$$\int_{-\infty}^{+\infty} S^{-1}(t)\{t[P_0^i,H_I(t)] - iP_I^i(t)\}S(t)dt=0\,.$$

Clearly this condition goes beyond the Poincaré algebra.

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[12]Throughout this work particles will be on the mass shell. The three-vector character of the momenta will be stressed sometimes by the explicit use of vector notation; see Eqs. (43) and (44).

[13]In the last terms of Eq. (47) the connectivity of the $\phi^{3\prime}$ interaction has been used to reduce the five-particle intermediate-state contribution.

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Feynman Rules for the Yang-Mills Field: A Canonical-Quantization Approach. II*

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The purpose of the present paper is to suggest noncovariant Feynman rules for the Yang-Mills field in the radiation gauge, to all orders with any number of loops using the methods of canonical quantization. It is shown that even though the interaction Hamiltonian in the radiation gauge has an infinite number of terms, the Feynman rules contain the usual three-vector and four-vector vertices, and a vector-scalar-scalar vertex, along with one propagator for the gauge particle and another propagator for the fictitious scalar particle. The fictitious particle, however, appears only within loops, and the scalar loop has an extra factor (−2) relative to the corresponding vector loop.

I. INTRODUCTION

In a recent paper,[1] the canonical quantization procedure was used to study the Yang-Mills field[2] in the radiation gauge to obtain the covariant Feynman rules[3] for the field. In paper I, we showed that even though the interaction Hamiltonian in the radiation gauge consists of an infinite number of terms, the noncovariant Feynman rules for the tree diagrams can be described by means of a three-vertex and a four-vertex along with two noncovariant propagators. With the help of these rules, the covariance of the tree diagrams was demonstrated to all orders. The purpose of the present paper is to prove the noncovariant Feynman rules to all orders in the coupling constant, with any number of loops. We present the detailed combinatorial analysis leading to these rules. In Sec. II, we present a theorem which we will prove in Secs. III and IV (for the tree diagrams in Sec. III and for the loops in Sec. IV). What the theorem says is that the noncovariant Feynman rules with any number of loops are the same as the ones for tree diagrams (paper I), except that the loops with all three vertices (n) and all propagators of type $-i\eta_\mu\eta_\nu/[k^2-(k\cdot\eta)^2]$ must be multiplied by a coefficient $1-1/2^{n-1}$; all other loops have the coefficients given by the symmetry of that loop. In Sec. V, we show that if we introduce a vector-scalar-scalar loop and a scalar propagator, the resulting noncovariant Feynman rules can be restated, giving the main result of our paper.

Feynman rules for the Yang-Mills field in the radiation gauge are, therefore, the following (see Fig. 1):

(1) Propagator for the gauge field [Fig. 1(a)]:

$$G^{ab}_{\mu\nu}(k)=\frac{-i\delta_{ab}}{k^2-i\epsilon}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu-k\cdot\eta(k_\mu\eta_\nu+k_\nu\eta_\mu)}{k^2-(k\cdot\eta)^2}\right).$$

(2) Three-vertex involving gauge fields only [Fig. 1(b)]:

$$\Gamma^{abc}_{\mu\nu\lambda}(p,k,q)=(-g)\epsilon_{abc}[\delta_{\mu\nu}(p-k)_\lambda+\delta_{\nu\lambda}(k-q)_\mu+\delta_{\lambda\mu}(q-p)_\nu].$$

(3) Four-vertex involving gauge fields only [Fig. 1(c)]:

$$\Gamma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4)=-ig^2[\epsilon_{aa_1a_2}\epsilon_{aa_3a_4}(\delta_{\mu_1\mu_3}\delta_{\mu_2\mu_4}-\delta_{\mu_1\mu_4}\delta_{\mu_2\mu_3})+\epsilon_{aa_1a_3}\epsilon_{aa_4a_2}(\delta_{\mu_1\mu_4}\delta_{\mu_3\mu_2}-\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4})$$
$$+\epsilon_{aa_1a_4}\epsilon_{aa_2a_3}(\delta_{\mu_1\mu_2}\delta_{\mu_4\mu_3}-\delta_{\mu_1\mu_3}\delta_{\mu_4\mu_2})].$$

(4) Vector-scalar-scalar vertex [Fig. 1(d)]:

$$\Gamma^{abc}_{\mu}(p,k,q)=-\tfrac{1}{2}g\epsilon_{abc}(k-q)_\nu(\delta_{\mu\nu}-\eta_\mu\eta_\nu)$$
$$=-\tfrac{1}{2}g\epsilon_{abc}\,\delta_{\mu i}(k-q)_i\,.$$

(5) Propagator for the scalar particle [Fig. 1(e)]:

$$G^{ab}(k)=\frac{-i\delta_{ab}}{k^2-(k\cdot\eta)^2},$$

where $\eta_\mu=(0,0,0,1)$ in the frame in which we are working.

No extra coefficient need multiply the diagrams, except that the scalar loop must be multiplied by a factor (−2) in all cases.

II. A THEOREM

It was noted in I that the interaction Hamiltonian for the Yang-Mills field in the interaction representation is given as follows:

$$\mathcal{H}_I=\tfrac{1}{2}g\,\vec{g}_{\mu\nu}\cdot\vec{b}_\mu\times\vec{b}_\nu+\tfrac{1}{4}g^2\vec{b}_\mu\times\vec{b}_\nu\cdot\vec{b}_\mu\times\vec{b}_\nu+\mathcal{H}_I'+\mathcal{H}_s, \tag{1}$$

where

$$\mathcal{H}_I'=-\tfrac{1}{2}g^2\sum_{n=0}^{\infty}(n+1)\vec{\chi}\cdot(-g\Delta^{-1}M)^n\Delta^{-1}\vec{\chi}, \tag{2}$$

$$\mathcal{H}_s=\tfrac{1}{8}g^2\epsilon_{abc}\,\partial_k\mathfrak{D}'_{cd}(x,x,t)\epsilon_{dbe}\partial_k\,\mathfrak{D}_{ea}(x,x,t), \tag{3}$$

where

$$\vec{\chi}=\vec{b}_\mu\times\partial_0\vec{b}_\mu,\quad M=\vec{b}_\mu\times\partial_\mu,\quad(\Delta+gM)\mathfrak{D}(x,y,t)=\delta^3(x-y), \tag{4}$$

and all the rest of the notation is the same as in I.[4] Note that $\mathcal{H}_s$ is the term, first discovered by Schwinger,[5] which is required if the field theory has to satisfy the requirements of Lorentz covariance.[6] In the present paper, we ignore the term $\mathcal{H}_s$ and concentrate on the rest of the interaction Hamiltonian. Moreover, making a free-field expansion for the fields in the interaction representation, we also showed that

$$\langle 0|T(b^a_\mu(x,t)b^b_\nu(x',t'))|0\rangle=\frac{-i\delta_{ab}}{(2\pi)^4}\int\frac{e^{ik\cdot(x-x')}}{k^2-i\epsilon}d^4k\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu-k\cdot\eta(k_\mu\eta_\nu+k_\nu\eta_\mu)+k^2\eta_\mu\eta_\nu}{k^2-(k\cdot\eta)^2}\right), \tag{5}$$

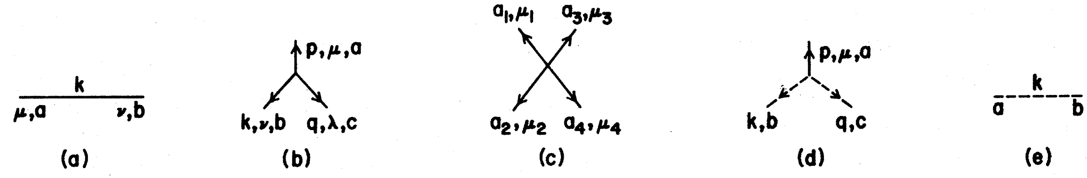

FIG. 1. Noncovariant Feynman rules for the Yang-Mills field in the radiation gauge are presented in (a)−(e). The scalar particle can occur only within a loop and this loop must be multiplied by a factor (−2).

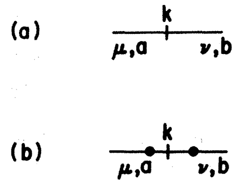

FIG. 2. These propagators are used for the sake of convenience in proving the theorem in Sec. II. In the text, (a) and (b) are referred to as D- and F-type propagators, respectively.

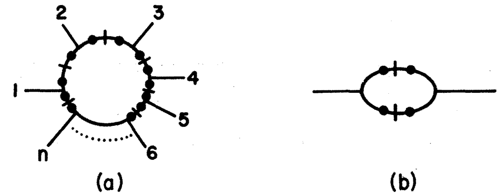

FIG. 3. (a) represents a closed loop with n three-vertices and all F-type propagators. Such a loop must be multiplied by $(1-1/2^{n-1})$ if $n>2$ and (b) must be multiplied by $\frac{1}{4}$.

$$\langle 0|T(\partial_\alpha b_\mu^a(x,t)b_\nu^b(x',t'))|0\rangle=\frac{\delta_{ab}}{(2\pi)^4}\int\frac{e^{ik\cdot(x-x')}}{k^2-i\epsilon}k_\alpha d^4k\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu-k\cdot\eta(k_\mu\eta_\nu+k_\nu\eta_\mu)+k^2\eta_\mu\eta_\nu}{k^2-(k\cdot\eta)^2}\right), \qquad (6)$$

$$\langle 0|T(\partial_\alpha b_\mu^a(x,t)\partial_\beta b_\nu^b(x',t'))|0\rangle=\frac{-i\delta_{ab}}{(2\pi)^4}\int\frac{e^{ik\cdot(x-x')}}{k^2-i\epsilon}d^4k(k_\alpha k_\beta-k^2\eta_\alpha\eta_\beta)\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu-k\cdot\eta(k_\mu\eta_\nu+k_\nu\eta_\mu)+k^2\eta_\mu\eta_\nu}{k^2-(k\cdot\eta)^2}\right), \qquad (7)$$

where $\eta_\mu=(0,0,0,1)$ and denotes the frame dependence of the propagator.

It is clear that the $\eta_\alpha\eta_\beta$ term in Eq. (7) will generate new terms when a Dyson-Wick expansion of the S matrix is performed. This extra term helps in the rearrangement of terms in the Dyson-Wick expansion so as to give the following intermediate result, from which in Sec. V we will prove the main result of the present paper, as described in the Introduction.

Theorem. The effective interaction Hamiltonian for the Yang-Mills field in the radiation gauge, ignoring $\mathcal{H}_s$, can be taken as

$$\mathcal{H}_I=-\mathcal{L}_{\text{int}}=\tfrac{1}{2}g\vec{g}_{\mu\nu}\cdot\vec{b}_\mu\times\vec{b}_\nu+\tfrac{1}{4}g^2\vec{b}_\mu\times\vec{b}_\nu\cdot\vec{b}_\mu\times\vec{b}_\nu\,, \qquad (8)$$

along with the following two propagators in momentum space:

$$D_{\mu\nu}^{ab}(k)=\frac{-i\delta_{ab}}{k^2-i\epsilon}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu-k\cdot\eta(k_\mu\eta_\nu+k_\nu\eta_\mu)+k^2\eta_\mu\eta_\nu}{k^2-(k\cdot\eta)^2}\right)\ \text{[shown in Fig. 2(a)]} \qquad (9)$$

and

$$F_{\mu\nu}^{ab}(k)=-i\delta_{ab}\frac{\eta_\mu\eta_\nu}{k^2-(k\cdot\eta)^2}\ \text{[shown in Fig. 2(b)]}. \qquad (10)$$

Furthermore, a diagram with all F propagators and n three-vertices and no four-vertex must be multiplied by a coefficient $1-1/2^{n-1}$ (shown in Fig. 3). It is, of course, well known that a loop with only two vertices is to be multiplied by an extra factor of $\frac{1}{2}$ if it contains either both F-type or both D-type propagators; the factor $\frac{1}{2}$ is the inverse of the so-called symmetry number of the diagram as defined in Ref. 7. (See Fig. 4.)

It must be stressed at this point that we have written the rules in the above way mainly to make the proof more understandable. This final set of rules given in the Introduction will be proved in Sec. V.

III. TREE DIAGRAMS

Our aim, in this section and Sec. IV, is to substantiate the noncovariant rules given in Sec. II. For that purpose, let us observe a few important things: Whereas the D-type propagator in a diagram arises when there is contraction between two field operators, in the Dyson-Wick expansion of the S matrix, the F-type

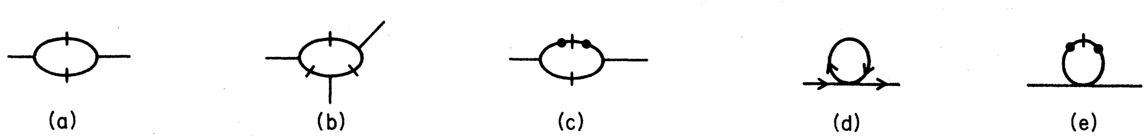

FIG. 4. Using the definition in Ref. 7, one can show that (a) has a symmetry number $S_n=2$ and, therefore, should be multiplied by $\frac{1}{2}$, whereas (b) and (c) have symmetry number $S_n=1$ and therefore should be multiplied by 1. Furthermore, whenever in a diagram loops of the type shown in (d) and (e) appear, each of them should be multiplied by a factor of $\frac{1}{2}$.

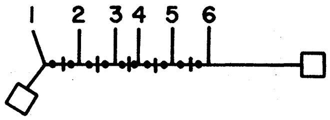

FIG. 5. A typical part of a Feynman diagram with five consecutive F-type propagators.

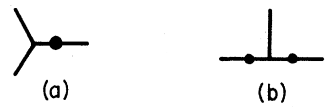

FIG. 6. (a) represents $\eta_\mu \Gamma^{abc}_{\mu\nu\lambda}(p,q,k)\,\epsilon_\lambda(k)\epsilon_\nu(q)$, and (b) represents $\eta_\mu\eta_\nu\Gamma^{abc}_{\mu\nu\lambda}(p,q,k)\epsilon_\lambda(k)$.

propagator arises solely from the inverse Laplacians contained in $\mathcal{H}_I'$. Therefore, in any Feynman diagram, a part which contains a chain of F-type propagators attached to three- and four-vertices comes entirely from one term in $\mathcal{H}_I'$. For example, if we have a series of five F-type propagators sandwiched between six three-vertices (Fig. 5), they can come from the following term:

$$C_4 g^6 \vec{\chi}\cdot(\Delta^{-1}M)^4\Delta^{-1}\vec{\chi}, \tag{11}$$

where C_4 is some numerical coefficient.

To understand this more clearly, we have to know what the momentum space representation of various factors like $\vec{\chi}$ and M are. Notice that

$$\vec{\chi} \equiv \vec{b}_\mu \times \partial_0 \vec{b}_\mu \tag{12}$$

in momentum space corresponds to $\epsilon_\nu(q)\epsilon_\lambda(k)\eta_\mu\Gamma^{abc}_{\mu\nu\lambda}(p,q,k)$ [shown in Fig. 6(a)] and M in momentum space[4] corresponds to $\frac{1}{2}\eta_\lambda\eta_\sigma\epsilon_\mu(p)\Gamma^{abc}_{\mu\lambda\sigma}(p,q,k)$ [shown in Fig. 6(b)]. Δ^{-1} corresponds to $1/[k^2-(k\cdot\eta)^2]$ in momentum space. Therefore, it is clear that, if $C_4=2^3$, then the expression (11) would generate diagrams like those in Fig. 5. [It might seem at first that $C_4=2^4$, but actually a factor of 2 appears because there are two equivalent ways of connecting the expression (11) to external lines or parts of a diagram.] Therefore, we can conclude at once that a diagram such as Fig. 5 with $n-1$ F-type propagators could come only from a term in $\mathcal{H}_I'$ of the following type (see Fig. 7):

$$2^{n-3}g^n\vec{\chi}\cdot(\Delta^{-1}M)^{n-2}\Delta^{-1}\vec{\chi}. \tag{13}$$

Looking at $\mathcal{H}_I'$ in Eq. (2), we find that even though there exists such a term in it, the coefficient of this term is $\frac{1}{2}(n-1)$ instead of 2^{n-3}. So, to prove the rules, we must show that the extra $\eta_\alpha\eta_\beta$ term in Eq. (7) really contributes in such a way as to generate this factor.

Moreover, according to the theorem, since the F-propagator can occur in conjunction with all types of vertices, it certainly can accompany a four-vertex. However, if only one leg of a four-vertex has an F-type propagator and the other three legs are either external particles or D-type propagators, using the expression in Fig. 1(c) for the four-vertex, one can see that the corresponding diagram will vanish, because

$$\eta\cdot\epsilon=0 \quad\text{and}\quad \eta_\mu D_{\mu\nu}(k)=0. \tag{14}$$

Similarly, a four-vertex which has three or four of its legs attached to F-type propagators can also be seen to vanish. However, when two legs have F-type propagators and the other two legs are either external or have D-type propagators (Fig. 8), the four-vertex makes a nonzero contribution. This, therefore, must come from $\mathcal{H}_I'$, only after using the extra term in Eq. (7), because the term whose momentum-space representation corresponds to Fig. 8 is

$$\Delta^{-1}\vec{b}_\mu\times(\vec{b}_\mu\times\Delta^{-1}), \tag{14$'$}$$

and clearly no such term occurs in $\mathcal{H}_I'$ as given in Eq. (2). Therefore, in general, diagrams of the type shown in Fig. 9 (where all propagators are F-type and, respectively, $r_1, r_2, r_3, \ldots, r_s$ three-vertices appear between four-vertices) can come from the following term:

$$g^{(r_1+r_2+\cdots+r_s+2s+2)}\,2^{r_1+r_2+\cdots+r_s-1}\int :\vec{\chi}\Delta^{-1}K(\Delta^{-1}M)^{r_1}\Delta^{-1}K(\Delta^{-1}M)^{r_2}\Delta^{-1}K\cdots(\Delta^{-1}M)^{r_s}\Delta^{-1}K\Delta^{-1}\vec{\chi}:\, d^4x, \tag{15}$$

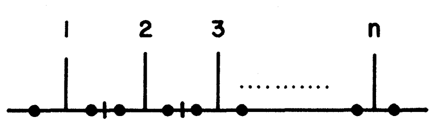

FIG. 7. Momentum-space representation of $2^{n-3}: -g(\Delta^{-1}M)^n:$.

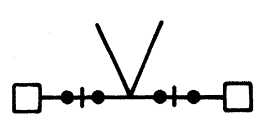

FIG. 8. Diagrammatic representation of $\dfrac{\eta_\alpha\eta_\lambda}{k^2-(k\cdot\eta)^2}\Gamma^{abcd}_{\lambda\mu\nu\sigma}\dfrac{\eta_\alpha\eta_\beta}{p^2-(p\cdot\eta)^2}$.

where

$$K\vec{f}=\vec{b}_\mu\times(\vec{b}_\mu\times\vec{f})\,. \tag{15'}$$

Now let us see where these terms come from. Notice that $\vec{\chi}=\vec{b}_\mu\times\partial_0\vec{b}_\mu$, i.e., it contains the time derivative of the field operator. Therefore, a time-ordered product which contains two $\vec{\chi}$'s will, after Wick expansion, always have a term where the $\partial_0\vec{b}_\mu$'s belonging to them are contracted. In that case, Eq. (7) tells us that apart from the first term, the $\eta_\alpha\eta_\beta$ term will also be nonzero. When this extra term is taken into account, one gets new types of terms as shown below. If we started with the time-ordered product

$$\int d^4x\, d^4y\, T(\vec{f}(x)\cdot\Delta^{-1}\vec{\chi}(x)\cdot\vec{h}(y)\Delta^{-1}\vec{\chi}(y)),$$

and in its Wick expansion kept only the above-mentioned extra $\eta_\alpha\eta_\beta$ factor, we would then get the following:

$$i\int d^4x{:}[\vec{f}(x)\cdot\Delta^{-1}K\Delta^{-1}\vec{h}(x)+\vec{f}(x)\cdot(\Delta^{-1}M)^2\Delta^{-1}\vec{h}(x)]{:}\,. \tag{16}$$

Notice that the first term in Eq. (16) will have a diagrammatic representation of the type shown in Fig. 8, where two legs of a four-vertex are attached to an F-type propagator. We call this term $T(1)$. The second term in Eq. (16), which we call $T(2)$, merely gives rise to the same type of diagrams as are already contained in $\mathcal{H}_I'$, i.e., chains of three-vertices attached to F-type propagators. Therefore $T(2)$ will add to $\mathcal{H}_I'$ and alter the numerical coefficients of various terms in its expansion. We will use this result below to prove that the Wick expansion of the S matrix contains only terms of the form shown in Eq. (13) and Eq. (15) and therefore substantiates the Feynman rules postulated in the theorem.

Let us suppose that we have a diagram where we have $n-1$ F-type propagators sandwiched between n three-vertices, the other legs of which are either external particles or D-type propagators. One typical diagram is shown in Fig. 10. This diagram will come from the Wick expansion of the following kind of T product, which again must come from the Dyson expansion of the S matrix:

$$(-g)^n\tfrac{1}{2}(n-1)\int d^4y_1\cdots d^4y_n\, d^4x\, T(\mathcal{L}_1(y_1)\mathcal{L}_2(y_2)\cdots\mathcal{L}_n(y_n)\vec{\chi}(\Delta^{-1}M)^{n-2}\Delta^{-1}\vec{\chi}(x)), \tag{17}$$

where the $\mathcal{L}_i(y_i)$'s are some parts of the full interaction Hamiltonian given in Eq. (1). As remarked earlier, the coefficient of the expression is different from Eq. (13). Notice, however, that in the Dyson expansion there will also exist a term like the following one:

$$(-g)^n\sum_{\substack{r_1,r_2,\ldots,r_s\\ r_1+\cdots+r_s=n-2s}}\frac{(r_1+1)(r_2+1)\cdots(r_s+1)}{2^s}\int d^4y_1\cdots d^4y_n d^4x_1\cdots d^4x_s$$
$$\times T(\mathcal{L}_1(y_1)\mathcal{L}_2(y_2)\cdots\mathcal{L}_n(y_n)\vec{\chi}(x_1)\cdot(\Delta^{-1}M)^{r_1}\Delta^{-1}\vec{\chi}(x_1)\vec{\chi}(x_2)\cdot(\Delta^{-1}M)^{r_2}\Delta^{-1}\vec{\chi}(x_2)\cdots\vec{\chi}(x_s)\cdot(\Delta^{-1}M)^{r_s}\Delta^{-1}\vec{\chi}(x_s))\,. \tag{18}$$

In the Wick expansion of Eq. (18), we will have a class of terms where the χ's at various points are contracted in such a way that in each of their contractions, if we kept $T(2)$-type terms of Eq. (16), then we would get $\vec{\chi}\cdot(\Delta^{-1}M)^{n-2}\Delta^{-1}\vec{\chi}(x)$. To carry out the actual contraction let us take a term in which there is no $\mathcal{L}_i(y_i)$. Then we will have in place of Eq. (17),

$$(-g)^n\tfrac{1}{2}(n-1)\int d^4x\, T(\vec{\chi}(x)\cdot(\Delta^{-1}M)^{n-2}\Delta^{-1}\vec{\chi}(x)), \tag{17'}$$

and in place of (18),

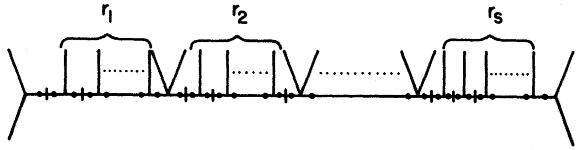

FIG. 9. Feynman diagram corresponding to Eq. (15).

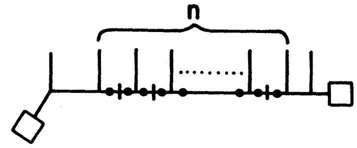

FIG. 10. A typical part of a Feynman diagram with n three-vertices and n F-type propagators.

$$(-g)^n \sum_{\substack{r_1, r_2, \ldots, r_s \\ r_1+\cdots+r_s=n-2s}} \frac{(r_1+1)(r_2+1)\cdots(r_s+1)}{2^s} \int d^4x_1 \cdots d^4x_s$$
$$\times T\big(\vec{\chi}(x_1)\cdot(\Delta^{-1}M)^{r_1}\Delta^{-1}\vec{\chi}(x_1)\vec{\chi}(x_2)\cdot(\Delta^{-1}M)^{r_2}\Delta^{-1}\vec{\chi}(x_2)\cdots\vec{\chi}(x_s)\cdot(\Delta^{-1}M)^{r_s}\Delta^{-1}\vec{\chi}(x_s)\big)\,. \tag{18'}$$

Let us contract the $\partial_0 b_\mu$'s from χ's at various space-time points in such a way that no two space-time points are connected by more than one contraction. Then, if we keep $T(2)$-type terms, we will obviously get $\vec{\chi}\cdot(\Delta^{-1}M)^{n-2}\Delta^{-1}\vec{\chi}$. Now, let us count the number of ways we can do this in Eq. (18′). Notice that, in the final result, two χ's must be free and they originally must have belonged to two space-time points. The number of ways those two space-time points can be chosen out of s points is $s!/(s-2)!\,2!$. Furthermore, the remaining $s-2$ pairs can be kept in any position with respect to each other, each position making an equal contribution. One typical contracted term is shown below:

$$:\vec{\chi}\cdot(\Delta^{-1}M)^{r_1}\Delta^{-1}\dot{\vec{\chi}}(x_1)\dot{\vec{\chi}}(x_2)\cdot(\Delta^{-1}M)^{r_2}\Delta^{-1}\dot{\vec{\chi}}(x_2)\dot{\vec{\chi}}(x_3)\cdot(\Delta^{-1}M)^{r_3}\Delta^{-1}\dot{\vec{\chi}}(x_3)\cdots\dot{\vec{\chi}}(x_s)\cdot(\Delta^{-1}M)^{r_s}\Delta^{-1}\vec{\chi}(x):\,, \tag{19}$$

where heavy superior dots indicate Wick contraction between the adjacent operators, and where contraction between two χ's means that the corresponding $\partial_0 b_\mu$'s are contracted and only the extra term is kept. We will always make contractions as above such that no two contractions overlap each other. So, different combinations correspond to different terms in the Wick expansion. There are $(s-2)!$ such combinations. Again, since each space-time point has two χ's, there will be 2^s contractions possible for each position of the various space-time points. Taking all these together, one can observe that the final coefficient accompanying $g^n\vec{\chi}\cdot(\Delta^{-1}M)^{n-2}\Delta^{-1}\vec{\chi}$ is

$$\sum_s \frac{s!(s-2)!2^s}{(s-2)!2!2^s}\,\frac{1}{s!}\sum_{r_i}\prod_{\substack{i=1 \\ r_1+\cdots+r_s=n-2s}}^{s}(r_i+1)=\tfrac{1}{2}\sum_s\sum_{r_i}\prod_{\substack{i=1 \\ r_1+\cdots+r_s=n-2s}}^{s}(r_i+1)\,. \tag{20}$$

This expression is nothing but the coefficient of x^n in the following sum:

$$\tfrac{1}{2}f(x)=\tfrac{1}{2}\sum_{m=1}^{\infty}\frac{x^{2m}}{(1-x)^{2m}}$$
$$=\tfrac{1}{2}\,\frac{x^2}{1-2x} \tag{21}$$
$$=\tfrac{1}{2}\sum_{n=2}^{\infty}2^{n-2}x^n\,. \tag{21'}$$

Therefore, the coefficient of x^n is 2^{n-3} and this is precisely the coefficient required [see Eq. (13)]. This proof can be trivially extended when the $\mathcal{L}_i(y_i)$'s in Eqs. (17) and (18) are also present. The only restriction is that the free χ's must be contracted with operators belonging to $\mathcal{L}$'s.

Next, let us try to show whether, using Eq. (16), one can prove that terms like Eq. (15) are also present in the Wick expansion with right coefficients. For that purpose, let us again look at Eq. (18′) and, in the contraction of χ's similar to that in Eq. (16), let us keep only $T(1)$-type terms. The reason why we keep only $T(1)$-type terms is that we want a diagram of the type shown in Fig. 9. If we kept both $T(1)$- and $T(2)$-type terms simultaneously, then the diagram would not be that in Fig. 9 anymore since there will be one part where, between two four-vertices, we will have r_i+r_j+2 three-vertices and we are not interested in this. In this case, our counting will be easier if we proceed in a somewhat different manner than we did above. Therefore, let us group the r_i's of Eq. (18′) into l different groups containing, respectively, $m_1, m_2, \ldots, m_l$ r_i's that are equal:

$$r_1, r_2, \ldots, r_{m_1}, \quad r_{m_1+1}, r_{m_1+2}, \ldots, r_{m_1+m_2}, \quad \ldots, \quad r_{n+1}, \ldots, r_s\,. \tag{22}$$

When we pick a term like Eq. (18′) in the Dyson expansion of the S matrix, satisfying Eq. (22), there will be $s!/m_1!\,m_2!\cdots m_l!$ different terms in the original expansion which become equal after the space-time points are suitably relabeled. Therefore, the above number should multiply Eq. (18′). This, of course, could have been done in the first case, when we picked only $T(2)$-type terms without altering anything. After we pick up $T(1)$-type terms after the Wick expansion, the term will look like this (we have omitted the numerical coefficient and coupling constants):

$$\int d^4x : \vec{\chi}\cdot(\Delta^{-1}M)^{r_1}\Delta^{-1}K(\Delta^{-1}M)^{r_2}\Delta^{-1}K\cdots(\Delta^{-1}M)^{r_s}\Delta^{-1}\vec{\chi}(x): . \tag{23}$$

To straighten the numerical coefficient, let us notice that we could interchange the pairs of χ's belonging to any m_i (i.e., with all r_i's equal) in Eq. (22), and there would be $m_i!$ ways of doing it and all would make equal contributions to Eq. (23). Therefore, Eq. (23) should be multiplied by a factor $m_1!m_2!m_3!\cdots m_l!$. Of course, a factor 2^s must also be multiplied since each space-time point has a pair of χ's. Collecting all these, we see that the numerical coefficient

$$\frac{1}{s!}\,\frac{s!2^s}{m_1!m_2!\cdots m_l!}\,m_1!m_2!\cdots m_l!\,\frac{(r_1+1)(r_2+1)\cdots(r_s+1)}{2^s}=(r_1+1)(r_2+1)\cdots(r_s+1) \tag{24}$$

should multiply Eq. (23). This factor is different from the one in Eq. (15). Therefore, we must collect other possible terms which, by virtue of Eq. (16), will also contribute to Eq. (23). We will first of all keep $r_2, r_3,\ldots,r_s$ fixed and concentrate on terms which, after using $T(2)$ in Eq. (16), will contribute to r_1. Actually, the same set of steps that led to Eq. (20) for the first case, will be repeated here except that, because of the presence of $r_2, r_3,\ldots,r_s$, the combinatorial factors are far from obvious. We will therefore go through the steps again. Let us therefore break up r_1 in such a way that

$$t_1+t_2+\cdots+t_i=r_1-2i+2\,. \tag{25}$$

In other words, we are considering the following terms in the Dyson expansion:

$$\frac{g^n}{(s+i-1)!}\,\frac{1}{2^{s+i-1}}\sum_{\substack{t_1+t_2+\cdots+t_i=r_1-2i+2\\ r_1+r_2+\cdots+r_s=n-2s}}(t_1+1)(t_2+1)\cdots(t_i+1)(r_2+1)\cdots(r_s+1)$$
$$\times\int d^4x_1\cdots d^4x_{s+i-1}T\big(\vec{\chi}(x_1)\cdot(\Delta^{-1}M)^{t_1}\Delta^{-1}\vec{\chi}(x_1)\cdots\vec{\chi}(x_i)\cdot(\Delta^{-1}M)^{t_i}\Delta^{-1}\chi(x_i)\,\vec{\chi}(x_{i+1})\cdot(\Delta^{-1}M)^{r_2}\Delta^{-1}\vec{\chi}(x_{i+1})\cdots\big)\,. \tag{26}$$

Now, to get the numerical factors right, let us first divide the t_2's into partitions, each containing all t_2's that are equal – the kth partition containing s_k t_2's:

$$t_1,\ldots,t_{s_1},\quad t_{s_1+1},\ldots,t_{s_1+s_2},\quad\ldots \tag{27}$$

Of course, the r's are partitioned in Eq. (22). It might happen that the t_i's in some partition (let us say s_j) are equal to the r_i's in one partition (let us say m_j). Then, these two partitions will be merged giving a new partition m_j+s_j. After this is done, we have the following partitions:

$$s_1,s_2,\ldots,s_{j-1},s_{j+1},\ldots,s_k,m_1,m_2,\ldots,m_{j-1},m_j+s_j,\ldots,m_l\,. \tag{28}$$

Therefore, after suitable relabeling of space-time points, it can be shown that there are

$$\frac{(s+i-1)!}{s_1!s_2!\cdots s_{j-1}!s_{j+1}!\cdots m_1!\cdots(m_j+s_j)!\cdots m_l!} \tag{29}$$

terms of the type in Eq. (26) in the S-matrix expansion which are equal. So, this factor should multiply Eq. (26). Let us then proceed with the Wick expansion. We will keep $T(2)$-type terms in the contraction among t_i's and $T(1)$-type terms in the contraction of t_i's with r_i's and among r_i's themselves. Here again a counting problem is involved, because the t_i's belonging to s_j-type partitions are mixed with r_j's and we have to choose s_j of those terms out of m_j+s_j such terms present in the initial T product. There are

$$(m_j+s_j)!/m_j!s_j! \tag{30}$$

ways of doing it. Now that we have all t_i's, we would like to keep only $T(2)$-type terms among them and proceeding as in Eq. (20), we have

$$i!2^{i-1} \tag{31}$$

ways of contracting the t_i's among themselves which will give the same term. Then, of course, we will count the number of various contractions that, after keeping $T(1)$-type terms, will lead us to Eq. (23). There are obviously

$$m_1!m_2!\cdots m_l!2^s \tag{32}$$

ways this can be done.

The final coefficient of Eq. (15), therefore, is [collecting Eqs. (29)–(32)]

$$g^n \frac{1}{(s+i-1)!} \frac{1}{2^{s+i-1}} \frac{(s+i-1)!\,2^s i!\,2^{i-1}}{s_1!\,s_2!\cdots\, m_1!\,m_2!\cdots(m_j+s_j)!\cdots m_l!} \frac{(m_j+s_j)!}{m_j!\,s_j!} m_1!\,m_2!\cdots m_l!$$
$$\times \sum_{t_1+t_2+\cdots+t_i=r_1-2i+2} (t_1+1)(t_2+1)\cdots(t_i+1)(r_2+1)\cdots(r_s+1) \tag{33}$$

which is equal to

$$g^n(r_2+1)\cdots(r_s+1) \sum_{t_1+t_2+\cdots+t_i=r_1-2i+2} \frac{i!}{s_1!\,s_2!\cdots s_k!}(t_1+1)(t_2+1)\cdots(t_i+1)\,. \tag{34}$$

In the second sum, it must be understood that only different combinations of $t_1, t_2, \ldots, t_i$ must be kept. [Note the difference between the summation over t's here and summation over r's in Eq. (20), where no such restriction on the r's exists. In fact, if we impose the restriction of including only different combinations in the sum, a factor $s!/m_1!\,m_2!\cdots m_l!$ would come out and, therefore, both these sums are actually equal if $r_1=n$.]

Now we must sum over i, the subscript of t, and then we would get for Eq. (34)

$$g^n(r_2+1)\cdots(r_s+1)\sum_i \sum_{\substack{t_1,t_2,\ldots \\ t_1+t_2+\cdots+t_i=r_1-2i+2}} \frac{i!}{s_1!\,s_2!\cdots s_k!}(t_1+1)(t_2+1)\cdots(t_i+1)$$
$$= g^n(r_2+1)\cdots(r_s+1)\ [\text{coefficient of } x^{r_1+2} \text{ in } f(x)]$$
$$= g^n(r_2+1)\cdots(r_s+1)\,2^{r_1}\,. \tag{35}$$

Similarly, we can proceed for $r_2, r_3, \ldots$, and our final coefficient will be

$$g^n 2^{r_1+r_2+\cdots+r_s}\,. \tag{36}$$

This is the same coefficient that occurs in Eq. (15). Therefore, we have shown that if one consistently keeps the effect of the $\eta_\alpha\eta_\beta$ term in Eq. (7), the correct vertices, i.e., those in Eqs. (13) and (15), emerge as a consequence. This therefore completes the proof of our Feynman rules for tree diagrams.

IV. LOOP DIAGRAMS

Loop diagrams can be of various types. There will be one type where all the propagators are D-type. Such loops will clearly emerge from T products containing only

$$\tfrac{1}{2}g\vec{b}_\mu\times\vec{b}_\nu\vec{g}_{\mu\nu}+\tfrac{1}{4}g^2\vec{b}_\mu\vec{b}_\nu\cdot\vec{b}_\mu\times\vec{b}_\nu\,,$$

where suitable contractions are taken. We will not devote any time to this, since rules for such loops are well known. There can be another kind of loop where F- and D-type propagators are mixed. In that case, again, we do not have any problem because we have shown in Sec. III that a chain of F-type propagators comes from $\mathcal{H}_I'$ alone and comes with correct numerical factors [see Eqs. (21′) and (36)]. Also, we observed that the way by which we arrived at Eqs. (21′) and (36) remains unchanged if the free χ's at the ends are contracted to a D-type propagator. Therefore a loop with both F-type and D-type propagators has always a coefficient of one. However, things are different when a loop has all F-type propagators.

These kinds of diagrams can be classified into three groups: (a) when all vertices in the loop are three-vertices; (b) when all vertices are four-vertices; (c) when both three- and four-vertices are mixed up.

Case (a). If we have n three-vertices, one term in the Dyson-Wick expansion that will contribute to it is the following:

$$-\tfrac{1}{2}i(-g)^n(n-1)\int d^4x\, T(\vec{\chi}(x)\cdot(\Delta^{-1}M)^{n-2}\Delta^{-1}\vec{\chi}(x))\,. \tag{37}$$

Because, in the Wick expansion of Eq. (37), there will be a term where the $\partial_0 b_\mu$'s in two χ's will be contracted and then the $\eta_\alpha\eta_\beta$ term in Eq. (7) will make an extra contribution, and if we keep $T(2)$-type terms in Eq. (16), we will get

$$+\tfrac{1}{2}(-g)^n(n-1)\int d^4x\!:(\Delta^{-1}M)^n:\,. \tag{38}$$

However, it is easy to convince oneself that there will be terms in the Wick expansion of (18′)

that will also contribute to the diagram in question and, therefore, we have to count as we did before arriving at Eq. (20). The terms that will contribute in this case can be symbolically denoted by Eq. (19), with the change that the χ's at the end are also contracted with each other. This extra contraction changes the number in Eq. (20). In fact, the total number of ways in which the various χ's in Eq. (18′) can be contracted to give Eq. (38) is

$$2^{s-1}(s-1)!\,. \tag{39}$$

Note that the corresponding number in Eq. (20) was $2^{s-1}s!$. If we use Eq. (39) in Eq. (18′), we get the following:

$$C_n(-g)^n\int d^4x\ :(\Delta^{-1}M)^n:\,, \tag{40}$$

where

$$C_n=\sum_{s=1}\frac{1}{2s}\sum_{\substack{r_1,r_2,\dots,r_s\\ r_1+r_2+\dots+r_s=n-2s}}\prod_{i=1}^{s}(r_i+1)\,. \tag{41}$$

One can see after a moment's thought that C_n is the coefficient of x^n in the following:

$$f(x)=\sum_{m=1}^{\infty}\frac{1}{2m}\frac{x^{2m}}{(1-x)^{2m}} \tag{41′}$$

$$=-\tfrac{1}{2}[\ln(1-2x)-2\ln(1-x)]\,. \tag{41″}$$

Therefore,

$$C_n=\frac{1}{n}(2^{n-1}-1)\,. \tag{42}$$

Since there are n powers of M, to get the correct vertex we must divide C_n by 2^n. To get the coefficient that should multiply the loop, as written down following the rules of the theorem, we must find the number of permutations of the external legs that are equivalent. One can see that there are $2n$ equivalent ways if $n>2$. So, the coefficient of such a Feynman diagram is

$$C_n\frac{2n}{2^n}=1-\frac{1}{2^{n-1}}\quad\text{if}\quad n>2\,. \tag{43}$$

If $n=2$, direct computation shows that the coefficient required is $\frac{1}{4}$.

Case (b). If there are n four-vertices and all F-type propagators, the only term in the Dyson expansion that will contribute to it after using $T(1)$-type terms of Eq. (16) is the following:

$$\frac{(i)^n}{n!}\frac{g^{2n}}{2^n}\int d^4x_1\cdots d^4x_n\,T(\vec{\chi}(x_1)\cdot\Delta^{-1}\vec{\chi}(x_1)\vec{\chi}(x_2)\cdot\Delta^{-1}\vec{\chi}(x_2)\cdots\vec{\chi}(x_n)\cdot\Delta^{-1}\vec{\chi}(x_n))\,. \tag{44}$$

As in case (a), the total number of ways of contracting the various $\vec{\chi}$'s, which will give rise to the same result, is $2^{n-1}(n-1)!$. Also, as in case (a), when we go from the Wick expansion to the Feynman diagram, there will be $2n$ (if $n>2$) terms that will make the same contribution. Therefore, on multiplying, we see that the coefficient of a diagram where $n>2$ is unity, whereas direct computation shows that for $n=2$ the coefficient is $\frac{1}{2}$. Incidentally, note that a similar diagram, where all propagators are D-type instead of F-type, also has the same coefficients as this case.

Case (c). In this case, the loop has all F-type propagators but there are both three- and four-vertices. This diagram will then come from an expression of the type shown below:

$$2^{r_1+r_2+\cdots+r_s}g^n\int :\Delta^{-1}K(\Delta^{-1}M)^{r_1}\Delta^{-1}K(\Delta^{-1}M)^{r_2}\times\Delta^{-1}K\cdots(\Delta^{-1}M)^{r_s}:d^4x\,. \tag{45}$$

This can come from Eq. (18′) if, in the Wick expansion of it, we pick up terms where all χ's are suitably contracted and $T(1)$-type terms are kept consistently.

In other words, expression (45) would arise from (18′) in the same way that (23) arose from (18′) except that all possible χ's are contracted. The counting procedure also remains the same except that, since all pairs are contracted, an extra factor of $\frac{1}{2}$ has to be multiplied. In other words, before Eq. (24), we said that "a factor 2^s must be multiplied since each space-time point has a pair of χ's"; instead, here the factor would be 2^{s-1} as can be easily checked. This factor of $\frac{1}{2}$ goes through and, finally, Eq. (36) should be multiplied by $\frac{1}{2}$; but, when we go from Eq. (45) to the actual Feynman diagram, an extra factor of 2 will come from each diagram because clockwise and counterclockwise configurations of the external connections are the same. This, therefore, completes our proof of Feynman rules for loops. Notice that when any of the external legs is connected to another part of a diagram, the counting is unaffected.

V. THE MAIN RESULT

The results obtained in Secs. III and IV will now be summarized in a fashion in which the ugly extra coefficient for case (a) of Sec. IV seems to disappear, if we accept a fictitious scalar loop and we rewrite the propagator for the Yang-Mills field in the Introduction. For this purpose, let us forget the $-1/2^{n-1}$ factor of Eq. (43) for a while. Then, one can see that all diagrams with both D- and F-type propagators have the same weight. Therefore, one can add the two types of propagators, as we did in paper I for tree diagrams, without any difficulty. Then the effective propagator becomes

$$G_{\mu\nu}^{ab}(k)=D_{\mu\nu}^{ab}(k)+F_{\mu\nu}^{ab}(k)$$
$$=\frac{-i\delta_{ab}}{k^2-i\epsilon}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu-k\cdot\eta(k_\mu\eta_\nu+k_\nu\eta_\mu)}{k^2-(k\cdot\eta)^2}\right). \tag{46}$$

Therefore, if the $-1/2^{n-1}$ factor in Eq. (43) were absent, we could describe the Feynman diagrams completely using this propagator and the vertices shown in Figs. 1(b) and 1(c). To accomodate this extra factor, we introduce a vector-scalar-scalar vertex (*VSS*)

$$\Gamma_\mu^{abc}(p,k,q)=-\tfrac{1}{2}g\epsilon_{abc}[(k-q)_\mu-(k-q)\cdot\eta\eta_\mu]$$
$$\equiv-\tfrac{1}{2}g\epsilon_{abc}\delta_{\mu i}(k-q)_i \tag{47}$$

and a propagator

$$G^{ab}(k)=\frac{-i\delta_{ab}}{k^2-(k\cdot\eta)^2}. \tag{48}$$

To see how we get this expression for the *VSS* vertex, note the following:

$$\Gamma_{\mu\nu\lambda}^{abc}(p,k,q)\eta_\nu\eta_\lambda=-g\epsilon_{abc}[\delta_{\mu\nu}(p-k)_\lambda+\delta_{\nu\lambda}(k-q)_\mu+\delta_{\lambda\mu}(q-p)_\nu]\eta_\nu\eta_\lambda$$
$$=-g\epsilon_{abc}[(k-q)_\mu-\eta_\mu(k\cdot\eta-q\cdot\eta)]$$
$$\equiv-g\epsilon_{abc}\delta_{\mu i}(k-q)_i. \tag{49}$$

Also, the loop we left out had only F-type propagators and out of each propagator the η_μ factors are now multiplied by the vertex as in Eq. (49). Therefore, the propagator for the scalar field has to be

$$\frac{-i\delta_{ab}}{k^2-(k\cdot\eta)^2}. \tag{50}$$

This then shows that if we use the *VSS* vertex in Eq. (47) and multiply any loop with the above-mentioned scalar vertices and scalar propagators by a factor (−2), we recover the diagrams that we omitted. This proves the set of rules described in the Introduction (Fig. 1). There is no extra factor multiplying a diagram now, except the usual $1/s_n$ described in Ref. 7. These rules are the same as the ones given by Fradkin and Tyutin,[8] who work in the radiation gauge but use a Lagrange-multiplier approach. The fictitious scalar particle seems to appear to all orders and may be a general characteristic of theories with non-Abelian gauge symmetry.

VI. DISCUSSION

In conclusion, we would like to stress that we have been able to obtain the noncovariant Feynman rules for the Yang-Mills field in the radiation gauge with any number of loops, using the conventional procedure of canonical quantization. One interesting feature of the rules is the appearance of the fictitious scalar particles to all orders. In obtaining the above rules, we ignored the extra term in the Hamiltonian given by Schwinger ($\mathcal{H}_s$). Even though it is possible to suggest a diagrammatic representation of $\mathcal{H}_s$ (the Schwinger contribution), we do not consider it here since our understanding of that contribution is far from clear. However, we suspect that it might become relevant when we want to prove the covariance of loop diagrams.

The method presented above, though difficult, has the appeal of being very straightforward. Another simple and straightforward approach to the present problem is one of taking the massless limit of the massive Yang-Mills field. (See Wong[3] and Veltman *et al.*[3]) However, it turns out that even if the external legs have transverse polarization only, the limit is highly singular and goes like m^{-2L+2}, where L is the number of loops, as $m\to0$, because, unlike quantum electrodynamics, the longitudinal modes do not decouple from the transverse ones in the massless limit. Therefore, if we want to understand the fictitious loops[3] in the covariant Feynman rules for the Yang-Mills field outside the framework of path-integral techniques, we are left with the only choice of working with a massless field in a suitable formalism from the beginning and using the canonical quantization procedure as we have done. If the lower-order calculations reported in paper I are representative, the fictitious scalar loops do indeed appear within our approach. Also, the tree diagrams can be made covariant (see I) and no extra vertex appears apart from the usual ones. Of course, if one believes, as Fradkin and Tyutin seem to,[9] that the Yang-Mills theory is nonanalytic at $g=0$ and therefore the perturbation expansion can at best be asymptotic, most of the work on Feynman rules and fictitious loops may not really be relevant. However, we think that, due to lack of any better alternative, the perturbative approach is still the most useful one from the practical point of view and, therefore, ought to be thoroughly explored.

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[4]We reproduce our notation in paper I: An arrow on a quantity denotes an isovector; $\mu, \nu, \alpha, \beta, \ldots$ stand for four Lorentz indices, with the convention that $x=(x_1, x_2, x_3, x_4)$, $x_4 = i x_0$, and $x^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$. Notice also that $\int f \cdot Mg\, d^3x = \int Mf \cdot g\, d^3x$. This property of the operator M can be made more explicit by writing M as follows: $f \cdot Mg = \frac{1}{2} \vec{f} \cdot \vec{b}_\mu \times \overleftrightarrow{\partial}_\mu g \equiv \frac{1}{2}(\vec{f} \cdot \vec{b}_\mu \times \partial_\mu \vec{g} - \partial_\mu \vec{f} \cdot \vec{b}_\mu \times \vec{g})$.

[5]J. Schwinger, Phys. Rev. 127, 324 (1962); S. I. Fickler and M. J. Russo, Phys. Rev. D 3, 1782 (1971).

[6]We will adopt the following attitude towards Lorentz covariance. If the Feynman rules to all orders with arbitrary number of loops can be written down in a covariant way and the on-shell amplitudes satisfy Weinberg's condition [S. Weinberg, Phys. Rev. 135, B1049 (1964)], i.e., when in an amplitude any one of the polarization vectors $\epsilon_{\mu_r}(k_r)$ is replaced by $(k_r)_{\mu_r}$, we get zero; then we will say that the S matrix is Lorentz-covariant.

[7]T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962). To obtain the symmetry number S_n of a loop with n vertices, label each of the internal lines with a different index and do all possible labelings with those indices. The number of diagrams, which are topologically equivalent after all the labeling is done, gives the symmetry number S_n, and the corresponding diagram should be multiplied by $1/S_n$.

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