Nonsingular potential model for heavy quarkonia

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A nonsingular potential model for heavy quarkonia is described which does not require recourse to an illegitimate perturbative treatment for the investigation of quarkonium spectra. Our overall results for the energy levels, the leptonic widths, and the $E1$ transition widths are in very good agreement with the experimental data for $b\bar{b}$ and they are reasonably good for $c\bar{c}$. We also find that the confining part of the quarkonium potential is scalar-vector exchange rather than the currently favored scalar exchange.

Potential models for quarkonia have been developed by various authors in recent years and in particular our quantum-chromodynamic potential model[1] has been remarkably successful[2] in accounting for the observed $b\bar{b}$ and $c\bar{c}$ spectra. Despite the success of our model, we have been aware that all existing quarkonium potential models including ours involve a fundamental difficulty due to the presence of highly singular interaction terms. The singular terms in the potential[3] make it impossible to obtain the energy levels by a nonperturbative treatment, while large contributions to the energy levels arising from these singular terms make the perturbative treatment questionable. The situation is more problematic here than in the case of positronium for which a perturbative treatment is adequate because of the small values of α and $\mathbf{v}^2/c^2$.

We have now made a breakthrough by developing a *genuine* potential model for heavy quarkonia which does not require recourse to an illegitimate perturbative treatment. For this purpose, we have used a nonsingular potential which was recently derived from the scattering operator by means of an improved quasistatic approximation.[4]

Our model is based on the semirelativistic Hamiltonian

$$H=2(m^2+\mathbf{p}^2)^{1/2}+V_p(\mathbf{r})+V_c(\mathbf{r}) ,$$

where V_p and V_c are the perturbative[5] and the confining potentials. Although there is general agreement that the spin-independent part of the confining potential is essentially linear, there is some controversy as to whether its spin dependence corresponds to scalar exchange, vector exchange, or something else. We have, therefore, made use of the mixed scalar-vector-exchange confining potential

$$V_c=(1-B)V_S+BV_V ,$$

where V_S and V_V are given in Ref. 4, and B is an arbitrary parameter. Furthermore, we have compared our results with those obtained by employing a confining potential with arbitrary spin-dependent terms of the form generated by the application of the improved quasistatic approximation to the scattering operator, which is given by

$$V_c=Ar+\frac{C_1}{m^2r}(1-e^{-2mr})\mathbf{S}_1\cdot\mathbf{S}_2+\frac{C_2}{m^2r}(1-\tfrac{1}{2}f_1)\mathbf{L}\cdot\mathbf{S}+\frac{C_3}{m^2r}(1-\tfrac{3}{2}f_2)S_{12} ,$$

where f_1 and f_2 are defined in Ref. 4, and C_1, C_2, and C_3 are arbitrary constants.

Our trial wave function and the formalism for the treatment of the semirelativistic Hamiltonian are described in an earlier paper.[6] We have also confirmed the results obtained by us with the use of another trial wave function,[7] which simplifies the computations.

Our results for the energy-level splittings in $b\bar{b}$ and $c\bar{c}$ are given[8] in Tables I and II, where the three sets of theoretical results correspond to the scalar exchange, the scalar-vector exchange, and the arbitrary forms of spin dependence in the confining potential. The scalar-vector-exchange case is clearly superior to the scalar-exchange case and it is remarkable that the arbitrary case yields practically the same results as the scalar-vector-exchange case for $b\bar{b}$ as well as for $c\bar{c}$. The scalar-vector mixing parameter B has the values 0.431 for $b\bar{b}$ and 0.258 for $c\bar{c}$, which shows that the confining potential is scalar-vector exchange rather than the currently favored scalar exchange, and that its spin structure is sensitive to the quark masses.[9]

In Tables III–V we give the results for the energy levels, the leptonic widths with radiative corrections, and

TABLE I. $b\bar{b}$ energy level splittings in MeV. Theoretical splittings and parameters correspond to the scalar exchange, the scalar-vector exchange, and the arbitrary forms of spin dependence in the confining potential.

	Scalar	Scalar-vector	Arbitrary	Expt.
Υ'-Υ	555.8	555.8	556.0	563.4±0.4
Υ''-Υ	897.4	897.8	897.6	895.5±0.6
$\chi_{b,\text{c.o.g.}}$-Υ	441.5	441.2	441.7	440.3±0.6
χ_{b2}-χ_{b1}	16.2	20.6	20.6	21.1±1.1
χ_{b1}-χ_{b0}	32.1	31.5	31.6	31.6±2.4
$\chi'_{b,\text{c.o.g.}}$-Υ	799.1	799.9	799.9	801.4±1.4
χ'_{b2}-χ'_{b1}	13.3	16.1	16.2	16 ±3
χ'_{b1}-χ'_{b0}	26.4	24.9	25.0	22 ±5
m_b (GeV)	5.234	5.559	5.481	
μ (GeV)	5.933	3.421	3.546	
α_S	0.255	0.280	0.280	
A (GeV^2)	0.184	0.187	0.186	

TABLE II. $c\bar{c}$ energy level splittings in MeV. Theoretical splittings and parameters correspond to the scalar exchange, the scalar-vector exchange, and the arbitrary forms of spin dependence in the confining potential. The experimental value of the Ψ'-η'_c splitting is not used for the determination of the $c\bar{c}$ parameters because of the uncertainty regarding the η'_c mass.

	Scalar	Scalar-vector	Arbitrary	Expt.
Ψ'-Ψ	597.6	592.8	591.7	589.1±0.1
Ψ-η_c	120.2	115.8	115.8	115.9±2
Ψ'-η'_c	74.1	70.6	70.2	(92 ±5)
$\chi_{\text{c.o.g.}}$-Ψ	427.0	425.4	425.6	428.5±0.3
χ_2-χ_1	29.1	46.5	45.5	45.6±0.6
χ_1-χ_0	88.7	94.8	95.6	95.8±1.2
m_c (GeV)	1.919	2.016	2.031	
μ (GeV)	3.173	2.626	2.177	
α_S	0.291	0.300	0.313	
A (GeV^2)	0.187	0.187	0.187	

TABLE III. $b\bar{b}$ energy levels in MeV with $m_b=5.56$ GeV, $\mu=3.42$ GeV, $\alpha_S=0.280$, $A=0.187$ GeV^2, and $B=0.431$.

State	Mass (theory)	Mass (expt.)
$1\,^3S_1(\Upsilon)$	9460.0	9460.0±0.7
$1\,^1S_0(\eta_b)$	9412.2	
$2\,^3S_1(\Upsilon')$	10015.8	10023.4±0.3
$2\,^1S_0(\eta'_b)$	9992.5	
$3\,^3S_1(\Upsilon'')$	10357.8	10355.5±0.5
$3\,^1S_0(\eta''_b)$	10339.7	
$1\,^3P_2(\chi_{b2})$	9913.9	9913.2±0.7
$1\,^3P_1(\chi_{b1})$	9893.3	9892.1±0.8
$1\,^3P_0(\chi_{b0})$	9861.8	9860.5±2.3
$1\,^1P_1(h_b)$	9900.1	
$2\,^3P_2(\chi'_{b2})$	10269.8	10271 ±2
$2\,^3P_1(\chi'_{b1})$	10253.7	10255 ±2
$2\,^3P_0(\chi'_{b0})$	10228.8	10233 ±5
$2\,^1P_1(h'_b)$	10259.0	
$1\,^3D_3$	10163.2	
$1\,^3D_2$	10152.5	
$1\,^3D_1$	10141.3	
$1\,^1D_2$	10154.0	

TABLE IV. $b\bar{b}$ leptonic widths in keV.

State	Γ_{ee} (theory)	Γ_{ee} (expt.)
$1\,^3S_1$	1.21	1.22 ± 0.05
$2\,^3S_1$	0.55	0.54 ± 0.03
$3\,^3S_1$	0.41	0.40 ± 0.03

TABLE V. $E1$ transition widths for $b\bar{b}$ in keV.

Transition	$J=2$	$J=1$	$J=0$	Total (theory)	Total (expt.)
$2\,^3S_1 \to 1\,^3P_J$	1.86	1.68	0.74	4.3	5.3 ± 1.3
$3\,^3S_1 \to 2\,^3P_J$	2.20	2.08	1.03	5.3	$4.3^{+3.7}_{-1.7}$
$3\,^3S_1 \to 1\,^3P_J$	0.36	0.05	0.01	0.4	
$1\,^3P_J \to 1\,^3S_1$	31.64	28.45	25.14	85.2	
$2\,^3P_J \to 2\,^3S_1$	14.72	13.02	11.20	38.9	
$2\,^3P_J \to 1\,^3S_1$	9.54	6.61	3.07	19.2	
$2\,^3P_J \to 1\,^3D_3$	2.08	0.00	0.00	2.1	
$2\,^3P_J \to 1\,^3D_2$	0.46	1.62	0.00	2.1	
$2\,^3P_J \to 1\,^3D_1$	0.04	0.69	1.47	2.2	
$1\,^3D_3 \to 1\,^3P_J$	22.89	0.00	0.00	22.9	
$1\,^3D_2 \to 1\,^3P_J$	5.05	18.03	0.00	23.1	
$1\,^3D_1 \to 1\,^3P_J$	0.49	8.83	14.89	24.2	

TABLE VI. $c\bar{c}$ energy levels in MeV with $m_c=2.02$ GeV, $\mu=2.63$ GeV, $\alpha_S=0.300$, $A=0.187\ \mathrm{GeV}^2$, and $B=0.258$.

State	Mass (theory)	Mass (expt.)
$1\,^3S_1(\Psi)$	3096.9	3096.9 ± 0.1
$1\,^1S_0(\eta_c)$	2981.1	$2981\ \pm2$
$2\,^3S_1(\Psi')$	3689.7	3686.0 ± 0.1
$2\,^1S_0(\eta_c')$	3619.1	$3594\ \pm5$
$1\,^3P_2(\chi_2)$	3553.5	3556.3 ± 0.4
$1\,^3P_1(\chi_1)$	3507.0	3510.7 ± 0.5
$1\,^3P_0(\chi_0)$	3412.2	3414.9 ± 1.1
$1\,^1P_1(h_c)$	3518.5	

TABLE VII. $c\bar{c}$ leptonic widths in keV.

State	Γ_{ee} (theory)	Γ_{ee} (expt.)
$1\,^3S_1$	5.57	4.75 ± 0.51
$2\,^3S_1$	2.87	2.05 ± 0.21

TABLE VIII. $E1$ transition widths for $c\bar{c}$ in keV.

Transition	J	Γ_{E1} (theory)	Γ_{E1} (expt.)
$2\,^3S_1 \to 1\,^3P_J$	2	25.4	17 ± 4
	1	29.4	19 ± 4
	0	19.8	20 ± 4
$1\,^3P_J \to 1\,^3S_1$	2	326.1	430 ± 270
	1	249.5	<700
	0	116.6	120 ± 40

the $E1$ transition widths for $b\bar{b}$ by using the scalar-vector-exchange confining potential, and the corresponding results for $c\bar{c}$ are given in Tables VI–VIII. The differences between these results and those obtained with a perturbative treatment[1] are more pronounced for $c\bar{c}$ than for $b\bar{b}$, and in particular the nonperturbative treatment yields much smaller values of the $E1$ transition widths for $c\bar{c}$. It is also noteworthy that the parameter A has the same value for $b\bar{b}$ and $c\bar{c}$, and that the values of α_S for the two systems are in fair agreement with the quantum-chromodynamic transformation relation.

Our overall results are in very good agreement with the experimental data for $b\bar{b}$, and they are reasonably good[10] for $c\bar{c}$. This is in accordance with the expectation that a model with a quasistatic potential will yield more accurate results for a heavier system than for a lighter one. We have thus found a potential model for heavy quarkonia which is satisfactory theoretically as well as experimentally.

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[1] S. N. Gupta, S. F. Radford, and W. W. Repko, Phys. Rev. D **26**, 3305 (1982); **34**, 201 (1986).

[2] See K. Berkelman, Cornell University Report No. CLNS-85/649, 1985 (unpublished); S. Cooper, Report No. SLAC-PUB-3555, 1985 (unpublished).

[3] For the purpose of solving the Schrödinger equation, we consider singular potential terms as those that are at least as singular as $1/r^2$ at the origin. See W. M. Frank, D. J. Land, and R. M. Spector, Rev. Mod. Phys. **43**, 36 (1971).

[4] S. N. Gupta, Phys. Rev. D **35**, 1736 (1987).

[5] Besides the second-order perturbative potential explicitly given in Ref. 4, we have also included the fourth-order potential by expressing it in the nonsingular form with the use of the improved quasistatic approximation.

[6] S. N. Gupta, S. F. Radford, and W. W. Repko, Phys. Rev. D **31**, 160 (1985).

[7] S. Jacobs, M. G. Olsson, and C. J. Suchyta III, Phys. Rev. D **33**, 3338 (1986).

[8] For experimental data we have relied on Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 1 (1986).

[9] We have assumed that effects such as couplings of the quark-antiquark system to dynamical glue and virtual decay channels are small below the bottom and charm thresholds.

[10] It is interesting that the most noticeable discrepancy in Table VI for $c\bar{c}$ is between the theoretical and the experimental values for the energy level η_c'. This state is also the one for which the experimental data are meager and subject to large uncertainty.