Flavor-changing neutral currents and seesaw masses for quarks

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The structure of flavor-changing neutral currents (FCNC's) which arise when one tries to give masses to quarks through the analogue of the seesaw mechanism for neutrino masses is studied. General conditions for the absence of FCNC's in the seesaw limit are derived. A specific model where these conditions are satisfied by the tree-level-quark mass matrices is identified. The FCNC's get generated in this model through radiative corrections to quark masses. These corrections lead to a hierarchy $F_{12} < F_{13} < F_{23}$ in typical strength F_{ij} of the neutral-current transition between ith and jth generations. Moreover the flavor-changing transitions between down quarks are suppressed compared to the corresponding transitions between up quarks. As a consequence, the FCNC's connecting d to s quark are greatly suppressed in conformity with observations such as K^0-$\bar{K}^0$ mixing and the decay $K_L \to \mu^+\mu^-$. In contrast, the transitions between c and t quarks could have observable magnitudes. The branching ratio for $Z \to \bar{t}c$ could be as large as 10^{-6}–10^{-7} in the model considered here. The D^0-$\bar{D}^0$ mixing could also be in the vicinity of the present experimental limits.

I. INTRODUCTION

The observed suppression of fermion masses relative to gauge-boson masses is attributed in the standard $SU(2)_L \times U(1)$ model to the presence of arbitrary small Yukawa couplings. The situation is most embarrassing in the case of neutrinos. If they are to obtain their mass in the same way as the other fermions do then one would require Yukawa couplings as small as 10^{-10}–10^{-11}. This is avoided by the seesaw mechanism of Gell-Mann, Ramond, Slansky, and Yanagida[1] where additional suppression in neutrino masses comes from the presence of vastly different mass scales in the theory. Small Yukawa couplings can be avoided this way also in the quark sector if one introduces additional SU(2)-singlet quarks having the same electric charges as the up and down quarks. Recently, such a seesaw mechanism for quark masses has been discussed by many authors.[2–8]

An immediate consequence of the seesaw mechanism for quark masses is the absence of the Glashow-Iliopoulos-Maiani (GIM) mechanism[9] which requires that all the quarks with given helicity and charge should transform identically under $SU(2)_L \times U(1)$. This results in the absence of flavor-changing neutral-current (FCNC) couplings of quarks to the Z boson in the $SU(2)_L \times U(1)$ model. In contrast, in the seesaw models,[2–8] additional $SU(2)_L \times SU(2)_R$-singlet quarks give rise to FCNC's through their mixing with the conventional quarks. Such couplings have to be adequately suppressed if the seesaw mechanism is to be viable.

This paper is addressed to a detailed discussion on the structure and magnitude of FCNC's in seesaw models. The basic objective is twofold. First, we wish to derive conditions under which the FCNC's are naturally absent to the leading order in the seesaw limit. A complete absence of FCNC's need not be required in practice. From the observational point of view it is sufficient to require that the flavor-changing couplings are adequately suppressed so as to be consistent with observations such as K^0-$\bar{K}^0$ mixing or the near absence of $K_L \to \mu^+\mu^-$ decay. Such couplings could be present in transitions involving higher generations. If this happens then the observation of such currents could in fact provide a window into the seesaw mechanism under study. The second objective of the paper is to study a specific seesaw model[8] which is shown to exhibit such hierarchy in FCNC transitions. In this model, the tree-level seesaw mass matrices do not give rise to any FCNC's. The radiative corrections to masses do generate FCNC's but they are such that one obtains a hierarchy in their structure. Specifically one finds $F_{12} < F_{13} < F_{23}$ where F_{ij} characterizes the strength of FCNC's between the ith and jth generation. Moreover, transitions in the down-quark sector are suppressed compared to the corresponding transitions in the up-quark sector. This pattern is consistent with observations and has interesting phenomenological consequences.

We proceed as follows. Section II contains a general discussion on structure of FCNC's in a class of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models which employ the seesaw mechanism to generate quark masses. Conditions for the natural absence of FCNC's in such models are derived in Sec. III to leading order in the seesaw limit. These are shown to be satisfied in specific models of Refs. 7 and 8 at the tree level. The radiative corrections to quark masses reintroduce FCNC's in these models. The structures of these induced FCNC's is studied in Sec. IV and V. In Sec. VI we discuss phenomenological consequences of this structure. Section VII contains a summary.

II. SEESAW MODELS AND STRUCTURE OF FCNC's

The seesaw mechanism can be naturally implemented in the left-right-symmetric[10] $SU(2)_L \times SU(2)_R$

$\times U(1)_{B-L} \equiv G$ model. However, the conventional[10] fermionic and bosonic content needs to be changed in order to implement it. In addition to n conventional quarks q_i ($i=1,\ldots,n$) one introduces m $SU(2)_L \times SU(2)_R$-singlet charged $-\frac{1}{3}$ ($\frac{2}{3}$) quarks N_α (P_α) ($\alpha=1,\ldots,m$) transforming as triplets of $SU(3)_c$. The conventional Higgs field transforming as $(\frac{1}{2},\frac{1}{2},0)$ is to be replaced by a pair of complex Higgs fields $\varphi_{L,R}$ transforming, respectively, like $(\frac{1}{2},0,1)$ and $(0,\frac{1}{2},1)$ under G. This has the consequence that the ordinary mass terms for quarks [transforming as $(\frac{1}{2},\frac{1}{2},0)$ under G] cannot be generated at the tree level. Instead the following seesaw form[2-8] results for the quark masses:

$$\mathcal{M}=\begin{bmatrix} 0 & m_1 \\ m_2 & M_H \end{bmatrix} . \tag{2.1}$$

Here m_1 (m_2) is an $n\times m$ ($m\times n$) matrix while M_H is an $m\times m$ matrix. The scale of m_1 [m_2] is determined by the vacuum expectation value of φ_L^0 [φ_R^0] and hence by the $SU(2)_L$- [$SU(2)_R$-]breaking scale. Thus for equal Yukawa couplings $m_1 \ll m_2$. The matrix M_H could result either from a bare mass term for the additional quarks or from their couplings to G-singlet scalar fields. Thus, the scale of M_H is arbitrary. The seesaw limit corresponds to $M_H \gg m_{1,2}$. In this limit, the matrix $\mathcal{M}$ generically possesses n light quarks with masses $\approx\langle\varphi_L^0\rangle\langle\varphi_r^0\rangle h^2/M$ and m heavy quarks with masses $\simeq M$. Here h and M correspond to typical Yukawa coupling and a typical scale in the matrix M_H. Because of the additional suppression $\langle\phi_R^0\rangle/M$ and because of the presence of h^2 instead of h, the required Yukawa couplings could be much larger than in the standard $SU(2)_L\times U(1)$ model.

The neutral-current couplings of the additional quarks N_α, P_α are different from that of the conventional quarks. The physical mass eigenstates involve both of them with the result that the Z^0 couplings to light quarks are not flavor diagonal, in general. Let us define the original weak eigenstate by $U'_{a,L,R}\equiv(u'_i,P'_\alpha)_{L,R} D'_{a,L,R} \equiv(d'_i,N'_\alpha)_{L,R}$ with $a=1,\ldots,n+m$. We shall sometimes denote them collectively by $G'_{a,L,R}\equiv(q'_i,H'_\alpha)_{L,R}$ with $q(H)$ representing $u(P)$ or $d(N)$. Physical mass eigenstates are determined by diagonalizing $\mathcal{M}$ having the form of Eq. (2.1) by a biunitary transformation. Thus we have, for the unprimed mass eigenstates,

$$(Q_a)_{L,R}=(A^q_{ab})_{L,R}(Q'_b)_{L,R} , \tag{2.2}$$

where A^u, A^d, respectively, diagonalize the charge $\frac{2}{3}$ and the charge $-\frac{1}{3}$ mass matrices. Explicitly,

$$A_{qL}\mathcal{M}_q A^\dagger_{qR} = \text{diagonal} . \tag{2.3}$$

The $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ model contains two massive neutral fields $Z_{1,2}$. Both of these will couple to FCNC's in models under consideration. Let us define[11] the states Z,D orthogonal to the photon field:

$$A_\mu=\sin\theta_w W^3_{L\mu}+\cos\theta_w B_\mu , \tag{2.4a}$$

$$Z_\mu=\cos\theta_w W^3_{L\mu}-\sin\theta_w B_\mu , \tag{2.4b}$$

$$D_\mu=\cos\phi W^3_{R\mu}-\sin\phi C_\mu , \tag{2.4c}$$

where θ_w is the weak-mixing angle. In terms of the gauge coupling $g_{L,R}$ [g_C] of $SU(2)_{L,R}$ [$U(1)_{B-L}$] we have $\tan\phi=g_C/g_R$, $\tan\theta_w=g'/g_L$, and $g'=g_R g_C/(g_R^2+g_C^2)^{1/2}$. The field B_μ appearing in Eqs. (2.4) is given by

$$B_\mu=\sin\phi W^3_{R\mu}+\cos\phi C_\mu . \tag{2.5}$$

The physical fields $Z_{1,2}$ are mixtures of fields Z and D:

$$Z_{1\mu}=\cos\beta Z_\mu+\sin\beta D_\mu , \tag{2.6a}$$

$$Z_{2\mu}=-\sin\beta Z_\mu+\cos\beta D_\mu . \tag{2.6b}$$

Given these definitions, it is easy to work out the couplings of weak eigenstate fermions $f'\equiv(u',d',P',N')$ to $Z_{1,2}$. We have

$$\mathcal{L}_{NC}=\frac{e}{\sin\theta_w\cos\theta_w}\left[\sum_{p=1,2} Z_{p\mu}(a^q_{pL}\bar{q}'_{Li}\gamma^\mu q'_{Li}+a^H_{pL}\bar{H}'_{\alpha L}\gamma^\mu H'_{\alpha L}+a^q_{pR}\bar{q}'_R\gamma^\mu q'_{Ri}+a^H_{pR}\bar{H}'_{\alpha R}\gamma^\mu H'_{R\alpha})\right] . \tag{2.7}$$

Here $e=g_L\sin\theta_w q'\sim(u',d)$ and $H'\sim(P',N')$ as before and, for any of these fermions,

$$a^f_{L1}=\cos\beta(T_{3L}-\sin^2\theta_w Q^{em})_{f_L}-\tfrac{1}{2}\sin\beta\sin\theta_w\tan\phi(B-L)_{f_L} , \tag{2.8a}$$

$$a^f_{R1}=-\cos\beta\sin^2\theta_w(Q^{em})_{f_R}-\tfrac{1}{2}\sin\beta\sin\theta_w\tan\phi[(B-L)_{f_R}-2\cot^2\phi(T_{3R})_{f_R}] , \tag{2.8b}$$

$a^{2f}_{L,R}$ are obtained from $a^{1,f}_{L,R}$ by replacing $\cos\beta\to-\sin\beta$, $\sin\beta\to\cos\beta$.

Equation (2.7) when reexpressed in terms of physical fields defined by Eq (2.2), contains flavor-nondiagonal terms between light quarks q_i. Using the unitarity of $A^q_{L,R}$ we obtain

$$\mathcal{L}_{FCNC}=\frac{e}{\sin\theta_w\cos\theta_w}\left[\sum_{\substack{p=1,2\\ i\neq j}} Z_{p\mu}[(a^q_p-a^H_p)_L F^q_{ijL}(\bar{q}_{Li}\gamma^\mu q_{L,j})+(a^q_p-a^H_p)_R F^q_{ijR}(\bar{q}_{Ri}\gamma^\mu q_{Rj})]\right] , \tag{2.9}$$

where

$$(F^q_{L,R})_{ij} \equiv (A^q_{ik} A^{q\dagger}_{kj})_{L,R} \tag{2.10}$$

with i,j,k running over $1,\ldots,n$. As would be expected, the FCNC's are absent if the additional quarks H do not mix with q (so that $F_{ij} \propto \delta_{ij}$) or if the former transform in the same way as q under G (so that $a^q_p = a^H_p$). The strength F_{ij} of FCNC's is fixed once the matrices $A^q_{L,R}$ are given. In the next section we work them out for the quark mass matrices of the form displayed in Eq. (2.1).

III. CONDITIONS FOR ABSENCE OF FCNC's

In this as well as the subsequent section, we shall work in the seesaw limit $M_H \gg m_1, m_2$. Moreover we shall assume that M_H appearing in Eq. (2.1) is invertible. The case[3] with $\det M_H = 0$ needs a separate treatment. In the seesaw limit, the matrix $\mathcal{M}$ [Eq. (2.1)] can be brought to a block-diagonal form by means of a biunitary transformation induced by[6]

$$U_{L,R} = \begin{bmatrix} 1-\frac{1}{2}\rho_{L,R}\rho^\dagger_{L,R} & -\rho_{L,R} \\ \rho^\dagger_{L,R} & 1-\frac{1}{2}\rho^\dagger_{L,R}\rho_{L,R} \end{bmatrix} + O(\rho^3_{L,R}) \;. \tag{3.1}$$

Here $\rho_{L,R}$ is an $n\times m$ matrix and it is to be chosen in such a way that $U_L \mathcal{M} U^\dagger_R$ does not contain any couplings between light (q) and heavy (H) quarks. Explicitly, one finds, to leading order,

$$\rho_L = m_1 M_H^{-1} \;, \tag{3.2a}$$

$$\rho^\dagger_R = M_H^{-1} m_2 \;, \tag{3.2b}$$

and

$$U_L \mathcal{M} U^\dagger_R = \begin{bmatrix} -m_1 M_H^{-1} m_2 & 0 \\ 0 & M_H \end{bmatrix} \left[I + O\left[\frac{m_{1,2}}{M_H}\right]\right] . \tag{3.3}$$

The first term on the right-hand side of the above equation can be diagonalized by a matrix

$$S_{L,R} = \begin{bmatrix} K_{L,R} & 0 \\ 0 & P_{L,R} \end{bmatrix} \tag{3.4}$$

with

$$K_L(-m_1 M_H^{-1} m_2) K^\dagger_R = m_l \;, \tag{3.5a}$$

$$P_L M_H P^\dagger_R = m_H \;, \tag{3.5b}$$

where m_l (m_H) are diagonal matrices with light (heavy) quark masses as entries. $K_{L,R}$ ($P_{L,R}$) are $n\times n$ ($m\times m$) unitary matrices. The matrix A connecting the physical and weak basis [Eq. (2.2)] is given by

$$A_{L,R} = S_{L,R} U_{L,R} = \begin{bmatrix} K(1-\frac{1}{2}\rho\rho^\dagger) & -K\rho \\ P\rho^\dagger & P(1-\frac{1}{2}\rho^\dagger\rho) \end{bmatrix}_{L,R} . \tag{3.6}$$

As a result, the strength of FCNC appearing in (2.10) is given as

$$(F_{L,R})_{ij} = [K(1-\tfrac{1}{2}\rho\rho^\dagger)(1-\tfrac{1}{2}\rho\rho^\dagger)K^\dagger]_{ij,L,R} \approx \delta_{ij} - (K_{L,R}\rho_{L,R}\rho^\dagger_{L,R}K^\dagger_{L,R})_{ij} + O(\rho^4_{L,R}) \;. \tag{3.7}$$

It follows from Eq. (3.7) that there are no FCNC-induced transitions in the seesaw limit if both $K_L\rho_L\rho^\dagger_L K^\dagger_L$ and $K_R\rho_R\rho^\dagger_R K^\dagger_R$ are diagonal. Conditions for this to happen are easy to derive. Remembering that $\rho_L = m_1 M_H^{-1}$ and $\rho^\dagger_R = M_H^{-1} m_2$, Eq. (3.5a) implies

$$K_L\rho_L m_2 m_2^\dagger \rho^\dagger_L K^\dagger_L = K_R\rho_R m_1^\dagger m_1 \rho^\dagger_R K^\dagger_R = \text{diagonal} \;. \tag{3.8}$$

Hence if $\rho_L m_2 m_2^\dagger \rho_L^\dagger$ ($\rho_R m_1^\dagger m_1 \rho_R^\dagger$) commute with $\rho_L\rho_L^\dagger$ ($\rho_R\rho_R^\dagger$) then $F_{L,R}$ given in Eq. (3.7) will be diagonal and there will not be any FCNC's to leading order in $m_{1,2}/M_H$. This can happen in a variety of ways but we point out two important classes of models where this happens. Consider a situation with

$$m_1 = \beta m_2^\dagger \tag{3.9a}$$

and

$$M_H = \gamma I \;, \tag{3.9b}$$

with β,γ being constants. Then $m_2 m_2^\dagger$ is proportional to $\rho_L^\dagger\rho_L$ with the result that $\rho_L\rho_L^\dagger$ commute with $\rho_L m_2 m_2^\dagger \rho_l^\dagger \propto (\rho_L\rho_L^\dagger)^2$. As a consequence, the matrix K_L which diagonalizes $\rho_L m_2 m_2^\dagger \rho_L^\dagger$ [see Eq. (3.8)] *also diagonalizes* $\rho_L\rho_L^\dagger$ and $(F_L)_{ij}$ in Eq. (3.7) are proportional to δ_{ij}. Similarly $(F_R)_{ij}$ are also proportional to δ_{ij} and flavor-changing terms are absent to leading order in Eq. (2.9). The second class of models where this happens corresponds to the addition of only one heavy quark coupled to light quarks of a given charge. In this case, $m_1^\dagger m_1$ and $m_2 m_2^\dagger$ are numbers and $\rho_L m_2 m_2^\dagger \rho_L^\dagger$ ($\rho_R m_1^\dagger m_1 \rho_R^\dagger$) trivially commute with $\rho_L\rho_L^\dagger$ ($\rho_R\rho_R^\dagger$) resulting in the absence of FCNC's as before. Recently proposed models of Refs. 7 and 8 fall in this category. The quark mass matrices at the tree level in these models have the structure displayed in Eq. (2.1) with $m=1$. They therefore do not contain any FCNC's at the tree level in the seesaw limit. As we shall demonstrate in Sec. V, this property is in fact more general and even when one does not take the seesaw limit, the flavor-changing transitions are absent at the tree level in these models.

So far our discussion assumed that there are no direct mass terms between light quarks. These may be generated by radiative corrections as in the models of Refs. 7 and 8. The general form of mass matrices one should consider then is

$$\mathcal{M} = \begin{bmatrix} \delta M & m_1 \\ m_2 & M_H \end{bmatrix} . \tag{3.10}$$

This can be brought into approximate block-diagonal form by the same $U_{L,R}$ as in Eqs. (3.1) and (3.2). But $K_{L,R}$ now satisfy

$$K_L(\delta M - m_1 M_H^{-1} m_2) K^\dagger_R = m_l \tag{3.11}$$

instead of Eq. (3.5a). The presence of δM could introduce now nontrivial FCNC's even if they are absent with $\delta M=0$. But the smallness of FCNC's would then be linked to the presumed smallness of δM. We will see this explicitly in the next section.

IV. FCNC's: SEESAW LIMIT

In this section we shall consider the seesaw limit and work out the structure of FCNC's in the model proposed by Balakrishna, Kagan, and Mohapatra.[8] In their model, only one additional heavy quark N or P is added to each charge sector. In addition, they introduce a color-triplet $SU(2)_L \times SU(2)_R$-singlet scalar field ω with $B-L=-\frac{2}{3}$. We shall explicitly consider the case of charged $\frac{2}{3}$ quarks. At the tree level, their masses are given by

$$\mathcal{M}_u^0=\begin{bmatrix} 0 & v_L|h_3\rangle \\ v_R\langle h_3| & M_P \end{bmatrix}, \tag{4.1}$$

where $|h_3\rangle$ is a column vector with n entries and $v_{L,R}\equiv\langle\varphi^0_{L,R}\rangle$. We shall restrict outselves to the case $n=3$. $\mathcal{M}_d^0$ is obtained from above by changing M_P to M_N and by changing Yukawa couplings $|h_3\rangle$. Following Ref. 8 we shall choose equal Yukawa couplings and hence the same $|h_3\rangle$ for both $\mathcal{M}_u^0$ and $\mathcal{M}_d^0$. In order to do this naturally, one needs to enlarge the gauge group to $SU(2)_L\times SU(2)_R\times U(1)_H\times U(1)_{B-L}$ as discussed in Ref. 8. This contains three neutral gauge bosons and one should include them in the analysis of the flavor-changing processes. For simplicity, we have chosen to work with the gauge group $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ but we choose equal Yukawa couplings in $\mathcal{M}_u$ and $\mathcal{M}_d$ by hand in what follows. The results so obtained would be expected to hold in the case with three gauge bosons provided the additional gauge boson does not have significant mixing with $Z_{1,2}$ considered in Sec. II.

Radiative corrections induce[7,8] the couplings between light quarks and change the structure of $\mathcal{M}_u^0$ to

$$\mathcal{M}_u=\begin{bmatrix} \delta M_u & v_L|\alpha\rangle \\ v_R\langle\alpha| & M_P \end{bmatrix}. \tag{4.2}$$

The δM_u and $|\alpha\rangle$ depend upon the details of the model but have the following generic form[8] at the ith loop level:

$$(\delta M_u)_i=\sum_{0\le k\le 2i} a_{i,kl}|H^k h_3\rangle\langle H^l h_3| , \tag{4.3a}$$

$$|\alpha\rangle_i=\sum_{0\le k<2i} b_{i,k}|H^k h_3\rangle , \tag{4.3b}$$

where H^k stands for the product $H^\dagger H H^\dagger H\cdots$ of k-symmetric matrices H_{ij} defining the couplings of light quarks to color-triplet field ω at the tree level.[7,8]

In the seesaw limit, the $\mathcal{M}_u$ can be brought to a block-diagonal form. The upper 3×3 block describing the effective light-quark masses is given [see Eq. (3.11)] by

$$M_u\equiv\delta M_u-a_0|\alpha\rangle\langle\alpha| \tag{4.4}$$

with $a_0=v_Lv_R/M_P$. At the tree level, this has only one nonzero eigenvalue corresponding to $m_t^0\equiv a_0h_3^2$ with $h_3^2\equiv\langle h_3|h_3\rangle$. The radiative corrections in the model are such[7] that the charm and up quarks receive their masses at the one- and two-loop levels, respectively.

In the absence of radiative corrections ($\delta M_u=0$) the FCNC's are absent as already discussed in Sec. III. With nontrivial δM_u, M_u and $\rho_{L,R}\rho^\dagger_{L,R}$ cannot be diagonalized simultaneously and the model contains FCNC's [see Eq. (3.7)]. In the present case, M_u is Hermitian and so $K_L=K_R\equiv K$ in Eq. (3.5a). The structure of K has been determined[7,8] perturbatively by expanding M_u into a parameter λ which counts the loop order. We briefly describe the procedure for completeness.

The radiative corrections [Eq. (4.3)] are expressed in terms of the column vectors $|h_3\rangle,|h_1\rangle\equiv|Hh_3\rangle$, $|h_2\rangle\equiv|H^\dagger Hh_3\rangle$, etc. We shall assume that $|h_i\rangle$ ($i=1,2,3$) form a linearly independent set and construct an orthonormal set of states from $|h_i\rangle$:

$$|\phi_3\rangle=\frac{1}{h_3}|h_3\rangle , \tag{4.5a}$$

$$|\phi_2\rangle=\frac{1}{N_2}\left[|\phi_3\rangle-\frac{1}{e_1}|h_1\rangle\right], \tag{4.5b}$$

$$|\phi_1\rangle=\frac{1}{N_1}\left[|\phi_3\rangle+\frac{e_3}{e_2}|\phi_2\rangle-\frac{1}{e_2}|h_2\rangle\right], \tag{4.5c}$$

where $e_1\equiv\langle\phi_3|h_1\rangle$, $e_2\equiv\langle\phi_3|h_2\rangle$, and $e_3\equiv\langle\phi_2|h_2\rangle$. $N_{1,2}$ are normalization constants.

The states $|\phi_i\rangle$ are eigenfunctions of M_u at the tree level[12] and therefore describe physical states in this approximation. In general,

$$|i\rangle^u=\lambda^m|m\rangle_i^u \tag{4.6}$$

and

$$m_i^u=\lambda^m m_m^u(i) \tag{4.7}$$

describe the eigenfunctions and eigenvalues of M_u up to $O(\lambda^m)$. We expand $|m\rangle_i^u$ in terms of eigenfunctions $|0\rangle_i\equiv|\phi_i\rangle$ of M_u at the tree level:

$$|m\rangle_i^u=x_{ij}^{m,u}|\phi_j\rangle,\quad i\neq j , \tag{4.8}$$

where x_{ij}^m are $O(\lambda^m)$. Then the standard perturbation theory up to $O(\lambda)$ gives[7,8]

$$x_{13}^{1u}=-x_{31}^{1u}=0 , \tag{4.9a}$$

$$x_{23}^{1u}=-x_{32}^{1u}=\frac{(M_1^u)_{32}}{m_t^0} , \tag{4.9b}$$

$$x_{12}^{1u}=-x_{21}^{1u}=-\frac{(M_2^u)_{21}}{m_c} \tag{4.9c}$$

Here $(M_i^u)_{kl}\equiv\langle\phi_k|M_i^u|\phi_l\rangle$. M_i^u corresponds to M^u at the ith loop level and nonzero quark masses up to $O(\lambda)$ are given by $m_c=(M_1^u)_{22}$ and $m_t^0=-(M_0^u)_{33}$.

The expression for x_{ij}^{1u} given above are sufficient to determine the F_{ij}^u up to $O(\lambda)$. We have, up to one-loop level,

$$(\rho_L\rho_L^\dagger)^u=\left[\frac{v_L}{v_R}\right]^2(\rho_R\rho_R^\dagger)^u$$
$$=\left[\frac{v_L}{M_P}\right]^2 h_3^2|\phi_3\rangle\langle\phi_3| \ . \quad (4.10)$$

The matrix K^u which connects the mass eigenstates $|i\rangle^u$ to the $|\phi_i\rangle$ basis is given by

$$K_{ij}^u=\delta_{ij}+\sum_{m=1}^{\infty} x_{ij}^{mu} \ . \quad (4.11)$$

As a consequence,

$$(F_L^u)_{ij}=(F_R^u)_{ij}\left[\frac{v_L}{v_R}\right]^2$$
$$=-(K\rho_L\rho_L^\dagger K^\dagger)_{ij}^u$$
$$=-\left[\frac{v_L}{M_P}\right]^2 h_3^2\left[\delta_{i3}+\sum_{m=1}^{\infty} x_{i3}^{mu}\right]$$
$$\times\left[\delta_{j3}+\sum_{n=1}^{\infty} x_{j3}^{nu}\right] \ . \quad (4.12)$$

It is clear from Eqs. (4.9a) and (4.12) that up to $O(\lambda)$ $(F_{L,R}^u)_{12}$, $(F_{L,R}^u)_{13}$ are zero and no flavor-changing transitions are induced between the first and the remaining generations. Only the nontrivial transition at this order is between the second and third generations and is given by

$$(F_L^u)_{23}^{\text{seesaw}}=\left[\frac{v_L}{v_R}\right]^2(F_R^u)_{23}^{\text{seesaw}}$$
$$=-\left[\frac{v_L}{M_p}\right]^2 h_3^2 x_{23}^{1u} \ . \quad (4.13)$$

We have explicitly added the label seesaw on F_{L23}^u to distinguish it from the exact result to be derived in the next section. The x_{23}^{1u} can be expressed in terms of the quark mass ratio m_c/m_t^0 which is $O(\lambda)$. In the present model,[8]

$$M_1^u=\delta M_1^u=a_{1,11}|h_1\rangle\langle h_1|+a_{1,00}|h_3\rangle\langle h_3| \ . \quad (4.14)$$

From Eqs. (4.9b) and (4.5) it follows that

$$x_{23}^{1u}=\frac{a_{1,11}}{m_t^0}(-N_2e_1^2)$$
$$=-\frac{m_c}{m_t^0}\frac{1}{N_2} \ , \quad (4.15)$$

where we have used the fact that $m_c=\langle\phi_2|M_1^u|\phi_2\rangle$ As a consequence of Eq. (4.15) the strength $(F_{23}^u)_{L,R}$ is essentially determined in terms of quark masses up to a normalization constant N_2 involving the Yukawa couplings. This arbitrariness can also be removed if both the u and d sectors contain the same Yukawa couplings $|h_3\rangle$ as assumed here. In this case, the element V_{cb} of Kobayashi-Maskawa (KM) matrix is given by[8]

$$V_{cb}=x_{32}^{d1}+x_{23}^{u1}=\frac{1}{N_2}\left[\frac{m_s}{m_b^0}-\frac{m_c}{m_t^0}\right] \ . \quad (4.16)$$

Equations (4.15) and (4.16) completely determine $(F_{23}^u)_{L,R}$ of Eq. (4.13) in terms of various masses:

$$(F_L^u)_{23}=\left[\frac{v_L}{v_R}\right]^2(F_F^u)_{23}$$
$$=-\left[\frac{v_L}{v_R}\right]^2 h_3^2\frac{V_{cb}}{1-\frac{m_s}{m_c}\frac{m_t^0}{m_b^0}} \quad (4.17)$$

with $h_3^2=M_Nm_b^0/v_Lv_R=M_Pm_t^0/v_Lv_R$. Analogously, the corresponding transition in the d sector is given by

$$(F_L^d)_{23}=\left[\frac{v_L}{v_R}\right]^2(F_R^d)_{23}$$
$$=-\left[\frac{v_L}{v_R}\right]^2 h_3^2\frac{V_{cb}}{1-\frac{m_c}{m_s}\frac{m_b^0}{m_t^0}} \ . \quad (4.18)$$

By carrying out this analysis further, one sees that transitions between 13 and 12 generations are of $O(\lambda^2)$ and $O(\lambda^3)$, respectively. In the seesaw limit, one has $m_t^0/m_b^0=M_N/M_P$. As a consequence $M_N \gg M_P$. It then follows that the FC transitions are more suppressed in the charged $-\frac{1}{3}$ sector compared to the charged $\frac{2}{3}$ sector. In fact, if M_P is not very different from v_L then $(F_L^u)_{23}$ may be quite substantial but Eq. (4.17) could be misleading in this regard, since one has already used the seesaw limit $M_{N,P}\gg v_{L,R}$. A general analysis of the model is needed if M_P is not very different from v_L. This we shall do in the following section.

V. FCNC's: Exact analysis

Now, we shall derive the general structure of FCNC's for the quark mass matrices of the form (4.2) without assuming the seesaw limit $M_{N,P}\gg v_{L,R}$. We present explicit results for the up-quark sector. Corresponding results for the down-quark sector are obtained by $M_N\leftrightarrow M_P$ and by interchanging the down- and up-quark masses. The structure of the matrix $A_{L,R}^q$ defined in Eq. (2.3) essentially follows from the analysis of Ref. 8. We summarize their results and use them to work out $(F_{L,R}^q)_{ij}$ of Eq. (2.10).

The matrices $A_{L,R}^u$ satisfy

$$A_L^uV_L^uA_l^{\dagger u}=A_R^uV_R^uA_R^{\dagger u}=\text{diagonal} \ . \quad (5.1)$$

Here $V_L\equiv\mathcal{M}_u\mathcal{M}_u^\dagger$ and $V_R\equiv\mathcal{M}_u^\dagger\mathcal{M}_u$ are to be expanded in powers of λ:

$$V_{L,R}^u=\sum_m\lambda^mV_{m,L,R}^u \ . \quad (5.2)$$

Let us first concentrate on determination of V_L^u. The eigenfunctions $|X_a\rangle_L^u$ of the zeroth-order matrix V_{0L}^u are given by

$$|X_{1,2}\rangle_L^u=\begin{bmatrix}|\phi_{1,2}\rangle\\0\end{bmatrix}, \quad |X_k\rangle_L^u=\frac{1}{N_{kL}^u}\begin{bmatrix}C_{kL}^u|\phi_3\rangle\\1\end{bmatrix} \ . \quad (5.3)$$

Here $|\phi_{1,2,3}\rangle$ are (3×1) vectors defined by Eq. (4.5). $k=3,4$ and $N_{kL}^{u2}=C_{kL}^{u\,2}+1$. C_{kL}^u satisfy

$$v_L^2h_3^2C_{kL}^u+M_Pv_Lh_3=E_0^u(k)C_{kL}^u\ , \tag{5.4a}$$

$$M_Pv_Lh_3C_{kL}^u+v_R^2h_3^2+M_P^2=E_0^u(k)\ . \tag{5.4b}$$

These equations determine $E_0(k)$ and C_{kL} separately. Explicitly

$$m_t^2\equiv E_0^u(3)=\tfrac{1}{2}(M_P^2+v^2h_3^2)[1-(1-\Delta)^{1/2}]\ , \tag{5.5a}$$

$$E_0^u(4)=\tfrac{1}{2}(M_P^2+v^2h_3^2)[1+(1-\Delta)^{1/2}]\ , \tag{5.5b}$$

with

$$\Delta\equiv\frac{4v_L^2v_R^2h_3^4}{(M_P^2+v^2h_3^2)^2}\le4\left[\frac{v_L}{v_R}\right]^2$$

and

$$v^2=v_L^2+v_R^2\ .$$

The matrix A_{0L}^u which diagonalizes V_{0L}^u is then given by

$$(A_{0L}^u)^\dagger=\begin{bmatrix}|\phi_1\rangle & |\phi_2\rangle & \dfrac{C_{3L}^u}{N_{3L}^u}|\phi_3\rangle & \dfrac{C_{4L}^u}{N_{4L}^u}|\phi_3\rangle\\ 0 & 0 & \dfrac{1}{N_{3L}^u} & \dfrac{1}{N_{4L}^u}\end{bmatrix}\ . \tag{5.6}$$

Just as in the previous section, we expand the general eigenfunctions $|a\rangle_L^u$ of V_L^u in terms of the eigenfunctions $|X_a\rangle_L^u$ of V_{0L}^u [compare Eq. (4.8)]:

$$|a\rangle_L^u=|X_a\rangle_L^u+\sum_{\substack{m=1\\a\neq b}}^{\infty}X_{Lab}^{mu}|X_b\rangle_L^u\ . \tag{5.7}$$

The coefficients X_{Lab}^{mu} are $O(\lambda^m)$ and can be determined perturbatively.[8] Below, we give the expressions of the ones which are required for our purpose:

$$X_{L1k}^{1u}=0\ , \tag{5.8a}$$

$$X_{Lk2}^{1u}=-X_{L2k}^{1u}=\frac{\langle X_2|\mathcal{M}_1^u\mathcal{M}_0^{u\dagger}|X_k\rangle}{E_0^u(k)}\ , \tag{5.8b}$$

$$X_{L1k}^{2u}=\frac{1}{E_0^u(k)}\left[\frac{\langle X_k|\mathcal{M}_0^u\mathcal{M}_1^{u\dagger}|X_2\rangle\langle X_2|\mathcal{M}_2^{u\dagger}|X_1\rangle}{m_c}-\langle X_k|\mathcal{M}_0^u\mathcal{M}_2^{u\dagger}|X_1\rangle\right]\ , \tag{5.8c}$$

$k=3,4$ in these expressions. The masses m_c and m_u are $O(\lambda)$ and $O(\lambda^2)$, respectively, and coincide[8] with their expressions in the seesaw limit:

$$m_c=\langle X_2|\mathcal{M}_1^u|X_2\rangle,\quad m_u=\langle X_1|\mathcal{M}_2^u|X_1\rangle\ . \tag{5.9}$$

Using Eqs. (5.6) and (5.7) the matrix A_L^u of Eq. (5.1) can be written as

$$(A_L^u)_{ab}=\left[\delta_{ac}+\sum_{m=1}^{\infty}X_{Lac}^{mu}\right](A_{0L}^u)_{cb}\ . \tag{5.10}$$

The expressions for the strength $(F_L^u)_{ij}$ [Eq. (2.10)] of FCNC's follow from Eq. (5.10). Remembering that $X_{L13}^u=X_{L14}^u=0$ we get, to leading order in λ,

$$(F_L^u)_{23}=-\frac{1}{(N_{3L}^u)^2}\left[X_{L23}^{1u}+\frac{N_{3L}^u}{N_{4L}^u}X_{L24}^{1u}\right]\ , \tag{5.11a}$$

$$(F_L^u)_{13}=-\frac{1}{(N_{3L}^u)^2}\left[X_{L13}^{2u}+\frac{N_{3L}^u}{N_{4L}^u}X_{L14}^{2u}\right]\ , \tag{5.11b}$$

$$(F_L^u)_{12}=-(N_{3L}^u)^2(F_L^u)_{23}(F_L^u)_{13}\ . \tag{5.11c}$$

The constants N_{kL}^u appearing in the equations above are determined from Eqs. (5.4) while X_L^u's are given in Eqs. (5.8). From Eqs. (5.11) it follows that $(F_L^u)_{ij}$ $(i\neq j)$ are zero at the tree level. This is a consequence of the structure of A_{0L}^u given by Eq. (5.6). Analogously $(F_R^u)_{ij}$ and $(F_{L,R}^d)_{,ij}$ also vanish at the tree level. Thus, the present model does not contain any FCNC's at the tree level even if the seesaw limit does not hold. Moreover, $(F_L^u)_{23}$, $(F_L^u)_{13}$, and $(F_L^u)_{12}$ are, respectively, of $O(\lambda)$, $O(\lambda^2)$, and $O(\lambda^3)$.

We shall now calculate $(F_L^u)_{ij}$ in terms of the basic parameters of the model. At the one-loop level, we have

$$\mathcal{M}_1^u=\begin{bmatrix}\delta M_1^u & 0\\ 0 & 0\end{bmatrix}\ ,$$

δM_1^u is given in Eq. (4.1). The X_{Lk}^{1u} of Eq. (5.8) are then related to the corresponding parameter x_{23}^{1u} in the seesaw limit [Eq. (4.9b)] as

$$X_{L2k}^{1u}=\frac{v_Rh_3}{N_{kL}^u}\frac{m_t^0}{E_0^u(k)}x_{23}^{1u}\ , \tag{5.12}$$

m_t^0 appearing here is the top-quark mass in the seesaw limit and is different from the exact m_t of Eq. (5.5a). In the following, we make two approximations. First, we neglect higher powers of $\Delta\le4(v_L/v_R)^2$ in Eq. (5.5). In this case one obtains

$$m_t^2=\frac{m_t^{02}}{\delta^u}\ , \tag{5.13}$$

where $\delta^u\equiv(1+v^2h_3^2/M_P^2)$. Second, we neglect terms of $O(m_t/M_P)$. With these approximations, the expression of C_{kL}^u following from Eqs. (5.4) lead to the following $(F_L^u)_{23}$ when use is made of Eqs. (5.11a) and (5.12):

$$(F_L^u)_{23}^{\rm ext}=C_L^u(F_L^u)_{23}^{\rm seesaw} \tag{5.14a}$$

with

$$C_L^u=\frac{\delta^u}{(\delta_L^u-1+\delta_R^{u2})^{3/2}}\left[1+\frac{\delta_R^u-1}{\delta_L^u\delta^{u2}}(\delta_L^u-1+\delta_R^{u2})\right]\ . \tag{5.14b}$$

In Eq. (5.14a), we have added a label "exact" to F_{L23}^u of Eq. (5.11a) to distinguish it from the corresponding expression in the seesaw limit given by Eq. (4.13). $\delta_{L,R}^u\equiv1+v_{L,R}^2h_3^2/M_P^2$, $\delta^u=\delta_L^u+\delta_R^u-1$. In the seesaw limit $\delta_{L,R}^u\to1$ and Eq. (5.14a) reduces to Eq. (4.13).

The evaluation of $(F_L^u)_{13}$ requires the analysis of the two-loop corrections having the general form given in Eqs. (4.3). In the model of Ref. 8 $b_{1,1}$, $a_{2,21}$, $a_{2,p0}$, and $a_{2,0p}$ with $p=2,3,4$ are zero. In addition, if one neglects the contributions of various $b_{2,i}$'s and of $a_{2,31}$ to $\mathcal{M}_2$ then simple expressions analogous to (5.14) follows for other transitions also. After some algebra one finds

$$X_{L1k}^{2u}=\frac{v_R h_3}{N_{kL}^u}\frac{m_t^0}{E_0^u(k)}x_{13}^{2u}\ , \tag{5.15a}$$

$$(F_L^u)_{13}^{\text{exact}}=C_L^u\left[\frac{v_L}{M_P}\right]^2 h_3^2 x_{13}^{2u}\ , \tag{5.15b}$$

where

$$x_{13}^{2u}=\frac{m_u}{m_t^0}\frac{1}{N_1}\left[1+\frac{e_3}{N_2 e_2}\right]. \tag{5.15c}$$

The parameters N_1, e_3, e_2 are related to the basic Yukawa couplings $|h_3\rangle$ and H_{ij} of the model and are defined in Eq. (4.5).

In an earlier section, we noticed that the unknown constant N_2 appearing in x_{23}^{1u} [Eq. (4.15)] can be determined in terms of the element V_{cb} of the KM matrix. This can be done in the present case also if we work in the limit $\delta_L^{u,d}\to 1$. $\delta_L^u=1+v_L^2h_3^2/M_P^2=1+(v_L/v_R)(m_t^0/M_P)$ where m_t^0 is the mass of the top quark in the seesaw limit. It then follows the even if M_P is not very different from v_L, δ_L^u is close to 1. δ_L^d is in fact smaller than δ_L^u. Thus, it is reasonable to assume $\delta_L^{u,d}\approx 1$. In this case, $A_{0L}^d A_{0L}^{u\dagger}\approx 1$ and the elements V_{ij} of the KM matrix are given by

$$V_{ij}\approx\sum_{c=1}^{4}\left[\delta_{jc}+\sum X_{Ljc}^{md}\right]\left[\delta_{ic}+\sum X_{Lic}^{mu}\right]. \tag{5.16a}$$

Explicitly one has to leading order

$$V_{cb}\approx X_{L32}^{1d}+X_{L23}^{1u}\ , \tag{5.16b}$$

$$V_{ub}\approx X_{L31}^{2d}+X_{L13}^{2u}\ , \tag{5.16c}$$

$$V_{us}\approx X_{L21}^{1d}+X_{L12}^{1u}\ . \tag{5.16d}$$

Within the approximations we are working in, we have, from Eqs. (5.12), (5.15), and their analogues in the down-quark sector,

$$X_{L23}^{1u,d}=\frac{\delta_{u,d}}{(\delta_R^{u,d^2}+\delta_L^{u,d}-1)^{1/2}}x_{23}^{1u,d}\approx x_{23}^{1u,d}\ , \tag{5.17a}$$

$$X_{L13}^{2u,d}=\frac{\delta_{u,d}}{(\delta_{Ru,d}^2+\delta_L^{u,d}-1)^{1/2}}x_{13}^{2u,d}\approx x_{13}^{2u,d}\ . \tag{5.17b}$$

Equations (5.16) and (5.17) can be used to eliminate the unknown Yukawa couplings appearing in $(F_L^u)_{23}$ and $(F_L^u)_{13}$ in favor of quark masses and KM matrix elements. As a result we obtain

$$F_{L23}^u=-C_L^u\left[\frac{v_L}{M_P}\right]^2\frac{V_{cb}h_3^2}{1-\frac{m_s}{m_b}\frac{m_t}{m_c}\left[\frac{\delta^u}{\delta^d}\right]^{1/2}}\ , \tag{5.18a}$$

$$F_{L13}^u=-C_L^u\left[\frac{v_L}{M_P}\right]^2\frac{V_{ub}h_3^2}{1-\frac{m_d m_t}{m_u m_b}\left[\frac{\delta^u}{\delta^d}\right]^{1/2}}\ , \tag{5.18b}$$

$$F_{L12}^u=-\frac{M_P^2}{v_L^2h_3^2}(\delta_R^{u2}+\delta_L^u-1)F_{L13}^uF_{L23}^u\ . \tag{5.18c}$$

The corresponding couplings in the right-handed sector can be obtained by diagonalizing $V_R^u\equiv\mathcal{M}_u^\dagger\mathcal{M}_u$. It is easy to see that they can be obtained from the above $(F_L^u)_{ij}$ by interchanging $v_L\leftrightarrow v_R$. As a result, $(F_R)_{ij}$ and $(F_L)_{ij}$ are related as follows:

$$(F_R^u)_{23}=\left[\frac{C_R^u}{C_L^u}\right]\left[\frac{v_R}{v_L}\right]^2 F_{L23}^u\ , \tag{5.19a}$$

$$(F_R^u)_{13}=\left[\frac{C_R^u}{C_L^u}\right]\left[\frac{v_R}{v_L}\right]^2 F_{L13}^u\ , \tag{5.19b}$$

$$(F_R^u)_{12}=\left[\frac{C_R^u}{C_L^u}\right]^2\left[\frac{v_R}{v_L}\right]^2\left[\frac{\delta_L^{u2}+\delta_R^u-1}{\delta_R^{u2}+\delta_L^u-1}\right](F_L^u)_{12}\ . \tag{5.19c}$$

C_R^u is obtained from Eq. (5.13b) by interchanging $v_L\leftrightarrow v_R$. In the $\delta_L^u\to 1$ limit $C_R^u/C_L^u\to(\delta_R^u)^{1/2}$. Finally we note that the $(F_L^d)_{ij}$ in the down sector is obtained by replacing M_P by M_N and interchanging $m_{u,c,t}$ and $m_{d,s,b}$ in Eqs. (5.18) and (5.19).

It is remarkable that the strength of the FCNC's is completely[13] fixed in terms of the masses and KM matrix elements and it displays the hierarchy present in elements V_{ij}. The only Yukawa coupling h_3^2 appearing in Eqs. (5.18) and (5.19) is also related to the masses. Using Eq. (5.13) and its analogue and the d sector one sees that

$$\left[\frac{M_N}{vh_3}\right]^2\geq\left[\frac{m_t}{m_b}\right]^2-1\ . \tag{5.20}$$

As a result δ^d is always close to 1. Therefore one has

$$h_3^2=\frac{M_N m_b^0}{v_L v_R}\approx\frac{M_N m_b}{v_L v_R}\ . \tag{5.21}$$

The M_N in fact gets fixed in terms of M_P as follows:

$$M_N\approx\tfrac{1}{2}\left\{(M_N)_{\min}+\left[(M_N)_{\min}^2+4\left[\frac{m_t}{m_b}\right]^2 M_P^2\right]^{1/2}\right\}, \tag{5.22a}$$

with

$$(M_N)_{\min}\equiv\frac{v_R}{v_L}m_b\left[\left[\frac{m_t}{m_b}\right]^2-1\right]. \tag{5.22b}$$

VI. DISCUSSION

A. Phenomenological implications

In this section we briefly discuss some of the consequences of FCNC's present in the seesaw model under construction. As already noted, the strength of flavor-changing transitions exhibit remarkable hierarchy. The strengths F_{23}, F_{13}, and F_{12} in both the left- and right-handed sectors are directly proportional to the KM matrix elements V_{cb}, V_{ub}, and $V_{cb}V_{ub}$. Furthermore, M_N is constrained to be much greater than M_P due to the hierarchy $m_t\gg m_b$. Since the former (latter) sets the scale of FCNC's in the down (up) sector, the flavor-changing transitions among d-type quarks are suppressed in comparison to the corresponding transitions among up quarks. As a consequence, the violation of the GIM mechanism is expected to be minimal in the d-s sector where it is already known to be very small due to the stringent limits coming from the $K_L \to \mu^+\mu^-$ decay and K^0-$\bar{K}^0$ mixing. In contrast, there could be a sizable departure from the GIM mechanism in transitions involving t and c quarks. This could result in an appreciable mixing among $t\bar{c}$ and $\bar{t}c$ states. Furthermore, if the top quark is light enough then the Z could decay into $t\bar{c}$ through FCNC's with comparatively large branching ratios. The mixing among u and c, although suppressed compared to u-t and t-c mixing, could still be appreciable and leads to a D^0-$\bar{D}^0$ mass difference which is close to the current experimental limit. Let us now turn to a quantitative analysis of these effects.

B. $Z \to t\bar{c}$ decay

There is no direct coupling between Z and $t\bar{c}$ in the standard model. This could be induced at the one-loop level. This induced coupling is very small because of the GIM mechanism. The branching ratio of $Z \to t\bar{c}$ decay in the $SU(2)_L \times U(1)$ model with three generations depends[14] on $(m_b/M_W)^4$ and is around 10^{-10}. In contrast, the FCNC's in the present case could lead to a much bigger branching ratio for $Z \to t\bar{c}$.

The model under consideration has two Z bosons. We shall concentrate on the decay of the lighter Z (namely, Z_1) into $t\bar{c}$. The coupling of Z_1 to $t\bar{c}$ can be read off from Eqs. (2.8) and (2.9). The $(F^u_{23})_{L,R}$ appearing in (2.9) are worked out in Eqs. (5.18a) and (5.19a). The actual strength of $t\bar{c}Z_1$ coupling depends upon the vacuum expectation values (VEV) of Higgs fields through angle β. In the present case with Higgs fields ϕ_L $[\varphi_R]$ transforming as doublets under $SU(2)_L$ $[SU(2)_R]$ one obtains

$$\sin\beta \approx -\left[\frac{v_L}{v_R}\right]^2 \frac{g_L}{\cos\theta_w}\left[\frac{g_L\tan\theta_w}{g_R}\right]^3 \frac{1}{g_C}, \qquad (6.1)$$

where v_L/v_R is assumed to be $\ll 1$. Using this, one could derive the decay width $\Gamma(z \to t\bar{c})$

$$\Gamma = \frac{\alpha M_z}{4\sin^2 2\theta_w}(1-x^2)(2+x^2)(F^u_{L23})^2 \times (1+\tan^4\theta_w \delta^u_R), \qquad (6.2)$$

where we have used Eq. (5.19a) and the limit $\delta^u_L \to 1$. F^u_{L23} is given in Eq. (5.18a). $x \equiv m_t/M_Z$ and we have neglected the charm-quark mass.

The branching ratio for $Z \to t\bar{c} + \bar{t}c$ is shown in Fig. 1 as a function of M_P/v_L for various values of m_t, m_s, and v_L/v_R. For m_t in the range 30–60 GeV the branching ratio is very sensitive to the assumed quark masses. The biggest uncertainty is in the strange-quark mass and we have displayed results for different values of m_s assuming $m_b = 5.3$ GeV, $m_c = 1.4$ GeV, Γ $(Z \to \text{all}) = 2.8$ GeV, and $\sin^2\theta_\omega = 0.226$. Note that the F^u_{L23} depends on the mass M_N also through $h^2_3 = m_b M_N / v_L v_R$. But given m_t and M_P, M_N gets fixed by Eq. (5.22).

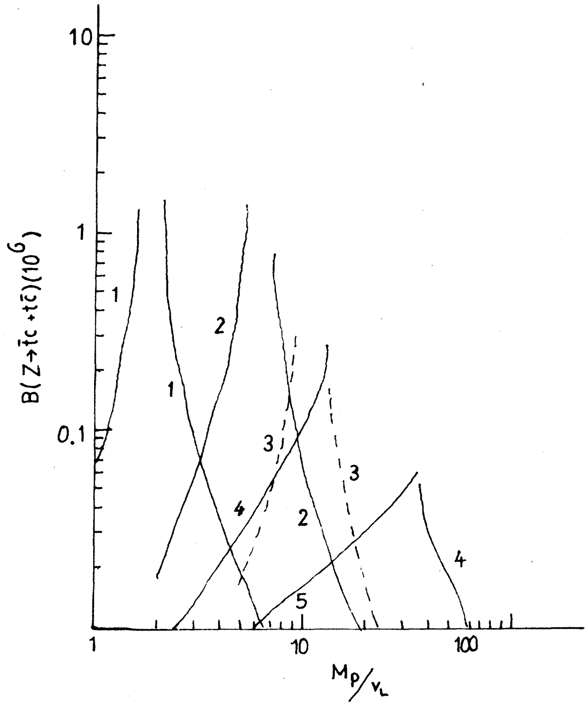

FIG. 1. Branching ratio for the decay $Z \to t\bar{c} + \bar{t}c$ shown as a function of M_P/v_L for various values of m_s and m_t. The labels on various curves correspond to different values (in GeV) of (m_s, m_t). 1, (0.13,30); 2, (0.12,45); 3,(0.13,45); 4, (0.15,45), 5, (0.12,60). The dashed (solid) lines are for $v_R/v_L = 20$ (15).

It is to be noted that for certain values of parameters the denominator in F^u_{L23} of Eq. (5.17a) becomes very small and as a result the branching ratio becomes extremely large. However, in this region the perturbative analysis leading to $(F^u_L)_{23}$ cannot be trusted since [as follows from Eqs. (4.15) and (4.16)] x^{1u}_{23} also becomes large for a fixed V_{cb}. Therefore we have displayed in Fig. 1 the branching ratios only for $x^{1u}_{23} < 1$. But even in this case, the branching ratio could be several orders of magnitude larger than predicted in the standard model[14] or its supersymmetric extensions.[15] In particular, for low m_t and v_R/v_L, the branching ratio reaches the observable[14] value 10^{-6}–10^{-7}. A more careful analysis which retains nonleading $O(\lambda^2)$ term in V_{cb} and x^{1u}_{23} is needed to predict branching ratios in the region for which $x^{1u}_{23} \approx 1$.

C. P^0-$\bar{P}^0$ mixing

Another distinctive feature of the model could be a sizable mixing among $P^0 \equiv u_i\bar{u}_j$ and $\bar{P}^0 \equiv \bar{u}_i u_j$ states. In the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model under construction, this mixing arises at the tree level from the exchange of neutral bosons $Z_{1,2}$ between u_i and $\bar{u}_j$. The expression for the effective Hamiltonian describing P^0-$\bar{P}^0$ mixing follows from the Lagrangian given in Eq. (2.9). In the limit $v_L \ll v_R$ one arrives at

$$\mathcal{H}_{\rm eff}=\sqrt{2}\,G_F\left[(F^u_{Lij})^2(\bar{u}_{Li}\gamma_\mu u_{Lj})^2+\left[\frac{v_L}{v_R}\right]^2(F^u_{Rij})^2(\bar{u}_{Ri}\gamma_\mu u_{Rj})^2\right]. \quad (6.3)$$

The first term in the above equation arises from the exchange of the Z_1 boson. In contrast, the second term describing transition among right-handed quarks gets a dominant contribution from Z_2 resulting in the relative factor $(v_L/v_R)^2$. This follows from the fact that purely at the $SU(2)_L\times U(1)$ level there are no FCNC's among right-handed fields since u_R and P_R transform identically under $SU(2)_L\times U(1)$. Thus, the right-handed transitions are caused either due to mixing among Z and D or due to exchange of Z_2 of which the latter dominates since $\sin^2\beta\sim(v_L/v_R)^4$.

Equation (6.3) leads to a splitting $(\Delta m)_{ij}$ between P^0 and $\bar{P}^0$:

$$(\Delta m)_{ij}=\frac{G_F}{\sqrt{2}}\frac{4}{3}f_P^2M_P\left[(F^u_{ijl})^2+\left[\frac{v_L}{v_R}\right]^2(F^u_{ijR})^2\right]. \quad (6.4)$$

Here, we have used the vacuum-saturation approximation[16] in writing the matrix element of the quark operator between P^0 and $\bar{P}^0$. f_P (M_P) denotes the decay constant (mass) of P^0.

In Fig. 2, we plot the D^0-$\bar{D}^0$ mass difference $(\Delta m)_{12}$ as a function of M_P/v_L for several values of m_t, m_s, v_L/v_R. Note that despite the factor $(v_L/v_R)^2$, the second term dominates the $(\Delta m)_{ij}$ in view of Eq. (5.19) and we have retained only this contribution in evaluating $(\Delta m)_{12}$. As in the previous case we display results in the case of $x^{1u}_{23}<1$ where the perturbative analysis could be trusted. As is clear, one obtains a D^0-$\bar{D}^0$ mixing which is several orders of magnitude larger than in the standard model.[17] But it still falls below the current experimental limit[18] of $(\Delta m)_{12}\lesssim 10^{-13}$ GeV at least by an order of magnitude.

We note that no significant limit on the value of M_P comes from the observed[19] limit $\leq 10^{-4}$ on the branching ratio for the process $D^0\to\mu^+\mu^-$. First, the rate of this process is suppressed by the muon (mass)^2. Second, the FC currents in the left- and right-handed sectors give comparable contribution to the decay unlike in the case of D^0-$\bar{D}^0$ mixing. One finds that even for the values for M_P and m_t leading to sizable $Z\to\bar{t}c$ branching ratio and $(\Delta m)_{12}$, F^u_{L12} is $\sim 10^{-6}$ leading to a much smaller branching ratio for $D^0\to\mu^+\mu^-$ than the current limit. With F^u_{L12} as small as 10^{-6}, one would obtain a much smaller branching ratio than the current experimental limit[19] even in case of inclusive flavor-changing decay $c\to u\mu^+\mu^-$.

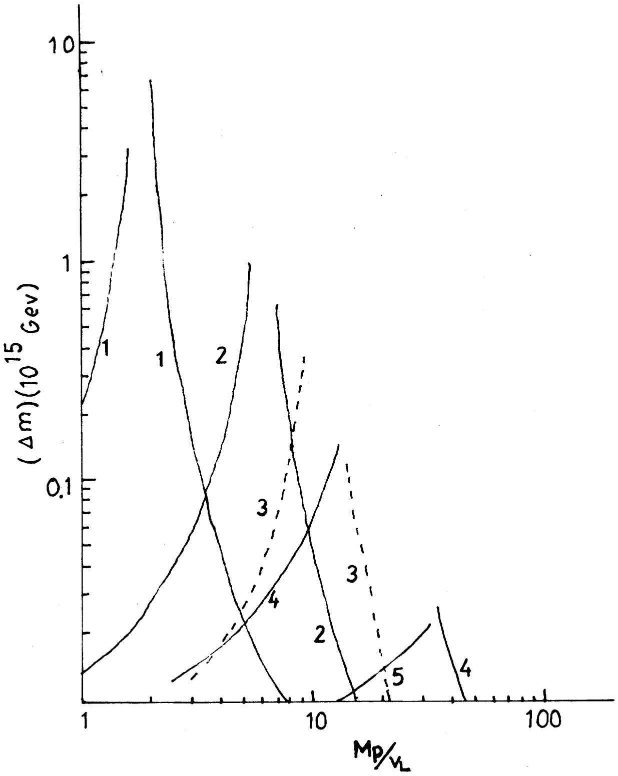

FIG. 2. The D^0-$\bar{D}^0$ mass difference Δm as a function of M_P/v_L. Labels are explained in the caption to Fig. 1.

We end this section with a brief comment on the d sector. In principle, the FCNC's could lead to mixing among K^0-$\bar{K}^0$ or B^0-$\bar{B}^0$ and to other flavor-changing processes such as $K_L\to\mu^+\mu^-$, $Z\to b\bar{s}$, etc. All these are suppressed here due to the fact that $v_L/M_N\ll 1$ [see Eq. (5.22)]. As a consequence $(F^d_L)_{ij}$ are very small. Numerically (see Table I) one finds that for $m_t\gtrsim 25$ GeV contribution of FCNC's to B^0_d-$\bar{B}^0_d$ mixing is much smaller than in the standard model. B^0_s-$\bar{B}^0_s$ mixing can at most get a

TABLE I. Strength of FCNC transitions in d-quark sector. The assumed values of various parameters are $m_s=0.15$ GeV, $M_p/v_L=1$, and $v_R/v_L=15$. $(\Delta m)_{\rm std}$ refers to the value of (Δm) in the $SU(2)_L\times U(1)$ model.

m_t (GeV)	$(\Delta m)^{K^0\text{-}\bar{K}^0}_{\rm FCNC}$ (GeV)	$\left[\frac{(\Delta m)_{\rm FCNC}}{(\Delta m)_{\rm std}}\right]^{B^0_s\text{-}\bar{B}^0_s}$	$\left[\frac{(\Delta m)_{\rm FCNC}}{(\Delta m)_{\rm std}}\right]^{B^0_d\text{-}\bar{B}^0_d}$	$(F^d_L)_{12}$
25	3.17×10^{-18}	1.4	0.07	3.18×10^{-7}
30	3.81×10^{-19}	0.12	0.024	1.1×10^{-7}
35	1.19×10^{-19}	0.028	0.0099	6.18×10^{-8}
50	1.58×10^{-20}	0.0019	0.0012	2.25×10^{-8}
60	6.61×10^{-21}	5.6×10^{-4}	4.11×10^{-4}	1.45×10^{-8}

comparable contribution from the FCNC's if m_t is ~ 25 GeV. Moreover, even in the most favorable case of $m_t \sim 25$ GeV, $M_P/v_L \sim 1$ and $v_L/v_R \sim \frac{1}{15}$, $F^d_{L12} \sim 3\times 10^{-7}$. This results in a $K_L \to \mu^+\mu^-$ branching ratio well below the current experimental limit $\le 10^{-9}$. Moreover, the contribution of FCNC's to K_L-K_S mass difference following from the formula analogous to Eq. (6.4) is also much smaller than the standard-model contribution (see Table I). Thus, the violation of the GIM mechanism is indeed very small in the charged $-\frac{1}{3}$ sector.

VII. CONCLUSIONS

We have made a general analysis of the expected FCNC's in a large class of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models[2–8] in which quarks obtain their masses through a seesaw mechanism. A detailed quantitative analysis of the strength of FCNC's is made in a specific model. It turns out that flavor-changing effects in the charge $-\frac{1}{3}$ sector are not significant. In contrast, the model studied could accommodate fairly large FCNC's in the up-quark sector. A particular transition studied is the flavor-changing decay of Z to $\bar{t}c$. This has a distinctive signature in the form of a fat jet originating from t balanced by a thin jet coming from c or in the form of two jets plus a hard lepton arising from the semileptonic decay of t (Ref. 14). For a light top quark[20] one obtains the observable[14] branching ratio in the range 10^{-6}–10^{-7}. This is unlike the three-generation standard model[14] or its supersymmetric generalizations.[15] Likewise, one predicts a D^0-$\bar{D}^0$ mixing which is much larger than in the standard model if the top quark is light. This is similar to the $SU(2)_L \times U(1)$ model with four generations in which case one also finds fairly large branching ratios[14] for $Z \to \bar{t}c$ decay and a large D^0-$\bar{D}^0$ mixing.[21]

We have not worked out detailed predictions for other rare processes induced by FCNC's, e.g., rare decays of B or D mesons. Although their rates have been argued to be less than the current experimental limit, it could still be more than the standard model. Also we have not worked out the strength of FCNC's in the leptonic sector. At the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ level, this will involve a new mass scale analogous to M_N or M_P but in some grand unified framework the leptonic FCNC's could be related to the FCNC's in the quark sector.

In this analysis we have assumed real mass matrices and CP conservation. The FCNC's connecting d and s although not sufficient to account for the real part of K^0-$\bar{K}^0$ mixing may provide a source of CP violation in such mixing. In this case, one would expect large CP-violating effects in mixings involving higher generations. This aspect is under study.

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[1] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, proceedings of the Stony Brook Workshop, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, New York, 1979); T. Yanagida, in *Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories*, Tsukuba, Japan, 1979, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979).

[2] D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **58**, 1600 (1987).

[3] A. Davidson and K. C. Wali, Phys. Rev. Lett. **59**, 393 (1987); **60**, 1813 (1988).

[4] S. Rajpoot, Phys. Lett. B **191**, 122 (1987); Phys. Rev. D **36**, 1479 (1987); Phys. Rev. Lett. **60**, 2003 (1988).

[5] F. del Augila, G. L. Kane, and M. Quiros, Phys. Lett. B **196**, 531 (1987).

[6] R. N. Mohapatra, Phys. Lett. B **198**, 69 (1987); **201**, 517 (1988).

[7] B. S. Balakrishna, Phys. Rev. Lett. **60**, 1602 (1988).

[8] B. S. Balakrishna, A. L. Kagan, and R. N. Mohapatra, Phys. Lett. B **205**, 345 (1988).

[9] S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).

[10] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* **11**, 566 (1975); R. N. Mohapatra and G. Senjanović, *ibid.* **12**, 1502 (1975); Phys. Rev. Lett. **44**, 912 (1980); Phys. Rev. D **23**, 165 (1981).

[11] J. K. Bajaj and G. Rajasekaran, Phys. Lett. **93B**, 461 (1980); Pramana **12**, 397 (1979); V. Barger, E. Ma, and K. Whisnant, Phys. Rev. D **26**, 2378 (1982).

[12] Note that because of the assumed equality of Yukawa couplings in the u and d sectors the states $|\phi_i\rangle$ are eigenfunctions also of M_d at the tree level.

[13] We should remember here that this is true for $(F_{L,R})_{13}$ only under the simplifying assumption of neglecting $a_{2,31}$ and $b_{2,2}$ in Eqs. (4.3). But the basic fact that $(F_{L,R})_{13}$ involves parameters of the same order as V_{ub} is independent of this.

[14] M. Clements *et al.*, Phys. Rev. D **27**, 570 (1983); V. Ganapathi *et al.*, *ibid.* **27**, 579 (1983); A. Axelrod, Nucl. Phys. **B209**, 349 (1982).

[15] B. Mukhopadhyaya and A. Raychaudhuri, University of Calcutta Report No.CUPP-88/2 (unpublished); M. J. Duncan, Phys. Rev. D **31**, 1139 (1985).

[16] B. McWilliams and O. Shankar, Phys. Rev. D **22**, 2853 (1980).

[17] A. Datta and D. Kumbhakar, Z. Phys. C **27**, 515 (1985); A. Datta and K. Niyogi, Phys. Rev. D **20**, 2441 (1979).

[18] A. Seiden, in *Proceedings of the European Physical Society High Energy Physics Conference [International Europhysics Conference on High Energy Physics]*, Uppsala, Sweden, 1987, edited by O. Botner (European Physical Society, Geneva, Switzerland, 1987), p. 368.

[19] P. Haas *et al.*, Phys. Rev. Lett. **60**, 1614 (1988).

[20] It has been observed by many authors that a large B^0_d-$\bar{B}^0_d$ mixing generally requires a heavy top quark. See G. Altarelli, in *Proceedings of the European Physical Society High Energy Physics Conference [International Europhysics Conference on High Energy Physics* (Ref. 18), p. 1002, for original references. However, a light top quark cannot be ruled out by B^0_d-$\bar{B}^0_d$ mixing alone, see, e.g., J. R. Cudell *et al.*, Phys. Lett. B **196**, 227 (1987).

[21] K. S. Babu *et al.*, Phys. Lett. B **205**, 540 (1988).