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[27]A vanishing of the three-Pomeranchukon vertex at $t=0$ should be distinguished from the identical vanishing of this vertex. The latter possibility was discussed in Sec. II when we discussed whether the Pomeranchukon pole and cuts interact.

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# Quark Model of Dual Pions*

A. Neveu† and John H. Schwarz
*Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540*
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Interacting pseudoscalar pions are incorporated into Ramond's model of free dual fermions. By considering the emission of $N-1$ pions and factorizing in the quark-antiquark channel, we recover the same $N$-pion amplitudes as were proposed in a previous paper.

Ramond[1] has recently proposed a model of free dual fermions related to the Dirac equation by a correspondence principle[2] analogous to one relating the conventional dual-resonance model to the Klein-Gordon equation. This fermion model possesses an infinite set of Ward identities that probably provides for the cancellation of all ghosts. Another recent development was the discovery of a dual model of pions[3] having a number of realistic features not shared by the conventional dual-resonance model. Subsequently, the algebraic properties responsible for the successes of this model (including the apparent absence of ghosts) have been obtained.[4]

In this paper we show that there is a deep connection between the fermion and pion models. Specifically, we construct the amplitude for emitting $N-1$ pions from a fermion line [Fig. 1(a)]. Requiring the gauge algebra of the fermion sector to hold in the presence of interaction imposes the condition $m_\pi{}^2=-\frac{1}{2}$, the same condition required for the gauges in the meson sector. By factorizing at the first pole in the quark-antiquark channel [Fig. 1(b)], we obtain the same $N$-pion amplitude as in Ref. 3.

Let us first review the algebra of Ramond's fermion model. In addition to the usual harmonic-oscillator operators[5] satisfying

$$[\alpha_m^\mu, \alpha_n^\nu]=-mg^{\mu\nu}\delta_{m,-n},$$

Ramond introduces anticommuting operators satisfying

$$\{d_m^\mu, d_n^\nu\}=-g^{\mu\nu}\delta_{m,-n}$$

and

$$[d_m^\mu, \alpha_n^\nu]=0,$$

where $m$ and $n$ are integers, $d_{-m}^\mu=d_m^{\mu\dagger}$, and $d_0^\mu=-(i/\sqrt{2})\gamma_5\gamma^\mu$. Then, introducing

$$P^\mu(\tau)=(\tfrac{1}{2})^{1/2}\sum_{n=-\infty}^{\infty}\alpha_n^\mu e^{-in\tau}$$

and

$$\Gamma^\mu(\tau)=i\sqrt{2}\,\gamma_5\sum_{n=-\infty}^{\infty}d_n^\mu e^{-in\tau},$$

one finds that the operators

$$L_n=-\langle e^{in\tau}\colon P^2(\tau)\colon\rangle+\tfrac{1}{4}i\left\langle e^{in\tau}\colon\Gamma(\tau)\cdot\frac{d}{d\tau}\Gamma(\tau)\colon\right\rangle$$

satisfy the Virasoro algebra[6]

$$[L_m, L_n]=(m-n)L_{m+n}.$$

The wave equation for a fermion state is

$$(F_0-m)|\psi\rangle=0,$$

where $m$ is the mass of the spin-$\frac{1}{2}$ ground-state fermion and

$$F_n=\langle e^{in\tau}\Gamma(\tau)\cdot P(\tau)\rangle.$$

Ramond's subsidiary ghost-eliminating conditions are

$$F_n|\psi\rangle=0,\quad n=1,2,3,\ldots \tag{1}$$

or, equivalently,

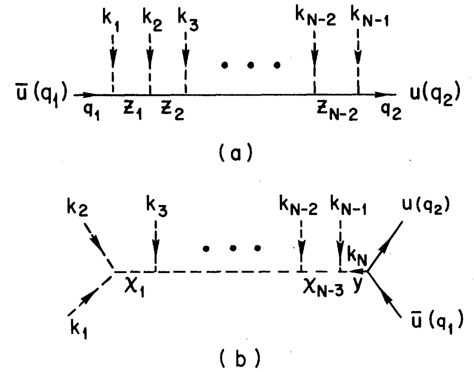


FIG. 1. (a) Emission of $N-1$ pions from a quark. (b) Factorization in the quark-antiquark channel.

$$L_n|\psi\rangle = 0, \quad n = 1, 2, 3, \ldots . \tag{2}$$

The remaining algebra of the $F_n$'s and $L_n$'s is

$$[L_m, F_n] = (\tfrac{1}{2}m - n)F_{m+n}, \tag{3}$$

$$\{F_m, F_n\} = -2L_{m+n}. \tag{4}$$

The key requirement for introducing interaction into Ramond's model is that it be compatible with the subsidiary conditions (1) and (2). The propagator for a fermion line will of course be $1/(F_0 - m)$. Therefore, in analogy with the conventional model, we seek off-shell gauge operators $X_n$ and a meson-emission vertex $V(k)$ such that

$$X_n \frac{1}{F_0 - m} V(k) = \frac{1}{F_0 - (m^2+n)^{1/2}} V(k) X_n . \tag{5}$$

If $X_n$ annihilates the on-shell fermion ground state, then Eq. (5) implies that it annihilates an arbitrary tree state. Using Eqs. (3) and (4) one can show that

$$X_n \frac{1}{F_0 - m} = [F_0 - F_n - m + c_n(L_n - L_0 - m^2)] \frac{1}{F_0 - m}$$
$$= \frac{1}{F_0 - (m^2+n)^{1/2}} [F_n - F_0 - m + c_n(L_n - L_0 - m^2 - \tfrac{1}{2}n)],$$

with

$$c_n = (2/n)[(m^2+n)^{1/2} + m] .$$

Therefore, since the ordinary vertex operator

$$V_0(k) = e^{ik\cdot x}\exp\left(\sqrt{2}\, k\cdot \sum_{n=1}^{\infty} \frac{\alpha_{-n}}{n}\right)\exp\left(-\sqrt{2}\, k\cdot \sum_{n=1}^{\infty} \frac{\alpha_n}{n}\right)$$

commutes with $F_0 - F_n$ and shifts $L_0 - L_n$ by $nk^2$, Eq. (5) can be satisfied by multiplying $V_0$ by an expression that anticommutes with $F_0 - F_n$. The simplest choice with this property[7] is

$$\Gamma_5 = \gamma_5 (-1)^{\sum_{n=1}^{\infty}(d_n^\dagger \cdot d_n)} .$$

Equation (5) is then satisfied if the emitted meson has $m^2 = -\frac{1}{2}$. It is interesting to note that the fermion model requires the emitted meson to be pseudoscalar, whereas in Ref. 3 the pion parity was undetermined.

The amplitude for the process shown in Fig. 1(a) is

$$B_{N-1} = \bar{u}(q_1) \left\langle 0 \left| \Gamma_5 V_0(k_1) \frac{1}{F_0 - m} \Gamma_5 V_0(k_2) \frac{1}{F_0 - m} \cdots \frac{1}{F_0 - m} \Gamma_5 V_0(k_{N-1}) \right| 0 \right\rangle u(q_2).$$

Rewriting the propagators in the form $-(F_0 + m)/(L_0 + m^2)$ and pushing the $\Gamma_5$'s to the left gives

$$B_{N-1} = (-1)^{(N/2)+1} \bar{u}(q_1)\gamma_5 \left\langle 0 \left| V_0(k_1) \frac{F_0 + m}{L_0 + m^2} V_0(k_2) \frac{F_0 - m}{L_0 + m^2} \cdots \frac{F_0 - m}{L_0 + m^2} V_0(k_{N-1}) \right| 0 \right\rangle u(q_2).$$

Next we take the $F_0 \pm m$ factors to the right through the vertices using

$$[F_0, V_0(k)] = -V_0(k) k\cdot \Gamma,$$

and dropping terms with cancelled propagators.[4] This procedure results in the formula

$$B_{N-1} = (-1)^{(N/2)+1} \bar{u}(q_1)\gamma_5 \left\langle 0 \left| V_0(k_1) \frac{1}{L_0 + m^2} k_2 \cdot \Gamma V_0(k_2) \frac{1}{L_0 + m^2} \cdots \frac{1}{L_0 + m^2} k_{N-1} \cdot \Gamma V_0(k_{N-1}) \right| 0 \right\rangle u(q_2). \tag{6}$$

The advantage of expressing $B_{N-1}$ in the form of Eq. (6) is that after substituting the standard integral representation

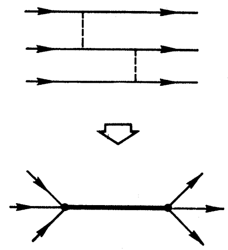

FIG. 2. Possible scheme for forming baryons from three quarks.

$$\frac{1}{L_0+m^2}=\int_0^1 z^{L_0+m^2-1}dz,$$

the algebra of $\alpha$ and $d$ modes separates. The contribution of the $\alpha$ modes to the integrand is essentially of the Bardakci-Ruegg form.[8] The algebra of $d$ modes, on the other hand, is similar to that of the $b$ modes occurring in the computation of the $N$-pion amplitude $A_N$ in the formulation of Ref. 4.

The last remark is the key to showing the connection between $A_N$ and $B_{N-1}$. First, one makes the standard change of variables to go from the configuration of Fig. 1(a) to the one of Fig. 1(b), namely,

$$x_i=\frac{1-z_1z_2\cdots z_i}{1-z_1z_2\cdots z_{i+1}},\quad i=1,2,\ldots,N-3$$

$$y=1-z_1z_2\cdots z_{N-2}.$$

Then factorizing in the quark-antiquark channel near the first pole $[(q_1-q_2)^2=m_\pi^2=-\frac{1}{2}]$, one finds that

$$B_{N-1}\underset{(q_1-q_2)^2\to m_\pi^2}{\sim}\bar{u}(q_1)\gamma_5u(q_2)\frac{1}{(q_1-q_2)^2-m_\pi^2}A_N(k_1,k_2,\ldots,k_N=q_1-q_2),$$

where, in the notation of Ref. 4,

$$A_N(k_1,k_2,\ldots,k_N)=\left\langle 0\left|k_2\cdot HV_0(k_2)\frac{1}{L_0-\frac{1}{2}}k_3\cdot HV_0(k_3)\cdots\frac{1}{L_0-\frac{1}{2}}k_{N-1}\cdot HV_0(k_{N-1})\right|0\right\rangle.$$

This proves our assertion that Ramond's fermion model leads to the dual-pion model in the meson sector.

In conclusion, we wish to point out that the connection between the amplitude depicted in Fig. 1(b) and the amplitudes of the dual-pion model has been demonstrated only at the first pole in the $q\bar{q}$ channel. It should, however, be possible to make an operator generalization to describe the coupling of an arbitrary meson state to the quark. Once this is done, one could attempt, for example, to construct baryons out of three quarks as shown in Fig. 2. In such a channel one would expect the pole positions to depend on the quark mass in contrast to what we have found for the $q\bar{q}$ channel. The quarks could be only mathematical since one would have a consistent scheme by considering just the states with zero triality. Clearly, many interesting problems remain to be studied.

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†NATO Fellow, on leave of absence from Laboratoire de Physique Théorique, Orsay, France.

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$$\alpha_n^\mu=-i\sqrt{n}\,a_n^\mu,\quad \alpha_{-n}^\mu=i\sqrt{n}\,a_n^{\mu\dagger},\quad \alpha_0^\mu=\sqrt{2}\,p^\mu.$$

Our metric and $\gamma$ matrices follow the conventions of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

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