Reply to "Comment on Jarlskog's conditions for CP invariance"

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It is shown that, using the flavor projection technique, one may deal also with the singular cases where the mass matrix $\hat{S}'$ contains zeros. The number of conditions, for CP invariance, in such cases is smaller than $(n-1)(n-2)/2$, where n is the number of families.

The first basis-independent condition for CP symmetry, in the standard electroweak model with $n=3$ (n being the number of families) was given by me.[1] I found that there is a unique condition for CP symmetry, viz.,

$$\det[S,S']=0 \ .$$

Here $S=mm^\dagger$ and $S'=m'm'^\dagger$, where m (m') is the 3×3 mass matrix of the up-kind (down-kind) quarks. Gronau, Kfir, and Loewy[2] considered the generalization of my condition to the case of $n>3$. They found that "The number of basis-independent conditions obtained for CP invariance is in general (except in the case $n=3$) larger than $(n-1)(n-2)/2$, the number of phases in the Kobayashi-Maskawa (KM) quark mixing matrix. Moreover, it is also larger than $n(n-1)(n-2)/6$ the number of distinct three-cycle products in a Hermitian mass matrix." This result was challenged by me[3] as follows.

The starting point was to go to a frame where $S\to$diagonal, whereby $S'\to\hat{S}'$. The conditions for CP symmetry, i.e., the requirement that $\hat{S}'$ be real are then easily written down. I then used the technique of "flavor projection operators" to translate the conditions to frame-independent statements. I found that the number of conditions thus obtained is $(n-1)(n-2)/2$ and, thus, equals the number of phases in the most general quark mixing matrix. Gronau and Loewy[4] object to my work because the "translation" procedure used by me may have singularities if there are zeros in the matrix $\hat{S}'$. The question is then what happens if there are zeros in the matrix $\hat{S}'$? Nevertheless, it is of interest to consider what happens if there are such zeros, as I shall now briefly discuss.

The first point to notice is that if there are zeros in $\hat{S}'$ the number of conditions for CP symmetry *must be smaller,* because there are fewer phases in $\hat{S}'$. The second point is that one can spot such zeros, in principle, and express them as basis-invariant statements. For example, $\hat{S}'_{ij}=0$ reads $\mathrm{tr}(P_iS'P_jS')=0$. One would, thus, start by checking whether or not there are zeros in $\hat{S}'$. If there are no such zeros, there are $(n-1)(n-2)/2$ conditions for CP symmetry and those are as given by me.[3] If there are zeros there will be fewer conditions. For example, in the "counterexample" given by Gronau and Loewy,[4] there is only one intrinsic phase in S' even though there seem to be four complex entries, because three of the phases may be removed by redefinition of the phases of the quark fields. There is, therefore, a single condition, for CP symmetry. Using the projection technique, the condition can be written as

$$\mathrm{Im}\,\mathrm{tr}(P_1S'P_2S'P_3S'^2)=0 \ .$$

Note that the vanishing of $\hat{S}'_{13}$ and $\hat{S}'_{24}$ automatically gives

$$|\mathrm{tr}(P_iS'P_jS'P_kS')|=0 \quad \forall i,j,k \ .$$

In conclusion, the flavor projection technique gives the "existence proof" that there are no more than $(n-1)(n-2)/2$ conditions and provides these conditions in closed forms. Unfortunately, however, we know from Abel's theorem that life will be very complicated if there would be five or more families in the standard model. The reason is that even if we are given the mass matrices m and m', in general, we cannot determine the quark masses in terms of elementary functions.

[1] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); Z. Phys. C **29**, 491 (1985).
[2] M. Gronau, A. Kfir, and R. Loewy, Phys. Rev. Lett. **56**, 1538 (1986).
[3] C. Jarlskog, Phys. Rev. D **36**, 2128 (1987).
[4] M. Gronau and R. Loewy, preceding Comment, Phys. Rev. D **39**, 986 (1989).

