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Symmetry-Breaking Role of the κ Meson

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We study the symmetry-breaking role of the κ meson in the context of the $(3,3^*)+(3^*,3)$ model and compare its role with that of the pseudoscalar mesons in the hypothesis of partial conservation of axial-vector current. In some dynamical schemes, field-divergence identities derived from the hypothesis of partial conservation of axial-vector current may play a parallel role to those derived from partial conservation of vector current, as is evident in many effective-Lagrangian and "hard-pion" calculations. However, the point of view that the vacuum is approximately SU(3)-symmetric, coupled with Dashen's counting of the order of chiral symmetry breaking, suggests a scheme in which κ plays a dynamically dissimilar role to that of the pion or kaon. We pursue this possibility and show that it is required if one demands that certain alternative definitions of the off-mass-shell amplitude are forced to coincide, at least to lowest order of chiral symmetry breaking. In such a dynamical scheme, which we find attractive, the κ cannot at small momentum transfer dominate the vector divergence in the same sense that the pseudoscalars dominate the axial-vector divergence. A κ effect can, however, contribute to matrix elements in a higher order of symmetry breaking, and we illustrate such a scheme in a simple Lagrangian model.

I. INTRODUCTION

The picture of chiral symmetry and chiral symmetry breaking which has evolved over the past few years has been dominated by the idea of spontaneous breakdown and the related idea of pole-dominated axial-vector divergences which have "smooth" matrix elements in momentum transfer. Practical calculations have leaned heavily on the smoothness approximation. The Goldberger-Treiman relation,[1] the Adler condition,[2] and the Adler-Weisberger condition[3] indicate that it is reliable at the 10% level for pion matrix elements. We shall be interested in the implications which follow if the smoothness approximation is also reliable at this level for kaon matrix elements.[4] Such smoothness conditions associated with partial conservation of axial-vector current (PCAC) also play a central role in the analysis of the transformation properties of chiral-symmetry-breaking and SU(3)-symmetry-breaking parts of the Hamiltonian density; for example, let us consider the Gell-Mann–Oakes–Renner and Glashow-Weinberg $(3, 3^*)+(3^*, 3)$ models.[5,6] In the Gell-Mann–Oakes–Renner (GOR) analysis,[5] summarized by

(i) PCAC for π and K mesons,

(ii) $\langle 0|u_0|0\rangle \neq 0$,

(iii) $\langle 0|u_8|0\rangle = 0$, (1.1)

(iv) $f_\pi = f_K$,

one is led to believe that these conditions hold to within the accuracy of the PCAC smoothness conditions. Notably absent from this scheme, but present in the Glashow-Weinberg (GW) analysis,[6] which also relies on smoothness conditions, is the κ PCVC (partial conservation of vector current) condition and the attendant modifications

(i) PCAC for κ and K mesons and PCVC for the κ meson,

(ii) $\langle 0|u_0|0\rangle \neq 0$,

(iii) $\langle 0|u_8|0\rangle \neq 0$, (1.2)

(iv) $f_\pi - f_K = f_\kappa$ (neglecting renormalization effects),

(v) SU(2)-symmetric vacuum.

In view of the experimental indications that a κ meson exists[7] but that it fails to dominate the scalar form factor in K_{l3} decay,[8] it is of interest to reconsider the role which κ may play in the scheme of approximate SU(3)×SU(3) symmetry. In particular, we present an attractive scheme in which the κ plays an essentially different dynamical role than the pseudoscalars and we correlate this dynamical difference with a group-theoretical difference in the way chiral symmetry is realized in the chiral-symmetry limit. We shall show that κ dominance fails, at least in connection with an approximate Goldstone role for κ, if certain alternative off-mass-shell extrapolations are required to coincide in lowest order of chiral symmetry breaking.

Dashen[9,10] has recently argued that it is very difficult to accommodate a κ-dominance relation and a Goldstone role for the κ in the chiral-symmetry limit. It is relatively easy, on the other hand, to construct models which mock either the GOR scheme or the GW scheme, and it seems useful to examine the two alternatives. Our point of view, in fact, is that the GOR scheme, the GW scheme, and the Dashen argument are really not inconsistent with each other, provided one suitably modifies the statements (i)–(v) by stipulations which make precise what limit of the theory one is discussing. Essential to this is the counting procedure of Dashen in which the order of symmetry breaking is parametrized in powers of δ, the chiral-symmetry-breaking parameter in the decomposition of the hadron energy density as $\mathcal{H}=\mathcal{H}_0-\delta\mathcal{H}'$, where $\mathcal{H}_0$ is a chiral singlet and $\delta\mathcal{H}'$ an SU(2) singlet. We shall argue that if the κ is to be accommodated without destroying a smoothness condition which requires alternative off-mass-shell definitions to coincide, then one must allow the vacuum $\Omega(\delta)$ of $\mathcal{H}(\delta)=\mathcal{H}_0-\delta\mathcal{H}'$, $\langle 0|u_0|0\rangle$, $\langle 0|u_8|0\rangle$, f_π, f_K, and f_κ to depend on δ in such a way that (with $\delta\mathcal{H}=u_0+cu_8$)

$$\begin{aligned}
&\lim_{\delta\to 0}\Omega(\delta)=\Omega_0,\quad \text{an SU(3) singlet},\\
&\lim_{\delta\to 0}\frac{\langle 0|u_0|0\rangle}{\delta}\neq 0,\quad \langle 0|u_0|0\rangle=O(\delta),\\
&\lim_{\delta\to 0}\frac{\langle 0|u_8|0\rangle}{\delta}=0,\quad \langle 0|u_8|0\rangle=O(\delta^2),\\
&\lim_{\delta\to 0}f_\pi=\lim_{\delta\to 0}f_K;\quad f_\pi, f_K=O(1),\\
&\lim_{\delta\to 0}f_\kappa{}^{-1}=0,\quad f_\kappa{}^{-1}=O(\delta),\\
&\lim_{\delta\to 0}m_\pi{}^2=0,\quad \lim_{\delta\to 0}m_K{}^2=0;\quad m_\pi{}^2, m_K{}^2=O(\delta),\\
&\lim_{\delta\to 0}m_\kappa{}^2\neq 0,\quad m_\kappa{}^2=O(1).
\end{aligned}\tag{1.3}$$

Thus the GW scheme [with formal κ-PCVC condition, SU(2) Ω, and $f_\pi\neq f_K$] can, in this context, be regarded as a more general description which tends in the chiral-symmetry limit to the GOR scheme. Note, however, that κ is *not* a Goldstone particle in this point of view and plays an essentially different role in nature from the pion and the kaon; even though it is coupled linearly to the vector divergence when $\delta\neq 0$, it decouples in the limit $\delta\to 0$ where the corresponding decay constant $f_\kappa{}^{-1}$ vanishes.

In this scheme the κ-PCVC condition and the other refinements of the GW scheme are of the order of PCAC smoothness corrections, on the same footing as corrections to the Goldberger-Treiman relation. Indeed we shall be led to this point of view by a brief study of pseudoscalar matrix elements of the $(3,3^*)+(3^*,3)$ densities in the $(3,3^*)+(3^*,3)$ model of chiral symmetry breaking. In a sense we shall merely be making explicit certain PCAC smoothness assumptions which are implicit in the GOR analysis and the work of Dashen. We shall see that these smoothness assumptions effectively make alternative off-mass-shell definitions of these scalar matrix elements coincide but also effectively deny the κ a Goldstone-like role in chiral- and SU(3)-symmetry breaking. This is the subject of Sec. II, in which we also comment on the feasibility of realizing the chiral-symmetry limit with the κ as a true Goldstone particle ($M_\kappa{}^2=0$, $f_\kappa{}^{-1}\neq 0$ in the chiral limit), concluding that the PCAC smoothness condition and the requirement that alternative off-mass-shell definitions coincide must then be relaxed. In Sec. III we construct an explicit model which spans the GW or GOR schemes and satisfies the limits in Eq. (1.3). In Sec. IV we summarize and comment briefly on the implications of our results.

II. PSEUDOSCALAR MATRIX ELEMENTS

Let us consider the matrix element

$$\langle P_i(p)|W_j(0)|P_k(p')\rangle\,,\tag{2.1}$$

where $P_i(p)$ is a pseudoscalar meson with octet index i and four-momentum p, and $W_j(0)$ is a scalar density. If we define the matrix element with one off-mass-shell pseudoscalar meson in the conventional manner[11] and take the soft limit $p\to 0$, the result is

$$\lim_{p\to 0}\langle P_i(p)|W_j(0)|P_k(p')\rangle=-2if_i\langle 0|[F_i^5, W_j]|P_k(p')\rangle\,,\tag{2.2}$$

where the PCAC constant f_i is defined by

$$\partial A_i(x)=\frac{m_i^2}{2f_i}\phi_i(x),\quad \partial A_i(x)\equiv\partial_\mu A_{\mu,i}(x)\,.\tag{2.3}$$

If now the second pseudoscalar is reduced and PCAC and a soft limit are taken, the result is

$$\lim_{p\to 0;p'\to 0}\langle P_i(p)|W_j(0)|P_k(p')\rangle=-2f_i2f_k\langle 0|[F_k^5,[F_i^5,W_j]]|0\rangle\,.\tag{2.4}$$

The limit, if the matrix element $\langle P_i(0)|W_j|P_k(0)\rangle$ is to have a definite meaning, should be independent of the order of reduction, and since

$$\lim_{p'\to 0;p\to 0}\langle P_i(p)|W_j(0)|P_k(p')\rangle=-2f_i2f_k\langle 0|[F_i^5,[F_k^5,W_j(0)]]|0\rangle\,,\tag{2.5}$$

we are required to conclude that

$$\langle 0|[F_k^5, [F_i^5, W_j]]|0\rangle = \langle 0|[F_i^5, [F_k^5, W_j]]|0\rangle . \qquad (2.6)$$

This is required if we define the double-soft off-mass-shell amplitude by reducing the pseudoscalars one at a time. The purpose of this section is to advocate that we may, under certain conditions, reach the same conclusion even if an alternative off-mass-shell definition is employed when $W_j = u_j$, a scalar density in the $(3, 3^*)+(3^*, 3)$ model of chiral symmetry breaking. The underlying assumption which supports this point of view is the smoothness of the matrix element

$$\langle 0|\partial A_i|P_i(p)\rangle = m_i^2/2f_i \qquad (2.7)$$

as $p\to 0$, a smoothness which *we* shall identify with the PCAC hypothesis. The important implication of Eq. (2.6) is that $\langle 0|u_8|0\rangle = 0$, where u_8 is the SU(3)-breaking part of the Hamiltonian density.

First, however, let us consider an alternate procedure for obtaining the double-soft limit. Define the tensor

$$M_{\mu\nu}(p, p') = \int d^4x \int d^4x' e^{ip'\cdot x'}\, e^{-ip\cdot x} \times \langle 0|T(A_{\mu,i}(x)W_j(0)A_{\nu,k}(x'))|0\rangle . \qquad (2.8)$$

Then we have

$$p_\mu p'_\nu M_{\mu\nu} = \int d^4x \int d^4x' e^{ip'\cdot x'}\, e^{-ip\cdot x} \times \partial_\mu\partial_\nu\langle 0|T(A_{\mu,i}\, W_j A_{\nu,k})|0\rangle . \qquad (2.9)$$

We can now differentiate in either order and use the generalized Ward-Takahashi identities, combined with the off-mass-shell definition

$$\langle P_i(p)|W_j(0)|P_k(p')\rangle = -(+i)^2(p'^2 + m_k^2)(p^2 + m_i^2)(4f_i f_k/m_i^2 m_k^2)\int d^4x' \int d^4x\, e^{ip'\cdot x'}\, e^{-ip\cdot x}\langle 0|T(\partial A_i W_j \partial A_k)|0\rangle \qquad (2.10)$$

to obtain the double-soft limits

$$\lim_{p\to 0, p'\to 0} \langle P_i(p)|W_j(0)|P_k(p')\rangle/4f_i f_k = \int d^4x' \int d^4x\langle 0|T(\partial A_i W_j \partial A_k)|0\rangle \qquad (2.11a)$$

$$= -\langle 0|[F_i^5, [F_k^5, W_j(0)]]|0\rangle + \int d^4x\langle 0|T(W_j[F_k^5, \partial A_i])|0\rangle \qquad (2.11b)$$

$$= -\langle 0|[F_k^5, [F_i^5, W_j(0)]]|0\rangle + \int d^4x\langle 0|T(W_j[F_i^5, \partial A_k])|0\rangle , \qquad (2.11c)$$

where the last equality rests on the assumption that the result is independent of the order of partial differentiation. Equations (2.4)–(2.6), (2.11b), and (2.11c) clearly do not necessarily lead to the same value for the double-soft limit and illustrate well-known ambiguities or freedom in defining matrix elements involving more than one off-mass-shell pseudoscalar meson. When $W_j(0) = \mathcal{H}(0)$, or any other operator which annihilates the vacuum, then Eqs. (2.11b) and (2.11c) reduce to Eqs. (2.4) and (2.5), respectively, because the second terms of (2.11b) and (2.11c) vanish and Eq. (6) is identically satisfied. Let us, however, consider the case where $W_j = \delta\mathcal{H}' = u_0 + cu_8$, the chiral- and SU(3)-symmetry-breaking part of the hadron energy density in the $(3, 3^*) + (3^*, 3)$ model, which does not necessarily annihilate the vacuum. Let us mean by PCAC the hypothesis that the matrix elements of $\partial A_i \equiv (m_i^2/2f_i)\phi_i$ are smooth enough in momentum transfer so that

$$\langle 0|\partial A_i(0)|P_j(p)\rangle = \frac{m_i^2}{2f_i}\delta_{ij}, \quad 0 \le -p^2 \le m_j^2 . \qquad (2.12)$$

Then

$$\begin{aligned}\langle P_i(0)|\delta\mathcal{H}'|P_k(p')\rangle &= -2if_i\langle 0|[F_i^5, \delta\mathcal{H}']|P_k(p')\rangle \\ &= 2f_i\langle 0|\partial A_i(0)|P_k(p')\rangle \\ &= m_i^2\delta_{ik} = 2f_i\langle 0|\partial A_i|P_k(0)\rangle \\ &= -4f_i f_k\langle 0|[F_k^5, [F_i^5, \delta\mathcal{H}']|0\rangle ,\end{aligned} \qquad (2.13)$$

where the second equality follows from the generalized Heisenberg equations of motion applied to the divergences and the remaining equalities follow from (2.12) and a second reduction. Similarly,

$$\begin{aligned}\langle P_i(p)|\delta\mathcal{H}'|P_k(0)\rangle &= 2f_k\langle P_i(p)|\partial A_k(0)|0\rangle = m_k^2\delta_{ik} \\ &= 2f_k\langle P_i(0)|\partial A_k|0\rangle \\ &= -4f_i f_k\langle 0|[F_i^5, [F_k^5, \delta\mathcal{H}']]|0\rangle .\end{aligned} \qquad (2.14)$$

Thus liberal use of (12) has led us to the strong smoothness conditions[12]

$$\begin{aligned}\langle P_i(0)|\delta\mathcal{H}'|P_k(p')\rangle &= \langle P_i(p)|\delta\mathcal{H}'|P_k(0)\rangle \\ &= \langle P_i(0)|\delta\mathcal{H}'|P_k(0)\rangle\end{aligned} \qquad (2.15)$$

requiring

$$\langle 0 \,|[F_k^5,[F_i^5,\delta\mathcal{H}']]\,|0\rangle = \langle 0\,|[F_i^5,[F_k^5,\delta\mathcal{H}']]\,|0\rangle\,, \tag{2.16}$$

or

$$0 = f_{ikl}\langle 0\,|[F_l,\delta\mathcal{H}']\,|0\rangle = f_{ikl}f_{l8m}\langle 0\,|u_m|0\rangle \tag{2.17}$$

which is certainly satisfied since the vacuum is invariant under space-time translation. Thus Eq. (2.6) is satisfied for the special case when $W_j = \delta\mathcal{H}'$. Also,

$$\langle P_i(p)\,|\delta\mathcal{H}'|\,P_j(p)\rangle$$

is clearly diagonal as a result of assumption (2.12) and takes the well-known form[13]

$$m_{ij}^2 = \delta_{ij}4f_i^2[\tfrac{2}{3} + (\tfrac{2}{3})^{1/2}cd_{ii8}] \times[\langle 0\,|u_0|\,0\rangle + (\tfrac{3}{2})^{1/2}d_{ii8}\langle 0\,|u_8|\,0\rangle]\,. \tag{2.18}$$

We also conclude that, if the PCAC smoothness approximation (12) is good, the second term in Eqs. (11b) and (11c) from the reduction of $\langle P_i\,|\delta\mathcal{H}'|\,P_k\rangle$ must be negligible, which is consistent with the counting of Dashen[9] and Dashen and Weinstein,[9,10] since they are of order δ^2 whereas the double-commutator terms are of order δ. This will be the case so long as there are no scalar poles with mass squared of $O(\delta)$ to reduce to δ the explicit δ^2 dependence of the second term in Eqs. (2.11a)–(2.11c) when $W_j = \delta\mathcal{H}'$. A scalar dilaton whose mass is of $O(\delta)$ would of course interfere with the smoothness.[14] At this stage there still seems to be support for a κ meson, even within the smoothness context, since $\langle 0\,|u_8|\,0\rangle$ is not obviously zero despite departures of (2.18) from the Gell-Mann–Okubo form. It is no coincidence that a singularity with κ quantum numbers cannot contribute to the matrix elements we have so far treated. When W_j, however, carries strangeness, there is a potential candidate for a scalar pole, namely, the κ particles[7] whose mass may be of $O(\delta)$.

To make the effect of the κ more explicit, we can use Eqs. (2.11b) and (2.11c), the Jacobi identities, and the charge algebra to obtain

$$i\delta(x_0)\langle 0\,|[F_m(x_0),\,W_j(0)]\,|\,0\rangle = \int d^4x\langle 0\,|\,T\{W_j(0)[\mathcal{H}(x),\,F_m(x_0)]\}\,|\,0\rangle. \tag{2.19}$$

Now assuming that the SU(3)-breaking part of the Hamiltonian density is the eighth component of an octet, we find

$$f_{mjr}\langle 0\,|\,W_r\,|\,0\rangle = if_{m8p}\int d^4x\langle 0\,|\,T(W_j(0)W_p(x))|0\rangle\,. \tag{2.20}$$

By inserting a complete set of intermediate states in the time-ordered product and saturating with a κ pole, we find the well-known relationship

$$\langle 0\,|\,W_8\,|\,0\rangle = \frac{4}{3}\frac{m_\kappa^{\,2}}{(2f_\kappa)^2} + \text{nonpole terms}\,. \tag{2.21}$$

Thus the difference between extrapolations (2.4), (2.5) and (2.11b), (2.11c) is a $\langle 0\,|\,W_8\,|\,0\rangle$ effect which persists or vanishes in the chiral-symmetry limit according to whether the κ is or is not, respectively, a Goldstone-like particle.

It is usually assumed, even in work where the κ pole is used to dominate the vector divergence,[15] that the (approximate) SU(3) symmetry is realized by an approximately symmetric vacuum, resulting in approximate SU(3) multiplets. In this case $\langle 0\,|\,W_8\,|\,0\rangle$ and $(1/2f_\kappa)^2$ are of second order in SU(3) breaking, while $m_\kappa^{\,2}$ is zeroth order in the breaking parameter. The difference between $1/f_\pi$ and $1/f_K$ is of order SU(3) breaking and one expects the Ademollo-Gatto theorem to apply to $f_+(0)$. In the κ Goldstone interpretation, on the other hand, $\langle 0\,|\,W_8\,|\,0\rangle$ and $m_\kappa^{\,2}$ are of first order in SU(3) breaking, while $1/f_\kappa$, the κ coupling to the weak current, does not vanish in the chiral-symmetry limit. It is of zeroth order in chiral-symmetry breaking as are the weak-decay constants of its Goldstone partners, the π and K (and η); these latter decay constants are not degenerate in the SU(3) limit where the vacuum has SU(2) symmetry, and the Ademollo-Gatto theorem does not apply to $f_+(0)$. Since the κ does not obviously dominate the K_{l3} scalar form factor,[8] there seems to be no utility in regarding it as a Goldstone boson, and we can effectively rule out the second possibility just discussed. The general conclusions regarding the first possibility hold whether or not the κ term is the only important one in Eq. (2.21). That is, the difference between the extrapolations (2.4), (2.5) and (2.11b), (2.11c) should be of order SU(3) $\times$SU(3) breaking, disappearing in the symmetry limit. This view that the PCAC ambiguity is a symmetry-breaking effect is quite compatible with the smoothness condition (2.12), as we shall now argue.

Consider the matrix elements

$$\langle P_i(p)\,|\,u_j(0)\,|\,P_k(p')\rangle\,, \tag{2.22}$$

where u_j is any scalar density belonging to the same $(3.3^*)+(3^*,3)$ representation as $\delta\mathcal{H}'$. We have the soft limit

$$\langle P_i(0)\,|\,u_j(0)\,|\,P_k(p')\rangle = -2if_i\langle 0\,|[F_i^5,u_j]|\,P_k(p')\rangle = -2f_id_{ijk}\langle 0\,|\,v_k|\,P_k(p')\rangle\,, \quad \text{no sum on } k\,. \tag{2.23}$$

In the $(3,3^*)+(3^*,3)$ model, however, v_k is proportional to the axial-vector divergence ∂A_k, where by the smoothness-condition equation (2.12) we have, analogous to conditions (2.15), the natural off-shell identifications

$$\langle P_i(0)|u_j(0)|P_k(p')\rangle = -2f_i d_{ijk}\langle 0|v_k|P_k(0)\rangle = \langle P_i(0)|u_j(0)|P_k(0)\rangle \quad (2.24)$$

and

$$\langle P_i(p)|u_j(0)|P_k(0)\rangle = -2f_k d_{ijk}\langle P_i(0)|v_i|0\rangle = \langle P_i(0)|u_j(0)|P_k(0)\rangle\,. \quad (2.25)$$

It is clear, however, that (2.24) and (2.25) are inconsistent with $\langle 0|u_8|0\rangle \neq 0$ and κ PCVC. One can see this formally by observing that to have (2.24) and (2.25) we must have (2.6) with $W_j = u_j$. But this implies that

$$0 = f_{ikl}\langle 0|[F_l, u_j]|0\rangle = -f_{ikl}f_{ljm}\langle 0|u_m|0\rangle\,,$$

which implies that $\langle 0|u_8|0\rangle = 0$,[16] to within the PCAC smoothness assumptions. Again the smoothness condition (2.12) has led us to drop the second terms in Eqs. (2.11b) and (2.11c), and has prevented κ from playing a Goldstone role with $m_\kappa^2 = O(\delta)$, $\langle 0|u_8|0\rangle = O(\delta)$, and $f_\kappa^{-1} = O(1)$.

We can understand this limitation on κ in another way by strongly restricting the momentum-transfer variation of vector-divergence matrix elements. Let us examine the matrix element

$$\langle K_i(p)|u_j(0)|\pi_k(p')\rangle\,, \quad p^2 = -m_k^2, \quad p'^2 = -m_\pi^2\,, \quad (2.26)$$

where K_i means kaon and π_i means pion.[17] κ PCVC would take the form

$$\partial V_j = \frac{m_\kappa^2}{2f_\kappa}\phi_j = c f_{j8l}u_l \quad (j=4,5,6,7)\,, \quad (2.27)$$

which implies

$$c f_{845}\langle K_i(p)|u_4|\pi_k(p')\rangle = \langle K_i(p)|\phi_\kappa^5|\pi_k(p')\rangle m_\kappa^2/2f_\kappa = \frac{m_\kappa^2}{2f_\kappa}\,\frac{\langle K_i(p)|J^\kappa(0)|\pi_k(p')\rangle}{m_\kappa^2+(p-p')^2}\,, \quad (2.28)$$

where J^κ is the κ source current whose matrix element should be slowly varying if κ PCVC is to be on the same footing as π and K PCAC. But this implies a large momentum dependence for (2.26), inconsistent with previous assumptions. There is no reason, however, which prevents a κ meson from existing and coupling linearly to the vector divergence provided its coupling vanishes and $\langle 0|u_8|0\rangle/\delta$ vanishes in the chiral-symmetry limit. A scheme consistent with Eq. (2.12), for example, is the following:

$$m_\kappa^2 = O(1)\,,$$
$$\langle 0|u_8|0\rangle = O(\delta^2)\,,$$
$$f_\kappa^{-1} = O(\delta)\,,$$

in which case κ effects are one order of δ down from PCAC effects, and the different off-mass-shell definitions coincide in lowest order. In such a picture, we have for the axial-vector divergences (following Dashen[9])

$$\langle A|\partial A_i|B\rangle = \frac{m_i^2}{m_i^2+q^2}\,\frac{G_{ABi}}{2f_i} + O(\delta) = \begin{cases} O(1) & q^2\to 0\,,\\ O(\delta) & \text{otherwise}\,,\end{cases} \quad (2.29)$$

whereas for vector-divergence matrix elements we have

$$\langle C|\partial V^\kappa|D\rangle = \frac{m_\kappa^2}{m_\kappa^2+q^2}\,\frac{G'_{CD\kappa}}{2f_\kappa} + O(\delta) = O(\delta) \text{ independent of } q^2\,. \quad (2.30)$$

In other words, even if the κ contribution varies rapidly for $q^2 \approx 0$, there is no reason to believe it is the dominant contribution to the matrix element for small q^2 and hence no reason to insist that the matrix element itself is varying rapidly. Thus for small δ in this scheme, the PCAC corrections, which are of order δ, can be neglected as $q^2\to 0$, in which limit the pole term is of order unity. The smoothness hypothesis is merely a statement that the correction term can be neglected even for $q^2 \neq 0$ but small. The matrix elements of the vector divergence, on the other hand, behave in an essentially different way in the chiral-symmetry limit. They are of order δ, independent of q^2, and are of the same chiral-symmetry-breaking order as PCAC correction terms such as 3π or $K\pi\pi$ continuum contributions. For internal consistency, therefore, κ or $\langle 0|u_8|0\rangle$ effects should be neglected in calculations in which strong smoothness conditions are invoked which require the off-mass-shell amplitude definitions to coincide and result in the "smoothness" of (2.24) and (2.25).

Of course, we cannot claim that we have derived the smoothness conditions or the requirement that off-mass-shell definitions coincide at the double-soft limit, but only that we have correlated them with a dynamical scheme in which the vacuum is approximately SU(3)-symmetric and in which κ effects are an order of chiral-symmetry breaking down from PCAC effects. [Alternatively, of course, one could reject the strong smoothness conditions and the concomitant requirement that the alternative off-mass-shell definitions coincide in lowest order, as well as the approximate SU(3) symmetry of the vacuum, lowering it even in the chiral-symmetry limit to SU(2).] We find our scheme, however, an attractive possibility consistent with the apparent asymmetry with which axial- and vector-charge conservation is achieved in the symmetry limit, and suggestive of a parallel asymmetry in the way the axial-vector and vector divergences behave when chiral symmetry is broken.

In Sec. III we pursue this picture by constructing an explicit model in which the κ meson behaves in this way.

III. A MODEL

Consider a Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu u_i \partial_\mu u_i + \partial_\mu v_i \partial_\mu v_i) - \mathcal{L}_{\text{stat}}(u_i, v_i), \tag{3.1}$$

where $\mathcal{L}_{\text{stat}}$ is a polynomial in the fields u_i and v_i, which span a $(3, 3^*) + (3^*, 3)$ representation[18] of $SU(3)\times SU(3)$; and suppose $\tilde{u}_i = \langle 0|u_i|0\rangle$ and $\tilde{v}_i = \langle 0|v_i|0\rangle$ are solutions to the extremum condition $\partial\mathcal{L}/\partial u_i = 0$, $\partial\mathcal{L}/\partial v_i = 0$. Then by familiar techniques[19] we have

$$\begin{aligned} \partial A_i &= (\tfrac{2}{3})^{1/2}(\langle 0|u_0|0\rangle + d_{8ii}\langle 0|u_8|0\rangle) m_i^2 v_i + \cdots, \\ \partial V_i &= f_{8ij}\langle 0|u_8|0\rangle M_j^2 u_j + \cdots, \end{aligned} \tag{3.2}$$

where only the terms linear in the fields have been kept and at most vacuum expectations of u_0 and u_8 have been allowed.[20] Identifying the decay constants we have

$$\begin{aligned} 1/2f_i &= (\tfrac{2}{3})^{1/2}\langle 0|u_0|0\rangle + d_{8ii}\langle 0|u_8|0\rangle, \\ 1/2f_\kappa &= \tfrac{1}{2}\sqrt{3}\,\langle 0|u_8|0\rangle. \end{aligned} \tag{3.3}$$

In Eq. (3.2) the pseudoscalar and scalar masses are given by

$$m_i^2 = \frac{\partial^2\mathcal{L}_{\text{stat}}}{\partial v_i \partial v_i}\,, \quad M_i^2 = \frac{\partial^2\mathcal{L}_{\text{stat}}}{\partial u_i \partial u_i}\,. \tag{3.4}$$

If $\mathcal{L}$ is linear in a chiral-symmetry-breaking parameter δ and if the ground-state solutions have the form

$$\begin{aligned} \langle 0|u_0|0\rangle &\equiv \lambda = O(1), \\ \langle 0|u_8|0\rangle &\equiv \lambda' = O(\delta), \end{aligned} \tag{3.5}$$

then the ground state is SU(3)-symmetric in the chiral limit but SU(2)-symmetric otherwise, and

$$\begin{aligned} 1/2f_i &= O(1), \\ 1/2f_i - 1/2f_j &= O(\delta), \\ 1/2f_\kappa &= O(\delta). \end{aligned} \tag{3.6}$$

This is the form of the solution we shall find, which also must have

$$\begin{aligned} m_i^2 &= O(\delta), \\ m_\kappa^2 &= O(1) \end{aligned} \tag{3.7}$$

in order that the divergences be of order δ. It is at this point necessary to distinguish between the basic fields u_i and v_i and the Hamiltonian densities which appear in the decomposition of $\mathcal{H}(x)$ in the $(3, 3^*) + (3^*, 3)$ model. We denote these by u_0' and u_8' so that

$$\mathcal{H} = \mathcal{H}_{\text{sym}} - (u_0' + u_8' c), \tag{3.8}$$

where $\mathcal{H}_{\text{sym}}$ is a chiral singlet. A polynomial Lagrangian model with the desired properties is given by

$$\begin{aligned} \mathcal{L}_{\text{stat}} &= \overline{\mathcal{L}} - \Delta\mathcal{L}, \\ \overline{\mathcal{L}} &= \alpha I_2 + \beta I_3^+ + \gamma I_4, \end{aligned} \tag{3.9}$$

where

$$I_2 = u_i^2 + v_i^2,$$

$$\begin{aligned} I_3^+ = {} & 4d_{ijk}(u_i u_j u_k - 3v_i v_j u_k) - 6\sqrt{6}\, u_0 u_i u_i \\ & + 6\sqrt{6}\, u_0 v_i v_i + 12\sqrt{6}\, v_0 v_i u_i + 6\sqrt{6}\, u_0^3 - 18\sqrt{6}\, v_0^2 u_0, \end{aligned}$$

$$\begin{aligned} I_4 = {} & 2(d_{ijn} d_{nkl} + i f_{ijn} d_{nkl} + i d_{ijn} f_{nkl} - f_{ijn} f_{nkl}) \\ & \times (u_i u_j u_k u_l + v_i v_j v_k v_l + 4u_i u_j v_k v_l - 2u_i v_j u_k v_l), \end{aligned}$$

$$\Delta\mathcal{L} = \delta[a(u_0 + c u_8) + d(U_0 + c U_8)],$$

where U_0 and U_8 are members of the $(3, 3^*) + (3^*, 3)$ decomposition[21] of the tensor product of the basic multiplet with itself,

$$\begin{aligned} U_i &= 18(\tfrac{2}{3})^{1/2}\,\delta_{i0}(u_0^2 - v_0^2) - 12(\tfrac{2}{3})^{1/2}(u_0 u_i - v_0 v_i) - 6(\tfrac{2}{3})^{1/2}\,\delta_{i0}(u_j^2 - v_j^2) + 4d_{ijk}(u_j u_k - v_j v_k), \\ V_i &= -36(\tfrac{2}{3})^{1/2}\,\delta_{i0} u_0 v_0 - 12(\tfrac{2}{3})^{1/2}\,\delta_{i0} u_j v_j - 12(\tfrac{2}{3})^{1/2}(u_i v_0 + u_0 v_i) + 8d_{ijk} u_j v_k. \end{aligned} \tag{3.10}$$

This model has been discussed elsewhere in the context of symmetry breaking with $\langle 0|u_8|0\rangle = 0$.[22] The extremum equations are

$$\begin{aligned} 0 &= 2\alpha\lambda + 4\sqrt{6}\,\beta\lambda^2 - 2\sqrt{6}\,\lambda'^2\beta + 8\gamma[\tfrac{2}{3}\lambda^3 + 2(\lambda')^2\lambda - \tfrac{1}{3}\sqrt{2}\,\lambda'^3] - \delta a - 8\delta d(\tfrac{2}{3})^{1/2}\lambda + 4\delta dc(\tfrac{2}{3})^{1/2}\lambda', \\ 0 &= 2\alpha\lambda' - 4\sqrt{6}\,\beta\lambda'\lambda - 12(\tfrac{1}{3})^{1/2}\beta\lambda'^2 + 8\gamma[2\lambda^2\lambda' - \sqrt{2}\,(\lambda')^2\lambda + \lambda'^3] - \delta ca + 4\delta d(\tfrac{2}{3})^{1/2}\lambda' + 4(\tfrac{2}{3})^{1/2}\delta dc\lambda + 8(\tfrac{1}{3})^{1/2}\delta dc\lambda', \end{aligned} \tag{3.11}$$

where $\lambda = \langle 0|u_0|0\rangle$ and $\lambda' = \langle 0|u_8|0\rangle$. The relations between (u_0', u_8') and (u_0, u_8) are

$$u_0' = \delta(au_0 + dU_0)\,,\qquad u_8' = \delta(au_8 + dU_8)\,. \tag{3.12}$$

Equations (3.11) have several solutions which differ in their residual symmetry in the $\delta = 0$ limit. The solution we are interested in $[\langle 0|u_8|0\rangle = O(\delta)]$, expanded to lowest order in δ, is

$$\lambda = \lambda_s - \frac{\delta}{4}\,\frac{a + 8d(\frac{2}{3})^{1/2}\lambda_s}{\alpha + \sqrt{6}\,\beta\lambda_s}\,,\qquad \lambda' = -\frac{\delta c}{4}\,\frac{a - 4d(\frac{2}{3})^{1/2}\lambda_s}{\alpha + 4\sqrt{6}\,\beta\lambda_s}\,, \tag{3.13}$$

where

$$\lambda_s = -\frac{1}{2}\left(\frac{3}{2}\right)^{3/2}\left[\frac{\beta}{\gamma} + \left(\frac{\beta^2}{\gamma^2} - \frac{2}{3}\frac{\alpha}{\gamma}\right)^{1/2}\right]\,.$$

It is clear that

$$\langle 0|u_0'|0\rangle = O(\delta)\,,\qquad \langle 0|u_8'|0\rangle = O(\delta^2)\,. \tag{3.14}$$

Moreover, one can readily verify that

$$m_i^{\,2} = (2f_i)^2[\tfrac{2}{3} + (\tfrac{2}{3})^{1/2}cd_{8ii}][\langle 0|u_0'|0\rangle + (\tfrac{3}{2})^{1/2}d_{8ii}\langle 0|u_8'|0\rangle]\,,$$
$$m_\kappa^{\,2} = \tfrac{3}{4}(2f_\kappa)^2\langle 0|u_8'|0\rangle\,. \tag{3.15}$$

It is clear that for this solution $m_i^2 = O(\delta)$, departures of m_i^2 from the Gell-Mann–Okubo form are $O(\delta^2)$, and $m_\kappa^{\,2} = O(1)$. The model solution has the GW structure but differs from the GOR scheme only by higher orders of the symmetry breaking. The κ mass persists in the model when the chiral symmetry breaking is turned off. It is not a Goldstone particle in this limit, and its mass stands in no particular relation to the pseudoscalar-meson masses, nor does its mass vanish or tend to ∞ in the SU(3) limit of $\mathcal{L}$ ($c = 0$), where the vacuum solution, Eq. (3.13), tends to SU(3) symmetry also. In this model solution the κ plays no special role in symmetry breaking in the symmetry limit[23] and its PCVC condition is therefore essentially different from the PCAC conditions in just the way which we discussed in Sec. II.

IV. CONCLUDING REMARKS

We studied the implications of a PCAC smoothness hypothesis and observed that it was not possible in this context to put κ PCVC on the same footing as PCAC, concluding that κ effects are of the same order as PCAC corrections. We discussed this situation in terms of vacuum expectation values of densities in the $(3, 3^*) + (3^*, 3)$ model and decided that, to within the smoothness approximation, κ and $\langle 0|u_8|0\rangle$ effects should be ignored, but that κ PCVC could be accommodated in a scheme in which it plays a subordinate role, contributing at the level of PCAC corrections. This discussion was facilitated by the counting procedure of Dashen[9,10] which enabled us to describe a chiral-symmetry-breaking scheme which takes the Gell-Mann–Oakes–Renner[5] form in lowest order of chiral-symmetry breaking and admits the κ effects in the next order. Finally we constructed a simple model for illustrative purposes in which the κ meson plays such a symmetry-breaking role. In this model the κ meson is not a Goldstone particle in the SU(3) or SU(3)×SU(3) limit, is still coupled linearly to the vector divergence, but does not dominate the vector divergence for small q^2 in the sense that the pion dominates the axial-vector divergence, even for small $m_\kappa^{\,2}$. This is compatible with the current experimental status of the scalar form factor for K_{l3} decay,[8] which does not seem to be rising at $q^2 = 0$ as it would if the κ were strongly influencing its behavior.[24]

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$$\langle P_i(p)|S(0)|P_k(p')\rangle = \langle P_i(0)|S(0)|P_k(p')\rangle\,,$$

with $p^2 = -m_i^{\,2}$ and $p'^2 = -m_k^{\,2}$ and where $S(0) = \delta\mathcal{H}'$, $u_i(0)$.

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[16]This vanishing of the vacuum expectation value of the SU(3)-breaking operator in the Hamiltonian is not dependent on the $(3^*,3)+(3,3^*)$ model.

[17]The matrix element (2.23) in the $(3^*,3)+(3,3^*)$ model is proportional to $F_0(q^2)/(m_K{}^2-m_\pi{}^2)$, where F_0 is the S-wave K_{l3} form factor.

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[20]It is clearly necessary to retain the symmetry-breaking piece in $\mathcal{L}_{\text{stat}}$ in order to obtain nonvanishing divergences. A less obvious point is that the symmetry breaking must be retained in solving the extremum conditions for the ground state if the dependence of the vacuum breaking on the Lagrangian symmetry-breaking piece is to be understood (see Ref. 22).

[21]A bilinear $(1,8)+(8,1)$ contribution to the symmetry-breaking part of the Lagrangian has recently been suggested as an interesting possibility [R. Arnowitt, M. A. Friedman, P. Nath, and R. Suitor, Phys. Rev. Letters 26, 104 (1971); W. F. Palmer, O. S. U. Report No. COO-1545-82 (unpublished)]. The possible roles of the κ in such a symmetry-breaking scheme are the same as in the model under discussion, and our remarks about the solutions characterized by (3.6) and 3.7) remain the same.

[22]W. F. Palmer, Ref. 21.

[23]Of course the κ may contribute a pole term in the vector-divergence form factor, but it does not dominate the form factor in the sense that the pseudoscalar poles dominate the axial-vector-divergence form factors.

[24]The indicated behavior for the scalar form factor is also in disagreement with the Dashen-Weinstein theorem (Ref. 10) if $f_\pi/f_K \equiv F_K/F_\pi > 1$, since the theorem then predicts a positive slope for the scalar form factor. The present experimental value $F_K/F_\pi\, f_+(0) = 1.23$ can only give $F_K/F_\pi < 1$, and thus agreement with the Dashen-Weinstein theorem, if $f_+(0) \approx 0.75$. In either case, large higher-order symmetry-breaking corrections are indicated and the utility of the Dashen-Weinstein idea of treating the SU(3)×SU(3) symmetry-breaking parameter in the strong Hamiltonian as a perturbative parameter is in doubt.

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Divergent Vertices and Anomalous Dimensions*

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A study of scale-invariance breaking and its connection with anomalous dimensions is presented. A new relation between the anomaly in scale dimension of the pion field and the low-energy form of its gravitational vertex function is obtained, without the use of perturbation theory.

I. INTRODUCTION

Recently, study of the behavior of strong interactions under scale transformations has shown that such analyses can be extremely useful in discussing problems in quantum field theory connected with the renormalization group and in elucidating the structure of the strong-interaction Hamiltonian.[1] Broken scale invariance at low energies has proceeded mainly by means of a Goldstone realization of the symmetry,[2-5] with one interesting exception.[6] A major aim of these authors has been to evaluate the dimension d of the scale-symmetry-breaking term in the strong-interaction Hamiltonian.[7] Study of a formal theory of scale-symmetry breaking by considering the high-energy behavior of Green's functions,[8-10] backed by perturbation-theory arguments, suggests that the scale dimension of the pion field differs from its canonical value of 1 in the presence of interactions and mass terms. The anomaly arises from the breakdown of certain theorems about the high-energy behavior of Green's functions. When modifications are made to the energy-momentum tensor,[11] it is shown that the dimension differs from the canonical value by a power series in the coupling con-