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QCD-based effective Lagrangian including quark mass effects: Calculation of f_K

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Quark mass effects are included in the construction of an effective Lagrangian with bilocal pion and kaon fields. We discuss the appropriate Schwinger-Dyson equations and the choice of pseudoscalar particle wave functions in the presence of explicit chiral-symmetry breaking. Identifying the pseudoscalar-mass terms in the effective Lagrangian, we reproduce the standard current-algebra mass formulas for charged pions and kaons. Approximate solutions to the *u*-, *d*-, and *s*-quark Schwinger-Dyson equations are constructed. The low-q^2 region is realistically modeled and the intermediate- and large-q^2 regions are represented by a superposition of the well-known "spontaneous" and "explicit" breaking asymptotic QCD solutions. We couple the *W*-boson gauge invariantly to the dynamical quarks and derive formulas for the pion and kaon decay constants f_π and f_K in the one-quark-loop approximation. With $f_\pi = 93$ MeV as input, we calculate the value $f_K/f_\pi = 1.13$, compared to 1.16 experimentally, for a representative value $\Lambda_{\text{QCD}} = 200$ MeV. In addition, we comment on the calculation of the pion electromagnetic form factor.

I. INTRODUCTION

Understanding of the way in which quark and gluon degrees of freedom can be replaced at low energy by composite meson degrees of freedom has progressed significantly in the past several years.[1] Various plausible lines of reasoning coupled with formal manipulations of field variables, especially as implemented in the framework of the functional integral formulation of the composite field's Lagrangian, have allowed a discussion of the Goldstone theorem in the context of bilocal field theory[2] and have illuminated the path by which QCD can lead to the chiral dynamics of Goldstone bosons. Physical quantities depend in a very detailed (often complicated) way on the pseudoscalar-meson wave functions, which in turn have been shown to be simple functions of the quark dynamical mass function in the spontaneous-chiral-breaking phase of the theory. Semiphenomenological results for zero quark mass and for nonzero *u*- and *d*-quark masses in the SU(2)-flavor sector of the chiral Lagrangian have been compared to experiment.[2–4] However, little attempt has yet been made to treat the quark self-mass $\Sigma(q)$, and, therefore, the pseudoscalar-meson wave functions, in a way fully consistent with asymptotic freedom, confinement, and explicit chiral-symmetry breaking by quark mass terms,[5,6] nor has the problem of handling the strange mesons and the large (u,d)-s quark mass splitting within the bilocal chiral Lagrangian dynamical framework been dealt with.

It is precisely these latter problems which we address in this paper: namely, the construction of a useful approximate solution to the Schwinger-Dyson equation consistent with asymptotic freedom and confinement of explicitly massive quarks (i.e., explicit symmetry-breaking solutions), and the inclusion of kaons in the problem, with attendant asymmetries in their wave functions due to the large mass difference between "bare" *s* and (u,d) quarks.

In Sec. II we outline the model for the gluon propagator which we adopt here and briefly review recent developments in understanding the roles of the "explicit" and the "spontaneous" symmetry-breaking solutions to the Schwinger-Dyson equations in QCD (Refs. 5 and 6) or, equivalently to the quark mass renormalization-group equations as originally formulated.[7]

In Sec. III we sketch the derivation of the pion mass terms in the effective Lagrangian and show that we obtain the standard current-algebra mass formula[8] in lowest order in explicit quark-mass-breaking factors. We follow this with a generalization to the kaon mass term and present our ansatz for the kaon wave function, involving as it does the strange and nonstrange quark dynamical masses. The kaon mass equation that results in lowest order in explicit quark masses is also the same as the standard current-algebra formula.[8]

In Sec. IV we derive the wave-function renormalization factors for the pion and kaon from the kinetic energy term, calculate the pion and kaon decay-constant formulas by introducing the gauge invariantly coupled *W* boson in the quark-meson Lagrangian, and fix the parameters in the model for the quark dynamical mass functions by using experimental values for m_π, m_K, and f_π, with f_π also used to set the scale of the long-distance part of the gluon propagator. The outputs are m_u $(=m_d$ by assumption), m_s, $\langle \bar{u}u \rangle$ $(=\langle \bar{d}d \rangle = \langle \bar{s}s \rangle$ in lowest order), and f_K. With[9] $f_\pi = 93$ MeV and a standard choice $\Lambda_{\text{QCD}} = 200$ MeV, we calculate $f_K = 105$ MeV or $f_K/f_\pi = 1.13$. This value compares well with $f_K/f_\pi = 1.16$ obtained from the measured rates[10] $\Gamma(K \to \mu\nu)$ and $\Gamma(\pi \to \mu\nu)$ and the value of the sine of the Cabibbo angle[9] $\sin\theta_C = 0.229$. Our f_K/f_π result is also close to the value 1.15 that one obtains from a strictly phenomenological analysis using a chiral Lagrangian.[11] We find the result very encouraging, containing as it does a large amount of detailed treatment of the kaon structure.

As an illustration of another low-energy parameter which is sensitive to the dynamics of the quark binding in the pseudoscalar wave functions, we compute the electromagnetic form factor of the pion in our formalism and check that its normalization is automatically correct when the kinetic energy of the pion field is properly normalized. The value for $\langle r_\pi^2 \rangle$ so obtained turns out to be much too small. This indicates that we ought to include at least the hadronic vector mesons, which are known phenomenologically to play a principal role in the description of form factors.[12] The formalism used in this paper indeed requires at appropriate energies the inclusion of bound states other than just the pseudoscalars, such as the vector mesons, and we plan to look further into this in a separate study.

In three appendixes we present a discussion of what we mean by the "Landau-type" gauge which we use in the text, we outline the argument involved in choosing the kaon wave function and approximating integrals, and we collect several detailed formulas—the kinetic energy normalization factors and the full pion form-factor expression—which are not explicitly needed in the text.

II. THE QUARK PROPAGATOR

Extremizing the effective action of Sec. III with respect to the translationally invariant bilocal field yields the Schwinger-Dyson equation in the ladder approximation. In Landau-type gauges (see Appendix A), this equation reads, in momentum space,

$$\Sigma(q^2)=\int G(k-q)\frac{\Sigma(k^2)}{k^2-\Sigma^2(k^2)}d^4k \ , \tag{1}$$

where G, the gluon potential, absorbs all constant factors.

We decompose $G(q)$ in a confining long-distance term proportional to $\delta^4(q)$ which is a regularization of the distribution q^{-4} and was employed by Munczek and Nemirovsky[13] with phenomenological success, and a Coulombic short-distance term, including a running coupling constant $\alpha_s(q^2)$ in a manner used extensively in the literature.[5,6,14,15] Specifically, we take

$$-G(k-q)=\eta^2\delta^4(k-q)+\frac{i}{\pi^2}\frac{\tilde{\lambda}(k-q)}{(k-q)^2} \ , \tag{2}$$

where, in Euclidean space, $\tilde{\lambda}(k-q)=\lambda(k^2)\theta(k^2-q^2)+\lambda(q^2)\theta(q^2-k^2)$ and $\lambda(q^2)=3[C_2(F)/4\pi]\alpha_s(q^2)$. $C_2(F)$ is the eigenvalue of the quadratic Casimir operator of the color group in the quark representation and $\alpha_s(q^2)$ is the one-loop running coupling constant of QCD. The approximation of breaking up α_s into $k^2>q^2$ and $q^2<k^2$ regions is standard in the literature, most recently appearing in technicolor applications,[15] and is adopted here. The dynamical mass equation now reads in Euclidean space

$$\Sigma(x)=m(\Lambda)+\eta^2\frac{\Sigma(x)}{x+\Sigma^2(x)}+\frac{\lambda(x)}{x}\int_0^x\frac{y\Sigma(y)dy}{y+\Sigma^2(y)}+\int_x^{\Lambda^2}\frac{\Sigma(y)\lambda(y)dy}{y+\Sigma^2(y)} \ , \tag{3}$$

where $x=p^2$ and we take $\lambda(x)=\lambda(x_0)$ when $x<x_0$. Here we have defined $\lambda(x)=(\pi b\ln x)^{-1}$, $b=(11N_c-2n_f)/12\pi$, with N_c the number of quark colors and n_f the number of flavors. $\Lambda_{\rm QCD}$ provides our scale and the arguments of the logarithms are understood to be made dimensionless by an implicit factor $\Lambda_{\rm QCD}^{-2}$. The Coulombic part has been given an infrared transition point x_0 which will be determined self-consistently within the approximation which we introduce later. The ultraviolet cutoff is necessary to define the bare quark mass with the potential chosen. This latter point has been discussed by Miransky[6] and also by Leung, Love, and Barden,[14] among others, and it is necessary in order to define the regularized product $m\langle\bar{q}q\rangle$ which determines the physical pseudoscalar masses. If there is no bare mass and the chiral breaking is purely spontaneous, i.e., $\langle\bar{q}q\rangle\neq 0$, while $\partial^u j_\mu^5=\lim m(\Lambda)\langle\bar{q}\gamma_5 q\rangle_\Lambda=0$, when the cutoff is taken to infinity, then the asymptotic form of the solution for $\Sigma(p^2)$ is $\sim 1/p^2(\ln p^2)^{-d+1}$, where $d=12/(33-2n_f)$ for three colors and n_f quark flavors. With an explicit breaking of chiral symmetry, one should include also a piece with the asymptotic behavior $\sim 1/[\ln(p^2)]^d$ so that $\partial^u j_\mu^5=m\langle\bar{q}\gamma_5 q\rangle\neq 0$ as the cutoff runs to infinity.[5,6,14]

It is our aim to include explicit chiral-symmetry-breaking mass terms for u, d, and s quarks, so we adopt a linear combination of the asymptotic forms $1/p^2(\ln p^2)^{-d+1}$ and $1/(\ln p^2)^d$ in our approximation to $\Sigma(p^2)$ for $p^2>p_0^2$, while adopting a linear approximation for $p^2<p_0^2$. We postpone further discussion of our (approximate) solutions to Eq. (3) until after our elaboration on the choice of (bilocal) pseudoscalar-meson wave function and its application to the pseudoscalar mass terms within our QCD-based effective Lagrangian approach.

III. PSEUDOSCALAR-MESON MASSES

A. Discussion

As developed in our earlier work, an effective bilocal Lagrangian in terms of bosonic degrees of freedom results from integrating the gluon and quark degrees of freedom. Schematically we have[2]

$$W_{\rm eff}[\omega]=-i\,{\rm Tr}\ln(i\partial\!\!\!/-\bar{\omega})+\tfrac{1}{2}{\rm Tr}(\omega\bar{\omega}) \ , \tag{4}$$

when there is no explicit chiral-symmetry breaking, where $\bar{\omega}(x,y)\equiv\gamma^\mu\omega(x,y)\gamma^\nu G_{\mu\nu}(x-y)$ and where Tr indicates the trace over discrete indices and integration over space-time variables. The bilocal field $\omega(x,y)$ has a translationally invariant piece and we define

$$\omega(x,y)\equiv -iS(x-y)+\phi(x,y) \ . \tag{5}$$

Minimizing $W_{\rm eff}[\omega]$ with respect to $\bar{S}$ and setting $\phi=0$ yields the ladder approximation to the Schwinger-Dyson equation

$$S^{-1}(x-y)=\langle x|(i\partial\!\!\!/+i\bar{S})|y\rangle \ , \tag{6}$$

whose momentum-space version we discussed in the preceding section. The bilinear terms in the expansion of $W_{\rm eff}[\omega]$ are given in momentum space by the expression

$$W_{\rm eff}^{(2)}=\frac{1}{2}\frac{1}{(2\pi)^8}\mathrm{tr}\left[i\int d^4p\int d^4q\left[S(q+\tfrac{1}{2}p)\bar{\phi}(p,q)S(q-\tfrac{1}{2}p)\bar{\phi}(-p,q)+\int d^4p\int d^4q\,\phi(p,q)\bar{\phi}(-p,q)\right]\right]. \quad (7)$$

The tr indicates tracing over discrete indices only.

The mass terms for the pseudoscalar fields are found by projecting out the pseudoscalar part $\gamma_5\phi_P(p,q)$ of $\phi(p,q)$, the Fourier transform of the field $\phi(x,y)$ defined by Eq. (5), and by setting $p=0$ in the fermion propagators, $S(q\mp\frac{1}{2}p)$. One obtains then the following expression for the pseudoscalar field mass terms:

$$W_{\rm mass}^{(2)}(\phi_P)=-\frac{2}{(2\pi)^8}\mathrm{tr}\int d^4p\int d^4q\left[\frac{i}{D(q)}\bar{\phi}_P(p,q)+\phi_P(p,q)\right]\bar{\phi}_P(-p,q), \quad (8)$$

where $D(q)=q^2-\Sigma^2(q)$ in Landau-type gauges. By definition,

$$\bar{\phi}_P(p,q)\equiv i\int\phi_P(p,k)G(q-k)d^4k\ , \quad (9)$$

where $G(q-k)$, proportional to the gluon two-point function in purely Yang-Mills theory, is defined so that gauge-dependent factors are absorbed in it. As in a previous paper,[2] we assume a factorized form for the bilocal field: namely,

$$\bar{\phi}_P(p,q)=\psi(p)\bar{\phi}(q)\ . \quad (10)$$

Also, as in Ref. 2, we choose for the pion wave function

$$\bar{\phi}(q)=i\Sigma(q)\ , \quad (11)$$

assuming u-d degeneracy. Furthermore, when there is explicit chiral-symmetry breaking the Schwinger-Dyson equation (1) is

$$\Sigma(q)=m+\int\frac{\Sigma(k)}{D(k)}G(q-k)d^4k\ . \quad (12)$$

Using the expressions above and keeping just the first order in m, the charged π mass term of Eq. (8), for example, has the form

$$W_{\rm mass}[\phi_\pi]=-4miN_c\left[\int\frac{d^4q}{(2\pi)^4}\int d^4k\,G^{-1}(k-q)\int d^4t\,G(q-t)\frac{\Sigma(t)}{D(t)}\right]\frac{1}{N}\int\frac{d^4p}{(2\pi)^4}\pi^+(p)\pi^-(-p)\ , \quad (13)$$

where $\pi(p)\equiv\psi(p)\sqrt{N}$. The normalization factor N will be discussed in Sec. IV in connection with the kinetic energy terms in the effective action, where it is shown that $N=f_\pi^2/2$.

We can rewrite Eq. (13) in the form

$$W_{\rm mass}[\phi_\pi]=-m_\pi^2\int d^4x\,\pi^+(x)\pi^-(x)\ , \quad (14)$$

with the standard current-algebra expression

$$m_\pi^2=2\frac{m}{f_\pi^2}\langle\bar{u}u\rangle\ , \quad (15)$$

where a sum over colors is understood.

As mentioned above, we did not consider isospin breaking here and took $m_u=m_d$. We consider the case of unequal quark masses in studying the kaon system.

As we discussed in the previous section, the product $m\langle\bar{q}q\rangle$, nonzero in the presence of explicit chiral-breaking mass terms in the QCD Lagrangian, must be treated by introducing a cutoff in the Schwinger-Dyson equation with the short-distance behavior governed by asymptotic freedom. The cutoff-dependent mass $m(\Lambda)$ goes to zero as $\Lambda\to\infty$, while $\langle\bar{q}q\rangle\simeq\int^{\Lambda^2}\Sigma(x)dx$ blows up as $\Lambda\to\infty$ with the product remaining finite.

B. Explicit SU(3)-flavor breaking

Introducing flavor breaking among the quark masses, we can study the mass, decay constant and form factors of the kaons. For example, we write the flavor space indices explicitly in the bilocal term:

$$\mathrm{tr}S\bar{\phi}S\bar{\phi}=S_a\delta_a^b\bar{\phi}_b^cS_c\delta_c^d\bar{\phi}_d^a=S_1\bar{\phi}_1^3S_3\bar{\phi}_3^1+\cdots \quad (16)$$

to obtain the bilinear charged-kaon term in the effective-action expansion:[16]

$$W_{\rm eff}^{(2)}(K^+)=\frac{1}{2}\frac{i}{(2\pi)}\mathrm{tr}\int d^4p\left[\int d^4q[S_u(q+\tfrac{1}{2}p)\gamma_5S_s(q-\tfrac{1}{2}p)\gamma_5\bar{\phi}_{K^+}(p,q)\bar{\phi}_{K^-}(-p,q)-\phi_{K^+}(p,q)\bar{\phi}_{K^-}(-p,q)]+[p\leftrightarrow-p]\right], \quad (17)$$

where $\bar{\phi}_1^3 \equiv -\gamma_5 \bar{\phi}_{K^+}$, for example.

Isolating the mass term gives

$$W_{\text{mass}}^{(2)}(K) = \frac{4N_c}{(2\pi)^8} i \int d^4p \int d^4q \left[\left[\frac{-q^2+\Sigma_u(q^2)\Sigma_s(q^2)}{D_u(q^2)D_s(q^2)} \bar{\phi}_{K^+}(p,q) - \phi_{K^+}(p,q) \right] \bar{\phi}_{K^-}(-p,q) + (u \to d, K^- \to K^0, K^+ \to \bar{K}^0) \right] . \quad (18)$$

The wave-function ansatz which we make to generalize the case where the constituent masses are equal is, taking K^-, for example,

$$\bar{\phi}_{K^-}(p,q) = i\psi_{K^-}(p)\tfrac{1}{2}[\Sigma_u(q)+\Sigma_s(q)] , \quad (19)$$

which can be inverted by using Eq. (9) and the Schwinger-Dyson equation (13) to obtain ϕ for use in the mass term, Eq. (18). We comment further on Eq. (19) in Appendix B.

Keeping leading terms in $\Delta\Sigma = \Sigma_s - \Sigma_u$ and explicit factors of m_s and m_u, we have the generalization of the mass term of the pseudo-Goldstone boson to the case of unequal quark mass constituents:

$$W_{\text{mass}}^{(2)}(K) = \int \frac{d^4p}{(2\pi)^4} \frac{K^+(p)K^-(-p)}{N} \left[(m_s+m_u) \left[\frac{-iN_c}{2} \right] \int \frac{d^4q}{(2\pi)^4} \left[\frac{1}{D_s(q^2)} + \frac{1}{D_u(q^2)} \right] [\Sigma_s(q^2)+\Sigma_u(q^2)] \right] + (u \to d, K^- \to K^0, K^+ \to \bar{K}^0) , \quad (20)$$

where to leading order in quark masses N is the same as in Eq. (13). Finally, when the zeroth-order expressions for Σ_u and Σ_s are used, namely, those for the massless quark case with spontaneous symmetry breaking, we recover the standard expression[8]

$$m_K^2 = \frac{m_s+m_u}{f_\pi^2} \langle \bar{q}q \rangle , \quad (21)$$

where $\langle \bar{q}q \rangle = \langle \bar{s}s \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$.

Let us consider the kinetic energy normalization next, including the $m_s \neq m_u, m_d$ effects and $\Sigma_s \neq \Sigma_{u,d}$ effects, since this will be needed in our next problem: namely, the determination of f_K.

IV. CALCULATION OF f_π and f_K

We need to normalize the pseudoscalar fields and calculate their mixing amplitudes with the W boson in order to evaluate f_π and f_K. Both the kinetic energy and W-π mixing terms are generated by the quark loop expansion, summarized now in the presence of W by the effective action term

$$W_{\ln} = -i \, \text{Tr} \ln[1 - S(WL + \bar{\phi})] , \quad (22)$$

where Tr denotes trace over spin and internal-symmetry indices and integration over space-time variables. The left-chirality projection operator is $L = (1-\gamma_5)/2$ and S is the full quark propagator. W is an appropriate matrix in flavor space for the charged weak boson. The field $\bar{\phi}$ will be given by our choice Eq. (11) for the pion and Eq. (19) for the kaon. The relevant effective action terms for the charged kaon kinetic energy and W mixing are

$$\frac{i}{2} N_c \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \text{tr}(S_u^+ \gamma_5 \bar{\phi}_{K^+}^{(+)} S_s^- \gamma_5 \bar{\phi}_{K^-}^{(-)}) \quad (23)$$

and

$$ig \frac{1}{\sqrt{2}} \sin\theta_C N_c \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \text{tr}[S_u^+ W^+(p) L S_s^- \gamma_5 \times \bar{\phi}_{K^-}(-p,q) + \text{H.c.}] \quad (24)$$

with tr designating the trace over spin, $S^\pm = S(q \pm \frac{1}{2}p)$ and $\bar{\phi}^{(\pm)} = \bar{\phi}(\pm p, q)$. Expanding the integrand in Eqs. (23) and (24), we keep the quadratic and linear terms in the variable p to identify the kinetic energy for K and W-K mixing. We omit the pseudoscalar–axial-vector-meson mixing in the approximation used here, and we keep the terms linear in (Σ_s-Σ_u) but drop higher powers of the mass-function difference. As outlined in Appendix B, the results can be conveniently expressed in the forms

$$W_{\text{KE}}(K) = \int d^4x \, \partial_\mu \psi_{K^+}(x) \partial^\mu \psi_{K^-}(x) \{ \tfrac{1}{2}[J(s)+J(u)] \} \quad (25)$$

and

$$W_{WK}(K) = \frac{1}{\sqrt{2}} g \sin\theta_C \int d^4x W_\mu^+ \partial^\mu \psi_{K^-}(x) \times \{ \tfrac{1}{2}[J(s)+J(u)] \} + \text{H.c.} , \quad (26)$$

where

$$J(q) = \frac{N_c}{8\pi^2} \int_0^\infty dx \frac{x \Sigma_q}{(x+\Sigma_q^2)^2} (\Sigma_q - \tfrac{1}{2} x \Sigma_q') , \quad (27)$$

with $q=u,s$. The equal-mass, pion, case can easily be retrieved by setting $s=d$ in Eqs. (25) and (26). The normalized kaon wave function is then given by

$$K(x)=\psi_K(x)\sqrt{N_K}\ ,$$

where

$$N_K=\tfrac{1}{2}[J(s)+J(u)]\ . \tag{28}$$

Referring to our previous work[2] for f_P identifications, where $P=\pi$ or K, we have

$$f_\pi=[2J(u)]^{1/2}\ , \tag{29}$$

$$f_K=[J(s)+J(u)]^{1/2}\ . \tag{30}$$

These results for f_π and f_K depend on our choices for the pseudoscalar-meson wave functions (11) and (19) which satisfy the requirements of the Goldstone theorem in the limit when the explicit quark masses vanish.[2] To make further progress, we must examine the solutions to the Schwinger-Dyson equations for $\Sigma_q(x)$ when explicit chiral-symmetry-breaking quark mass terms are included, as discussed in Sec. II.

Practically speaking, we only need reasonable approximations to the solutions to Eq. (3) for the strange- and nonstrange-quark cases. The function

$$\begin{aligned}\Sigma_q(x)&=\Sigma_q(0)-\frac{1}{2}\frac{x}{\Sigma_q(0)},\quad x<(x_0)_q\ ,\\ \Sigma_q(x)&=\frac{C_q}{x(\ln x)^{(1-d)}}+\frac{\kappa_q/C_q}{(\ln x)^d},\quad x>(x_0)_q\ ,\end{aligned} \tag{31}$$

with $\Sigma(x)$ and $\Sigma'(x)$ required to be continuous at $x=x_0$, reproduces main features of the low-energy behavior of Eq. (3) as well as the well-known asymptotic behavior for the explicit breaking case.[5,6] Since the integrals which determine f_π and f_K, Eq. (29) and Eq. (30), converge slowly, we use the value $d=\frac{12}{21}$ corresponding to six flavors.

Our mass formulas (15) and (21) enable us to determine κ_q for $q=u$ and s, respectively. To zeroth order in explicit mass breaking we have, for its divergent part,

$$\begin{aligned}\langle\bar{q}q\rangle&\simeq\frac{N_c}{4\pi^2}\int^{\Lambda^2}\Sigma_q(x)dx\simeq\frac{N_cC_q}{4\pi^2}\int^{\Lambda^2}\frac{dx}{x(\ln x)^{1-d}}\\ &=\frac{N_cC_q}{4\pi^2 d}(\ln\Lambda^2)^d\ .\end{aligned}$$

On the other hand, the explicit mass parameter itself depends on the cutoff as [see Eqs. (3) and (31)]

$$m_q(\Lambda)\simeq\frac{d\kappa_q}{C_q}\int_{\Lambda^2}^{\infty}\frac{dy}{y(\ln y)^{1-d}}=\frac{\kappa_q}{C_q}\frac{1}{(\ln\Lambda^2)^d}\ .$$

The product is cutoff independent, and using Eqs. (15) and (21) we obtain

$$\kappa_u=f_\pi^2 m_\pi^2\frac{2\pi^2}{N_c}d \tag{32a}$$

and

$$\kappa_s=f_\pi^2 m_K^2\frac{4\pi^2}{N_c}d\ , \tag{32b}$$

where $m_u(\Lambda)\ll m_s(\Lambda)$ allows one to neglect terms of order m_u/m_s in (32b).

Our functions Σ_u and Σ_s are not yet completely determined. With Eqs. (32) determining κ_q, we have the three parameters C, $\Sigma(0)$, and x_0 to be determined by the continuity of $\Sigma(x)$ and $\Sigma'(x)$ at $x=x_0$ plus one more condition. We choose to use the equation for f_π, Eq. (29), to provide the needed information. The parameters are summarized in Table I. The values for Σ_s follow by noticing that in Eq. (3) the parameter η^2 represents confining, purely gluonic effects and is flavor independent. We should have then

$$\eta^2=\Sigma_q^2(0)-\frac{1}{\pi}\Sigma_q(0)\int_0^\infty\frac{\alpha_s(y)\Sigma_q(y)}{y+\Sigma_q^2(y)}dy\ , \tag{33}$$

where, as in Sec. II, we have

$$\alpha_s(x)=\frac{d}{(\ln x_0)_u},\quad x<(x_0)_u\ ,$$

$$\alpha_s(x)=\frac{d}{\ln x},\quad x>(x_0)_u\ .$$

Equation (33) determines η^2 given that $\Sigma_u(x)$ is determined by the value of f_π plus the continuity requirements, subsequently we can determine $(x_0)_s$ from (33). The functions $\Sigma_s(x)$ and $\Sigma_u(x)$ are plotted in Fig. 1. We find that in the range $0<x<1000$ (in units of $\Lambda_{\rm QCD}^2$) the original integral equation (3) is typically satisfied by our approximate solutions at the 10–20 % level.

With the dynamical mass functions just described and displayed in Fig. 1, predictions for f_K and $\langle r_\pi^2\rangle$, the pion charge radius, can be make within our QCD-based effective Lagrangian framework. We find

$$\frac{f_K}{f_\pi}=1.13\ . \tag{34}$$

The value for f_K/f_π as extracted from experiment depends somewhat on assumptions about universality, axial-vector renormalization, Cabibbo angles in mesonic versus baryonic processes, and so forth, but a commonly used value[17] $\sin\theta_C=0.229$ yields $f_K/f_\pi=1.16$, so we find our result (34) very satisfying. For the pion charge radius we find, using the expression given in Appendix C, the value $\langle r_\pi^2\rangle=0.026$ fm^2 to be compared with the experimental result,[18] $\langle r_\pi^2\rangle=0.454$ fm^2. The fact that the

TABLE I. Values for the parameters appearing in Eqs. (3) and (31). κ, η^2, Σ, x_0, and C are of mass dimension four, two, one, two, and three, respectively. The values are given in units of $\Lambda_{\rm QCD}$, chosen to be 0.200 GeV.

q	κ	η^2	$\Sigma(0)$	x_0	C
u	0	5.46	2.75	8.82	14.01
u	0.398	5.46	2.75	8.22	14.13
s	9.92	5.46	2.92	7.69	9.69

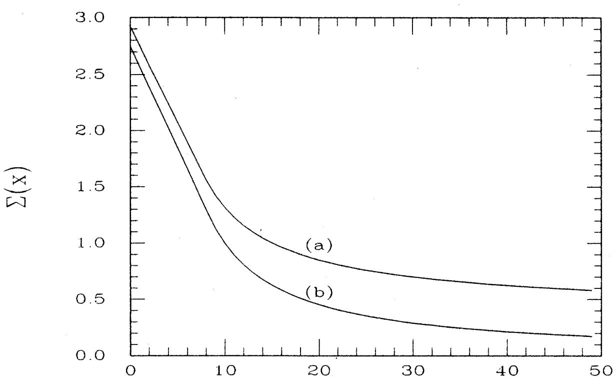

FIG. 1. Representation of $\Sigma(x)$ given by Eq. (31). Curve (a) is for the s quark. Curve (b) corresponds to the u quark both for $\kappa=0$ and 0.398 (within the accuracy of the drawing). $\Sigma(x)$ and x, the momentum squared, are given in units of $\Lambda_{\rm QCD}$, chosen to be 0.200 GeV. Table I gives the values of the parameters for each curve.

value calculated is so small is to be expected since, as is well known, vector-meson dominance of the electromagnetic current provides a good quantitative description of the pion form factor and the vector mesons are not yet accounted for in our treatment, which explains the discrepancy. An analogous situation should arise in trying to calculate K_{l3} form factors, for example. On the other hand, our result for f_K indicates that the kaon wave function may be a good approximation to use in other low-energy mixing, or two-point functions calculations, e.g., $\Delta S=2$ processes.

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APPENDIX A

The ladder approximation to the Schwinger-Dyson equation is commonly studied in the Landau gauge because only the mass term of the second-order perturbative proper self-energy part obeys a nontrivial equation in this gauge. In this appendix we show that a "Landau-type" gauge can always be chosen in such a way that the ladder Schwinger-Dyson equation similarly simplifies to a single equation for the self-mass.

We work in configuration space, where the argument is quite simple. We can choose a gauge in which the gauge field two-point function is

$$G_{\mu\nu}(x)=G^T_{\mu\nu}(x)+x_\mu x_\nu g(x^2)$$

or

$$G_{\mu\nu}(x)=g_1(x^2)g_{\mu\nu}+g_2(x^2)x_\mu x_\nu\ , \tag{A1}$$

where $\partial^\mu G^T_{\mu\nu}(x)=0$ and $g(x^2)$ can be an arbitrary function.[19] The proper self-energy part $\Sigma(x)=\not\partial\Sigma_1(x^2)+\Sigma_2(x^2)$ and the fermion propagator $S(x)=\not\partial S_1(x^2)+S_2(x^2)$ are related by the Schwinger-Dyson equation

$$\Sigma(x)=-i\gamma_\mu S(x)\gamma_\nu G^{\mu\nu}(x) \tag{A2}$$

in the ladder approximation. Constant factors have been absorbed into the definition of G in order to simplify notation.

The equation for $\Sigma_1(x^2)$ is readily found to be

$$-i[-2g_1(x^2)+x^2g_2(x^2)]\not\partial S_1(x^2)=\not\partial\Sigma_1(x^2)\ , \tag{A3}$$

so the gauge choice such that

$$g_1(x^2)=\frac{x^2}{2}g_2(x^2) \tag{A4}$$

allows one to set $\Sigma_1(x^2)$ equal to zero.

APPENDIX B

We sketch here the arguments which lead us to the kaon wave function choice and to the approximate forms for the kaon kinetic energy and W-mixing terms used in the text.

Concerning $\bar\phi_K$, we first note that it should reduce to the Goldstone-boson form $\bar\phi_K=i\Sigma=\bar\phi_\pi$ when explicit quark mass terms are zero[2] ($\Sigma_s=\Sigma_u\equiv\Sigma$ in this case). The kaon effective action is symmetric in s and u or s and d, and we choose $\bar\phi_K$ to be accordingly symmetric. Furthermore, we shall only keep terms linear in $\Delta\Sigma\equiv\Sigma_s-\Sigma_u$ since this is consistent with working to first order in explicit chiral-breaking quark masses. It follows that Eq. (19) is essentially unique in satisfying the aforementioned requirements.

Concerning the integral which appears in the kinetic energy, for example, we note that the integrand is symmetric in the interchange of Σ_s and Σ_u given the above symmetry in $\bar\phi_K$. We can think of the integrand (integral over four-momentum), as a function $F(\Sigma_s,\Sigma_u)=F(\Sigma_u,\Sigma_s)$ which we can expand in powers of $\Delta\Sigma\equiv\Sigma_s-\Sigma_u$. We write

$$F(\Sigma_s,\Sigma_u)=F(\Sigma_u,\Sigma_u)+\frac{d}{d\Sigma_s}F(\Sigma_s,\Sigma_u)\bigg|_{\Sigma_s=\Sigma_u}(\Sigma_s-\Sigma_u)+\cdots$$

or

$$F(\Sigma_s,\Sigma_u)=F(\Sigma_s,\Sigma_s)+\frac{d}{d\Sigma_u}F(\Sigma_s,\Sigma_u)\bigg|_{\Sigma_u=\Sigma_s}(\Sigma_u-\Sigma_s)+\cdots\ .$$

We can therefore rewrite $F(\Sigma_s,\Sigma_u)$ as

$$F(\Sigma_s,\Sigma_u)=\tfrac{1}{2}[F(\Sigma_s,\Sigma_s)+F(\Sigma_u,\Sigma_u)]+\Delta\Sigma[F'(\Sigma_u,\Sigma_u)-F'(\Sigma_s,\Sigma_s)]+O(\Delta\Sigma)^2\ .$$

But $F'(\Sigma_u,\Sigma_u)-F'(\Sigma_s,\Sigma_s)$ is itself of order $\Delta\Sigma$, so up to terms of order $(\Delta\Sigma)^2$ we have

$$F(\Sigma_s,\Sigma_u)=\tfrac{1}{2}[F(\Sigma_s,\Sigma_s)+F(\Sigma_u,\Sigma_u)]\ .$$

The above argument is the basis for our approximations for the kaon normalization factor and for the K-W mixing factor.

APPENDIX C

In this appendix, we offer some of the details of the pion form-factor calculation. The electromagnetic potential A_μ is coupled gauge invariantly to the quarks, and after integrating out gluons and quarks the effective action can be written now to include A_μ in Eq. (22) as follows:

$$W_{\ln}=-i\,\mathrm{Tr}\ln[1-S(QA+WL+\bar{\phi})]\ , \tag{C1}$$

where Q is the quark charge matrix. Expanding Eq. (C1) to third order, one obtains the charged pion coupling to A_μ in the form

$$W[\pi,\pi,A]=-i\frac{N_c}{N}e\int d^4x\ A^\lambda(x)\int d\bar{k}_1 d\bar{k}_2\pi^+(k_1)\pi^-(-k_2)\frac{(k_1+k_2)_\lambda}{2}e^{-x\cdot(k_1-k_2)}I(k_1,-k_2) \tag{C2}$$

$$\equiv\int d^4x\ A^\lambda(x)j_\lambda(x)\ . \tag{C3}$$

We use the condensed notation $d^4k/(2\pi)^4\equiv d\bar{k}$. We can read off the expression for the current operator:

$$j_\lambda(0)=-i\frac{N_c}{N}e\int d\bar{k}_1\int d\bar{k}_2\pi^+(k_1)\pi^-(-k_2)\frac{(k_1+k_2)_\lambda}{2}I(k_1,-k_2)\ . \tag{C4}$$

The factor N is the pion wave-function normalization factor and the integral $I(k_1,-k_2)$ can be shown to depend only on $(k_1-k_2)^2\equiv q^2$, and we work in the massless quark and massless pion limit in the following formulas. The expression for $I(q^2)$ then is

$$I(q^2)=4\int d^4\bar{l}\frac{\Sigma\left[l+\frac{k_1}{2}\right]\Sigma\left[l-\frac{k_2}{2}\right]}{D(l+k_1)D(l)D(l-k_2)}\times\left[\frac{k\cdot l}{k^2}[-l^2+k^2-q^2/4+\Sigma(l)\Sigma(l-k_2)+\Sigma(l)\Sigma(l+k_1)-\Sigma(l+k_1)\Sigma(l-k_2)]\right.$$
$$\left.-2l^2-2l\cdot k+\Sigma(l)[\Sigma(l-k_2)+\Sigma(l+k_1)]\right]\ , \tag{C5}$$

with $D(p)\equiv p^2-\Sigma^2(p)$ and $k\equiv k_1+k_2/2$. Taking the matrix element of $j_\lambda(0)$ between pseudoscalar states, we identify $F_\pi(q^2)$ from the expression

$$\langle p_2|j^\mu(0)|p_1\rangle=\frac{-e(p_1+p_2)^\mu}{(2\pi)^3(4\omega_1\omega_2)^{1/2}}F_\pi(q^2)\ , \tag{C6}$$

where $q^2=(p_1-p_2)^2$ here and the normalization is that of Bjorken and Drell.[20] The result is

$$F_\pi(q^2)=i\frac{N_c}{N}\frac{I(q^2)}{2}\ , \tag{C7}$$

and one can check that $I(0)=-i2(J/N_c)=-i2(N/N_c)$, so that $F_\pi(0)=1$ as it must. The integral J is given in Eq. (27).

The charge radius, obtained by expanding Eq. (C7) in powers of q^2, retaining the leading term and using the definition

$$\langle r_\pi^2\rangle=\frac{1}{6}\frac{d}{dq^2}F_\pi(q)^2\bigg|_{q^2=0}\ ,$$

is given by the following expression:

$$\langle r_\pi^2\rangle=\frac{1}{6}\frac{1}{I(0)}\left[\frac{1}{16\pi^2}\int_0^\infty dx\frac{x^2}{D^2(x)}[E_1(x)+E_2(x)+E_3(x)]\right]\ , \tag{C8}$$

where $D(x)=x+\Sigma^2(x)$ and the functions $E_i(x)$ are

$$E_1=2\frac{D'}{D}\Sigma'\Sigma-4\Sigma^2\frac{(D')^2}{D^2}-(\Sigma')^2\ ,$$

$$E_2=x\left[\frac{\Sigma^2\Sigma'}{D^2}\left[\frac{D''}{2}+\frac{2(D')^2}{D}\right]\frac{20}{3}+\frac{8(D')^3\Sigma^2}{D^3}-\tfrac{5}{12}\Sigma'\Sigma''+\frac{2}{3}\frac{\Sigma\Sigma'}{D}\left[4D''-\frac{(D')^2}{D}\right]+\frac{4D'}{3D}[\Sigma\Sigma''-\tfrac{5}{4}(\Sigma')^2]\right]\ ,$$

and

$$E_3=-\frac{\Sigma}{D}\left[-4\Sigma\Sigma'\frac{D'}{D}+2(\Sigma')^2-\tfrac{10}{3}\Sigma'\Sigma''x\right]\ .$$

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[16]Throughout the paper we use momentum-space variables p and q which are conjugate to the configuration-space variables $z=(x+y)/2$ and $t=x-y$, respectively, where the quark field variables in the composite fields ϕ are x and y. For unequal-mass quarks in the composite field, as in the kaon case, one could choose variables $z=\alpha x+\beta y$ and $t=x-y$, with $\alpha\neq\beta$ and $\alpha+\beta=1$, reflecting the shifted center of mass in the nonrelativistic sense. The corresponding change in the kinetic energy formula is that $S_u(q+\frac{1}{2}p)\rightarrow S_u(q+\alpha p)$ and $S_s(q-\frac{1}{2}p)\rightarrow S_s(q-\beta p)$, and the corrections to the following formulas are all proportional to $\alpha\beta$, which one can see by symmetry arguments. Writing $\alpha=\frac{1}{2}-\epsilon$, $\beta=\frac{1}{2}+\epsilon$, $\alpha\beta=\frac{1}{4}-\epsilon^2$, shows that quark mass difference effects, parametrized by ϵ, show up only in quadratic terms, which we have consistently dropped in the present work. To estimate the size of the corrections, one can take constituent-quark masses $M_u\approx\frac{1}{2}m_\rho$ and $M_s\approx\frac{1}{2}m_\phi$, appropriate to the *bound*-state effective masses. One finds $\epsilon=0.07$ and $\epsilon^2=0.005$, truly negligible. We might also remark that in dynamical calculations of meson mass spectra, it is found that results are insensitive to the choice of center-of-mass coordinate, i.e., choice of α and β [Munczek and Nemirovsky (Ref. 13)].

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