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Underlying event multiplicity in jet and vector-boson production: A measure of quark and gluon spatial distributions

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In nucleon-nucleon collisions, constituent-constituent interactions, such as the production of jets, W's, Z's, Drell-Yan pairs, etc., favor small nucleon impact parameters with high spatial overlap, thereby enhancing the production of associated low-p_t hadrons which make up the "underlying" event. We use a simple model, which reproduces the measured minimum-bias multiplicity distributions, to determine the multiplicity–impact-parameter relationship and to estimate the underlying event multiplicity in constituent-constituent interactions as compared with the minimum-bias multiplicity. Measurement of these distributions will reflect the spatial distribution of the hadron constituents.

I. INTRODUCTION

Most of the cross section in proton-proton collisions is accounted for by the production of hadrons via low-p_t interactions. These processes, precisely because they are low p_t, are poorly understood. The hadrons that are produced do not come from a single constituent-constituent (c-c) collision, but from interactions among many of the constituents. A small part of the pp cross section, arising from single quark-quark, quark-gluon, or gluon-gluon interactions producing jets, W's, Z's, Drell-Yan pairs, etc., is calculable via QCD.

Since it is not possible to turn off the low-p_t processes responsible for the "minimum-bias" event structure, when particles of high invariant mass or high p_t are produced by c-c interactions, an "underlying structure" of low-p_t hadrons is observed to accompany the particles that are produced in the hard process. In this paper we define the "associated" hadrons as those which are not produced by the primary c-c collision that produced the jet, W, Z, etc. Among these particles are the so-called "spectator contributions." The associated hadrons are most cleanly distinguished experimentally from the c-c-produced hadrons by sampling the multiplicity at polar and azimuthal angles removed from the jet (or W or Z) axes.

We believe that important information about hadron structure is contained in the multiplicity of the accompanying hadrons. This paper uses a simple model which simultaneously accounts for the minimum-bias multiplicity distributions and predicts the multiplicity distribution of underlying particles in events containing jets, W's, Z's, etc.

The basis for this study is that very-small-cross-section quark-quark or quark-gluon collisions in a spatially extended hadron favor zero-impact-parameter collisions with maximum spatial overlap of the interacting constituents. If this is indeed the case, the production of accompanying hadrons should be enhanced in comparison with minimum-bias collisions, which are unbiased with respect to impact parameter. This correlation effect should be present in any reaction. In principle, measurements of the correlated event structure should provide information on the spatial distribution of quarks and gluons within the extended hadron. For example, if the spatial distributions of u or d quarks and gluons are not the same, one should be able to discover the differences by studying the multiplicity distributions of the accompanying particles in events produced by the interactions of different constituents.

Unfortunately, without a QCD theory applicable to low-p_t processes, a quantitative description is not presently possible. Nevertheless, as we do in this paper, it is worthwhile to study *models* of the interplay between soft and high-Q^2 particle production since, without guidance from them, one might overlook geometric or kinematic effects that might appreciably affect the interpretation of the observed cross sections. The work in this paper is an extension of an earlier study.[1] In a detailed study of multiplicity distributions using the Lund parton model, Sjöstrand and Van Zijl[2] have also examined the question of impact-parameter correlations.

To carry out our program, we must first develop a model of low-p_t interactions that correlates impact parameter with low-p_t particle production. In particular, we wish to determine how selection of an event of multiplicity m is correlated with the impact parameter b. Although our main purpose is not to produce an impact-parameter model of minimum-bias particle production, we shall see, in fact, that it is easy to produce a good representation of the multiplicity distribution data.

II. MODEL CALCULATIONS OF THE MINIMUM-BIAS MULTIPLICITY

To study the magnitude of the impact-parameter correlation, we make use of an idea advanced by Barshay,[3] that the multiplicity of hadrons produced in pp collisions is correlated with the spatial overlap of the protons: i.e., the volume swept out geometrically by protons colliding with impact parameter b. Taking a clue from the analogous ultrarelativistic nuclear case, where it is known experimentally that the multiplicity is proportional to the number of participants rather than the number of scatters,[4] we assume that it is the swept-out volume $V(b)$ that determines the multiplicity. Our first assumption, therefore, is that the *mean* multiplicity at an impact parameter b is given by

$$\langle m(b)\rangle = A + BV(b) \ . \tag{1}$$

This means that any *inelastic pp* interaction produces a minimum mean multiplicity A, the mean multiplicity increasing with the swept-out volume $V(b)$. A cannot be zero since an inelastic scatter produces at least one new particle. Further, to relate the impact parameter to the multiplicity of a particular event, which is the measured quantity, we need to relate the multiplicity of an event with impact parameter b to the *mean* for all events at that impact parameter: i.e., we need to determine $P(\langle m(b)\rangle, m)$.

Our second assumption[5] is that the multiplicity *distribution* for a particular impact parameter b is a Poisson distribution, $P(\langle m(b)\rangle, m)$, about the mean multiplicity, $\langle m(b)\rangle$. This is a desirable choice since the Poisson distribution does not introduce any additional arbitrary parameters.

Accordingly, we first investigate how well this model reproduces the multiplicity distribution measured, for example, in $|y| < 1.0$ in *pp* collisions at $\sqrt{s} = 63$ GeV in Ref. 6. For this study, we have investigated the results for two different assumed radial distributions within the proton: (a) uniform density, $\rho(r) = \text{const}$, and (b) Gaussian density, with $\rho(r)$ falling off to $1/e^2$ at the nucleon edge.

[The choice of a truncated Gaussian is not intended as representation of the proton but only to allow calculation of a spatial distribution that does not differ too much from the flat distribution. From these choices we can test the sensitivity of our calculations to $\rho(r)$.]

What is needed in the general case is the "number" of constituents swept out from both nucleons at impact parameter b. For the volume swept out of a single spherical proton with uniform constituent density and unit diameter we use[7] the approximation

$$V(b) = \frac{\pi}{6}(1-b)^2(1+ab), \quad a = \frac{3}{\sqrt{2}} - 1 \ , \tag{2}$$

where $0 < b < 1$ is the normalized impact parameter.

The normalized "minimum-bias" multiplicity distribution $P(m)$ is then

$$P(m) = \int P(\langle m(b)\rangle, m) b\, db \Big/ \int b\, db$$
$$= \int \frac{e^{-\langle m(b)\rangle}\langle m(b)\rangle^m}{m!} b\, db \Big/ \int b\, db \ . \tag{3}$$

To see how sensitive our results are to the choice of distribution function, we have also investigated the use of another function $H(\langle m(b)\rangle, a, m)$ in place of the Poisson function $P(\langle m(b)\rangle, m)$ in Eq. (3):

$$P(m) = \int H(\langle m(b)\rangle, a, m) b\, db$$
$$= \int \frac{a(am)^{a\langle m(b)\rangle - 1} e^{-am}}{\Gamma(a\langle m(b)\rangle)} b\, db \Big/ \int b\, db \ . \tag{4}$$

$H(\langle m(b)\rangle, a, m)$, which is similar to the Γ distribution, includes a free parameter a, unlike the zero parameter Poisson function.

Figure 1 shows the comparison of these models with the multiplicity distribution at $\sqrt{s} = 63$ GeV in $|y_{\text{c.m.}}| < 1.0$ (Ref. 6), using the two spatial distributions and the Poisson and H functions. It is clear that the uniform and truncated-Gaussian spatial distributions produce almost identical fits, and that the use of the H function, with an extra parameter a, makes only a minor improvement to the fits at high multiplicity. Since we do not know the true spatial distribution, it is gratifying that the fits to the minimum-bias distribution are not sensitive to the choice of ρ.

Figure 2 shows the probability $P(b)$ of getting an impact parameter b for events with multiplicities of $m = 1$, 4, and 16. This curve is plotted for constant ρ with the Poisson fit. A strong correlation is seen between m and b, the prerequisite for utilizing impact-parameter correlations to study the spatial distributions of hadrons.

III. MEAN ASSOCIATED MULTIPLICITIES

The mean multiplicity for minimum-bias events is given by

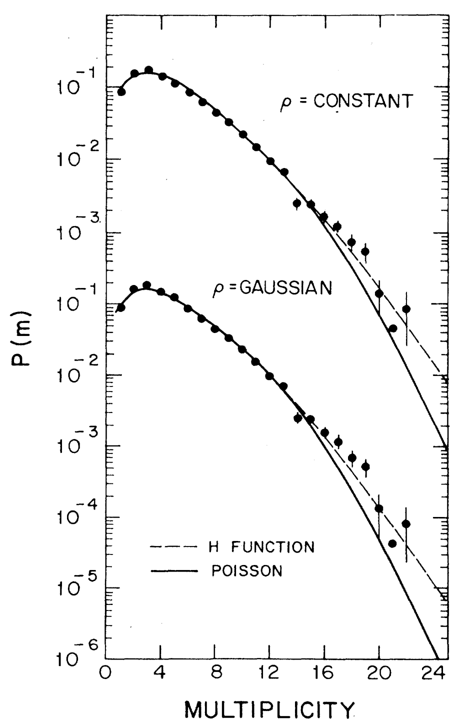

FIG. 1. Comparison of $P(m)$, for $\rho(r) = \text{const}$ and Gaussian using both the Poisson and H functions, with the minimum-bias multiplicity distribution extracted from Ref. 6.

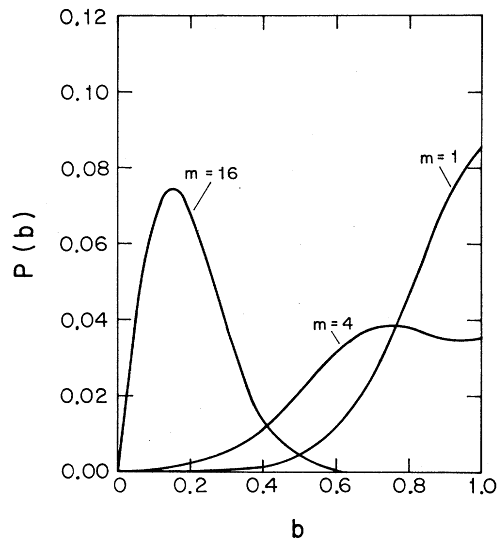

FIG. 2. The $P(b)$ distribution for selected multiplicities, $m=1$, 4, and 16, using the Poisson distribution and ρ=const.

$$\langle m_{\text{min-bias}} \rangle = \int [A + BV(b)] b\, db \Big/ \int b\, db \; . \quad (5)$$

In a constituent-constituent interaction, we assume that the cross section depends on the product of the number of interacting constituents in the swept-out regions of both nucleons, which is proportional to $V^2(b)$. It is this assumption which allows us to calculate the impact-parameter correlation between jets, W's, etc., and accompanying low-p_t hadrons. Thus, the mean associated multiplicity is given by

$$\langle m_{\text{assoc}} \rangle = \int V^2(b)[A + BV(b)] b\, db \Big/ \int V^2(b) b\, db \; . \quad (6)$$

It is the ratio of the associated to the minimum-bias multiplicity, given by Eqs. (5) and (6), that can be extracted directly from experiment. [One could, of course, calculate these ratios from the multiplicity distributions which we generated in the previous section, using the Poisson (or H) distribution assumption. But in calculating mean values, only the parametrization of the mean multiplicity enters, via the quantities A and B.]

The associated multiplicities are best sampled at azimuthal angles of 90° with respect to the jet axes and at rapidities well removed from that of the jet, W, or Z. "Minijets," resulting from initial- or final-state gluon bremsstrahlung, should also be removed from the sample. Table I shows the values of the enhancement, $\epsilon = \langle m_{\text{assoc}} \rangle / \langle m_{\text{min-bias}} \rangle$, found in several experiments.[8,9] It also indicates how the different production processes might depend on the different proton constituents. The values of A and B for each model, as determined by fits to the minimum-bias multiplicity distribution, are shown in Table II.

It would be valuable to have more precise measurements of the associated multiplicities to see if there are indeed differences that make it possible to distinguish the u, d, q, $\bar{q}$, and gluon distributions in the proton. To do this, the associated multiplicities should be compared at the appropriate $\sqrt{s}$, obtained by subtracting the energy of the jets or W's, etc., from the incident energy.

All we can conclude from the data in Table I is that the associated multiplicities are generally enhanced at the level predicted by the impact-parameter correlation. New UA1, UA2, and CDF data and analyses should be available within the next year.

TABLE I. Enhancement values for several experiments and predictions.

Reaction	Energy (GeV)	Constituents	Products	ϵ
pp	63	uu, dd, ud	Jets	$\simeq 1.5$
$p\bar{p}$	540	gluon-gluon	Jets	$\simeq 2.0$
$p\bar{p}$	540	$u\bar{d}, d\bar{u}$	W	$\simeq 1.1$
$p\bar{p}$	540	$u\bar{u} + d\bar{d}$	Z	$\simeq 1.7$
	Theory	$\rho(r)$	ϵ	
	Poisson	Constant	1.69	
	Poisson	Gaussian	1.71	
	H function	Constant	1.58	
	H function	Gaussian	1.61	

IV. ASSOCIATED MULTIPLICITY DISTRIBUTIONS

We now turn to the associated multiplicity distributions in jet, W, etc., events, and to the measurement of the cross section as a function of the associated multiplicity. The conditional probability of a constituent-constituent interaction at impact parameter b, $P_p(b)$, is given by

$$P_p(b) = \sigma \rho^2 V^2(b) b\, db / N_e \; , \quad (7)$$

with $N_e = \int \sigma \rho^2 V^2(b) b\, db$.

Equation (7) states that the cross section depends on the constituent-constituent cross section σ and on the square of $\rho V(b)$, the number of constituents in the swept-out region of each nucleon.

The associated multiplicity accompanying the detection of a direct constituent-constituent interaction is then

$$P_d(m) = \int P(\langle m(b) \rangle, m) P_p(b) b\, db$$
$$= \frac{1}{N_e} \int \frac{e^{-\langle m(b) \rangle} \langle m(b) \rangle^m}{m!} \rho^2 \sigma V^2(b) b\, db \; . \quad (8)$$

TABLE II. Values of fitted parameters for multiplicity distributions.

$\rho(r)$	Function	A	B	a
Constant	Poisson	2.4	15.5	
Gaussian	Poisson	2.7	13.3	
Constant	H	2.8	13.6	0.91
Gaussian	H	3.0	11.7	0.91

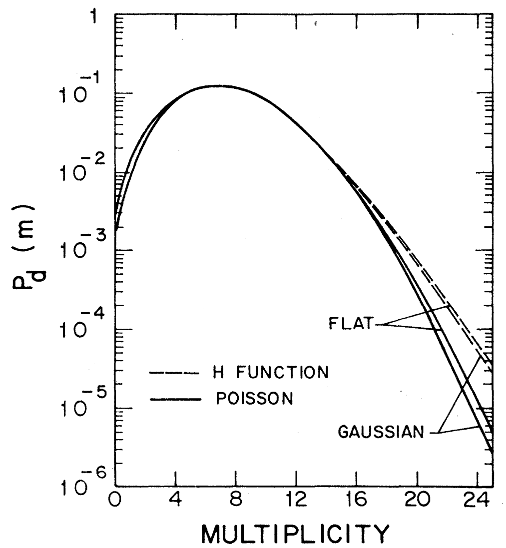

FIG. 3. $P_d(m)$ for constant and Gaussian distributions and for the Poisson and H functions.

Either the Poisson (as shown) or H distributions can be used in Eq. (8).

Figure 3 shows $P_d(m)$ for the four calculational variations whose parameters were set by the multiplicity distribution fits in Fig. 1. The curves are normalized to unity.

Figure 4 shows the ratio $P_d(m)/P(m)$, which illustrates the difference between the associated multiplicity distribution and the unbiased multiplicity distribution. This ratio is our basic result, since it shows the enhancements at high multiplicities arising from the impact-parameter correlation.

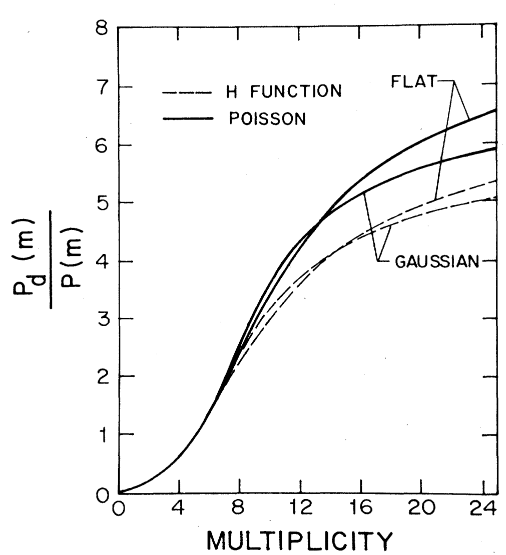

FIG. 4. $P_d(m)/P(m)$ for the Poisson and H functions, showing the strong variation with multiplicity.

V. ESTIMATING DIFFERENCES IN MULTIPLICITY DISTRIBUTIONS FOR DIFFERENT PRODUCTION PROCESSES

Suppose that the spatial distributions for quarks and gluons were different, one being constant and the other a truncated Gaussian. Figure 5 indicates the difference in the number of events as a function of multiplicity that would be observed in q-q and g-g collisions. We plot $R=(N_{\rm con}-N_{\rm Gau})/(N_{\rm con}+N_{\rm Gau})$ versus multiplicity for two samples with the same number of events. The observation of such a shape for R in comparisons such as $(N_{\rm jet}-N_W)/(N_{\rm jet}+N_W)$ would be an indication of different spatial distributions for the constituents.

In our calculations, the large difference in R at high multiplicity merely reflects the fact that the tail of the multiplicity distribution is more sensitive to the form of the Poisson or H function than to differences in $\rho(r)$. However, the rise at high multiplicities will occur, but will reflect the real fluctuations of multiplicities about the mean, rather than our uncertain estimates using the Poisson and H distributions.

VI. DISCUSSION

There are several remarks about the underlying event structure and our calculations that merit attention.

(1) We have used a geometrical model that assumes point interactions of the constituents. Any smearing of the sharp $V(b)$ boundaries will smear the multiplicity-impact-parameter correlation. Thus the enhancements we have calculated are likely to be upper limits.

(2) Although we have assumed that the dependence of $\langle m(b)\rangle$ on $V(b)$ for minimum-bias events is linear in V, other functional forms [e.g., $A+BV^2(b)$] could have been employed. Such a choice will affect the magnitude of the impact-parameter correlation effects, but cannot remove them.

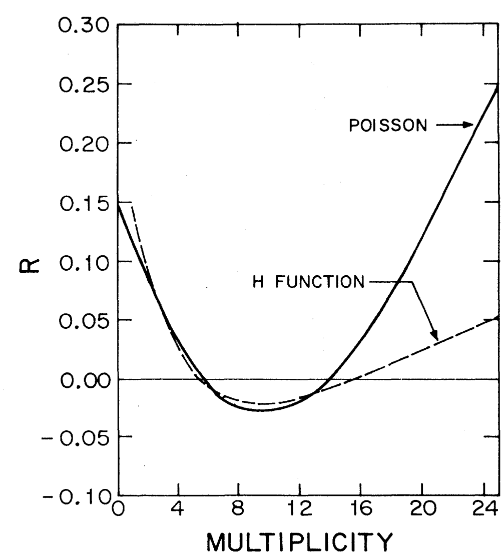

FIG. 5. $R=(N_{\rm con}-N_{\rm Gau})/(N_{\rm con}+N_{\rm Gau})$ vs multiplicity.

(3) We have assumed that the mean multiplicity in minimum bias varies as the swept-out volume. Changing this b dependence could change the relative shapes of the associated and mean multiplicity distributions. This emphasizes the fact that it is *differences* in the event structure for different reactions that are the interesting feature.

(4) The associated multiplicity distribution should not be expected to scale with the minimum-bias multiplicity distribution as $\langle m \rangle P(m/\langle m \rangle)$.

(5) Four-jet events that arise from gluon bremsstrahlung in a single c-c interaction will have a different underlying structure than those originating from two correlated c-c interactions. The latter "multiparton" events are proportional to $V^4(b)$, and will have a larger mean multiplicity: using constant density and a Poisson distribution, we find $\epsilon = 1.92$ for multiparton events and 1.69 for gluon bremsstrahlung events, which are proportional to $V^2(b)$. Figure 6 compares $P_d(m)/P(m)$ for the two cases, and shows that very high multiplicities should be richer in multiparton four-jet events.

(6) The impact-parameter correlations we have described also will produce particle ratio effects: Since it is known that particle ratios vary with multiplicity in minimum-bias events (e.g., about 30% in the K/all ratio[10] as one proceeds from low to high multiplicities), and since the mean multiplicities are affected by the impact-parameter correlation, one should find that the underlying structure would not have the same underlying particle ratios as minimum-bias events.

(7) An understanding of the nature of the underlying structure should be useful in practical ways: for example, clean separation of the "background" under the jets themselves can be aided by use of the information resting in the whole underlying structure. The usual method employs a jet-finding algorithm to search for jets in a region $\Delta R = \Delta\eta\Delta\phi$. The choice of ΔR represents a trade-off between losing jet particles and including particles from the underlying event. A more suitable method[11] might be to use a larger area to contain the jet and subtract, *on an event-by-event bias,* an appropriate number of underlying particles or E_t obtained by sampling the underlying structure, distant in both η and ϕ from the jet axes. This method would be superior if the multiplicity of hadrons has a strong correlation; i.e., the underlying multiplicity within the jet region is correlated with the overall underlying event multiplicity sampled in a large region away from the jet. In experiments with rapidity coverage large compared with $\Delta\eta$, one would expect the correlation to be large. In addition, such an event-by-event subtraction would allow binning of background subtracted events, essentially deconvoluting the background from the E_t of the jet.

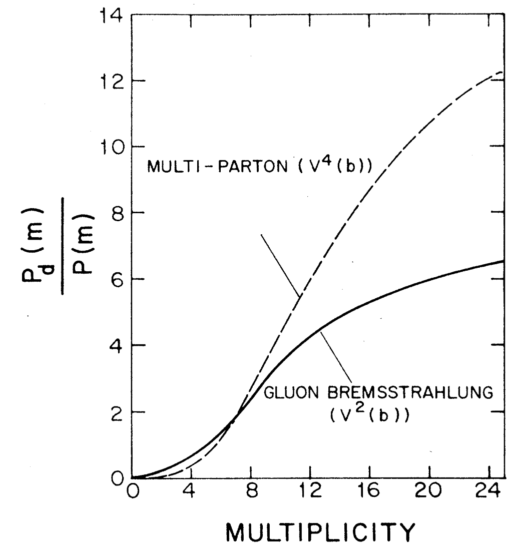

FIG. 6. $P_d(m)/P(m)$ for four-jet events.

While event structure data are soon to be available from CERN and Fermilab collider experiments, an application of our theory to the multiplicity dependence of a special case, low-p_t prompt positron production,[12] has been carried out[13] and will be published elsewhere.

VII. CONCLUSIONS

We have seen that several variations of a simple model for minimum-bias multiplicity distributions allow one to correlate impact-parameter and multiplicity distributions in pp collisions. We have used this to estimate the conditional probability of observing low-p_t particles in the underlying event accompanying constituent-constituent interactions such as jet or W production. We find that we can account qualitatively for the enhanced mean multiplicity (over that in "minimum-bias" events) found in jet, W, and Z events. The existence of an impact-parameter correlation can be demonstrated experimentally by showing that the cross sections increase with the associated multiplicity. Then the observation of different associated multiplicity distributions for W's, Z's, jets, and Drell-Yan pairs will indicate different spatial distributions for the various proton constituents. Hopefully such a demonstration will also focus attention on the role these events can play in understanding low-p_t hadron physics.

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