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PHYSICAL REVIEW D

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Second-Class Currents, Neutral Currents, and the Strangeness-Conserving Nonleptonic Weak Interactions*

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The isospin structure of the strangeness-conserving nonleptonic weak Hamiltonian constructed from products of octets of vector and axial-vector currents is investigated in the presence of charged and neutral currents of both regular and irregular behavior under charge conjugation. It is shown that in a CP -invariant theory an isospin-invariant Hamiltonian is possible only in the absence of charged second-class currents.

Although the description of weak interactions is fairly successful in terms of the present theory, there are certain deficiencies, along with some other open problems, in our knowledge of the behavior of the hadronic current with respect to the symmetry transformations of strong interactions. While the transformation properties with respect to the Lorentz group, parity, $SU(3)$, strangeness, and isospin are now rather well established, the question of whether currents with "irregular" behavior under charge conjugation¹ exist has not been yet answered.² Similarly, not much is known about the possibility of neutral currents participating in the weak interactions.³ In the case of charge-conjugation transformation properties, this lack of information is due to the circumstance that in the experimentally best accessible semi-leptonic reactions the contributions of irregular currents, even if permitted by selection rules, is kinematically suppressed.⁴ The search for possible neutral hadronic currents is even more difficult in view of the great variety of ways in which such currents can be incorporated into the theory. Certain types of neutral currents are already severely constrained by the available data; others are still quite unrestricted by them.³ The latter is true in particular for models where the neutral hadronic currents are not accompanied by leptonic neutral currents or in which the neutral leptonic currents are decoupled from the hadronic ones.⁵

In such a case the nonleptonic processes constitute the only source of information.

The nonleptonic weak interactions, kinematical limitations being here absent, could also be of importance for the study of the question of the existence of irregular currents.⁶ To trace the consequences of the presence of a particular type of current in the nonleptonic weak interactions is, of course, an extremely difficult, if not impossible task. The exceptional situation is the one in which the presence of such current leads to consequences that are independent of the dynamics of strong interactions. The strangeness-changing nonleptonic weak processes are not particularly useful in this respect: The empirically well obeyed $\Delta T = \frac{1}{2}$ sum rules, for example, can be satisfied with any Hamiltonian, by invoking octet dominance if necessary. However, in the $\Delta S = 0$ nonleptonic weak interactions⁷ the isospin structure is richer, in the current \times current theory $\Delta T = 0, 1, 2$, in general, and even if the octet part of the Hamiltonian dominates, the various models can still differ in the content of the corresponding isoscalar and isovector parts. An example is provided by the effects of charged second-class currents introduced into the standard CP -invariant current \times current Hamiltonian: In the absence of second-class currents the effective coupling constant of the isovector interaction is $G \sin^2 \theta$ ($\theta \equiv$ Cabibbo angle); interference between first- and second-class currents introduces an ad-

ditional isovector term which is proportional to $G \cos^2 \theta$.⁸ This could provide a way of testing the existence of second-class currents. Such a test is not unambiguous, however, since certain possible theories that include neutral hadronic currents also contain an isovector term of the same order of magnitude: In the absence of second-class currents, an isovector interaction with an effective coupling constant of the order of $G \cos^2 \theta$ is evidence for the existence of neutral currents.⁹ The converse of this statement does not hold: There are possible theories involving neutral currents which do not contain a large isovector interaction. As shown in a recent study by Albright and Oakes¹⁰ of the isospin structure of a general nonleptonic weak Hamiltonian constructed from products of regular currents, a suitable choice of neutral currents can eliminate the isovector part altogether. At the same time it is possible to modify the Hamiltonian further so as to cancel also the isotensor part, obtaining thus an isospin invariant $\Delta S = 0$ nonleptonic weak Hamiltonian.¹¹

In the present note we investigate the isospin structure of the strangeness-conserving nonleptonic weak Hamiltonian under more general circumstances, namely, when both regular and irregular, charged as well as neutral vector and axial-vector octet currents participate.

Let us assume that the existing differences in the ft values of mirror β decays² are not explicable in terms of isospin-conservation-violating effects. The strangeness-conserving hadronic current must be then of the form¹²

$$J^{\Delta S=0} = (J_{110}^R + \alpha J_{110}^I) \cos \theta, \quad (1)$$

where α is a complex number with a nonvanishing real part and J_{110}^R, J_{110}^I satisfy¹³

$$\begin{aligned} GPJ_{110}^R(GP)^{-1} &= -J_{110}^R, \\ GPJ_{110}^I(GP)^{-1} &= +J_{110}^I; \end{aligned} \quad (2)$$

$$\begin{aligned} e^{i\pi T_2} J_{110}^R e^{-i\pi T_2} &= -J_{110}^{R*}, \\ e^{i\pi T_2} J_{110}^I e^{-i\pi T_2} &= +J_{110}^{I*}. \end{aligned} \quad (3)$$

More generally, we introduce a regular and an irregular octet¹ $J_{(\nu)}^R = V_{(\nu)}^R - A_{(\nu)}^R$ and $J_{(\nu)}^I = V_{(\nu)}^I - A_{(\nu)}^I$ satisfying

$$\begin{aligned} J_{(\nu)}^{R*} &= (-1)^{Q\nu} J_{(-\nu)}^R, \\ J_{(\nu)}^{I*} &= (-1)^{1+Q\nu} J_{(-\nu)}^I, \end{aligned} \quad (4)$$

and, by definition,

$$\begin{aligned} CPJ_{(\nu)}^R(CP)^{-1} &= (-1)^{Q\nu} J_{(-\nu)}^R, \\ CPJ_{(\nu)}^I(CP)^{-1} &= (-1)^{1+Q\nu} J_{(-\nu)}^I, \end{aligned} \quad (5)$$

and take the $\Delta Q = 1$ hadronic weak current to be¹⁴

$$\begin{aligned} J^{\Delta Q=1} &= (J_{110}^R + \alpha J_{110}^I) \cos \theta + (J_{\frac{1}{2} \frac{1}{2} 1}^R + \alpha' J_{\frac{1}{2} \frac{1}{2} 1}^I) \sin \theta \\ &\equiv (\alpha_+ J_{110+} + \alpha_- J_{110-}) \cos \theta \\ &\quad + (\alpha'_+ J_{\frac{1}{2} \frac{1}{2} 1+} + \alpha'_- J_{\frac{1}{2} \frac{1}{2} 1-}) \sin \theta, \end{aligned} \quad (6)$$

where

$$\begin{aligned} \alpha_{\pm} &= \frac{1}{2}(1 \pm \alpha), \quad \alpha'_{\pm} = \frac{1}{2}(1 \pm \alpha'), \\ J_{(\nu)\pm} &= J_{(\nu)}^R \pm J_{(\nu)}^I. \end{aligned}$$

It is plausible to assume that the nonleptonic Hamiltonian in this case consists of a current \times current interaction built up from $J^{\Delta Q=1}$, to which we add a term constructed from the neutral components of the regular and irregular octets:

$$\begin{aligned} H_{NL} &= -\frac{G}{\sqrt{2}} \frac{1}{2} \{ J^{\Delta Q=1}, J^{\Delta Q=1*} \}_+ \\ &\quad - \frac{G}{\sqrt{2}} \sum_{i,j=\pm} \sum_{m,n=3,6,7,8} h_{ij}^{(mn)} J_{(m)i} J_{(n)j}. \end{aligned} \quad (7)$$

Hermiticity, CPT invariance, and CP invariance impose the following conditions on the coefficients $h_{ij}^{(mn)}$:

$$h_{ij}^{(mn)*} = h_{-j-i}^{(nm)}, \quad (8a)$$

$$h_{ij}^{(mn)*} = h_{-i-j}^{(mn)}, \quad (8b)$$

$$h_{ij}^{(mn)*} = \epsilon(m)\epsilon(n)h_{-i-j}^{(mn)}, \quad (8c)$$

where $(-i) = \pm$ for $i = \mp$, $\epsilon(m) = 1$ for $m = 1, 3, 4, 6, 8$, and $\epsilon(m) = -1$ for $m = 2, 5, 7$.¹

The $SU(3)$ structure of (7) is a mixture of $\{1\}$, $\{8s\}$, $\{8a\}$, $\{10\}$, $\{\bar{10}\}$, and $\{27\}$, that is, of all representations contained in the direct product $\{8\} \times \{8\}$.¹⁵ The isospin content of $H_{NL}^{\Delta S=0}$ is $T = 0, 1, 2$.

The conditions for $H_{NL}^{\Delta S=0}$ to be free of an isotensor, isovector, or isoscalar part are as follows.

No $\Delta T = 2$:

$$\begin{aligned} h_{++}^{(33)} &= \frac{1}{4}(1 - |\alpha|^2) \cos^2 \theta, \\ h_{+-}^{(33)} &= \frac{1}{4}(1 + |\alpha|^2) \cos^2 \theta. \end{aligned} \quad (9)$$

No $\Delta T = 1$:

$$\begin{aligned} h_{++}^{(38)} &= h_{+-}^{(38)} = 0, \\ h_{++}^{(86)} + h_{++}^{(77)} &= \frac{1}{2}(1 - |\alpha'|^2) \sin^2 \theta, \\ h_{+-}^{(86)} + h_{+-}^{(77)} &= \frac{1}{2}(1 + |\alpha'|^2) \sin^2 \theta, \\ h_{+-}^{(67)} &= -\frac{1}{2}i \operatorname{Re} \alpha' \sin^2 \theta, \end{aligned} \quad (10)$$

$$\operatorname{Re} \alpha \cos^2 \theta = 0.$$

No $\Delta T = 0$:

$$\begin{aligned} h_{++}^{(88)} &= h_{+-}^{(88)} = 0, \\ h_{++}^{(33)} &= -\frac{1}{2}(1 - |\alpha|^2)\cos^2\theta, \\ h_{+-}^{(33)} &= -\frac{1}{2}(1 + |\alpha|^2)\cos^2\theta, \\ h_{++}^{(66)} + h_{+-}^{(77)} &= -\frac{1}{2}(1 - |\alpha'|^2)\sin^2\theta, \\ h_{+-}^{(66)} + h_{+-}^{(77)} &= -\frac{1}{2}(1 + |\alpha'|^2)\sin^2\theta, \\ h_{+-}^{(67)} &= \frac{1}{2}i \operatorname{Re}\alpha' \sin^2\theta. \end{aligned} \quad (11)$$

As seen from the last relation in (10), the neutral currents do not cancel the isovector term introduced by the charged second-class currents. Consequently, in addition to the nonexistence, in the presence of charged currents, of a $\Delta S = 0$ Hamiltonian satisfying a $\Delta T = 1$ or a $\Delta T = 2$ selection rule,¹⁰ a pure isoscalar $\Delta S = 0$ Hamiltonian is not possible either if charged second-class currents participate.

Next we consider a more general Hamiltonian in which $V+A$ as well as $V-A$ currents are involved for both the regular and the irregular octets:

$$H_{NL} = -\frac{G}{\sqrt{2}} \sum_{k,l=\pm} \sum_{i,j=\pm} \sum_{m,n=1}^8 h_{ij}^{(mn)kl} J_{(m)l}^k J_{(n)j}^l, \quad (12)$$

where

$$\begin{aligned} J_{(m)+}^{\pm} &= (V^R \pm A^R) + (V^I \pm A^I), \\ J_{(m)-}^{\pm} &= (V^R \pm A^R) - (V^I \pm A^I), \end{aligned}$$

and

$$V^R \pm A^R, \quad V^I \pm A^I$$

satisfy the relations (4) and (5). We assume H_{NL} to be CPT -invariant, but allow for possible CP violation.

It is convenient to introduce the quantities

$$\begin{aligned} M^{(mn)} &= h_{++}^{(mn)--} - h_{++}^{(mn)++}, \\ N^{(mn)} &= h_{+-}^{(mn)--} - h_{+-}^{(mn)++}, \\ L^{(mn)} &= h_{+-}^{(mn)-+} - h_{+-}^{(mn)+-}, \\ Q^{(mn)} &= h_{++}^{(mn)+-} - h_{++}^{(mn)-+}. \end{aligned} \quad (13)$$

The conditions for the absence of various isospin components of $H_{NL}^{\Delta S=0, P=-1}$ can be written in the following concise form:

No $\Delta T = 2$:

$$\Gamma^{11} = \Gamma^{33}. \quad (14)$$

No $\Delta T = 1$:

$$\begin{aligned} \Gamma^{38} &= 0, \\ \Gamma^{44} &= \frac{1}{2}(\Gamma^{66} + \Gamma^{77}), \\ \Omega^{45} &= \Omega^{67}, \\ \Omega^{12} &= 0. \end{aligned} \quad (15)$$

No $\Delta T = 0$:

$$\begin{aligned} \Gamma^{68} &= 0, \\ \Gamma^{11} &= -\frac{1}{2}\Gamma^{33}, \\ \Gamma^{44} &= -\frac{1}{2}(\Gamma^{66} + \Gamma^{77}), \\ \Omega^{45} &= -\Omega^{67}. \end{aligned} \quad (16)$$

In (14), (15), and (16), Γ stands for $\operatorname{Re}M$, $\operatorname{Im}M$, $\operatorname{Re}N$, and $\operatorname{Im}N$ and Ω stands for $\operatorname{Re}Q$, $\operatorname{Im}Q$, $\operatorname{Re}L$, and $\operatorname{Im}L$.

The implications of (14), (15), and (16) can be summarized as follows:

(a) A CP -invariant $\Delta S = 0$ nonleptonic weak interaction satisfying a $\Delta T = 0$ rule is possible only in the absence of charged second-class currents. This conclusion is not affected by the presence of $V+A$ currents.¹⁶

(b) If we allow for CP violation, the condition $\Omega^{12} = 0$ can be satisfied, and consequently a pure isoscalar $\Delta S = 0$ nonleptonic weak Hamiltonian can be constructed, even in the presence of charged second-class currents. An example of such a theory is given by the Hamiltonian (7) built up from the hadronic current (6) in which α is taken to be pure imaginary (in this case $\operatorname{Im}Q^{12} = \operatorname{Re}Q^{12} = \operatorname{Re}L^{12} = \operatorname{Im}N^{12} = 0$).¹⁷

To answer the question of the existence of a $\Delta T = 1$ component in the parity-violating nuclear forces with as high as possible accuracy, is therefore of great importance.¹⁸ The absence of such interaction is evidence against the existence of charged second-class currents of the type capable of producing a difference between the matrix elements of mirror β transitions.¹⁹ On the other hand, an unambiguous interpretation of a $\Delta T = 1$ strangeness-conserving weak interaction, if found, will require additional information about the currents entering the nonleptonic weak interactions.

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¹An octet of vector or axial-vector currents $J_{(\nu)\mu}$

$\equiv J_{(TT_3Y)\mu}$ ($\mu = 1, 2, 3, 4$) which goes into itself under charge conjugation satisfies (omitting in the following the Dirac indexes)

$$CPJ_{(\nu)}(CP)^{-1} = \eta(J)(-1)^{Q_{\nu}}J_{(-\nu)},$$

where $(-\nu) \equiv (T - T_3 - Y)$, $Q_{\nu} = T_3 + \frac{1}{2}Y$, and $\eta(J) = \pm 1$ and is the same for all components of a given octet [Y. Dot-han, *Nuovo Cimento* **30**, 399 (1963); M. Gell-Mann, *Phys. Rev. Letters* **12**, 155 (1964)]. $J_{(\nu)} = V_{(\nu)} \pm A_{(\nu)}$ will be called *regular* (J^R) if $\eta = +1$, and *irregular* (J^I) when $\eta = -1$. Under G parity

$$GPJ_{(1T_3^0)}^R(GP)^{-1} = -J_{(1T_3^0)}^R$$

and

$$GPJ_{(1T_3^0)}^I(GP)^{-1} = J_{(1T_3^0)}^I,$$

so that $J_{(1T_3^0)}^R$ is a first-class and $J_{(1T_3^0)}^I$ a second-class current [S. Weinberg, *Phys. Rev.* **112**, 1375 (1959)]. Regarding $SU(3)$ we use the notation and conventions of J. J. de Swart, *Rev. Mod. Phys.* **35**, 916 (1963). Cartesian components will be denoted by Latin indexes: $J_{(m)}$ ($m = 1, 2, \dots, 8$).

²Recent studies of differences in the ft values of mirror β decays are suggestive of the possibility of the existence of second-class currents: Cf. R. J. Blin-Stoyle and M. Rosina, *Nucl. Phys.* **70**, 321 (1965); D. H. Wilkinson, *Phys. Letters* **31B**, 447 (1970); D. H. Wilkinson and D. E. Alburger, *Phys. Rev. Letters* **24**, 1134 (1970); *Phys. Letters* **32B**, 190 (1970). See, however, J. Delorme and M. Rho, CERN report, 1970 (unpublished) and also D. H. Wilkinson and D. E. Alburger, *Phys. Rev. Letters* **26**, 1127 (1971).

³A general analysis of interactions involving neutral currents is given in C. H. Albright and R. J. Oakes, *Phys. Rev. D* **2**, 1883 (1970).

⁴S. Weinberg, *Phys. Rev.* **112**, 1375 (1959).

⁵M. L. Good, L. Michel, and E. de Rafael, *Phys. Rev.* **151**, 1199 (1966).

⁶It could, of course, happen that the irregular currents are present in the semileptonic weak interactions while absent in the nonleptonic ones or vice versa. In addition, the nonleptonic weak interactions may involve other types of currents, not present in the semileptonic weak interactions, for example, charged $V+A$ currents or scalar and tensor currents [cf. F. Zachariasen and G. Zweig, *Phys. Rev. Letters* **14**, 794 (1965)]. Finally, the nonleptonic weak interactions may not be of a current \times current form. An example is the theory of Nishijima [K. Nishijima, in *Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January 1968*, edited by A. Perlmutter, C. A. Hurst, and B. Kurşunoğlu (Benjamin, New York, 1968), p. 175]. The standard current \times current theory, applied to the nonleptonic weak interactions, is reviewed by S. Pakvasa and S. P. Rosen, in a paper presented to the symposium "The Past Decade in Particle Theory," University of Texas at Austin, 1970 (unpublished).

⁷There is good evidence now for the existence of $\Delta S = 0$ nonleptonic weak interactions from observations of parity nonconservation in nuclear transitions. The present experimental situation is reviewed by F. Boehm, in an invited paper presented at the International Conference on Angular Correlation in Nuclear Disintegration, Delft, The Netherlands, 1970 (unpublished). Recent reviews of the theory are R. J. Blin-Stoyle, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969*, edited by J. Prentki and J. Steinberger (CERN,

Geneva, 1969), p. 495; E. M. Henley, *Ann. Rev. Nucl. Sci.* **19**, 367 (1969). See also papers presented at the *International Conference on High-Energy Physics and Nuclear Structure, Columbia University, 1969*, edited by S. Devons (Plenum, New York, 1970).

⁸R. J. Blin-Stoyle and P. Herczeg, *Phys. Letters* **23**, 376 (1966).

⁹R. F. Dashen, S. C. Frautschi, M. Gell-Mann, and Y. Hara, *The Eightfold Way* (Benjamin, New York, 1964), p. 254.

¹⁰C. H. Albright and R. J. Oakes, *Phys. Rev. D* **3**, 1270 (1971).

¹¹However, a Hamiltonian satisfying a $\Delta S = 0$, $\Delta T = 1$, or a $\Delta S = 0$, $\Delta T = 2$ selection rule is not possible, unless the charged currents are absent (Ref. 10).

¹²See, for example, N. Cabibbo, in *Particle Symmetries, Brandeis University Summer Institute in Theoretical Physics, 1965*, edited by M. Chrétien and S. Deser (Gordon and Breach, New York, 1966), p. 1.

¹³Note that Eqs. (3) imply $J_{(110)\mu}^{R*} = -J_{(1-10)\mu}^R$ and $J_{(110)\mu}^{I*} = J_{(1-10)\mu}^I$ ($J_{(\nu)\mu}^* = J_{(\nu)\mu}^I$ for $\mu = 1, 2, 3$ and $J_{(\nu)4}^* = -J_{(\nu)4}^I$; $\dagger \equiv$ Hermitian conjugation), i.e., the Cartesian components of J^R and J^I are Hermitian and anti-Hermitian, respectively. If $\text{Im}\alpha = 0$, the semileptonic weak Hamiltonian corresponding to (1) is CP -invariant.

¹⁴There is no information at present on $\Delta S = 1$ irregular currents. The identification of such currents is complicated by the $SU(3)$ -breaking strong interactions which are capable of inducing second-class form factors even in the absence of irregular currents. Cf. L. Wolfenstein, *Phys. Rev.* **135**, B1436 (1964).

¹⁵Cf. P. Herczeg, *Nucl. Phys.* **B4**, 153 (1967). In the standard theory not involving irregular currents only the symmetric combinations $\{1\}$, $\{8_s\}$, and $\{27\}$ enter.

¹⁶Note also that the nonexistence in the presence of charged currents of a Hamiltonian satisfying a $\Delta T = 1$ or a $\Delta T = 2$ selection rule¹⁰ persists in the presence of $V+A$ currents. The condition $\Gamma^{11} = \Gamma^{33} = 0$, for example, required to satisfy a $\Delta T = 1$ rule, eliminates again the corresponding charged-current products.

¹⁷Note that for $\text{Re}\alpha = 0$ the matrix elements of mirror β transitions are equal. This is the theory of CP violation proposed by N. Cabibbo [*Phys. Letters* **12**, 137 (1964)]. It could lead to a large muon polarization perpendicular to the decay plane in $K_{\mu 3}$ decays. Although the experimental results are consistent with zero polarization [K. K. Young *et al.*, *Phys. Rev. Letters* **18**, 806 (1966); D. Bartlett *et al.*, *ibid.* **16**, 282 (1966); J. Bettels *et al.*, *Nuovo Cimento* **56A**, 1106 (1968); M. J. Longo, K. K. Young, and J. A. Helland, *Phys. Rev.* **181**, 1808 (1969)], this does not necessarily rule out the theory [cf. N. Cabibbo, *Phys. Rev. Letters* **14**, 965 (1965); L. Maiani, *Phys. Letters* **26B**, 538 (1968); C. W. Kim and H. Primakoff, *Phys. Rev.* **180**, 1502 (1969)].

¹⁸In the experiments performed so far, the $\Delta T = 1$ channel is not isolated (Ref. 7). Suggestions of experiments in which the isovector component can be separated from the isoscalar and isotensor parts have been made [cf. Ref. 9 and G. S. Danilov, *Phys. Letters* **18**, 40 (1965); D. Tadić, *Phys. Rev.* **174**, 1694 (1968); E. M. Henley, *Phys. Letters* **28B**, 1 (1968)]. The existence of an isoscalar component in the parity-nonconserving nuclear force has been recently established in studies of parity-forbidden α decays [H. Hättig, K. Hünchen, and

W. Wäffler, Phys. Rev. Letters **25**, 941 (1970); E. L. Sprenkel-Segel, R. E. Segel, and R. H. Siemssen, in *Proceedings of the International Conference on High-Energy Physics and Nuclear Structure, Columbia University, 1969*, edited by S. Devons (Plenum, New York, 1970), p. 763; Wen-Kwei Cheng, E. Fischbach,

H. Primakoff, D. Tadić, and K. Trabert, Phys. Rev. D **3**, 2289 (1971)].

¹⁹As seen from the relations (15), neutral currents must in this case also exist, unless the charged $\Delta S=1$ regular currents are absent in the nonleptonic weak interactions.

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Proposed Experiment to Test Local Hidden-Variable Theories

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It is shown that by making a plausible assumption concerning the behavior of linear polarizers, it is possible to relax the experimental condition of Clauser, Horne, Shimony, and Holt for testing local hidden-variable theories.

Bell recently proved that any local hidden-variable theory cannot contain all of the predictions of quantum mechanics.¹ The proof is based on a Bohm² variant of the Einstein-Podolsky-Rosen experiment.³ As a result of this proof, in a recent letter Clauser, Horne, Shimony, and Holt (CHSH) proposed an experimental test of local hidden-variable theories.⁴ The proposed experiment is that of Kocher and Commins⁵ - requiring, however, high-sensitivity high-transmission polarizers at appropriate relative orientations. We show that the high-transmission condition, which makes an experimental test extremely difficult, is unnecessary.

In the Kocher and Commins's experiment the linear polarization correlation was examined of two successive photons emitted in the cascade $6^1S_0 \rightarrow 4^1P_1 \rightarrow 4^1S_0$ in calcium. Each of the two detectors consisted of a linear polarizer, followed by a wavelength filter and photomultiplier. We define ϵ_i^p as the transmission of the polarizers, where $i=M, m$ designates light polarized parallel or perpendicular to the polarizer axis and $p=I, II$ designates the polarizer station. We will examine, as do CHSH, the case for $\epsilon_M^p \gg \epsilon_m^p$. We define t^p as the product of the transmissivity of the filter and the photomultiplier efficiency.

The quantity of interest for local hidden-variable experiments is the correlation function $P(a, b)$, defined as^{1,4}

$$P(a, b) = \int d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda), \quad (1)$$

where a and b are the angle of the polarizers at stations I and II, respectively. The variables $A, B = \pm 1$ designate the detection (+1) or nondetection (-1) of photons at stations I, II. Since t^p is

relatively small, A and B were reinterpreted by CHSH as the emergence or nonemergence of photons from the polarizers, which defines a new correlation function $P'(a, b)$.

When $\epsilon_M^p \gg \epsilon_m^p$, each polarizer can be described as a perfect polarizer ($\epsilon_m = 0, \epsilon_M = 1$) followed by a gray filter of transmissivity ϵ_M^p . Reinterpreting A and B now as the emergence or nonemergence of photons from the respective perfect polarizers, we obtain another correlation function $P''(a, b)$.

Evaluating (1) we obtain for $P'(a, b)$ and $P''(a, b)$

$$P'(a, b) = 4\epsilon_M^I \epsilon_M^{II} \bar{R}(a, b) - \epsilon_M^I - \epsilon_M^{II} + 1 \quad (2)$$

and

$$P''(a, b) = 4\bar{R}(a, b) - 1, \quad (3)$$

where $\bar{R}(a, b)$ is the emergence coincidence rate from the perfect polarizers. The relation between $\bar{R}(a, b)$ and the measured coincidence rate from the photomultipliers, $R(a, b)$, is

$$\bar{R}(a, b) = R(a, b) / \epsilon_M^I \epsilon_M^{II} t^I t^{II}. \quad (4)$$

In examining experimental tests of local hidden-variable theories, we obtain relationships between different correlation functions.^{1,4} CHSH using $P'(a, b)$ obtained a minimum condition for $\epsilon_M^I = \epsilon_M^{II} = \epsilon_M$. They obtained

$$\epsilon_M > 2/(1 + \sqrt{2}) \approx 83\% \quad (5)$$

[Eq. 4, Ref. 4, $F(\theta) = 1$]. Since $P''(a, b)$ could just as well have been used, comparing (2) and (3) we obtain

$$1 > 2/(1 + \sqrt{2}) \quad (6)$$

which is always true.

From the above we observe that an experimental