizes of the $|\Delta I|=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ decay amplitudes. We have expanded the previous calculations by including the $\Delta S=\pm\Delta Q$ amplitudes of leptonic kaon decays, which various authors[3] have used to find an approximate value for $\mathrm{Re}\epsilon$. Since it is impossible at this time to ascertain which of the available numerical values to use for the phase-shift difference $\delta_2-\delta_0$ between $I=2$ and $I=0$ s-wave scattering for π-π interactions, the results are given with both Walker's[4] value and Marateck's[5] value for $\delta_2-\delta_0$ as inputs.

We begin with Roper's[2] formulation of $K\to 2\pi$ decays. By means of the experimental values of various quantities we are able to derive values of the ratio $\bar{b}_3$ of the complex $|\Delta I|=\frac{3}{2}$ reduced matrix element to the $|\Delta I|=\frac{1}{2}$ reduced matrix element, the ratio $\bar{b}_5$ of the complex $|\Delta I|=\frac{5}{2}$ reduced matrix element to the $|\Delta I|=\frac{1}{2}$ reduced matrix element, the complex K^0-$\bar{K}^0$ mixing parameter ϵ, the complex ratio x of the $\Delta S=-\Delta Q$ amplitude to the $\Delta S=+\Delta Q$ amplitude in leptonic kaon decays, and the π-π phase-shift difference $\delta_2-\delta_0$.

The above values were found by using the four solutions found by Roper[2] as inputs in a least-squares fit to the data by means of the exact equations. The four solutions reduced to two solutions, one of which satisfies the $|\Delta I|=\frac{1}{2}$ dominance rule and the other of which does not.

By assuming very precise values for ϕ_{00} we show that even great precision may not allow one to distinguish between the two solutions obtained here.

II. THEORY

We write the isotopic-spin amplitudes as

$$\langle 0,0|A|K^0\rangle=\beta_0,\quad \langle 0,0|A|\bar{K}^0\rangle=\beta_0\,,$$

$$\langle 2,0|A|K^0\rangle=\beta_2,\quad \langle 2,0|A|\bar{K}^0\rangle=\beta_2^*\,,$$

$$\langle 2,1|A|K^+\rangle=\beta_1,\quad \langle 2,-1|A|K^-\rangle=\beta_1^*\,,$$

where

$$\beta_0=b_1/\sqrt{2}\,,$$

$$\beta_2=(b_3+b_5)/\sqrt{2}\,,$$

and

$$\beta_1=(\tfrac{3}{4})^{1/2}(b_3-\tfrac{2}{3}b_5).$$

Here b_n is the reduced matrix element of $|\Delta I| = \frac{1}{2}n$. We have chosen the arbitrary phase such that β_0 (or b_1) is real. *CPT* conservation is assumed. For *CP* conservation β_2 and β_1 are real.[6] Thus, $\mathrm{Im}\beta_2$ and $\mathrm{Im}\beta_1$ are *CP*-nonconserving parameters.

We define

$$\bar{b}_3 = b_3/b_1, \quad \bar{b}_5 = b_5/b_1, \quad \bar{\beta}_2 = \beta_2/\beta_0 = \bar{b}_3 + \bar{b}_5, \quad \bar{\beta}_1 = (\tfrac{2}{3})^{1/2}\beta_1/\beta_0 = \bar{b}_3 - \tfrac{2}{3}\bar{b}_5 .$$

From these definitions we can write

$$(\tfrac{3}{5}a)^2 = (\mathrm{Re}\bar{b}_3 - \tfrac{2}{5}b)^2 + (\mathrm{Im}\bar{b}_3 - \tfrac{2}{5}c)^2 , \tag{1a}$$

$$(\tfrac{3}{5}a)^2 = (\mathrm{Re}\bar{b}_5 - \tfrac{3}{5}b)^2 + (\mathrm{Im}\bar{b}_5 - \tfrac{3}{5}c)^2 , \tag{1b}$$

where

$$a = |\bar{\beta}_1|, \quad b = \mathrm{Re}\bar{\beta}_2, \quad \text{and} \quad c = \mathrm{Im}\bar{\beta}_2 .$$

The reader is referred to Ref. 2 for the derivation[7] of Eqs. (2)–(7).

$$\bar{R} = \frac{|\langle \pi^+\pi^-|A|K_S\rangle|^2}{|\langle \pi^0\pi^0|A|K_S\rangle|^2} = \frac{A - 2\mathrm{Im}\epsilon\,\mathrm{Re}\bar{\beta}_2\mathrm{Im}\bar{\beta}_2 - 2\sqrt{2}\,\mathrm{Im}\bar{\beta}_2(g\,\mathrm{Re}\epsilon + f\,\mathrm{Im}\epsilon) + |\epsilon|^2(\mathrm{Im}\bar{\beta}_2)^2}{B - 4\mathrm{Im}\epsilon\,\mathrm{Re}\bar{\beta}_2\mathrm{Im}\bar{\beta}_2 + 2\sqrt{2}\,\mathrm{Im}\bar{\beta}_2(g\,\mathrm{Re}\epsilon + f\,\mathrm{Im}\epsilon) + 2|\epsilon|^2(\mathrm{Im}\bar{\beta}_2)^2} , \tag{2}$$

where

$$f = \cos(\delta_2 - \delta_0), \quad g = \sin(\delta_2 - \delta_0), \quad A = 2 + (\mathrm{Re}\bar{\beta}_2)^2 + 2\sqrt{2}f\,\mathrm{Re}\bar{\beta}_2, \quad \text{and} \quad B = 1 + 2(\mathrm{Re}\bar{\beta}_2)^2 - 2\sqrt{2}f\,\mathrm{Re}\bar{\beta}_2 ;$$

$$\bar{X} = \frac{|\langle \pi^+\pi^-|A|K_S\rangle|^2}{|\langle \pi^+\pi^0|A|K^+\rangle|^2} = \frac{4}{9}\,\frac{A - 2\mathrm{Im}\epsilon\,\mathrm{Re}\bar{\beta}_2\mathrm{Im}\bar{\beta}_2 - 2\sqrt{2}\,\mathrm{Im}\bar{\beta}_2(g\,\mathrm{Re}\epsilon + f\,\mathrm{Im}\epsilon) + |\epsilon|^2(\mathrm{Im}\bar{\beta}_2)^2}{(1 + |\epsilon|^2)|\bar{\beta}_1|^2} ; \tag{3}$$

$$|\eta_{+-}|^2 = \frac{|\langle \pi^+\pi^-|A|K_L\rangle|^2}{|\langle \pi^+\pi^-|A|K_S\rangle|^2} = \frac{|\epsilon|^2 A + 2\mathrm{Im}\epsilon\,\mathrm{Re}\bar{\beta}_2\mathrm{Im}\bar{\beta}_2 - 2\sqrt{2}\,\mathrm{Im}\bar{\beta}_2(g\,\mathrm{Re}\epsilon - f\,\mathrm{Im}\epsilon) + (\mathrm{Im}\bar{\beta}_2)^2}{A - 2\mathrm{Im}\epsilon\,\mathrm{Re}\bar{\beta}_2\mathrm{Im}\bar{\beta}_2 - 2\sqrt{2}\,\mathrm{Im}\bar{\beta}_2(g\,\mathrm{Re}\epsilon + f\,\mathrm{Im}\epsilon) + |\epsilon|^2(\mathrm{Im}\bar{\beta}_2)^2} ; \tag{4}$$

$$|\eta_{00}|^2 = \frac{|\langle \pi^0\pi^0|A|K_L\rangle|^2}{|\langle \pi^0\pi^0|A|K_S\rangle|^2} = \frac{|\epsilon|^2 B + 4\mathrm{Im}\epsilon\,\mathrm{Re}\bar{\beta}_2\mathrm{Im}\bar{\beta}_2 - 2\sqrt{2}\,\mathrm{Im}\bar{\beta}_2(g\,\mathrm{Re}\epsilon - f\,\mathrm{Im}\epsilon) + (\mathrm{Im}\bar{\beta}_2)^2}{B - 4\mathrm{Im}\epsilon\,\mathrm{Re}\bar{\beta}_2\mathrm{Im}\bar{\beta}_2 + 2\sqrt{2}\,\mathrm{Im}\bar{\beta}_2(g\,\mathrm{Re}\epsilon + f\,\mathrm{Im}\epsilon) + 2|\epsilon|^2(\mathrm{Im}\bar{\beta}_2)^2} ; \tag{5}$$

$$\phi_{+-} = \tan^{-1}\left(\frac{\mathrm{Im}\,\eta_{+-}}{\mathrm{Re}\,\eta_{+-}}\right) = \tan^{-1}\left[\frac{(1 - |\epsilon|^2)\mathrm{Im}\bar{\beta}_2(\mathrm{Re}\bar{\beta}_2 + \sqrt{2}f) + A\,\mathrm{Im}\epsilon - (\mathrm{Im}\bar{\beta}_2)^2\mathrm{Im}\epsilon}{-(1 + |\epsilon|^2)\sqrt{2}\,g\,\mathrm{Im}\bar{\beta}_2 + A\,\mathrm{Re}\epsilon + (\mathrm{Im}\bar{\beta}_2)^2\mathrm{Re}\epsilon}\right] ; \tag{6}$$

and

$$\phi_{00} = \tan^{-1}\left(\frac{\mathrm{Im}\,\eta_{00}}{\mathrm{Re}\,\eta_{00}}\right) = \tan^{-1}\left[\frac{(1 - |\epsilon|^2)\mathrm{Im}\bar{\beta}_2(2\mathrm{Re}\bar{\beta}_2 - \sqrt{2}f) + B\,\mathrm{Im}\epsilon - 2(\mathrm{Im}\bar{\beta}_2)^2\mathrm{Im}\epsilon}{(1 + |\epsilon|^2)\sqrt{2}\,g\,\mathrm{Im}\bar{\beta}_2 + B\,\mathrm{Re}\epsilon + 2(\mathrm{Im}\bar{\beta}_2)^2\mathrm{Re}\epsilon}\right] . \tag{7}$$

The physical amplitudes for leptonic kaon decays are[8]

$$\langle \pi^- e^+ \nu|H|K^0\rangle = F, \quad \langle \pi^+ e^- \bar{\nu}|H|\bar{K}^0\rangle = F^*, \quad \langle \pi^- e^+ \nu|H|\bar{K}^0\rangle = G, \quad \langle \pi^+ e^- \bar{\nu}|H|K^0\rangle = G^* .$$

The F amplitudes correspond to $\Delta S = \Delta Q$ and the G amplitudes correspond to $\Delta S = -\Delta Q$. We define

$$x \equiv G/F .$$

Clearly x is the amount by which the $\Delta S=\Delta Q$ rule is violated.

The exact theoretical expression for the charge asymmetry in leptonic kaon decays is

$$\delta_L=\frac{\Gamma_{e^+}-\Gamma_{e^-}}{\Gamma_{e^+}+\Gamma_{e^-}}=\frac{2\mathrm{Re}\epsilon}{1+|\epsilon|^2}\,\frac{1-|x|^2}{1+|x|^2-2(1+|\epsilon|^2)^{-1}C}\,, \tag{8}$$

where

$$\Gamma_{e^+}=|\langle\pi^- e^+\nu|H|K_L\rangle|^2\,,\quad \Gamma_{e^-}=|\langle\pi^+ e^-\bar\nu|H|K_L\rangle|^2\,,\quad \text{and}\quad C=(1-|\epsilon|^2)\mathrm{Re}x+2\mathrm{Im}\epsilon\,\mathrm{Im}x\,.$$

We have seven equations involving eight parameters: $\mathrm{Re}\bar\beta_2$, $\mathrm{Im}\bar\beta_2$, $|\bar\beta_1|$, $\mathrm{Re}\epsilon$, $\mathrm{Im}\epsilon$, $\delta_2-\delta_0$, $\mathrm{Re}x$, and $\mathrm{Im}x$. [Note that Eq. (3) is the only equation which contains $\mathrm{Re}\bar\beta_1$ and $\mathrm{Im}\bar\beta_1$ and only in the form of $|\bar\beta_1|$; therefore, we can only determine $|\bar\beta_1|$.] However, three of the parameters can be found experimentally from other physical processes. These three parameters are $\delta_2-\delta_0$, $\mathrm{Re}x$, and $\mathrm{Im}x$. We consequently "hardened" these three parameters by allowing them to be functions also. By this means, the parameters were allowed to vary only within the experimental errors as determined by the other physical processes. Thus, we have ten experimental numbers (equations) and eight parameters; the expected χ^2 is 2.

III. DATA

The K_S branching ratio[9] is given by the experimental quantity

$$R=\bar R\,\frac{\rho_{+-}^{(S)}}{\rho_{00}^{(S)}}=2.196\pm 0.049\,,$$

where the phase-space ratio[10] is

$$\frac{\rho_{00}^{(S)}}{\rho_{+-}^{(S)}}=\left(\frac{M_S{}^2-4m_0{}^2}{M_S{}^2-4m_+{}^2}\right)^{1/2}=1.015\pm 0.000\,06$$

in terms of the K_S mass M_S, the π^+ mass m_+, and the π^0 mass m_0.

The ratio[9] of $K_S\to\pi^++\pi^-$ to $K^+\to\pi^++\pi^0$ partial decay rates is

$$X=\bar X\,\frac{\rho_{+-}^{(S)}}{\rho_{+0}}=470.2\pm 8.7\,,$$

where the phase-space ratio[10] is

$$\frac{\rho_{+0}}{\rho_{+-}^{(S)}}=\frac{M_S}{M_+{}^2}\left\{\frac{[M_+{}^2-(m_+-m_0)^2][M_+{}^2-(m_++m_0)^2]}{M_S{}^2-4m_+{}^2}\right\}^{1/2}$$

$$=1.004\pm 0.0002\,,$$

M_+ being the K^+ mass.

Table I gives the values used for Eqs. (2)–(8), as well as the input values fixed for $\mathrm{Re}x$, $\mathrm{Im}x$, and $\delta_2-\delta_0$. Since there were no means of determining which of the reported values[4,5] of $\delta_2-\delta_0$ to use, we ran the fits twice using both reported values of $\delta_2-\delta_0$.

IV. RESULTS AND DISCUSSION

A least-squares-fit routine was used to determine the eight parameters. The four solutions found by Roper[2] were used as starting points for these parameters. It was found that two of these solutions collapsed into the other two solutions, and we were left with two solutions, one which satisfies the $|\Delta I|=\frac12$ dominance rule and one which does not, cf. Table II. The $|\Delta I|=\frac12$ is slightly, but not significantly, preferred on the basis of χ^2 values for the fits.

In all solutions certain qualitative features can be seen. First, the absolute value of the real part of the $I=2$ isotopic-spin amplitude is much greater than the absolute value of the imaginary part; i.e., $|\mathrm{Re}\bar\beta_2|\gg|\mathrm{Im}\,\bar\beta_2|$. Furthermore, the imaginary part of $\bar\beta_2$ could be zero within the error. Despite this situation, nothing can be said about the relative sizes of the real and imaginary parts of the $|\Delta I|=\frac32$ and $\frac52$ reduced matrix elements for the $|\Delta I|=\frac12$ dominance solution. By means of relations (1a) and (1b) we show a plot of these quantities with their attendant uncertainties in Fig. 1. For the $|\Delta I|\neq\frac12$ solution obtained with $\delta_2-\delta_0=-30°\pm 10°$, we see

TABLE I. Experimental values for the exact theoretical functions.

	Experimental value	Reference
$\bar R$	2.229 ± 0.050	9
$\bar X$	472.1 ± 8.7	9
$10^3\lvert\eta_{+-}\rvert$	1.92 ± 0.05	9
$10^3\lvert\eta_{00}\rvert$	2.5 ± 0.8	9
ϕ_{+-}	$44°\pm 5°$	9
ϕ_{00}	$23°\pm 32°$	9
$10^3\,\mathrm{Re}x$	21 ± 36	9
$10^3\,\mathrm{Im}x$	-99 ± 47	9
$10^3\delta_L$	2.7 ± 0.3	a
$\delta_2-\delta_0$	$-52°\pm 11°$	4
$\delta_2-\delta_0$	$-30°\pm 10°$	5

[a]World average given by J. Steinberger, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969,* edited by J. Prentki and J. Steinberger (Ref. 1), p. 297.

TABLE II. Calculated theoretical and experimental values for $K\to 2\pi$ decays and leptonic kaon decays.

Parameter	$\delta_2-\delta_0=-30°\pm10°$ $\|\Delta I\|=\frac{1}{2}$ dominance Solution 1	$\|\Delta I\|\neq\frac{1}{2}$ Solution 2	$\delta_2-\delta_0=-52°\pm11°$ $\|\Delta I\|=\frac{1}{2}$ dominance Solution 1	$\|\Delta I\|\neq\frac{1}{2}$ Solution 2
		Theoretical value		
$10^3\|\bar{\beta}_1\|$	44.17 ± 0.44	109 ± 10	44.19 ± 0.44	82.6 ± 15.4
$10^3\mathrm{Re}\bar{\beta}_2$	29.3 ± 6.8	2250 ± 240	41.8 ± 13.9	1580 ± 410
$10^3\mathrm{Im}\bar{\beta}_2$	-0.12 ± 0.32	-0.50 ± 2.46	-0.099 ± 0.278	-0.35 ± 1.16
$10^3\mathrm{Re}\epsilon$	1.40 ± 0.13	1.39 ± 0.13	1.41 ± 0.14	1.41 ± 0.14
$10^3\mathrm{Im}\epsilon$	1.43 ± 0.25	1.49 ± 0.19	1.40 ± 0.19	1.48 ± 0.46
$\delta_2-\delta_0$	$-30.0°\pm10.0°$	$-30.2°\pm10.0°$	$-52.0°\pm11.0°$	$-52.1°\pm11.0°$
$10^3\mathrm{Re}x$	12.5 ± 33.1	13.4 ± 33.4	11.6 ± 33.9	11.6 ± 34.3
$10^3\mathrm{Im}x$	-102 ± 47	-102 ± 47	-102 ± 47	-102 ± 47
		Experimental value		
$\bar{R}$	2.229	2.229	2.229	2.229
$\bar{X}$	472.1	472.1	472.1	472.1
$10^3\|\eta_{+-}\|$	1.92	1.92	1.92	1.92
$10^3\|\eta_{00}\|$	2.2	1.9	2.1	2.0
ϕ_{+-}	44.9°	44.8°	45.1°	45.1°
ϕ_{00}	46.6°	39.6°	44.2°	38.9°
$10^3\delta_L$	2.8	2.8	2.8	2.8
$10^3\mathrm{Re}x$	12.5	13.4	11.6	11.6
$10^3\mathrm{Im}x$	−102	−102	−102	−102
$\delta_2-\delta_0$	−30.0°	−30.2°	−52.0°	−52.1°
χ^2	0.938	1.033	0.933	0.980

that $\mathrm{Re}b_5>\mathrm{Re}b_3$, $\mathrm{Re}b_5>b_1$, $\mathrm{Re}b_3\gg\mathrm{Im}b_3$, and $\mathrm{Re}b_5\gg\mathrm{Im}b_5$, whereas with $\delta_2-\delta_0=-52°\pm11°$, we can only state that $\mathrm{Re}b_3<b_1$, $\mathrm{Re}b_3\gg\mathrm{Im}b_3$, and $\mathrm{Re}b_5\gg\mathrm{Im}b_5$. Secondly, both the modulus and argument of ϵ, η_{+-}, and η_{00} as calculated from the parameters are almost equal (cf. Table III), in good agreement with the superweak theory.[11]

An effort was made to determine how accurately one would have to measure ϕ_{00} in order to discriminate between Solutions 1 ($\Delta I=\frac{1}{2}$) and 2 ($\Delta I\neq\frac{1}{2}$) of Table II. Since the calculated value of ϕ_{00} is ~45° for Solution 1 and ~40° for Solution 2, we use these two values with errors of 2° as hypothetical data. The results are given in Table IV. For the "datum" $\phi_{00}=45°\pm2°$, the $|\Delta I|=\frac{1}{2}$ solution is slightly, but not significantly, preferred. For the "datum"

TABLE III. Calculated values for ϵ, η_{+-}, η_{00}.

	$\delta_2-\delta_0=-30°\pm10°$ Solution 1	Solution 2	$\delta_2-\delta_0=-52°\pm11°$ Solution 1	Solution 2
$10^3\|\epsilon\|$	2.00 ± 0.20	2.04 ± 0.54	1.99 ± 0.17	2.04 ± 0.35
$10^3\|\eta_{+-}\|$	1.92	1.92	1.92	1.92
$10^3\|\eta_{00}\|$	2.2	1.9	2.1	2.0
ϕ_ϵ	$45.6°\pm5.7°$	$47.0°\pm14.3°$	$44.8°\pm4.8°$	$46.4°\pm9.4°$
ϕ_{+-}	44.9°	44.8°	45.1°	45.1°
ϕ_{00}	46.6°	39.6°	44.2°	38.9°

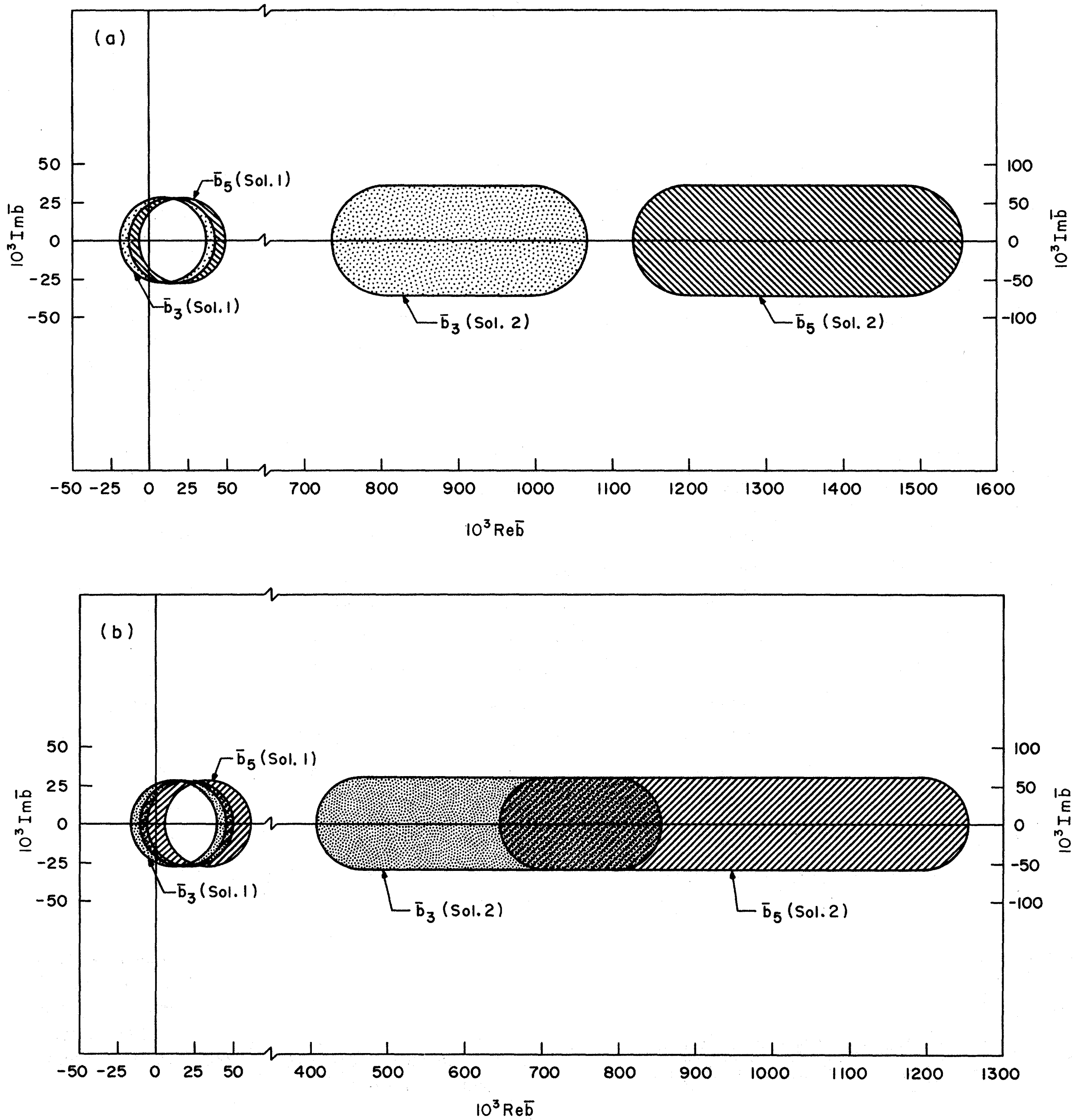

FIG. 1. (a) Plot in the complex plane of the values and uncertainties (shaded areas) for the reduced matrix elements $\bar{b}_3$ and $\bar{b}_5$ for both solutions found with $\delta_2-\delta_0=-30°\pm10°$. (b) Plot in the complex plane of the values and uncertainties (shaded areas) for the reduced matrix elements $\bar{b}_3$ and $\bar{b}_5$ for both solutions found with $\delta_2-\delta_0=-52°\pm11°$.

$\phi_{00}=40°\pm2°$, the $|\Delta I|\neq\frac{1}{2}$ is significantly preferred on the basis of χ^2 comparisons.

Another computer run was made leaving the π-π phase-shift difference free; i.e., $\delta_2-\delta_0$ was not "hardened" in the manner described above. This resulted in enormous errors on all parameters, especially $\delta_2-\delta_0$, such that the data could be fitted by choosing most of the parameters to be equal to zero.

V. CONCLUSIONS

By using exact equations for various experimental quantities in terms of theoretical parameters, two solutions were obtained for these theoretical parameters. One solution satisfies the $|\Delta I|=\frac{1}{2}$ dominance rule; the other does not. In either case one cannot determine whether or not $|b_3|>|b_5|$ (except for the case of Solution 2 for $\delta_2-\delta_0$

TABLE IV. Theoretical solutions from pseudodata for ϕ_{00}.

Parameter	$\delta_2-\delta_0=-30°\pm10°$ $\lvert\Delta I\rvert=\frac{1}{2}$ dominance Solution 1	$\lvert\Delta I\rvert\neq\frac{1}{2}$ Solution 2	$\delta_2-\delta_0=-52°\pm11°$ $\lvert\Delta I\rvert=\frac{1}{2}$ dominance Solution 1	$\lvert\Delta I\rvert\neq\frac{1}{2}$ Solution 2
χ^2	0.505	0.592	0.496	0.600
ϕ_{00}	45° ± 2°	45° ± 2°	45° ± 2°	45° ± 2°
$10^3\lvert\bar\beta_1\rvert$	44.17 ± 0.43	109 ± 10	44.19 ± 0.44	82.7 ± 15.4
$10^3\text{Re}\bar\beta_2$	29.4 ± 6.8	2260 ± 240	41.3 ± 13.4	1580 ± 410
$10^3\text{Im}\bar\beta_2$	−0.06 ± 0.20	0.06 ± 0.48	−0.09 ± 0.28	0.02 ± 0.28
$10^3\text{Re}\epsilon$	1.40 ± 0.13	1.367 ± 0.09	1.39 ± 0.09	1.36 ± 0.08
$10^3\text{Im}\epsilon$	1.38 ± 0.08	1.33 ± 0.24	1.41 ± 0.18	1.34 ± 0.21
$\delta_2-\delta_0$	−30° ± 10°	−30° ± 10°	−52° ± 11°	−52° ± 11°
$10^3\text{Re}x$	13 ± 33	16 ± 32	13 ± 32	16 ± 31
$10^3\text{Im}x$	−102 ± 47	−101 ± 47	−102 ± 47	−101 ± 47
χ^2	1.196	0.750	1.501	0.700
ϕ_{00}	40° ± 2°	40° ± 2°	40° ± 2°	40° ± 2°
$10^3\lvert\bar\beta_1\rvert$	44.17 ± 0.44	108.9 ± 9.6	44.20 ± 0.44	826 ± 15.4
$10^3\text{Re}\bar\beta_2$	29.0 ± 6.6	2250 ± 240	44.9 ± 15.6	1580 ± 410
$10^3\text{Im}\bar\beta_2$	0.11 ± 0.16	−0.45 ± 0.48	−0.12 ± 0.27	−0.28 ± 0.29
$10^3\text{Re}\epsilon$	1.38 ± 0.12	1.387 ± 0.087	1.49 ± 0.11	1.40 ± 0.08
$10^3\text{Im}\epsilon$	1.236 ± 0.06	1.47 ± 0.24	1.52 ± 0.17	1.45 ± 0.21
$\delta_2-\delta_0$	−30° ± 10°	−30° ± 10°	−52° ± 11°	−52° ± 11°
$10^3\text{Re}x$	15 ± 33	14 ± 32	2 ± 32	13 ± 31
$10^3\text{Im}x$	−101 ± 47	−102 ± 47	−106 ± 47	−102 ± 47

$=-30°\pm10°$; cf. Fig. 1). In all cases we find that $\epsilon\cong\eta_{+-}\cong\eta_{00}$.

Since $\epsilon\cong10^{-3}$, one can approximate

$$\eta_{+-}=\frac{ie^{i(\delta_2-\delta_0)}\text{Im}\,\bar\beta_2+\epsilon(\sqrt2+e^{i(\delta_2-\delta_0)}\text{Re}\bar\beta_2)}{\sqrt2+e^{i(\delta_2-\delta_0)}\text{Re}\bar\beta_2+i\epsilon e^{i(\delta_2-\delta_0)}\text{Im}\,\bar\beta_2}$$
$$\cong\epsilon\left[1+\frac{e^{2i(\delta_2-\delta_0)}(\text{Im}\,\bar\beta_2)^2}{(\sqrt2+e^{i(\delta_2-\delta_0)}\text{Re}\bar\beta_2)^2}\right]+i\,\frac{e^{i(\delta_2-\delta_0)}\text{Im}\,\bar\beta_2}{\sqrt2+e^{i(\delta_2-\delta_0)}\text{Re}\bar\beta_2}\tag{9}$$

and

$$\eta_{00}=\frac{i\sqrt2\,e^{i(\delta_2-\delta_0)}\text{Im}\,\bar\beta_2+\epsilon(-1+\sqrt2\,e^{i(\delta_2-\delta_0)}\text{Re}\bar\beta_2)}{-1+\sqrt2\,e^{i(\delta_2-\delta_0)}\text{Re}\bar\beta_2+i\sqrt2\,\epsilon e^{i(\delta_2-\delta_0)}\text{Im}\,\bar\beta_2}$$
$$\cong\epsilon\left[1+\frac{2e^{2i(\delta_2-\delta_0)}(\text{Im}\,\bar\beta_2)^2}{(-1+\sqrt2\,e^{i(\delta_2-\delta_0)}\text{Re}\bar\beta_2)^2}\right]+i\sqrt2\,\frac{e^{i(\delta_2-\delta_0)}\text{Im}\,\bar\beta_2}{-1+\sqrt2\,e^{i(\delta_2-\delta_0)}\text{Re}\bar\beta_2},\tag{10}$$

without being very much in error. However, the further approximation that $|\bar\beta_2|\ll1$, the $|\Delta I|=\frac{1}{2}$ rule, is not justified by the numerical analyses given above. One of the possible solutions does not satisfy the rule. The $|\Delta I|=\frac{1}{2}$ approximation yields the often-quoted equations

$$\eta_{+-}=\epsilon+e^{i(\pi/2+\delta_2-\delta_0)}\text{Im}\,\bar\beta_2/\sqrt2\equiv\epsilon+\epsilon'$$

and

$$\eta_{00}=\epsilon-2\epsilon'.$$

The $|\Delta I|=\frac{1}{2}$ solution is, of course, the well-known Wu-Yang solution[12] which predicts $\bar R\cong2$ and $\bar X$ to be large. The $|\Delta I|\neq\frac{1}{2}$ solution gives the same results by means of fortuitous cancellations among the $|\Delta I|=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ amplitudes.

We were unable to ascertain a choice for the value for $\delta_2-\delta_0$, the π-π phase-shift difference, from the literature. An effort was made to determine this phase-shift difference from the exact equations derived above. However, the results were inconclusive, due to the fact that $\text{Im}\bar{\beta}_2 \cong 0$ for all solutions and $\delta_2 - \delta_0$ always occurs in terms multiplied by $\text{Im}\bar{\beta}_2$ in Eqs. (2), (3), (9), and (10).

By using hypothetically precise "data" for ϕ_{00} we have shown that such precise data may not enable one to determine whether $|\Delta I| = \frac{1}{2}$ is dominant in $K\to 2\pi$ decays.

Since completion of the above analysis, new results have appeared:

$\phi_{00} = 51° \pm 23°$ (Ref. 13),

$|\eta_{00}| = (2.09 \pm 0.23)\times 10^{-3}$ (Ref. 14).

Note from Tables I and II that both of these values differ from the previous values in the direction that our two solutions require. We redid the analysis with these new results and obtained the results shown in Table V. It is seen that there are still two solutions ($|\Delta I| = \frac{1}{2}$ and $|\Delta I| \neq \frac{1}{2}$), neither of which is significantly preferred. Again from Table V the predicted values of ϕ_{00} greatly differ for the two solutions. By assuming that $\phi_{00} = 45° \pm 2°$, $50° \pm 2°$, $60° \pm 2°$ an attempt was made to distinguish between the two solutions. The salient features of this analysis are shown in Table VI.[15] Note that for higher values of ϕ_{00}, the $|\Delta I| \neq \frac{1}{2}$ solution is significantly preferred (by a factor of 10 in χ^2) over the $|\Delta I| = \frac{1}{2}$ solution.

TABLE V. Calculated theoretical and experimental values for $K\to 2\pi$ decays and leptonic kaon decays for $\phi_{00} = 51° \pm 23°$ and $10^3|\eta_{00}| = 2.09 \pm 0.23$.

	$\delta_2-\delta_0=-30°\pm10°$		$\delta_2-\delta_0=-52°\pm11°$	
Parameter	$\lvert\Delta I\rvert=\frac{1}{2}$ dominance Solution 1	$\lvert\Delta I\rvert\neq\frac{1}{2}$ Solution 2	$\lvert\Delta I\rvert=\frac{1}{2}$ dominance Solution 1	$\lvert\Delta I\rvert\neq\frac{1}{2}$ Solution 2
		Theoretical value		
$10^3\lvert\bar{\beta}_1\rvert$	44.17 ± 0.44	109.8 ± 9.3	44.19 ± 0.44	83.4 ± 15.4
$10^3\text{Re}\bar{\beta}_2$	29.2 ± 6.7	2280 ± 230	41.5 ± 13.7	1600 ± 410
$10^3\text{Im}\bar{\beta}_2$	-0.17 ± 0.33	1.6 ± 2.5	-0.09 ± 0.28	0.36 ± 1.2
$10^3\text{Re}\epsilon$	1.40 ± 0.13	1.30 ± 0.13	1.39 ± 0.14	1.32 ± 0.15
$10^3\text{Im}\epsilon$	1.47 ± 0.25	0.89 ± 0.73	1.41 ± 0.19	1.22 ± 0.45
$\delta_2-\delta_0$	$-29.6°\pm10.0°$	$-29.6°\pm10.0°$	$-51.5°\pm11.0°$	$-51.5°\pm11.0°$
$10^3\text{Re}x$	12.4 ± 33.1	22.8 ± 33.6	12.9 ± 33.9	20.7 ± 34.3
$10^3\text{Im}x$	-102 ± 47	-98.4 ± 46.8	-102 ± 47	-99.1 ± 46.8
		Experimental value		
$\bar{R}$	2.229	2.229	2.229	2.229
$\bar{X}$	472.1	472.1	472.1	472.1
$10^3\lvert\eta_{+-}\rvert$	1.92	1.92	1.92	1.92
$10^3\lvert\eta_{00}\rvert$	2.3	2.1	2.1	1.9
ϕ_{+-}	45.5°	43.6°	45.5°	44.2°
ϕ_{00}	47.8°	58.5°	44.7°	50.7°
$10^3\delta_L$	2.8	2.7	2.8	2.7
$10^3\text{Re}x$	12.4	22.8	12.9	20.6
$10^3\text{Im}x$	−102	−98.4	−102	−99.1
$\delta_2-\delta_0$	−29.6°	−29.2°	−51.5°	−51.5°
χ^2	0.383	0.302	0.531	0.566

TABLE VI. Features of the theoretical solutions from pseudodata for ϕ_{00}.

Parameter	$\delta_2-\delta_0=-30°\pm10°$ $\|\Delta I\|=\frac{1}{2}$ dominance Solution 1	$\delta_2-\delta_0=-30°\pm10°$ $\|\Delta I\|\neq\frac{1}{2}$ Solution 2	$\delta_2-\delta_0=-52°\pm11°$ $\|\Delta I\|=\frac{1}{2}$ dominance Solution 1	$\delta_2-\delta_0=-52°\pm11°$ $\|\Delta I\|\neq\frac{1}{2}$ Solution 2
χ^2	0.503	0.441	1.85	0.566
ϕ_{00}	$50°\pm2°$	$50°\pm2°$	$50°\pm2°$	$50°\pm2°$
$10^3\|\bar\beta_1\|$	44.19 ± 0.44	109.3 ± 9.5	44.2 ± 0.4	83 ± 15
$10^3\mathrm{Re}\bar\beta_2$	28.8 ± 6.4	2260 ± 230	44 ± 15	1600 ± 410
$10^3\mathrm{Im}\bar\beta_2$	-0.25 ± 0.23	0.59 ± 0.52	0.07 ± 0.24	0.32 ± 0.30
$10^3\mathrm{Re}\epsilon$	1.40 ± 0.13	1.35 ± 0.08	1.25 ± 0.07	1.33 ± 0.08
$10^3\mathrm{Im}\epsilon$	1.55 ± 0.12	1.17 ± 0.25	1.40 ± 0.19	1.23 ± 0.21
χ^2	3.95	0.217	5.20	0.673
ϕ_{00}	$60°\pm2°$	$60°\pm2°$	$60°\pm2°$	$60°\pm2°$
$10^3\|\bar\beta_1\|$	44.2 ± 1.1	109.8 ± 9.2	44.2 ± 0.5	85 ± 15
$10^3\mathrm{Re}\bar\beta_2$	26.4 ± 5.4	2280 ± 230	58 ± 25	1640 ± 400
$10^3\mathrm{Im}\bar\beta_2$	-0.62 ± 0.23	1.81 ± 0.71	0.38 ± 0.16	0.94 ± 0.45
$10^3\mathrm{Re}\epsilon$	1.38 ± 0.12	1.30 ± 0.08	1.12 ± 0.07	1.26 ± 0.08
$10^3\mathrm{Im}\epsilon$	1.86 ± 0.13	0.83 ± 0.27	1.25 ± 0.17	1.01 ± 0.22

[1]J. C. Chollet *et al.*, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969* (CERN, Geneva, 1969), p. 309.

[2]L. D. Roper, Phys. Rev. 176, 2120 (1968).

[3]For example, see S. Bennett *et al.*, Phys. Rev. Letters 19, 997 (1967).

[4]W. D. Walker *et al.*, Phys. Rev. Letters 18, 630 (1967).

[5]S. Marateck *et al.*, Phys. Rev. Letters 21, 1613 (1968).

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[7]ϕ_{+-} (this paper) $=\theta_{+-}$ (Roper) and ϕ_{00} (this paper) $=\theta_{00}$ (Roper). Also, Eqs. (5) and (6) of Roper's paper have been corrected in this paper. The physical amplitudes $A_{00}^{(S)}$ and $A_{00}^{(L)}$ in Roper's paper should read

$$A_0^{(S)}=-\alpha[\beta_0-\sqrt{2}\,e^{i(\delta_2-\delta_0)}\,\mathrm{Re}\beta_2+\epsilon(-\sqrt{2}\,ie^{i(\delta_2-\delta_0)}\mathrm{Im}\beta_2)],$$

$$A_0^{(L)}=\alpha[-\sqrt{2}\,ie^{i(\delta_2-\delta_0)}\,\mathrm{Im}\beta_2+\epsilon(\beta_0-\sqrt{2}\,e^{i(\delta_2-\delta_0)}\,\mathrm{Im}\beta_2)].$$

[8]S. Bennett *et al.*, Phys. Rev. Letters 19, 997 (1967).

[9]World average from A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. 42, 87 (1970).

[10]W. Koch and O. Skjeggestad, in Proceedings of the 1964 Easter School for Physics, CERN, Geneva (unpublished).

[11]L. Wolfenstein, Phys. Rev. Letters 13, 562 (1964); see also J. Steinberger, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969*, Ref. 1, p. 302.

[12]T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964).

[13]J. C. Chollet *et al.*, Phys. Letters 31B, 658 (1970).

[14]V. V. Barmin *et al.*, in *Proceedings of the Fifteenth International Conference on High-Energy Physics, Kiev, U.S.S.R., 1970* (Atomizdat, Moscow, to be published).

[15]The values of the parameters found for $\phi_{00}=45°\pm2°$ are the same as the values listed in Table IV for the same value of $45°\pm2°$.