Spontaneous breaking of Lorentz symmetry in string theory

V. Alan Kostelecký
Physics Department, Indiana University, Bloomington, Indiana 47405

Stuart Samuel
Physics Department, City College of New York, New York, New York 10031

(Received 27 June 1988)

The possibility of spontaneous breakdown of Lorentz symmetry in string theory is explored via covariant string field theory. A potential mechanism is suggested for the Lorentz breaking that may be generic in many string theories.

The basic bosonic string has a highly constrained structure that for consistency requires a 26-dimensional spacetime. Similarly, the superstring requires ten dimensions. A dramatic metamorphosis must therefore take place if strings are to describe a world with four flat dimensions. One appealing idea is that the extra dimensions compactify. For this to happen, the 26- or 10-dimensional Poincaré symmetry must be broken. In most approaches, the occurrence of compactification must be assumed because there is no known mechanism for the breaking.

In this paper we investigate the possibility that Lorentz-symmetry breakdown is natural when the perturbative string vacuum is unstable. We present a potential mechanism for the breaking that may be generic in many string theories.

The basic idea is that Lorentz invariance can be spontaneously broken by the generation for Lorentz tensors of negative square masses. Whether this happens is most easily analyzed using covariant string field theory. In this paper we examine the covariant field theory of open bosonic strings[1] to determine whether the right couplings are present for spontaneous Lorentz-symmetry breaking. Support for this idea also comes from the σ-model approach.[2]

In a particle field theory, spontaneous symmetry breakdown occurs when symmetries of the Lagrangian are not respected by the ground state of the theory. This situation arises if the naive perturbative vacuum, in which all fields have zero expectation value, is unstable. In a true vacuum, some fields acquire nonzero expectation values and any symmetries of the Lagrangian not leaving invariant these values are spontaneously broken.

In covariant string field theory, the same ideas apply. For spontaneous breaking of Lorentz symmetry, the fields of interest are ones transforming nontrivially under the Lorentz group. These are tensor fields. We denote them generically by $T^M(x)$, where the composite index $M=\mu\nu\cdots\rho$ consists of one or more Lorentz vector indices. If any quadratic term in T^M acquires a negative coefficient in the potential, some of the components of T obtain nonzero expectation values and the Lorentz group undergoes spontaneous symmetry breakdown.[3]

Consider the Witten field theory of open bosonic strings.[1] As the tachyon field $\phi(x)$ has the wrong sign for its mass squared, the naive perturbative vacuum is unstable. These are three possibilities: $\langle\phi\rangle$ is infinite and the theory is ill defined; $\langle\phi\rangle$ is nonzero and positive; or $\langle\phi\rangle$ is nonzero and negative. Below, we suggest that in the latter case the coefficient of the quadratic term for the massless vector field $A^\mu(x)$ in the potential becomes nonzero and negative, whereupon Lorentz-symmetry breakdown takes place.

The expectation $\langle\phi\rangle$ is difficult to calculate even at the tree level. The minimum of the static effective potential $V(\phi)$ for ϕ must be found. Since at the tree level $V(\phi)$ receives equally important contributions to all orders in the coupling and no systematic truncation is known, its derivation requires knowledge of the arbitrary n-point off-shell tachyon amplitude. Only recently have the off-shell four-tachyon amplitude[4] and the four-point contribution to $V(\phi)$ (Ref. 5) been obtained. At present, the higher-order tree contributions involve arduous computation. Nevertheless, we show that the occurrence of spontaneous Lorentz-symmetry breaking is both possible and natural in string theories.

The static potential in the Witten string field theory has the form

$$V(\{S^i\},\{T^i_M\})=\tfrac{1}{2}\sum_{i,j} m^2_{ij}S^iS^j+\tfrac{1}{2}\sum_{ij} M^2_{ij}T^i_MT^{jM} +\frac{1}{3!}\sum_{i,j,k} g^{SSS}_{ijk}S^iS^jS^k +\tfrac{1}{2}\sum_{i,j,k} g^{STT}_{ijk}S^iT^j_MT^{kM}+(TTT\text{ term}) ,\qquad(1)$$

where S^i denotes a generic scalar field, m^2_{ij} and M^2_{ij} are the scalar and tensor mass-squared matrices, and g^{SSS}_{ijk}, g^{STT}_{ijk} are coupling constants determined in Ref. 6. The TTT term involves three tensor fields with tensor indices appropriately contracted.

In the Siegel-Feynman gauge,[7] the string field Ψ satisfies $b_0\Psi=0$, where b_0 is the zero mode of the antighost $b(z)$ (Ref. 8). This yields the expansion

$$\Psi=[\phi(x)+\alpha^\mu_{-1}A_\mu(x)+\cdots]|0\rangle ,\qquad(2)$$

where $|0\rangle$ is the first-quantized string vacuum. Equation (1) becomes

$$V(\phi)=-\frac{\phi^2}{2\alpha'}+\frac{\bar{g}}{6}\phi^3+\tfrac{8}{27}\bar{g}A_\mu A^\mu\phi+\cdots\ , \tag{3}$$

where $\bar{g}$ is the three-tachyon coupling at zero momentum. It is related to the on-shell three-tachyon coupling g via

$$\bar{g}=\left[\frac{3\sqrt{3}}{4}\right]^3 g\ . \tag{4}$$

The perturbative vacuum is unstable because of the first term in Eq. (3). The expectation $\langle\phi\rangle$ must be nonzero. If $\langle\phi\rangle$ is finite, the vacuum is stabilized. Ignoring loop effects and tree-level contributions from other scalars, the mass M_A of A^μ is given by

$$M_A^2=\tfrac{16}{26}\bar{g}\langle\phi\rangle\ . \tag{5}$$

For the case of negative $\langle\phi\rangle$, this expression is negative and spontaneous symmetry breaking of the Lorentz group occurs.

One way of determining $\langle\phi\rangle$ from the functional integral is to integrate over all fields S^i and T_M^i except $\phi(x)$. This generates a tree-level effective potential for ϕ. The static potential $V(\phi)$ is then found by setting $p=0$ in $\tilde{\phi}(p)$, the Fourier transform of $\phi(x)$. An off-shell calculation is required because $p^2=0$ rather than $p^2=1/\alpha'$. Via this procedure, the ϕ^4 term in $V(\phi)$ was obtained in Ref. 5. To determine $\langle\phi\rangle$ definitively, explicit knowledge is needed of the coefficients v_n in

$$V(\phi)=\sum_{n=2}^{\infty}v_n\phi^n\ . \tag{6}$$

Note that for $\bar{g}>0$ if $\langle\phi\rangle$ is finite and random values of v_n are assigned then a negative expectation $\langle\phi\rangle$ is more likely than a positive one. This is due to the sign of the ϕ^3 term in Eq. (3), which favors a negative value of $\langle\phi\rangle$. Hence, spontaneous Lorentz-symmetry breakdown is also favored. The same conclusion holds for $\bar{g}<0$.

The key reason why tensor fields can acquire negative quadratic couplings in the potential is the presence of static scalar-tensor-tensor couplings in Eq. (1). This is a string effect; such couplings are not possible in renormalizable particle theories. For example, renormalizable theories of scalars and vectors in four dimensions are necessarily gauge theories, as gauge invariance is needed to remove unphysical degrees of freedom. The trilinear terms in the Lagrangian are of the form $A^{i\mu}\partial_\mu\phi^j\phi^k$; they do not contribute to the static potential due to the presence of the derivative. Although these theories also contain scalar-scalar-vector-vector couplings, such terms lead only to positive quadratic coefficients for the vector fields when the scalars acquire vacuum expectation values.

The open bosonic string avoids these constraints. First, the theories are well behaved at short distances because the string is an extended object. This can be seen in the string field theory; the fields $\tilde{f}(x)$ that enter in the trilinear interaction term are related to the basic local fields $f(x)$ by

$$\tilde{f}(x)=\left[\frac{3\sqrt{3}}{4}\right]^{\alpha'\partial^\mu\partial_\mu}f(x)\ ; \tag{7}$$

that is, they are smeared out over a distance $1/\sqrt{\alpha'}$.

Second, the presence of tensor fields necessitates an enormous gauge group. The field theory has such gauge invariances:

$$\delta\Psi=Q\Lambda+\Psi*\Lambda-\Lambda*\Psi\ , \tag{8}$$

where Q is the Becchi-Rouet-Stora-Tyutin (BRST) operator, $*$ is the Witten star product, and Λ is a string field that contains an infinite number of gauge parameters. These invariances include $\delta A^\mu=\partial^\mu\lambda+\cdots$, where the gauge parameter λ is the first component in $\Lambda=[\lambda(x)b_{-1}+\cdots]|0\rangle$ and b_{-1} is a component of the antighost $b(z)$ (Ref. 8). The other fields in Λ generate other gauge invariances. Remarkably, terms such as $A^\mu A_\mu\phi$ in Eq. (3) are compatible with these gauge transformations. The infinite number of particle fields in Ψ and the infinite number of trilinear interactions make this possible.

In summary, the constraints of renormalizability and gauge invariance rule out spontaneous symmetry breaking of the Lorentz group in a particle field theory. However, this does not apply to string theories even though they are well behaved at short distances and possess gauge invariances. Lorentz-symmetry breaking can arise from static tensor-tensor-scalar couplings that are allowed because strings are extended objects containing an infinite number of particle modes.

If the Lorentz breaking occurs, it is nonperturbative in g. Replace

$$S^i\to\frac{\bar{S}^i}{\alpha'\bar{g}}\ ,\qquad T_M^i\to\frac{\bar{T}_M^i}{\alpha'\bar{g}}\ , \tag{9}$$

and set

$$m_{ij}^2=\frac{c_{ij}^{SS}}{\alpha'},\quad M_{ij}^2=\frac{C_{ij}^{TT}}{\alpha'}\ ,\qquad g_{ijk}^{SSS}=c_{ijk}^{SSS}\bar{g},\quad g_{ijk}^{STT}=c_{ijk}^{STT}\bar{g}\ . \tag{10}$$

Since g_{ijk}^{SSS} and g_{ijk}^{STT} are proportional to g, c_{ijk}^{SSS} and c_{ijk}^{STT} are pure numbers. The potential in Eq. (1) becomes

$$V(\{\bar{S}^i\},\{\bar{T}_M^i\})=\frac{1}{\alpha'^3\bar{g}^2}\left[\tfrac{1}{2}\sum_{i,j}c_{ij}^{SS}\bar{S}^i\bar{S}^j+\tfrac{1}{2}\sum_{i,j}C_{ij}^{TT}\bar{T}_M^i\bar{T}^{jM}+\frac{1}{3!}\sum_{i,j,k}c_{ijk}^{SSS}\bar{S}^i\bar{S}^j\bar{S}^k+\tfrac{1}{2}\sum_{i,j,k}c_{ijk}^{STT}\bar{S}^i\bar{T}_M^j\bar{T}^{kM}+(\bar{T}\bar{T}\bar{T}\ \text{term})\right]\ . \tag{11}$$

Equation (11) shows that the breaking is determined in terms of numerical coefficients independent of the couplings. This means that systematic approximation is difficult. The equation also implies that if ϕ or any other scalar field has a nonzero vacuum expectation value then it is of order[9] $1/(\alpha' g)$ and the contribution to the cosmological constant is of order $1/(\alpha'^3 g^2)$.

The above analysis focused on the dynamical effects of the tachyon. There is another approach to analyzing the vacuum physics. Instead of using the effective tachyon potential, one can find solutions to the equation of motion. They are obtained by varying Eq. (1) with respect to the various fields. A solution in terms of constants corresponds to a set of vacuum expectation values. If $\langle \phi \rangle \neq 0$ then $\langle S^i \rangle \neq 0$ for the other scalars because whenever $g_i^{\phi\phi S} \neq 0$ a term proportional to $\langle \phi \rangle^2 S^i$ is generated. These linear terms in S^i drive further instabilities. If the open bosonic theory is well defined and $\langle \phi \rangle$ is finite, the theory is radically different from the first-quantized approach. Virtually every scalar acquires a nonzero vacuum expectation value. The resulting mass matrix for the tensor fields is

$$\mathcal{M}_{ij}^2 = M_{ij}^2 + \sum_k g_{kij}^{STT} \langle S^k \rangle \ . \tag{12}$$

If $\mathcal{M}_{ij}^2$ has any negative eigenvalues then spontaneous breaking of Lorentz symmetry occurs.

What are the possibilities for Lorentz-symmetry breakdown in string theories other than the basic open bosonic one? The simplest extension is to incorporate internal symmetries by adding Paton-Chan factors. The above analysis of Lorentz-symmetry breakdown carries over to this more involved case. There is now also the possibility of spontaneous breaking of the internal symmetry, which was studied prior to the existence of an off-shell formalism in Refs. 10. It may be that both Lorentz and internal symmetries are broken in the presence of the Paton-Chan factors.

One can also consider the closed bosonic string. Its Lagrangian in terms of particle fields has not yet been derived but there is no reason to expect that nonderivative scalar-tensor-tensor couplings are absent. Since there is a tachyon, the perturbative vacuum is unstable. When the tachyon acquires a vacuum expectation value, the mass spectrum changes radically. Scalar-tensor-tensor couplings of the appropriate sign would again generate Lorentz-symmetry breaking.

This mechanism might also work for the spinning string. In contrast, superstrings lack a tachyon field. However, the role of the tachyon might be played by the dilaton Φ. In principle, a nonzero vacuum expectation value $\langle \Phi \rangle$ and appropriate interactions can lead to negative quadratic couplings for some tensors resulting in spontaneous symmetry breakdown. However, in addition to the three cases enumerated above, a zero value of $\langle \Phi \rangle$ is possible since the first term in Eq. (3) is absent. This case corresponds to a stable perturbative vacuum in which Lorentz-symmetry breaking is unlikely.

Other string theories that are good candidates for spontaneous Lorentz-symmetry breaking include the heterotic strings with tachyons.[11] This large subset of heterotic theories may also have stable vacua. This idea is made more plausible by the existence of compactified solutions that do not contain tachyons even though the parent theories do.[12] Internal symmetries may also be spontaneously broken in this cases.

There remain several interesting areas for exploration. For example, a nonperturbative argument demonstrating spontaneous Lorentz-symmetry breaking would be satisfying. Another intriguing question concerns the nature of the breaking. In principle, there might be several minima of the potential leading to different possibilities for Lorentz-symmetry breakdown. Even for the attractive case of only one global minimum, the presence of higher-energy local minima could affect, for example, string-based cosmology and the evolution of the Universe. Finally, it is unclear whether negative quadratic couplings for tensor fields in the potential correspond to compactification. Presumably, when gravity is incorporated certain components of the graviton acquire masses in analogy with the Higgs mechanism. The effect of this on the spacetime manifold needs to be investigated.

This work was supported in part by the United States Department of Energy under Contracts Nos. DE-AC02-82ER40125 and DE-AC02-83ER40107.

[1]E. Witten, Nucl. Phys. **B268**, 253 (1986).

[2]E. S. Fradkin and A. A. Tseytlin, Nucl. Phys. **B261**, 1 (1985).

[3](Note that the temporal component of, for example, any vector field has a quadratic term in the potential appearing with negative coefficient due to the Minkowski metric. However, gauge symmetries must be present to eliminate these negatively normed states. To circumvent these complications, we work in Euclidean space.

[4]J. H. Sloan, Princeton Report No. PUPT-1075, 1987 (unpublished); S. Samuel, Nucl. Phys. **B308**, 285 (1988).

[5]V. A. Kostelecký and S. Samuel, Phys. Lett. B **207**, 169 (1988).

[6]D. Gross and A. Jevicki, Nucl. Phys. **B283**, 1 (1987); E. Cremmer, A. Schwimmer, and C. Thorn, Phys. Lett. B **179**, 57 (1986); S. Samuel, *ibid.* **181**, 255 (1986).

[7]W. Siegel, Phys. Lett. **142B**, 276 (1984); **151B**, 391 (1985); **151B**, 396 (1985).

[8]D. Friedan, E. Martinec, and S. Shenker, Nucl. Phys. **B271**, 93 (1986).

[9]In 26 dimensions, mass dimensionalities are $\dim S^i = 12$, $\dim T_M^i = 12$, $\dim g = -10$, and $\dim \alpha' = -2$.

[10]K. Bardakçi, Nucl. Phys. **B68**, 331 (1974); **B70**, 397 (1974); **B133**, 297 (1978); K. Bardakçi and M. Halpern, *ibid.* **B73**, 295 (1974); **B96**, 285 (1975); Phys. Rev. D **10**, 4230 (1974).

[11]L. J. Dixon and J. A. Harvey, Nucl. Phys. **B274**, 93 (1986); N. Seiberg and E. Witten, *ibid.* **B276**, 272 (1986); L. Alvarez-Gaumé, P. Ginsparg, G. Moore, and C. Vafa, Phys. Lett. B **171**, 155 (1986).

[12]K. S. Narain and M. H. Sarmadi, Phys. Lett. B **184**, 165 (1987).