Current-quark masses from a relativistic constituent-quark model

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We consider a relativistic version of the constituent-quark model which gives sharply defined and model-independent values of constituent-quark masses and gives a good fit to the ground-state 35 mesons and to the excited states of ρ and K^* mesons. The isospin-breaking *u*-*d* constituent-quark mass difference is determined from the mass splittings of K and K^* mesons after appropriate electromagnetic corrections. The constituent-quark mass is written as the sum of a flavor-independent dynamical mass (which is determined by using the Goldstone theorem guaranteeing vanishing pseudoscalar-meson masses in the chiral limit) and a flavor-dependent part which is a function of current-quark mass. Depending on the model used, we get $m_s/m = 6.8$ to 19.1 and values of $(m_d - m_u)/(m_d + m_u)$ which range between 0.06 and 0.11 if we consider K mesons and between 0.22 and 0.40 if we consider K^* mesons.

I. INTRODUCTION

If, as is generally believed, quantum chromodynamics[1,2] (QCD) is the true theory of strong interactions, it would be desirable to make an accurate determination of the parameters that it contains. These parameters are, essentially, the QCD scale parameter Λ and the masses (of quarks of various flavors) which enter the QCD Lagrangian in appropriate mass terms. There are various ways to determine the QCD scale parameter and there is fair agreement among them because QCD perturbation theory can be trusted for short distances or high momenta. The determination of the masses (of quarks), on the other hand, is not simple. One method is via lattice gauge theory[3] in its various versions, but, at this point in time, it is still not free of uncertainties. Then there are models, especially current algebra. An early determination of the masses using current algebra was made by Weinberg[4] and it is common to call these masses current-quark masses to distinguish them from the constituent-quark masses which enter dynamical models of bound $q\bar{q}$ and qqq states. Nonrelativistic models of this type have considerable antiquity,[5,6] their successes are impressive, and they suggest a picture of low-lying baryons and mesons made up of quarks (and antiquarks in the case of mesons) almost at rest. However, it would be difficult to understand the large excitation energies of the baryons and mesons (of the order of a constituent-quark mass) if this were so. It is far more likely that the quarks in these low-lying mesons and baryons have their large kinetic energies delicately balanced by their large negative potential energies. If this is so, a relativistic picture is called for.

The aim of this paper is to construct a relativistic model of bound quarks, extract constituent-quark masses, and use them to determine current-quark masses. There have been other attempts at constructing a relativistic quark model[7,8] but the constituent-quark masses that they obtain are strongly model dependent. (Reference 7 gives 94 MeV for the mass of the u/d quark while Ref. 8 gives 671 MeV.) Furthermore, these values are not sharply defined, even in the context of one single method, because a different value for the mass can usually be obtained by adding a constant term to the potential. (This is also true of nonrelativistic models.) Since the zero of a confining potential cannot be defined unambiguously, this difficulty is serious. The basic problem is, of course, the definition of the mass of a confined particle. This problem will be solved in Sec. II, and constitutent-quark masses which are model independent and sharply defined will be extracted in the limit in which we ignore the *u*-*d* mass difference. (We shall not consider here the heavy *c*, *b*, or *t* quarks.) We shall exhibit and give the parameters for a model which predicts not only the ground-state mesons accurately but also the excited states of ρ and K^*. (We have selected these because they contain numerous orbital and radial excitations.) In Sec. III we use this model and the Fermi-Breit interaction of electromagnetism to calculate the small *u*-*d* (constituent-quark) mass difference from K^0 and $K^{\pm}$ splitting and from K^{*0} and $K^{*\pm}$ splitting. (They do not quite agree.) Finally, in Sec. IV we construct simple interpolating formulas for the dependence of the constituent-quark mass on the current-quark mass depending on one parameter and calculate the latter in terms of this parameter (i.e., we determine two of the *u*-,*d*-,*s*-quark masses in terms of the third). Although these turn out to be strongly model dependent, they do, for example, unambiguously rule out a vanishing mass for the *u* quark.

II. A RELATIVISTIC QUARK MODEL

The four-momentum of a free particle is constrained by the familiar relation $p^2 = m^2$ which defines the mass of the particle. This mass is also the expectation value of the energy of the particle in the (non-normalizable) $\mathbf{p} = 0$ state of the particle. In theories of directly interacting particles,[9] one introduces, for two particles, mass-shell constraints of the type

$$\phi_a = m_a^2 - p_a^2 + V(q_1 - q_2, p_1, p_2) \approx 0 , \quad (1)$$

for $a=1,2$ where the particular form of the potential is subject to the requirement that the constraints ϕ_1 and ϕ_2 should have zero Poisson bracket (or commute in quantum mechanics). This fixes that the dependence of V on the coordinates can arise only in the combination

$$(q_1-q_2)^2 - \frac{[(q_1-q_2)\cdot(p_1+p_2)]^2}{(p_1+p_2)^2}$$

and ensures that ϕ_1 and ϕ_2 are first-class constraints, and their vanishing is implemented in quantum mechanics by demanding that they be annihilators of physical states. If two particles interact according to (1), the mass operator of the composite system can be obtained as follows (**r** and **p** are position and momentum operators in the c.m. frame):

$$M = p_1^0 + p_2^0 \quad \text{(in the c.m. frame)}$$
$$= [\mathbf{p}^2 + V(\mathbf{r},\mathbf{p}) + m_1^{*2}]^{1/2} + [\mathbf{p}^2 + V(\mathbf{r},\mathbf{p}) + m_2^{*2}]^{1/2} . \quad (2)$$

(From now on m_a^* will mean the constituent mass of the particle a.) Equation (2) is the most general formula possible for two spin-zero particles. If the potential V is of confining type, its zero is arbitrary and, as mentioned earlier, this arbitrariness is reflected in the value of m_a^* that one may get in a particular model. We shall, therefore, adopt the definition that the mass of the confined quark a is the energy p_a^0 in the c.m. frame in a state in which $\mathbf{p}^2+V$ has the smallest eigenvalue which will be adjusted to be zero by adding a suitable constant term to the potential V. This has the consequence that the mass of the lowest-lying state of two confined particles (without spin) always comes out to be $m_1^*+m_2^*$. Masses extracted in this way will be model independent and sharply defined.

(i) Mass formula for $(6,\bar{6})^-$ *mesons.* That a formula of this type is approximately valid for the (spin-averaged) masses of the ground state 0^- and 1^- mesons was first stated by Rosner,[10] who wrote the mass formula

$$M_{ab} = m_a^* + m_b^* + A\frac{\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b}{m_a^* m_b^*} \quad (3)$$

and used these masses to calculate $M1$ transition rates for the various mesons (and magnetic moments in the case of baryons). The last term arises from a short-ranged color-magnetic hyperfine interaction.[6] We found a much improved fit with an additional term corresponding to an additional short-ranged spin-independent interaction:

TABLE I. Calculated masses of ground state $(6,\bar{6})$ mesons for $m^*=305$ MeV, $m_s^*=480$ MeV, and $A=157.3\times(305)^2$ MeV^3.

π	138.1 MeV
ρ	767.3 MeV
K	495.6 MeV
K^*	895.4 MeV
ϕ	1023.5 MeV
η (pure octet)	559.0 MeV
η' (pure singlet)	348.5 MeV

$$M_{ab} = m_a^* + m_b^* + \frac{A}{2}\left[\frac{1}{m_a^*} - \frac{1}{m_b^*}\right]^2 + \frac{A\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b}{m_a^* m_b^*} . \quad (4)$$

With this formula, choosing the rounded-off values $m^*=305$ MeV (for u or d quark), $m_s^*=480$ MeV, and $A=157.3\times(305)^2$ (MeV)^3, we get the values listed in Table I. The values of π, K, and K^* are more or less the input, so the accuracy of the model is close to 3 parts per thousand (ppt). (The cases of η and η' and ω are exceptional.[11]) That we are able to fit closely the masses of five mesons with a three-parameter formula suggests relations among the meson masses. Indeed there exist two very closely satisfied relations. One is the familiar formula

$$\rho^+ + \phi = \overline{K^{*0}} + K^{*+} , \quad (5)$$

which is experimentally satisfied within less than 1 ppt (we let the particle names stand for their masses). As written, the same q (or $\bar{q}$) appears on both sides, and since the charges of d and s quarks are equal, the electromagnetic energy shifts should be the same on both sides provided the distortion due to SU(3) breaking in the corresponding wave functions is ignored. For this reason, we expect that Eq. (5) should be almost exactly satisfied in QCD and we have incorporated it exactly in (4) by the specific choice of the coefficient ($A/2$) multiplying the third term there. The other relation is[12]

$$\frac{\rho-\pi}{K^*-K} = \frac{\dfrac{3K^*+K}{4} - \dfrac{3\rho+\pi}{8}}{\dfrac{3\rho+\pi}{8}} \quad \text{(2 ppt)} , \quad (6)$$

which is essentially the condition that the hyperfine splitting should be inversely proportional to the product of constituent-quark masses. However, this is only approximately incorporated in Eq. (4), though it would hold exactly if Eq. (3) were true.

(ii) A general formula. We shall continue to ignore tensor and spin-orbit forces in the interest of simplicity.[13] We write for the mass operator

$$M_{ab} = (\mathbf{p}^2+V+m_a^{*2})^{1/2} + (\mathbf{p}^2+V+m_b^{*2})^{1/2}$$
$$+ B\left\{\frac{1}{2}\left[\frac{1}{(\mathbf{p}^2+V+m_a^{*2})^{1/2}} - \frac{1}{(\mathbf{p}^2+V+m_b^{*2})^{1/2}}\right]^2 + \frac{\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b}{(\mathbf{p}^2+V+m_a^{*2})^{1/2}(\mathbf{p}^2+V+m_b^{*2})^{1/2}}, \delta(\mathbf{r})\right\} . \quad (7)$$

In Eq. (7), it is understood that the last term will be treated by first-order perturbation theory [because of the appearance of $\delta(\mathbf{r})$]. This formula is obviously designed to reproduce Eq. (4) and needs some physical justification to stand on its own, especially for the last line. Essentially we are assuming that the color hyperfine term is of the contact type with the color-magnetic moment being inversely proportional to the energy of the quark in the center-of-mass frame. Examples can be given where this is so (e.g., a free Dirac particle) but no relativistic derivation of the contact term is known, so Eq. (7) is in a sense largely phenomenological. Because of the way we have defined the constituent-quark masses, the expression for the $q\bar{q}$ potential (and hence the ground state and other wave functions) is determined only by the excited states of the $q\bar{q}$ system. Specifically, we looked at ρ and K^* mesons because they have many radial and orbital excitations. We find a reasonable fit provided the eigenvalues ϵ_{nl} of $\mathbf{p}^2+V$ are $(n+l)\Omega^2$, where n is the radial and l the orbital quantum number, and both are zero for the ground state. Essentially we find that radial and orbital excitations require about the same energy. By perturbing about large circular orbits we can show quite generally that no purely local potential V can have these eigenvalues for all n and l. A one-parameter model potential which gives these eigenvalues is

$$V=\tfrac{1}{16}\Omega^4 r^2+\frac{3L^2-2(L^2+\frac{1}{4})^{1/2}+1}{r^2}-\tfrac{3}{4}\Omega^2 \tag{8}$$

($\mathbf{L}=\mathbf{r}\times\mathbf{p}$). It is a one-parameter, harmonic-oscillator potential with an additional centrifugal term and with zero-point energy subtracted.[14] Its normalized eigenfunctions are

$$\psi_{nlm}(\mathbf{r})=\frac{\Omega^{3/2}}{2\Gamma(2l+\frac{3}{2})}\left[\frac{\Gamma(2l+\frac{3}{2}+n)}{\Gamma(n+1)}\right]^{1/2} \times(\tfrac{1}{4}\Omega^2 r^2)^l e^{-\Omega^2 r^2/8} \times {}_1F_1(-n,2l+\tfrac{3}{2};\tfrac{1}{4}\Omega^2 r^2)Y_{lm}(\hat{r}) \tag{9}$$

where

$${}_1F_1(\alpha,\gamma;z)=1+\frac{\alpha}{\gamma}\frac{z}{1}+\frac{\alpha(\alpha+1)}{\gamma(\gamma+1)}\frac{z^2}{2!}+\cdots$$

is the confluent hypergeometric function. This leads to the mass formula[15]

$$M_{ab}=(\epsilon_{nl}+m_a^{*2})^{1/2}+(\epsilon_{nl}+m_b^{*2})^{1/2}+\frac{A\Gamma(n+\frac{3}{2})}{\Gamma(n+1)\Gamma(\frac{3}{2})} \times\left[\frac{1}{2}\left[\frac{1}{(\epsilon_{nl}+m_a^{*2})^{1/2}}-\frac{1}{(\epsilon_{nl}+m_b^{*2})^{1/2}}\right]^2 +\frac{\sigma_a\cdot\sigma_b}{(\epsilon_{nl}+m_a^{*2})^{1/2}(\epsilon_{nl}+m_b^{*2})^{1/2}}\right]\delta_{l,0}\,. \tag{10}$$

A comparison of the calculated masses of ρ and K^* families with the experimentally observed masses is given in Table II. Considering the simplicity of the model the agreement is quite remarkable, both for K^* and for ρ. The flavor dependence of the interaction is worth commenting. Partly this is a reduced-mass effect, the nonrelativistic analog of potential energy is not V but

$$\frac{1}{2}\left[\frac{1}{m_1^*}+\frac{1}{m_2^*}\right]V\,.$$

On the other hand, exact flavor independence is probably untrue and is not expected because of flavor-SU(3) breaking.

III. ELECTROMAGNETIC SPLITTING

To proceed further and extract the u-d mass difference it is necessary to include the effect of purely electromagnetic interactions on meson masses. We shall work in the one-photon-exchange approximation. As the quarks are relativistic, substantial corrections are expected to the usual Coulomb energy shift. The most appropriate method would be to construct a Bethe-Salpeter amplitude in the instantaneous approximation and use a standard formula[18] for the first-order energy shift due to the one-photon-exchange kernel, but its gauge invariance is not obvious to us. We therefore work with the Fermi-Breit interaction:

TABLE II. Comparison of theoretical predictions with experimental results in MeV for ρ and K^* families. Experimental data are from Ref. 16 except for K_5^* which is from Ref. 17. We have introduced a prime to denote a radial excitation. We have chosen A_1 among the ^3P excitations of ρ since it is presumably close to the center of gravity of the ^3P level.

ρ family $\Omega^2=2.58\times10^5$ (MeV)^2		Theory	Expt.	K^* family $\Omega^2=3.17\times10^5$ (MeV)^2		Theory	Expt.
ρ	$n=0,\ l=0$	767.3	770	K^*	$n=0,\ l=0$	896[a]	896
ρ'	$n=1,\ l=0$	1247.5	1250	K_2^*	$n=0,\ l=1$	1380	1423
A_1	$n=0,\ l=1$	1216.6	1270				
ρ''	$n=2,\ l=0$	1605	1590±20	$K^{*\prime}$	$n=1,\ l=0$	1427	1410
$\rho_3(g)$	$n=0,\ l=2$	1561	1675±11	K_3^*	$n=0,\ l=2$	1782	1780
ρ'''	$n=3,\ l=0$	1899		$K^{*\prime\prime}$	$n=2,\ l=0$	1818	1790
ρ''''	$n=4,\ l=0$	2153	2150	K_4^*	$n=0,\ l=3$	2108	2060
ρ_3''	$n=2,\ l=2$	2121	2250	K_5^*	$n=0,\ l=4$	2391	2382
ρ_5'	$n=1,\ l=4$	2352	2350				

[a]Input.

$$V_\gamma = \alpha Q_a Q_b \left[\frac{1}{r} + \frac{1}{2m_a^* m_b^*} \left[\frac{1}{r} p^2 + \frac{\mathbf{r}\cdot(\mathbf{r}\cdot\mathbf{p})\mathbf{p}}{r^3} \right] - \frac{\pi}{2}\delta(\mathbf{r}) \left[\frac{1}{m_a^{*2}} + \frac{1}{m_b^{*2}} + \frac{4\sigma_a\cdot\sigma_b}{3m_a^* m_b^*} \right] \right] . \quad (11)$$

The expectation value of this is easily calculated following Itoh.[19] The total energies are given in Table III.

The meson masses corresponding to pure QCD are obtained by subtracting these from the experimental meson masses. For the Coulomb-corrected mass differences for K and K^* we get $\Delta K_C = K_C^0 - K_C^\pm = 5.50 \pm 0.30$ MeV and $\Delta K_C^* = 5.30 \pm 0.9$ MeV using the experimental masses from Ref. 16 and adding an uncertainty of 0.25 MeV arising from the Coulomb-shift estimation. (For K^* masses we have used the mean values quoted in Ref. 16 for individual particles and not the value quoted for K^* mass differences.) We shall not use the π^0-$\pi^\pm$ mass difference because it contains an unknown contribution arising from isospin breaking in the annihilation diagram.[11] Using Eq. (4) and the known value of $m^* = \frac{1}{2}(m_u^* + m_d^*)$, we get $m_d^* - m_u^* = 3.0 \pm 0.3$ MeV from K-meson masses and $m_d^* - m_u^* = 10.9 \pm 1.5$ MeV from K^*-meson masses. This discrepancy can ultimately be traced to the fact that Eq. (4) predicts much larger K-meson splittings than K^*-meson splittings because of the contrasting behavior of the hyperfine term in the two cases. Explicitly, we find from Eq. (4) that a small u-d mass difference is amplified (reduced) by the largely attractive (repulsive) color-magnetic hyperfine term in producing K (K^*) mass differences. This is also what one would naively expect from the Goldstone theorem, which suggests that pseudoscalar-meson masses are a more rapidly varying function of quark masses than are the vector-meson masses. It is possible, however, that the experimental value for K^* mass splitting and the estimated electromagnetic corrections are in error to such an extent as to make the two values of $m_d^* - m_u^*$ compatible. For the present, we shall use both of them separately to calculate the u-d mass difference.

IV. EXTRACTION OF CURRENT-QUARK MASSES

The general expression for the constituent-quark mass must have the form $m_a^* = m_0^* + f(m_a)$ where m_a is the current-quark mass of flavor a and f is a function which vanishes with its argument. m_0^* is, then, the dynamically generated mass which is the constituent-quark mass in the chiral limit $m_a \to 0$. m_0^* is easily determined from the formula (4) if we use the Goldstone theorem that the pseudoscalar-meson mass vanishes in the chiral limit. This gives

$$0 = 2m_0^* - \frac{3A}{m_0^{*2}} , \quad (12)$$

which determines m_0^* to be 280 MeV. (We have here used the fact that the coefficient A, arising from short-distance gluonic effects, does not change much as $m_a \to 0$.) The most important problem then is to determine $f(m_a)$. The precise form of $f(m_a)$ is determined, at least for small m_a, by the manner in which chiral symmetry is broken in the real world. For large m_a, we expect $f(m_a) \simeq m_a$, and $f(m_a) = m_a$ may be considered our first model. It implies $M_{ab} \propto (m_a + m_b)$ for the pseudoscalar-meson masses near the chiral limit. It gives $m = \frac{1}{2}(m_u + m_d) = 25$ MeV and $m_s = 200$ MeV. Even if these values are not correct, their ratio $m/m_s = 1/8$ may be so. [It amounts to assuming $f(m_a) = cm_a$ where c is an unknown constant.] Similarly, we get $(m_d - m_u)/(m_d + m_u) = 0.06$ from K-meson masses and $(m_d - m_u)/(m_d + m_u) = 0.22$ from K^* masses (after correction for electromagnetic effects). As we have already pointed out, we do not know how these values can be made compatible. If we simply take the average of the two values, we get $m_u/m_d = 0.76$. The precise value of m_u/m_d is, of course, very model dependent. But Eqs. (4) and (12) show that $[f(m_d) - f(m_u)]/[f(m_d) + f(m_u)]$ has the value 0.06 if we take K mass data and the value 0.22 if we take K^* masses. Either value completely rules out $m_u = 0$, in which case the fraction would be unity. We also note that Eq. (4) predicts that the charged-kaon mass for vanishing up-quark mass ($m_u = 0$) would be 449.6 MeV.

There is a theoretical model which gives rather similar results. Elias and Scadron[20] study the mass in the quark propagator and making a renormalization-group analysis, express it as the sum of a flavor-dependent Lagrangian mass and the flavor-independent dynamical mass, all of which run

$$m(p^2) = m_{\text{cur}}(p^2) + m_{\text{dyn}}(p^2) \quad (13)$$

for all quark flavors. The constituent-quark mass is defined by

$$m_{\text{con}} = m(p^2 = m_{\text{con}}^2) \quad (14)$$

and the dynamical mass satisfies

$$m_{\text{dyn}}(M^2) \simeq m_{\text{dyn}}^3 / M^2 . \quad (15)$$

This gives

$$\frac{m}{m_s} = \frac{m_{\text{con}}^3 - m_{\text{dyn}}^3}{m_{s,\text{con}}^3 - m_{\text{dyn}}^3} \frac{m_{s,\text{con}}}{m_{\text{con}}} \simeq \frac{1}{6.8} , \quad (16)$$

where $m_{\text{dyn}} = m_0^*$ in our notation, $m_{\text{con}} = m^*$, and $m_{s,\text{con}} = m_s^*$.

A different picture emerges in current algebra. A

TABLE III. Contribution of the three terms in V_γ. The meson masses corresponding to pure QCD are obtained by subtracting these from the experimental meson masses.

	I (MeV)	II (MeV)	III[a] (MeV)	III[b] (MeV)	Total (MeV)
K^0	−0.258	−0.139	0.051	−0.139	−0.485
K^{*0}	−0.258	−0.139	0.051	+0.046	−0.300
K^+	+0.515	+0.279	−0.102	+0.279	0.971
K^{*+}	0.515	0.279	−0.102	−0.093	0.599

[a]Spin-independent terms.

[b]Spin-dependent terms.

TABLE IV. Values of Λ and of the ratios m_s/m and $(m_d-m_u)/(m_d+m_u)$ for some typical choices of m.

m (MeV)	Λ (MeV)	m_s (MeV)	$\frac{m_s}{m}$	$(m_d-m_u)/(m_d+m_u)$ From K	From K^*
7	82.3	134.0	19.14	0.11	0.40
11	45.8	159.4	14.49	0.10	0.37
15	26.7	175.1	11.67	0.09	0.32

straightforward application of partial conservation of axial-vector current (PCAC) gives[2]

$$M_{ab}^2 \propto (m_a+m_b) \tag{17}$$

for the square of the pseudoscalar-meson mass M_{ab}. This result is valid only to first order in m_a, m_b. It is not possible to reproduce this result in the model of Sec. II. The best we can do is

$$M_{ab} \propto (\sqrt{m_a}+\sqrt{m_b}) \ . \tag{18}$$

A very simple functional relation between the constituent-quark mass and current-quark mass reproduces this result and interpolates to the linear dependence expected for very large current-quark mass:

$$m_a^* = m_0^* + (\Lambda m_a + m_a^2)^{1/2} \ , \tag{19}$$

where Λ is an unknown parameter which is not simply related to the QCD scale parameter Λ. The values of Λ and of the ratios m_s/m, $(m_d-m_u)/(m_d+m_u)$ for some typical choices of m are given in Table IV.

V. CONCLUSION

A relativistic quark model is set up from which sharply defined, model-independent, constitutent-quark masses are extracted after approximate inclusion of electromagnetic effects. The model gives good fit of the ground-state mesons and of the excited states of ρ and of K^*. Current-quark masses can then be obtained from these constituent-quark masses in a model-dependent manner. Depending on the model we get a value between 6.8 and 19.1 for m_s/m. For $(m_d-m_u)/(m_d+m_u)$ we get a value between 0.06 and 0.11 from K-meson masses and a value between 0.22 and 0.40 from K^*-meson masses.

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[11]Let β_{ij} denote the contribution to the mass matrix of the annihilation-with-recombination diagrams $q_i\bar{q}_i \rightarrow$gluons$\rightarrow q_j\bar{q}_j$. The β_{ij} is independent of i and j in the limit of perfect SU(3)-flavor symmetry and contributes to the mass of η' only. Because of flavor SU(3) and isospin breaking, small contributions to π^0, ρ^0, ω, η also occur from annihilation diagrams either directly or indirectly through modification of the wave functions; see Ref. 6. The agreement for η is exact if the η-η' mixing is about 12° in magnitude.

[12]While Eq. (5) is quite familiar, especially in its flavor-averaged form (being a consequence of ideal mixing), Eq. (6) has not occurred in the literature as far as we know.

[13]There is a formalism wherein spin-dependent and spin-independent forces can be introduced in an arbitrary combination. This formalism uses a covariant Hamiltonian in the center-of-mass frame in Foldy-Wouthuysen representation with appropriate constraints. See J. Thakur, Pramana (J. Phys.) **27**, 731 (1986).

[14]Since this potential appears under a square-root sign it corresponds to an interaction energy between quarks which increases linearly at large distances. Note that the second term in Eq. (8) is a kinetic rather than potential term and is of obscure origin, and also that if the zero-point energy $(\frac{3}{4}\Omega^2)$ is not subtracted out in the definition of V in Eq. (8), the constituent-quark masses come out to be imaginary because $m^{*2}, m_s^{*2} < \frac{3}{4}\Omega^2$.

[15]In Eq. (10), A is related to the B occurring in Eq. (7) by $A=2B\Omega^3$. To keep contact with Eq. (4) we have assumed that A is constant.

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