Rising cross sections in QCD and the cosmic-ray data

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Novel features of the QCD-implied growth of the total cross sections are discussed. Specifically, Lipatov's QCD Pomeron is a series of poles above unity, so that the center of gravity of the QCD Pomeron—the effective intercept—moves to higher j with rising energy. The higher the energy the steeper is the predicted rate of growth of the cross section, until unitarity enforces saturation of the growth rate and the onset of the Froissart asymptotics at extremely high energies. We demonstrate that the QCD-predicted rapid rise of the $p(\bar{p})p$ total cross section agrees perfectly with the recent CERN S$\bar{p}p$S datum on the phase of the $\bar{p}$-p forward amplitude. Further evidence for the QCD-predicted rise of the total cross section comes from the published Akeno and Fly's Eye data on the extensive air showers. Our principal observation is that in view of the unitarity-enforced slope–cross-section correlation one can determine proton-proton inelastic cross section from the proton-air absorption cross section to a very high accuracy. The published Fly's Eye result $\sigma_{abs}(p\text{-air})=540\pm40$ mb corresponds to $\sigma_{in}(p\text{-}p)=113^{+16}_{-15}$ mb and $\sigma_{tot}(p\text{-}p)=164^{+32}_{-27}$ mb. All of the previous determinations of $\sigma_{tot}(p\text{-}p)$ from $\sigma_{abs}(p\text{-air})$ have grossly underestimated $\sigma_{tot}(p\text{-}p)$. We present predictions for Fermilab Tevatron, $\sqrt{s}=1.8$ TeV, and Superconducting Super Collider, $\sqrt{s}=40$ TeV, energies. We give a functional form of correlation of the scaling violations in proton-air collisions with $\sigma_{abs}(p\text{-air})$, allowance for which could greatly improve the reliability of determination of $\sigma_{abs}(p\text{-air})$ from data on extensive air showers.

I. INTRODUCTION

The conventional phenomenology of the soft hadronic scattering at high energies is based on Reggeon field theory with the supercritical Pomeron having the intercept $\alpha_P(0)-1=0.07-0.1$ (Refs. 1 and 2). The model is consistent with the s- and t-channel unitarity constraints[1] and, complemented by the Abramovsky-Gribov-Kancheli (AGK) cutting rules,[3] provided consistent description of elastic scattering and multiproduction processes. Among its irrefutable successes are predictions of a specific pattern of geometric[1] and Koba-Nielsen-Olesen (KNO) scalings,[4] perfectly confirmed at the CERN S$\bar{p}p$S. Yet, its principal assumption, that the Pomeron is a simple isolated pole in the complex angular momentum plane, lacks any field-theoretic justification.

As Lipatov has shown,[5] perturbative QCD predicts quite a different Pomeron: a series of poles in the complex j plane above unity at

$$1<j<1+\Delta\ , \tag{1}$$

which accumulate at $j=1$. A pole with intercept j contributes to the total cross section a term $\propto E^{j-1}$. Henceforth, a novel feature of the QCD Pomeron is that its center of gravity moves to higher intercepts with rising energy, so that the higher the energy the faster the total cross section rises until unitarity saturates the growth rate and enforces the true Froissart regime at extremely high energies.

One cannot tell the difference between the single-pole and QCD Pomerons from the data in the limited energy range.[6] The purpose of this paper is to demonstrate that the recent S$\bar{p}p$S data[7] on the phase of the $\bar{p}p$ forward amplitude and the published measurements[8,9] of the cross section of absorption of the superhigh-energy cosmic-ray protons in the Earth's atmosphere give strong evidence for the QCD Pomeron with asymptotic intercept $\Delta=0.1-0.3$.

Regarding the cosmic-ray data our principal finding is that if $\sigma_{abs}(p\text{-air})$ is known, then $\sigma_{in}(p\text{-}p)$ can be determined virtually free of uncertainty: $\sigma_{in}(p\text{-}p)=(100\text{ mb})[\sigma_{abs}(p\text{-air})/507\text{ mb}]^{1.89}$ at $\sigma_{abs}(p\text{-air})>300$ mb. This is a consequence of the unitarity enforced slope–cross-section correlation in the p-p elastic-scattering amplitude. The often-quoted previous determinations[9,10] of $\sigma_{tot}(p\text{-}p)$ from the same cosmic-ray data grossly underestimate the proton-proton total cross section and are quite wrong.

The further presentation is as follows. In Sec. II we describe the salient properties of the QCD Pomeron and discuss unitarization of the scattering amplitude with allowance for the diffraction-dissociation processes. In Sec. III we present fits to the accelerator data on the (anti)proton-proton diffraction slopes and total cross sections and extrapolations of $\sigma_{tot}(p\text{-}p)$ and B_{pp} beyond the accelerator energies. We calculate the phase of the $p(\bar{p})p$ forward-scattering amplitude and conclude that the QCD-predicted rapid rise of the total cross section agrees perfectly with the recent finding[7] that $\rho_{\bar{p}p}=\text{Re}F_{\bar{p}p}(t=0)/\text{Im}F_{\bar{p}p}(t=0)=+0.24\pm0.04$. The subject of Sec. IV is determination of the proton-proton total cross section from the proton-air absorption cross section measured in extensive-air-shower experiments.[8,9] Our major observation is that the proton-proton inelastic cross section can be determined from the proton-air absorption cross section virtually free of uncertainty, whereas determination of the total cross section is less reliable in view of uncertainties with reconstruction of the p-p elastic cross section on the basis of $\sigma_{\rm in}(p\text{-}p)$.

An analysis of the extensive air showers (EAS's) requires certain assumptions on the scaling violations, specifically in the fragmentation region. In Sec. V we observe that by virtue of the AGK cutting rules[3] there exists a strong correlation between the cross-section growth and the rise of the inelasticity coefficient $K_{\rm in}$. We present a functional form of this correlation, allowance for which could improve the reliability of EAS determinations of $\sigma_{\rm abs}(p\text{-air})$ and, henceforth, $\sigma_{\rm in}(p\text{-}p)$ and $\sigma_{\rm tot}(p\text{-}p)$.

In the Conclusions we summarize our basic results. Some of the results of this study have been presented elsewhere.[11,12]

II. THE QCD POMERON, DIFFRACTION DISSOCIATION, AND UNITARIZATION

In quantum chromodynamics the Pomeron is generated by exchange of glueballs—bound states of gluons—in the t channel. Remarkably, the lowest-order, two-gluon-($2G$-) exchange diagram ($\Delta=0$, fixed pole at $j=1$) nicely reproduces the constant part of the hadron-nucleon total cross sections at moderate energies.[13,14] Higher-order perturbation-theory diagrams, explicitly containing the multiproduction of gluons,[15] give rise to a series of poles in the j plane in the range $1<j<1+\Delta$. This series of poles accumulates at $j=1$. The simplest perturbative calculations give the intercept[16]

$$\Delta(\mathbf{q}^2)=[\alpha_s(\mathbf{q}^2)/\pi]12\ln 2\ . \tag{2}$$

Strictly speaking, QCD perturbation-theory considerations only refer to the large-momentum-transfer region, where the strong-interaction coupling $\alpha_s(\mathbf{q}^2)$ is small. We are not in the position to extrapolate reliably the large-$\mathbf{q}^2$ perturbative QCD amplitudes down to $\mathbf{q}^2=0$. Yet, the fundamentals of the ultimate QCD phenomenology of the high-energy diffraction scattering of hadrons are obvious.

Namely, as energy goes up, the perturbation-theory diagrams of higher and still higher order come in, so that dominance of the lowest-order $2G$ exchange and the constant cross section (we omit for time being secondary Regge poles) are superseded by the rising contribution of poles with $j-1>0$: $\sigma_{\rm tot}\propto E^{j-1}$. The higher the energy the larger is the relative contribution of the rightmost singularity, so that the effective intercept of the QCD Pomeron increases with energy.

A crude approximation to a complete QCD phenomenology of the high-energy diffraction scattering of hadrons is then the two-pole amplitude of the form $[\xi=\ln(s/s_0)-i\pi/2,\ s=2m_pE,\ s_0=1\ \text{GeV}^2]$:

$$f(\mathbf{q})=ih_{2G}(\mathbf{q})+ih_{\mathbb{P}}(\mathbf{q})\exp(\Delta\xi-\alpha'_{\mathbb{P}}\xi\mathbf{q}^2)\ . \tag{3}$$

Hereafter, Δ is the intercept at $t=0$.

The residue of the $2G$-exchange contribution to the p-p amplitude which dominates at moderate energies can explicitly be computed in terms of the quark wave function of the interacting protons

$$h_{2G}(\mathbf{q})=\tfrac{16}{3}\alpha_s^2\int\frac{d^2\mathbf{k}[F(\mathbf{q})-D(\mathbf{k}^2+\mathbf{q}^2/4)]^2}{[(\mathbf{k}+\mathbf{q}/2)^2+m_G^2][(\mathbf{k}-\mathbf{q}/2)^2+m_G^2]}\ . \tag{4}$$

Here $F(\mathbf{q})$ and $D(\mathbf{k}^2+\mathbf{q}^2/4)$ are the single-quark (the charge form factor) and the two-quark vertex functions of the proton, m_G is an effective mass of the gluon. Equation (4) gives the right magnitude of the slope at moderate energies.[14] However, since in our crude approximation the terms $2G$ and $\mathbb{P}$ do rather comprise contributions of many poles in the j plane, we use the simple Gaussian parametrization

$$h_{2G}(\mathbf{q})=\sigma_{2G}\exp(-B_0^2\mathbf{q}/2)\ . \tag{5}$$

Lipatov's QCD Pomeron possesses specific conformal properties in the impact-parameter space,[5] in view of which all the residues are basically controlled by the quark wave functions of hadrons. Henceforth, the residues $h_{2G}(\mathbf{q})$ and $h_{\mathbb{P}}(\mathbf{q})$ are expected to exhibit similar t dependence ($t=-\mathbf{q}^2$) and for the beginners we simply set

$$R=h_{2G}(\mathbf{q})/h_{\mathbb{P}}(\mathbf{q})=\text{const}\ . \tag{6}$$

As we have mentioned above, the $2G$ amplitude as given by Eq. (4) is calculable at all $\mathbf{q}^2$ including $\mathbf{q}^2=0$. Technically, it is the driving term that generates the $\mathbb{P}$ amplitude too. This is true for the perturbative domain of large $\mathbf{q}^2$ and small strong-interaction coupling $\alpha_S(\mathbf{q}^2)\ll 1$. Unfortunately, there are no reliable prescriptions how to extrapolate the perturbative considerations down to $\mathbf{q}^2=0$ that we are interested in. For this reason we regard R and Δ rather as free parameters and accept for the residue $h_{2G}(\mathbf{q})$ the Gaussian parametrization. We emphasize that the case of $R=0$ is inconsistent with the QCD pattern of the Pomeron. We retain it just for purposes of comparison with the old-fashioned Pomeron models.

In the single-pole approximation the effective intercept $\Delta_{\rm eff}$ defined as $\Delta_{\rm eff}=d[\ln f(\xi,\mathbf{q}=0)]/d\xi$ is a constant equal to the conventional intercept. The two-pole approximation (4) produces the effective intercept

$$\Delta_{\rm eff}=\frac{\Delta}{1+R\exp(-\Delta\xi)}\ , \tag{7}$$

which rises with energy, thus reproducing a major novel feature of the QCD Pomeron.

The resulting partial-wave amplitude in the impact-parameter space

$$\begin{aligned}u(\mathbf{b})&=-\frac{i}{2}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\exp(-i\mathbf{q}\cdot\mathbf{b})f(\mathbf{q})\\&=\sigma_{2G}\left[\frac{1}{4\pi B_0}\exp\left[-\frac{\mathbf{b}^2}{2B_0}\right]\right.\\&\qquad\left.+\frac{1}{R}\frac{\exp(\Delta\xi)}{4\pi B_{\mathbb{P}}}\exp\left[-\frac{\mathbf{b}^2}{2B_{\mathbb{P}}}\right]\right]\ ,\end{aligned} \tag{8}$$

where $B_{\mathbb{P}}=B_0+2\alpha'_{\mathbb{P}}\xi$, overshoots the unitarity bound $u(\mathbf{b})\le 1$ at high energies, when $\Delta\xi\gg 1$. The unitarity is restored summing up all the eikonal multi-Pomeron exchange amplitudes, when the full partial amplitude $F(\mathbf{b})$ takes the form

$$F(\mathbf{b})=1-\exp[-u(\mathbf{b})] \,. \tag{9}$$

An inherent feature of the high-energy scattering of hadrons is diffraction dissociation (DD). The presence of DD implies that diffraction scattering of hadrons is a multichannel process and the so-called inelastic shadowing (IS) coming from the DD transitions such as $h \rightarrow h^* \rightarrow h$ affects unitarization of the elastic scattering amplitude.

A convenient approach to IS is a method of the diffraction-scattering eigenstates (DSE's) which can be summarized as follows.[17-20] Diffraction-scattering eigenstates form a basis $|\alpha\rangle$ in which the matrix of the diffraction amplitudes $\hat{\mathbf{F}}$ is diagonal:

$$\langle\beta|\hat{\mathbf{F}}|\alpha\rangle=F_\alpha\delta_{\alpha\beta} \,. \tag{10}$$

The real hadrons, the mass-matrix eigenstates, are superpositions of DSE's: $|h\rangle=\sum_\alpha C_\alpha^h|\alpha\rangle$, so that

$$\begin{aligned}\langle h|\hat{\mathbf{F}}|h\rangle&=\sum_\alpha |C_\alpha^h|^2F_\alpha(\mathbf{b})=\langle F_\alpha(\mathbf{b})\rangle_\alpha\\ &=1-\langle\exp[-u_\alpha(\mathbf{b})]\rangle_\alpha \,.\end{aligned} \tag{11}$$

The single-channel unitarized amplitude corresponds to $\exp[-\langle u_\alpha(\mathbf{b})\rangle_\alpha]$, i.e., to computing first the average of the eikonal. Since $\langle \exp(z)\rangle > \exp(\langle z\rangle)$, the net effect of IS is that hadrons become less opaque.

To give an idea of significance of IS corrections consider the simplest $2G$-exchange amplitude in the pion-nucleon scattering. Here the separation of pion's quark and antiquark in the impact-parameter space $\boldsymbol{\rho}$ is frozen in the scattering process and is the diffraction eigenvalue parameter. In view of the color cancellations[19] the meson-nucleon scattering amplitude will be proportional to $\boldsymbol{\rho}^2$:

$$u_\rho(\mathbf{b})\simeq(\boldsymbol{\rho}^2/\langle\boldsymbol{\rho}^2\rangle)u(\mathbf{b}) \tag{12}$$

with the DSE expansion coefficients given by the meson's wave function, which for the sake of simplicity can be taken from the Gaussian form

$$\begin{aligned}|C_\rho^h|^2&=|\Psi_h(\boldsymbol{\rho})|^2\\ &=(1/\pi\langle\boldsymbol{\rho}^2\rangle)\exp(-\boldsymbol{\rho}^2/\langle\boldsymbol{\rho}^2\rangle) \,.\end{aligned} \tag{13}$$

Then the expectation value (11) is readily computed:

$$F(\mathbf{b})=\langle F_\rho(\mathbf{b})\rangle_\rho=u(\mathbf{b})/[1+u(\mathbf{b})] \,. \tag{14}$$

Although being of a different functional form, Eqs. (9) and (14) are broadly alike in that both exhibit the black-disc behavior $F(\mathbf{b})=1$ in the high-energy limit of $u(\mathbf{b})\gg 1$.

A more realistic model should allow for DD of both the projectile and target. In view of the factorization property of the Regge residues the $|\alpha\rangle|\beta\rangle$ scattering amplitude can be written as

$$u_{\alpha\beta}(\mathbf{b})=g_\alpha g_\beta u(\mathbf{b}) \,, \tag{15}$$

where the coefficients g_α and g_β on the right-hand side (RHS) of Eq. (15) stand for the corresponding relative residues, with normalization $\langle g_\alpha\rangle_\alpha=\langle g_\beta\rangle_\beta=1$. Then, by virtue of Eq. (11), the net effect of IS is that the ν-fold scattering amplitude $\propto u(\mathbf{b})^\nu$ will be enhanced by a factor $K_\nu=\langle g_\alpha^\nu\rangle_\alpha\langle g_\beta^\nu\rangle_\beta$, where (we suppress the eigenvalue indices when it does not lead to confusion)

$$\begin{aligned}\langle g^\nu\rangle&=(1+\Delta g)^\nu\\ &=1+(1/2!)\nu(\nu-1)\langle\Delta g^2\rangle\\ &\quad+(1/3!)\nu(\nu-1)(\nu-2)\langle\Delta g^3\rangle+\cdots \,.\end{aligned} \tag{16}$$

Here, $\Delta g=g-\langle g\rangle=g-1$. The second moment $\langle\Delta g^2\rangle$ can be related to the inclusive forward DD cross section:[20,21]

$$\begin{aligned}\left.\frac{d\sigma^{\rm DD}}{dt}\right|_{t=0}&=\int dM^2\left.\frac{d\sigma^{\rm DD}}{dt\,dM^2}\right|_{t=0}\\ &=\frac{\langle\sigma_\alpha^2(1+\rho_\alpha^2)\rangle_\alpha-|\langle\sigma_\alpha(1-i\rho_\alpha)\rangle_\alpha|^2}{16\pi}\\ &=\langle\Delta g^2\rangle\left.\frac{d\sigma_{\rm el}}{dt}\right|_{t=0} \,,\end{aligned} \tag{17}$$

where

$$\sigma_\alpha(1-i\rho_\alpha)=2\int d^2\mathbf{b}\,F_\alpha(\mathbf{b})$$

and σ_α and ρ_α are the total cross section and Re/Im for the interaction of the eigenstate $|\alpha\rangle$ with the target.

Parametrization (15) corresponds to roughly the same diffraction slopes for all DSE's. Since the IS corrections to the total cross sections we are interested in only depend on the inclusive forward DD cross section (17), which is basically insensitive to the slope fluctuations (for detailed discussion see Ref. 18), for purposes of the present analysis (15) is justified. We notice that the enhancement coefficient K_ν and the moments $\langle\Delta g^\nu\rangle$ do not depend on energy. In what follows we shall retain only the lowest-order IS corrections $\propto\langle\Delta g^2\rangle$, borrowing $\langle\Delta g^2\rangle=0.35$ from an analysis[21] of IS corrections to p-D and p-He total cross sections. The effects of higher-order moments $\langle\Delta g^\nu\rangle$ with $\nu\geq 3$ were estimated in Refs. 20–22 and found to be much smaller than that of the leading-order term $\propto\langle\Delta g^2\rangle$.

III. QCD PREDICTIONS FOR $p\,(\bar{p})$ SCATTERING AT SUPERHIGH ENERGIES

Our Pomeron is described by five free parameters: the overall normalization σ_{2G}, the radius squared B_0 of the residue (the low-energy diffraction slope), the Pomeron trajectory slope $\alpha'_{\rm P}$, the asymptotic intercept Δ, and the parameter R. We have fitted the accelerator data[23-26] on $\sigma_{\rm tot}(p\text{-}p)$ and p-p diffraction slope at $|t|=0.02$ $({\rm GeV}/c)^2$, adding in the Regge term $\sigma_R\cdot s^{-0.5}$ on top of the Pomeron cross section given by amplitude (11) with the IS enhancement coefficient

$$K_\nu=[1+\nu(\nu-1)\langle\Delta g^2\rangle/2]^2 \,.$$

TABLE I. Sets of fitted parameters of the bare QCD Pomeron at different choices of the ratio R and intercept Δ.

R	Δ	σ_{2G} (mb)	B_0 $(\mathrm{GeV}/c)^2$	$\alpha'_{\mathbb{P}}$ $(\mathrm{GeV}/c)^2$	σ_R (mb)
36	0.32	48.7	10.15	0.105	8.3
8	0.22	37.9	9.88	0.132	31.5
0	0.097	28.7[a]	8.87	0.141	65.0

[a]This entry for the single-pole fit is the residue of the pole at $j=1+\Delta$.

In addition, we have assumed the conventional Regge-type vanishing crossing-odd cross section

$$\Delta\sigma_{\mathrm{tot}}=\sigma_{\mathrm{tot}}(\bar{p}\text{-}p)-\sigma_{\mathrm{tot}}(p\text{-}p) \simeq 70s^{-0.56}\ \mathrm{mb}\ . \tag{18}$$

The fitted parameters are listed in Table I. The parameters R and Δ are very strongly correlated and cannot be fixed uniquely from existing accelerator data judging from χ^2 alone. The entries in Table I are the results of fits with fixed R. The found R-Δ correlation is shown in Fig. 1.

An important advantage of our treatment of DD and IS is that our elastic and DD amplitudes have the built-in s-channel unitarity property. Moreover, our model gives the DD cross section with full allowance for the numerically very important absorption corrections. Unlike the conventional naive $\ln(s)$ parametrization of the DD cross section, our model gives very slowly rising single-diffraction cross section σ_{SD}: it rises by about 50% from $\sqrt{s}=20$ GeV to $\sqrt{s}=1$ TeV, then saturates and starts decreasing slowly beyond $\sqrt{s}=10$ TeV.

The results of fits to the total cross section and slope and predictions at superhigh energies are shown in Figs. 2 and 3. We notice that the higher the intercept Δ the lower is the Pomeron's slope $\alpha'_{\mathbb{P}}$. This correlation can easily be understood: the higher Δ, the steeper rises the total cross section and the larger is the rising component of the diffraction slope coming from the unitarity-implied slope–cross-section correlation.

Shown on the cross-section plot are also our determinations of $\sigma_{\mathrm{tot}}(p\text{-}p)$ from the cosmic-ray data on $\sigma_{\mathrm{abs}}(p\text{-air})$, described in detail in the next section. We remind the readers that previous single-pole Pomeron fits to the total cross section up to the CERN ISR energies, $\sqrt{s}=60$ GeV, have resulted in $\Delta=0.07$ (Ref. 1). Adding into the data set the S$\bar{p}p$S data[25] we find in the single-pole case $\Delta=0.097$ (see entries for $R=0$ in Table I), but even so enlarged intercept grossly underestimates the total cross section beyond $\sqrt{s}=1.5-2$ TeV suggested by the cosmic-ray data.[8,9]

If taken literally, the cosmic-ray data[8,9] are consistent with only the QCD fits with $\Delta=0.2-0.3$. Regarding the asymptotic properties of the total cross section, the S$\bar{p}p$S energy range proves precisely the transient region from the dominance of the approximately constant cross section ($\Delta=0$) to the large-Δ regime.

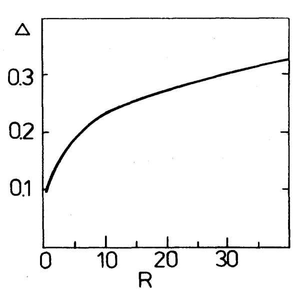

FIG. 1. A dependence of the intercept Δ on the parameter R.

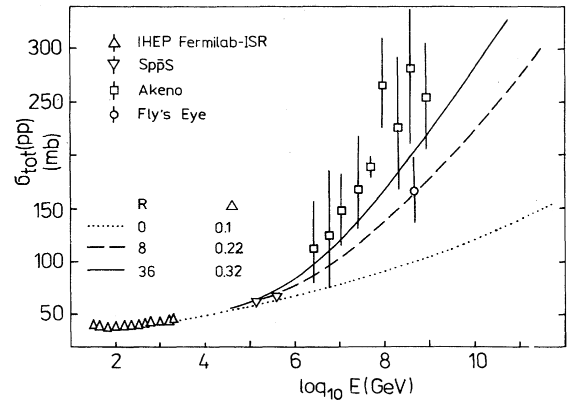

FIG. 2. The energy dependence of the pp total cross section vs the intercept Δ: solid curve, $\Delta=0.32$, $R=36$; dashed curve, $\Delta=0.22$, $R=8$; dotted curve, $\Delta=0.097$, $R=0$. Shown also are the fitted accelerator data (Ref. 23) on $\sigma_{\mathrm{tot}}(pp)$, the S$\bar{p}p$S data (Ref. 25) on $\sigma_{\mathrm{tot}}(\overline{p}p)$, and the values of $\sigma_{\mathrm{tot}}(pp)$ determined by us from the Akeno (Ref. 8) and Fly's Eye (Ref. 9) data on absorption of protons in the Earth's atmosphere.

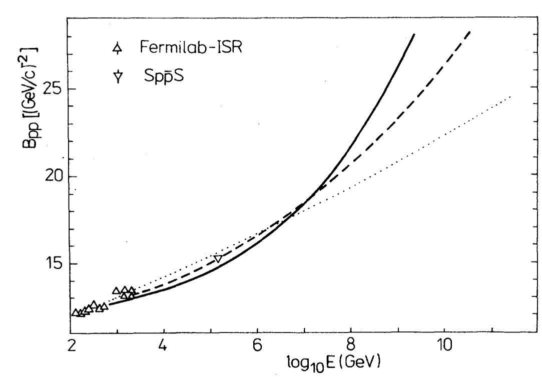

FIG. 3. The energy dependence of the diffraction slope B_{pp} vs the intercept Δ: solid curve, $\Delta=0.32$; dashed curve, $\Delta=0.22$; dotted curve, $\Delta=0.097$, $R=0$. Shown are the fitted experimental data on the p-p slope (Ref. 23) and the S$\bar{p}p$S result (Ref. 25) on $\bar{p}$-p slope at $\sqrt{s}=540$ GeV.

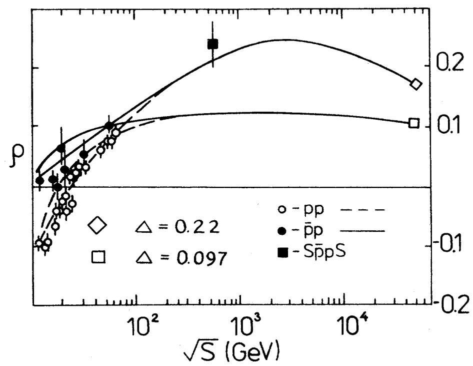

FIG. 4. The energy dependence of the ratio $\rho = \text{Re}F(t=0)/\text{Im}F(t=0)$ for the $\bar{p}p$ (solid curve) and pp (dashed curve) forward elastic-scattering amplitude. Shown also are the S$\bar{p}p$S result (Ref. 7) for $\sqrt{s}=540$ GeV and the lower-energy data compiled by Camillieri (Ref. 27).

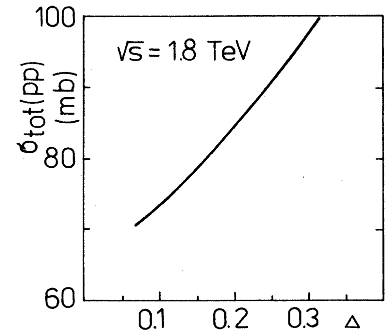

FIG. 5. Predictions of the $\bar{p}p$ total cross section at the Fermilab Collider, $\sqrt{s}=1.8$ TeV vs the intercept Δ.

By virtue of the dispersion relations the ratio $\rho = \text{Re}F(t=0)/\text{Im}F(t=0)$ for the forward-scattering amplitude is sensitive to the energy dependence of $\sigma_{\text{tot}} = \text{Im}F(t=0)$ at energies higher than ρ was measured at. Specifically, the higher the cross-section growth rate the bigger and positive is ρ. In Fig. 4 we present our predictions for ρ_{pp} and $\rho_{\bar{p}p}$. On top of the crossing-even Pomeron+Regge amplitude we have included the real part coming from the above specified crossing-odd p-p and $\bar{p}$-p cross-section difference (18). We emphasize that we did not fit the real parts of the forward amplitude and our results are genuine predictions. Obviously, the S$\bar{p}p$S result[7] for $\rho_{\bar{p}p}$ strongly favors the QCD Pomeron with large Δ. Even the SPS-Fermilab-ISR data on ρ_{pp} and $\rho_{\bar{p}p}$ are reproduced somewhat better, namely, by the QCD model.

Here, we have subscribed to the conventional Regge-type vanishing difference of the antiproton-proton and proton-proton total cross sections. The novel feature of QCD is a possibility of the so-called odderon—the crossing-odd singularity at $j=1$ (Ref. 28). Such an odderon might give a nonvanishing contribution to the real parts of forward amplitudes, particularly to ρ_{pp}-$\rho_{\bar{p}p}$ (Ref. 29). However, the recent perturbative QCD analysis[30] has shown that odderon's contribution to the forward-scattering amplitudes is numerically very small and it can safely be neglected for purposes of our analysis.

Notice that ρ_{pp} saturates at energy of a few TeV's and then slowly goes down. This is an onset of the Froissart regime of $\sigma_{\text{tot}} \propto \ln^2(s)$ and $\rho \propto 1/\ln(s)$, which remains elusive even at the highest energies accessible in the extensive-air-shower experiments.

The asymptotic behavior of the p-p and $\bar{p}$-p scattering amplitudes could much better be constrained at the Fermilab Tevatron. In Fig. 5 we present our predictions for the $\bar{p}$-p total cross section versus the intercept Δ at $\sqrt{s}=1.8$ TeV. It proves to be a fairly steep function of the intercept Δ, typical QCD prediction for the Fermilab Tevatron, $\sqrt{s}=1.8$ TeV, being 85–95 mb (Ref. 11). Similar estimations were recently obtained by Leader[31] on the basis of the S$\bar{p}p$S measurement of $\rho_{\bar{p}p}$ using the derivative analyticity relations.[32]

IV. RELIABLE DETERMINATION OF THE PROTON-PROTON CROSS SECTION FROM COSMIC-RAY DATA

Our principal concern in this section is how one can infer the proton-proton cross sections from the data on absorption of the superhigh-energy cosmic-ray protons in the Earth's atmosphere. Until the Superconducting Super Collider (SSC) becomes a reality, the data on the extensive air showers will remain the sole source of information on the proton-proton total cross section in the multi-TeV c.m.-system energy range.

We remind the readers that the quantity measured in the cosmic-ray experiments is the so-called absorption or production cross section

$$\sigma_{\text{abs}} = \sigma_{\text{tot}} - \sigma_{\text{el}} - \sigma_{\text{Qel}} , \tag{19}$$

where σ_{Qel} is a cross section of the quasielastic scattering off the target nucleus $pA \to pA^*$ not followed by production of the new particles, so that the scattered proton is retained in the incident flux.

At moderate energies the standard Glauber-Sitenko-Gribov multiple-scattering theory[33,34] gives

$$\sigma_{\text{abs}}(pA) = \int d^2\mathbf{b} \left[1 - \left[1 - \frac{1}{A}\sigma_{\text{in}}(pp)T(\mathbf{b}) \right]^A \right] , \tag{20}$$

where

$$T(\mathbf{b}) = \int dz\, d^2\mathbf{c}\, \rho_A(z,\mathbf{c}) \frac{1}{2\pi B_{pp}} \exp\left[-\frac{(\mathbf{b}-\mathbf{c})^2}{2B_{pp}} \right] \tag{21}$$

and $\rho_A(\mathbf{r})$ is the nuclear-matter density.

We have to derive the correct extension of (20) and (21) valid at superhigh energies when the proton-proton scattering has the non-Gaussian profile and IS corrections are to be included with allowance for DD of both projectile and target nucleons, as was explicitly done in the proton-proton scattering. We shall follow the general method developed in Ref. 20.

Let us start with the proton-nucleus total cross section. For interaction of DSE $|\alpha\rangle$ with the nucleus one finds in a usual way

$$\sigma_{\rm tot}(\alpha A)=2\int d^2\mathbf{b}\{1-\langle 1,{\rm in}|\langle \exp[-g_\alpha g_\beta f(\mathbf{b}-\mathbf{c})]\rangle_\beta|1,{\rm in}\rangle^A\} . \tag{22}$$

This can be derived as follows: Start with the state of the nucleus in which the nucleons N_i are in DSE $|\beta_1\rangle,\ldots,|\beta_A\rangle$. Then one has to take the matrix element of the phase operator

$$\prod_{i=1,A}\exp[-u_{\alpha\beta_i}(\mathbf{b}-\mathbf{c}_i)] \tag{23}$$

over the nuclear wave function, integrating over the coordinates of nucleons $\mathbf{c}_i$ and including IS correction for DD of the target nucleons, i.e., taking the averages $\langle\ \rangle_{\beta_i}$ which can be carried out independently for the different nucleons. With the usually assumed factorized nuclear wave function one then gets a product of the matrix elements over the single-particle nuclear wave function $|1,{\rm in}\rangle$ and readily obtains Eq. (22). Finally, one has to average over the eigenstates of the projectile proton:

$$\sigma_{\rm tot}(pA)=2\int d^2\mathbf{b}\{1-\langle[\langle 1,{\rm in}|\langle \exp[-g_\alpha g_\beta f(\mathbf{b}-\mathbf{c})]\rangle_\beta|1,{\rm in}\rangle]^A\rangle_\alpha\} . \tag{24}$$

A similar, though much more tedious, derivation of the absorption cross section yields

$$\sigma_{\rm abs}(pA)=\int d^2\mathbf{b}\{1-\langle\langle[\langle 1,{\rm in}|\langle\langle \exp[-g_\alpha g_\beta f(\mathbf{b}-\mathbf{c})-g_\gamma g_\delta f(\mathbf{b}-\mathbf{c})]\rangle\rangle_{\beta\delta}|1,{\rm in}\rangle^A]\rangle\rangle_{\alpha\gamma}\} . \tag{25}$$

We emphasize that IS corrections to the nuclear cross sections are computed here on the same footing as to the elementary proton-proton cross sections: one cannot switch off DD in the proton-nucleus amplitude retaining it in the proton-proton amplitude and vice versa.

In our QCD analysis of the proton-nucleus scattering we retain in (25) only the corrections $\propto\langle\Delta\alpha^2\rangle$. We have used the conventional Gaussian parametrization of the nuclear-matter density in ^{16}O and ^{14}N nuclei with the charge radii $\langle R_{\rm ch}^3\rangle^{1/2}=2.72$ and 2.54 fm, respectively.[35] A word of caution is necessary here: in order to compute the nuclear-matter density one has to subtract from the charge radii of nuclei the contributions of the charge radii of nucleons. Otherwise $\sigma_{\rm abs}(p\text{-air})$ will be overestimated by up to 7%, depending on energy.

One often simplemindedly uses formulas (20) and (21) even at very high energies. Indeed, at moderate energies the slope B_{pp} and the total/inelastic cross section are two independent parameters and the proton-nuclei cross sections depend on both. The trouble with the Gaussian parametrization is that the s-channel unitarity constraint for the partial-wave amplitudes is violated as soon as $\sigma_{\rm el}>\sigma_{\rm tot}/4$, precisely what happens with most extrapolations of the slope and total cross section to superhigh energies of interest. To this end, our formula (25) is exact and enables us to take correctly into account both an onset of the black-disc regime in the proton-proton scattering and IS corrections.

With onset of the black-disc regime in the p-p amplitude the nuclear absorption cross section will also approach the black-disc limit

$$\sigma_{\rm abs}(p\text{-air})=\pi(R_A+\sqrt{2B_{pp}})^2 \tag{26}$$

and will be basically a function of the slope B_{pp} rather than $\sigma_{\rm tot}(p\text{-}p)$. However, by virtue of the same s-channel unitarity, which enforces the black-disc asymptotics, the proton-proton slope and total cross section become strongly correlated. In Fig. 6 we show this correlation in the B_{pp}-$\sigma_{\rm in}(p\text{-}p)$ plot at different values of the intercept Δ. The naive geometric scaling $\sigma_{\rm in}/B=$const is elusive. Yet, in view of this correlation the nuclear absorption cross section exhibits a remarkable "scaling" property of being virtually a function of only $\sigma_{\rm in}(p\text{-}p)$. This scaling property is clearly seen in Fig. 7. Although the fits with different intercepts Δ give rather different energy dependence of $\sigma_{\rm abs}(p\text{-air})$, shown in Fig. 8, the scattering of the same curves when drawn in the $\sigma_{\rm abs}(p\text{-air})$-$\sigma_{\rm in}(p\text{-}p)$ plot, is virtually negligible. Using this plot, we find the proton-proton inelastic interaction cross sections shown in Fig. 9 alongside the theoretical predictions with different intercepts.

The above $\sigma_{\rm abs}(p\text{-air})$-$\sigma_{\rm in}(p\text{-}p)$ relationship can analytically be parametrized as

$$\sigma_{\rm in}(p\text{-}p)=(100\ {\rm mb})[\sigma_{\rm abs}(p\text{-air})/507\ {\rm mb}]^{1.89} . \tag{27}$$

The formula (27) is valid to a few percent accuracy in the range of $\sigma_{\rm abs}(p\text{-air})>300$ mb and/or $\sigma_{\rm in}(p\text{-}p)>37$ mb.

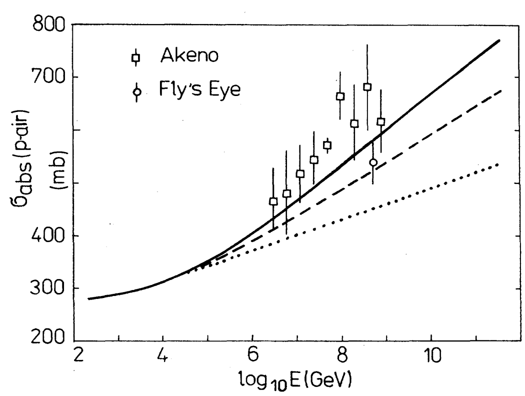

FIG. 6. Predicted energy dependence of the cross section of absorption of protons in air vs the intercept Δ: solid curve, $\Delta=0.32$; dashed curve, $\Delta=0.22$; dotted curve, $\Delta=0.097$, $R=0$. Shown are the Akeno (Ref. 8) and Fly's Eye (Ref. 9) determinations of $\sigma_{\rm abs}(p\text{-air})$ from the data on extensive air showers.

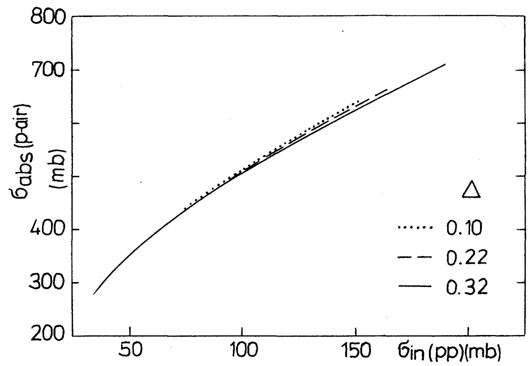

FIG. 7. The plot of the proton-air absorption cross section vs the proton-proton inelastic cross section at different intercepts Δ: solid curve, $\Delta=0.32$; dashed curve, $\Delta=0.22$; dotted curve, $\Delta=0.097$.

Once $\sigma_{in}(p\text{-}p)$ is known, further determination of the total cross section requires a knowledge of the elastic cross section. The predicted energy dependence of the ratio σ_{el}/σ_{in} is shown in Fig. 10. We conclude that the uncertainty in reconstruction of the elastic cross section would amount to up to 5–10 % theoretical uncertainty in the total-cross-section determination. It is entirely due to a very feeble form of the geometric scaling when it comes to the B_{pp}-$\sigma_{el}(p\text{-}p)$ correlation. Consequently, the $\sigma_{abs}(p\text{-air})$-$\sigma_{tot}(p\text{-}p)$ relationship, shown in Fig. 11 is much less stringent than that of Fig. 7 for the inelastic cross section.

An analysis of the Fly's Eye results on the longitudinal development of the extensive air showers has led[9] to $\sigma_{abs}(p\text{-air})=540\pm40$ mb at $E=4.5\times10^8$ GeV. According to the above considerations, our result is

$$\sigma_{in}(p\text{-}p)=113^{+16}_{-15}(\text{stat})\pm5(\text{syst})\text{ mb},$$
$$\sigma_{tot}(p\text{-}p)=164^{+32}_{-27}(\text{stat})\pm10(\text{syst})\text{ mb}. \quad (28)$$

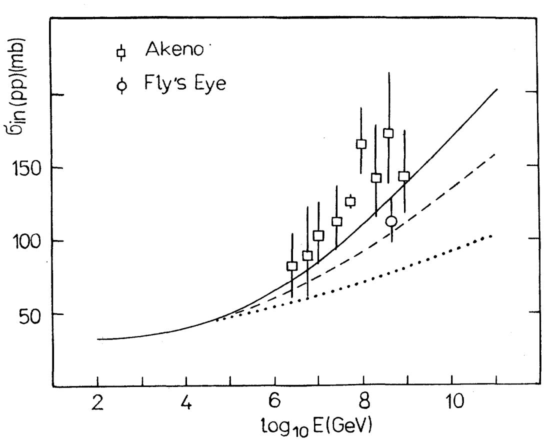

FIG. 8. The energy dependence of the proton-proton inelastic cross section vs the intercept Δ: solid curve, $\Delta=0.32$, $R=36$; dashed curve, $\Delta=0.22$, $R=8$; dotted curve, $\Delta=0.097$, $R=0$. Shown also are our determinations of $\sigma_{in}(p\text{-}p)$ from the Akeno (Ref. 8) and Fly's Eye (Ref. 9) data on $\sigma_{abs}(p\text{-air})$.

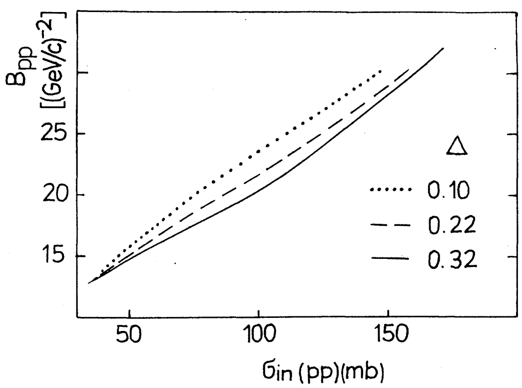

FIG. 9. Correlation of the proton-proton diffraction slope and inelastic cross section at different intercepts Δ: solid curve, $\Delta=0.32$; dashed curve, $\Delta=0.22$; dotted curve, $\Delta=0.097$.

Here we presented our educated guess of the systematic theoretical uncertainties of determination of the p-p cross sections from the p-air absorption cross section. Similar numbers for the Akeno result $\sigma_{abs}(p\text{-air})=570\pm12$ mb at energy $E=4.7\times10^7$ GeV are (we suppress our estimation of the theoretical systematic errors) $\sigma_{in}(p\text{-}p)=126\pm5$ mb and $\sigma_{tot}(p\text{-}p)=188\pm10$ mb.

Previous determinations[9,10] of the p-p cross section from the same cosmic-ray data have produced much lower p-p cross sections. Apparently, the major source of error was an imprudent extension of the geometric scaling up to the superhigh energies, though it is badly broken already at $Sp\bar{p}S$ energies.[36] Enforcing it one would ascribe to the slope a growth rate much higher than that suggested by any sensible model and, henceforth, underestimate $\sigma_{in}(p\text{-}p)$ by about 15–20 %. This estimation follows from comparison of Takagi's results[10] with ours. On top of that, putting $\sigma_{el}(p\text{-}p)/\sigma_{tot}(p\text{-}p)=$const will obviously underestimate the elastic cross section (see Fig. 10). Hence the grossly underestimated $\sigma_{tot}(p\text{-}p)=120\pm10$ mb inferred from the Fly's Eye result. To that we must add that the neglecting by IS corrections in papers[9,10] also leads to some underestimation of the proton-proton cross section.

Much closer in spirit to ours is an analysis in the recent paper by Gaisser, Sukhatme, and Yodh.[37] Using the Chou-Yang model suggested relationship between the

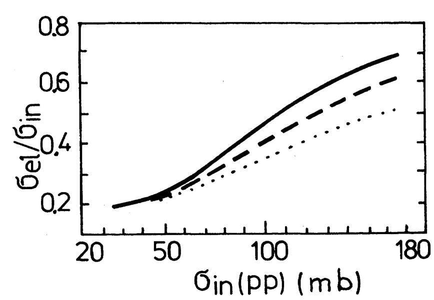

FIG. 10. The ratio of the proton-proton elastic and inelastic cross section at different intercepts Δ: solid curve, $\Delta=0.32$; dashed curve, $\Delta=0.22$; dotted curve, $\Delta=0.097$.

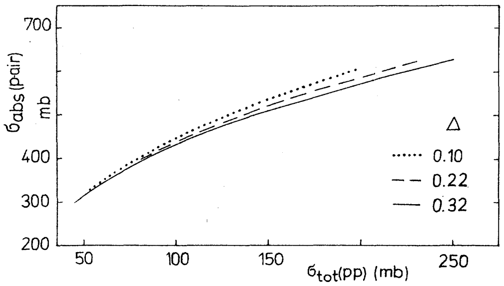

FIG. 11. The plot of the proton-air absorption cross section vs the proton-proton total cross section at different intercepts Δ: solid curve, $\Delta=0.32$; dashed curve, $\Delta=0.22$; dotted curve, $\Delta=0.097$.

slope and total cross section these authors have concluded that the Fly's Eye result corresponds to $\sigma_{tot}(p\text{-}p)=175^{+40}_{-27}$ mb. This $\sigma_{tot}\text{-}B_{pp}$ relationship is close to that in our QCD-suggested models and since it is the principal issue in inferring the elementary cross section from the nuclear data, their result is close to ours. Gaisser, Sukhatme, and Yodh correctly criticized the geometric-scaling-suggested extrapolations but have overlooked that the $\sigma_{abs}(p\text{-air})\text{-}\sigma_{in}(p\text{-}p)$ relationship is virtually model independent. To this criticism we add that their treatment of the IS corrections and of the unitarization of the p-p scattering amplitude is incomplete and the employed extrapolation of the DD cross section leads to slightly overestimated IS correction to the absorption cross section. Besides, their formula for the IS correction to the absorption cross section does not contain the numerically very important IS correction to the quasielastic scattering[20] and they use the Gaussian p-p amplitude beyond the applicability region. Yet, as all these factors are much less significant than using the right relationship between the slope and total cross section, the conclusions by Gaisser, Sukhatme, and Yodh[37] do broadly agree with those of our earlier paper.[11]

Thus, our principal conclusion is that once absorption of the superhigh-energy protons in the Earth's atmosphere has been measured, the further conversion of $\sigma_{abs}(p\text{-air})$ into the proton-proton inelastic cross section is virtually model independent and the data on extensive air showers are a reliable source of information on the proton-proton scattering up to $E=10^9-10^{10}$ GeV.

V. RISING CROSS SECTIONS, SCALING VIOLATIONS, AND MEASUREMENTS OF $\sigma_{abs}(p\text{-air})$

Direct measurements of the depth of absorption of the cosmic-ray protons would have given a straightforward determination of $\sigma_{abs}(p\text{-air})$. However, practical determination of the absorption cross section requires a very involved analysis of the development of the extensive air showers. We cannot go into details, but in fact experimentally one rather determines a depth of the maximum of the electromagnetic component of the EAS. Hence, in a crude approximation, a quantity of prime interest is a rate of energy transfer to the secondary particles, primarily π^0's. Theoretical predictions for the fragmentation spectra are model dependent,[38] but the gross features of the energy dependence of the inelasticity coefficient $K_{in}(p\text{-air})$ are well reproduced by the so-called leading-particle-cascade (LPC) model, long discussed by cosmic-ray physicists.[39] Our major observation is that $K_{in}(p\text{-air})$ is strongly correlated with $\sigma_{abs}(p\text{-air})$.

Let $L_1(x)=(x/\sigma_{in})(d\sigma/dx)$ be the inclusive spectrum of the leading nucleon for one cut Pomeron (one multiperipheral chain production process). The LPC model gives the following recurrence relation for the same spectrum in the production process with ν cut Pomerons (production of ν multiperipheral chains of secondary particles):

$$L_{\nu+1}(x)=\int\frac{dz}{z}L_1(z)L_\nu\left[\frac{x}{z}\right]. \tag{29}$$

Once the weights $w_\nu=\sigma_\nu/\sigma_{in}$ are known, the resulting leading particle spectrum will be

$$L_{p\text{-}p}(x)=\sum_\nu w_\nu L_\nu(x). \tag{30}$$

For the purposes of the scaling-violation analysis we can neglect the IS corrections. Then

$$\sigma_{in}(p\text{-}p)=\int d^2\mathbf{b}\{1-\exp[-2\,\mathrm{Re}f(\mathbf{b})]\} \tag{31}$$

and, by virtue of the AGK cutting rules,[3] the ν chain production cross section equals

$$\sigma_\nu=\int d^2\mathbf{b}\frac{[2\,\mathrm{Re}f(\mathbf{b})]^\nu}{\nu!}\exp[-2\,\mathrm{Re}f(\mathbf{b})]. \tag{32}$$

Recalling the above-discussed slope–cross-section correlation it can easily be understood that the weights w_ν are almost uniquely determined by the magnitude of $\sigma_{in}(p\text{-}p)$. The implication is that such parameters as the elasticity coefficient $K_{el}(p\text{-}p)=\langle x\rangle$ will, to a large extent, only depend on $\sigma_{in}(p\text{-}p)$.

Some numerical estimations are in order. Taking the leading particle spectrum for the single Pomeron of the form

$$L_1(x)=\mu x^\mu \tag{33}$$

we observe that the resulting spectrum (30) follows very closely the same parametrization (33) with renormalized exponent μ. The exponents μ of the spectrum (30) and the elasticity coefficient $K_{el}(p\text{-}p)$ in p-p collisions at different energies and/or different p-p inelastic cross sections are related by (the cross sections are in mb's)

$$\mu_{p\text{-}p}=\mu_{p\text{-}p}(E=100\text{ GeV})\exp\left[-\frac{\sigma_{in}(p\text{-}p)-32}{120}\right], \tag{34}$$

$$K_{p\text{-}p}=\frac{\mu_{p\text{-}p}}{1+\mu_{p\text{-}p}}. \tag{35}$$

Similar but tedious analysis can be repeated for the proton-air interactions. The result for the nuclear spec-

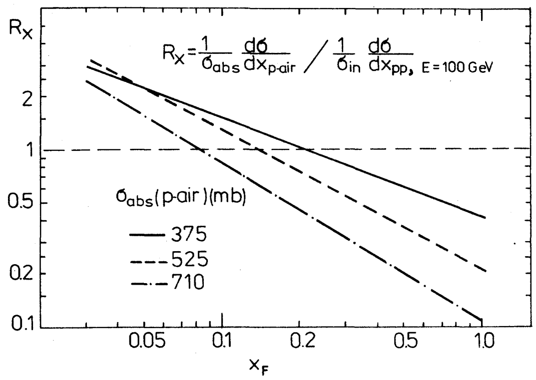

FIG. 12. The pattern of the scaling violation in the leading proton spectrum in the proton-air collisions as a function of the proton-air absorption cross section. The quantity plotted is a ratio of the leading proton spectrum in p-air collisions to that in p-p collisions at $E_{\text{lab}} = 100$ GeV.

trum exponent is

$$\mu_{p\text{-air}} = 0.3\mu_{p\text{-}p}(E = 100\,\text{GeV}) \times \exp\left[-\frac{\sigma_{\text{abs}}(p\text{-air}) - 500}{250}\right]. \tag{36}$$

Remarkably, the parametrization (33) for the single-cut Pomeron spectrum reproduces the overall leading-particle spectra in p-p and p-air interactions to 5% accuracy. The typical pattern of the scaling violation in the p-air leading-particle spectrum is shown in Fig. 12. The ratio R_x would have been unity were it not for the intranuclear absorption of the leading particle and the increase of the absorption probability, i.e., decrease of w_1 with rising $\sigma_{\text{in}}(p\text{-}p)$. The larger $\sigma_{\text{in}}(p\text{-}p)$, the larger is $\sigma_{\text{abs}}(p\text{-air})$ and the stronger is the nuclear attenuation of fast leading nucleons.

Finally, the elasticity coefficient $K_{\text{el}}(p\text{-air})$ can be parametrized as

$$K_{\text{el}}(p\text{-air}) = K_{\text{el}}(p\text{-}p, E = 100\ \text{GeV})0.45 \times \exp\left[-\frac{\sigma_{\text{abs}}(p\text{-air}) - 500}{300}\right]. \tag{37}$$

The parametrizations (36) and (37) hold at $\sigma_{\text{abs}}(p\text{-air}) > 350$ mb.

As one observes from Fig. 12, the scaling violation in p-air collisions is numerically very significant. For instance, if one starts with the p-p inelasticity coefficient of $K_{\text{in}}(p\text{-}p, E = 100\ \text{GeV}) = 0.5$, one will wind up with $K_{\text{in}}(p\text{-air}) = 0.78$ if $\sigma_{\text{abs}}(p\text{-air}) = 500$ mb. This will increase the energy dissipation rate and an analysis of the EAS data with allowance for such scaling violations will result, apparently, in somewhat lower values of $\sigma_{\text{abs}}(p\text{-air})$ than those found with $K_{\text{in}}(p\text{-air}) = \text{const}$.

A few comments on other fragmentation models are in order. The dual topological unitarization models[40] give slightly different prescriptions for the energy partition between the cut Pomerons but the final scaling violation is very close to that described above. The additive quark model[38] gives somewhat weaker scaling-violation effects but still not much different from the LPC model predictions.

VI. CONCLUSIONS

We have developed a QCD-motivated description of (anti)proton-proton diffraction scattering at superhigh energies. Our basic results are as follows.

(i) We have developed unitary description of the diffraction scattering of hadrons at superhigh energies based on the QCD model of the Pomeron with full allowance for the IS corrections coming from the diffraction dissociation processes.

(ii) The QCD Pomeron—a series of poles in the complex angular momentum plane with intercepts above unity—predicts that the total-cross-section growth rate increases with energy up to $\sqrt{s} = 2$–3 TeV. QCD predicts the p-p total cross section of 85–95 mb at the Tevatron, $\sqrt{s} = 1.8$ TeV, and 170–220 mb at the SSC energy of $\sqrt{s} = 40$ TeV.

(iii) QCD Pomeron predicts a rapid rise of the phase of the forward (anti)proton-proton scattering in perfect agreement with the recent measurement[7] of $\sigma_{\bar{p}\text{-}p}$ at $S\bar{p}pS$.

(iv) We have derived the exact formula for the cross section of absorption of the high-energy protons by nuclei with full allowance for the inelastic shadowing corrections due to inelastic intermediate states coming from the diffraction dissociation of both projectile and target nucleons.

(v) We have found that in view of the unitarity suggested correlation between the diffraction slope and inelastic cross section of p-p scattering, much weaker than suggested by the geometric scaling though, there exists nearly model-independent relationship between $\sigma_{\text{abs}}(p\text{-air})$ and $\sigma_{\text{in}}(p\text{-}p)$. We have obtained simple equation, $\sigma_{\text{in}}(p\text{-}p) = 100\ \text{mb}\,[\sigma_{\text{abs}}(p\text{-air})/507\ \text{mb}]^{1.89}$, valid to a few percent accuracy at $\sigma_{\text{abs}}(p\text{-air}) > 300$ mb. This relationship implies that p-p inelastic cross section can be deduced from the cosmic-ray data to a very high accuracy and, considering that till the post-SSC era the extensive-air-shower experiments remain the sole source of information on the proton-proton interactions at superhigh energies, justifies efforts in pursuing the extensive-air-shower studies.

(vi) We demonstrate that previous determinations of the proton-proton total cross section from the cosmic-ray data were wrong and present new results for the total and inelastic p-p cross sections from correct analysis of the published Akeno and Fly's Eye data[8,9] on absorption of the cosmic-ray protons in the Earth's atmosphere. These cross sections give strong evidence for the QCD suggested asymptotic growth of the proton-proton total cross section.

(vii) We have shown that the inelasticity coefficient in the proton-proton and proton-nucleus scattering is strongly correlated with the p-p inelastic and p-air absorption cross sections and found the functional form of this correlation. Such a correlation, when properly included into the codes for analysis of the longitudinal development of the extensive air showers, could greatly improve a reliability of determinations of $\sigma_{abs}(p\text{-air})$.

(viii) We comment that the energy range from ISR up to the Fermilab Collider proves precisely the transient region from the approximately constant cross section to the unitarity enforced true Froissart asymptotics at energies not accessible even with cosmic rays. High-precision measurements of the $\bar{p}$-p cross section at the Fermilab Collider could put stringent constraints on parameters of the QCD Pomeron.

Note added. After this manuscript was submitted for publication, we became aware of the paper by Loyal Durand and Hong Pi, Phys. Rev. D **38**, 78 (1988). These authors claim that the Fly's Eye result corresponds to $\sigma_{tot}(p\text{-}p)=106\pm20$ mb, in strong disagreement with our conclusions. The origin of discrepancy is that Durand and Pi use the non-Glauber approach to the high-energy proton-nucleus scattering: Namely, in their formulation the nuclear S matrix is a product of the proton-nucleon S matrices with the phase (eikonal) given by convolution of the proton-nucleon phase (eikonal) with the single-particle nuclear density. In the Glauber formulation of the multiple-scattering theory, one should rather compute convolution with the nuclear-matter density of the exponent of the proton-nucleon eikonal. As a result, Durand and Pi obtained larger proton-nucleus cross sections and, vice versa, have obtained a much lower proton-proton cross section.

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[16]Strictly speaking, formula (2) gives the intercept of the moving cut in the j plane which roughly reproduces a sum of contributions of all the poles; for more details see Lipatov's paper of 1986 (Ref. 5).

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