P- and CP-odd terms in the photon self-energy within a medium

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(Received 6 July 1988)

For the propagation of electromagnetic fields within a medium (e.g., a plasma or a heat bath at finite temperature), we write down the most general expression for the vacuum-polarization tensor that is consistent with gauge and Lorentz invariance in four dimensions. The expression contains a term which signals parity and CP violation either in the Lagrangian, or in the background, or both. The effect of this term is to split the otherwise degenerate transverse degrees of polarization of the photon. The physical implications of this term are discussed.

I. INTRODUCTION

A quantum-field-theoretical study of the propagation of the electromagnetic field within a medium is important for a variety of physical reasons. It is relevant, for example, in discussing photon propagation within a plasma,[1–3] which has been used[4] to calculate cross sections for particle interactions within a star. Another important application is in discussing finite-temperature field theory,[5,6] where the medium acts as a heat bath at a specific temperature.

Despite its importance, attempts at a covariant formulation[2–4] of the propagation of electromagnetic fields within a medium are quite recent and by no means complete. It has been demonstrated, using field-theoretic methods, that within a medium the photon develops a longitudinal degree of freedom in addition to the two transverse degrees present in the vacuum. Moreover, all these modes exhibit some effective mass,[2–4,7] which explains why electromagnetic information travels slower within a medium compared to in a vacuum. The resulting dielectric constant and permeability, however, have been calculated only in the simplest situations.[3,4]

In this paper, we want to formulate an important aspect of the propagation of photons within a medium. We write down the most general expression of the vacuum-polarization tensor for four space-time dimensions that is consistent with electromagnetic gauge invariance and Bose-Einstein symmetry. The expression contains a term, proportional to the antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$, whose effects have never been discussed before, at least in the context of a covariant formulation. Such a term arises due to the virtual effects of weak interaction involving the particles constituting the medium and breaks some discrete symmetries of the Lorentz group. Therefore, the presence of this term affects, in an important way, the description of photon propagation in a medium. For example, as we show later, the two transverse states of polarization travel with different speeds, that is to say with different dielectric constants, within a medium.[8] The Feynman rules for calculating diagrams involving photons are also affected, as shown later.

The paper is organized as follows. In Sec. II we discuss the symmetry properties of the vacuum-polarization tensor $\pi_{\mu\nu}$ and the most general form of $\pi_{\mu\nu}$ dictated by them. In Sec. III we show that the general form obtained in Sec. II violates discrete symmetries such as parity and CP. The photon propagator and the dispersion relations in the presence of these terms have been derived in Sec. IV. While the propagator is useful for internal photon lines in a Feynman diagram, the corresponding Feynman rules for an external photon line is discussed in Sec. V. In Sec. VI we make contact between the formalism and the macroscopic formulation of electrodynamics within a medium, thereby obtaining expressions for macroscopic properties such as the dielectric constant. Section VII presents some discussions.

II. PHOTON SELF-ENERGY WITHIN A MEDIUM

In the vacuum, the photon self-energy $\pi_{\mu\nu}$ is a tensor constructed out of the virtual-photon momentum k^μ and the metric $g_{\mu\nu}$. As such, it turns out to be symmetric under the interchange $\mu\leftrightarrow\nu$ and, separately, under $k\rightarrow -k$. This property of $\pi_{\mu\nu}$ is not dictated by any principle, but follows from the fact that $\pi_{\mu\nu}$ depends only on k^μ and that space-time is four dimensional. The condition that follows from Bose-Einstein statistics and crossing symmetry is that $\pi_{\mu\nu}$ must be symmetric under the *simultaneous* interchange $\mu\leftrightarrow\nu$ and $k\rightarrow -k$, i.e.,

$$\pi_{\nu\mu}(-k,s)=\pi_{\mu\nu}(k,s) \ , \tag{2.1}$$

where we have indicated explicitly the dependence on the variable

$$s=k^2+i\epsilon \ . \tag{2.2}$$

In addition, electromagnetic gauge invariance implies that

$$k^\mu\pi_{\mu\nu}(k)=k^\nu\pi_{\mu\nu}(k)=0 \ , \tag{2.3}$$

so that we obtain

$$\pi_{\mu\nu}=\pi(s)\tilde{g}_{\mu\nu} \quad (2.4)$$

for an arbitrary number of dimensions of space-time,[9] where

$$\tilde{g}_{\mu\nu}=g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2} . \quad (2.5)$$

With this form, $\pi_{\mu\nu}$ satisfies not only Eq. (2.1), but also the conditions

$$\pi_{\mu\nu}(-k,s)=\pi_{\mu\nu}(k,s)=\pi_{\nu\mu}(k,s) , \quad (2.6)$$

which, we stress, do not follow from any principle in contrast with Eq. (2.1).

Finally, Hermiticity of the interaction Lagrangian implies that

$$\pi^*_{\nu\mu}(k,s^*)=\pi_{\mu\nu}(k,s) . \quad (2.7)$$

This can easily be derived diagrammatically. Taking the complex conjugate of any diagram that contributes to $\pi_{\mu\nu}$ reverses the direction of the internal and external momentum lines and therefore gives a diagram that contributes to $\pi_{\nu\mu}$; but it also changes the $i\epsilon$ in the internal propagators to $-i\epsilon$. Changing s to s^* takes this effect into account, and summing all the diagrams yields Eq. (2.7). It can also be derived by considering $\pi_{\mu\nu}$ as the expectation value of the time-ordered product of two currents. The complex-conjugation operation reverses the order of the two currents and changes the time-ordered product to an antitime-ordered product, which also has the effect of replacing the $i\epsilon$ by $-i\epsilon$ in the propagators, so that we again obtain Eq. (2.7). The upshot of this equation for the invariant function $\pi(s)$ defined in Eq. (2.5) is that

$$\pi^*(s^*)=\pi(s) . \quad (2.8)$$

In other words, the dispersive part of $\pi(s)$ is real and if the absorptive part is zero, then $\pi(s)$ is real.

Let us now discuss the situation in a medium. The effects of the medium will introduce an additional dependence of $\pi_{\mu\nu}$ on the velocity four-vector u^μ of the background. In any space-time dimensions, one can now form two tensors[2-4,7] which are orthogonal to both k^μ and k^ν:

$$Q_{\mu\nu}=\frac{\tilde{u}_\mu\tilde{u}_\nu}{\tilde{u}^2}, \quad R_{\mu\nu}=\tilde{g}_{\mu\nu}-Q_{\mu\nu} , \quad (2.9)$$

where

$$\tilde{u}_\mu=\tilde{g}_{\mu\nu}u^\nu . \quad (2.10)$$

However, we want to point out that for the particular case of four-dimensional space-time, one can have another tensor which has the same property:[10]

$$P_{\mu\nu}=\frac{i}{\kappa}\epsilon_{\mu\nu\alpha\beta}k^\alpha u^\beta , \quad (2.11)$$

where κ is a Lorentz-invariant quantity defined by

$$\kappa=(\omega^2-k^2)^{1/2}, \quad \omega=k\cdot u . \quad (2.12)$$

The most general form of $\pi_{\mu\nu}$ consistent with gauge invariance [Eq. (2.3)] in four dimensions is therefore given by

$$\pi_{\mu\nu}(\kappa,\omega)=\pi_T R_{\mu\nu}+\pi_L Q_{\mu\nu}+\pi_P P_{\mu\nu} , \quad (2.13)$$

where we have explicitly indicated the dependence on the scalar variables κ and ω defined in Eq. (2.12). In fact, π_T, π_L, and π_P are also functions of these variables. We will write this dependence explicitly whenever we need it.

For future convenience, let us note the following properties of the tensors $R_{\mu\nu}$, $Q_{\mu\nu}$, and $P_{\mu\nu}$:

$$\begin{aligned} &R_{\mu\nu}R^{\nu\rho}=R_\mu{}^\rho, \quad R_{\mu\nu}Q^{\nu\rho}=0 , \\ &Q_{\mu\nu}Q^{\nu\rho}=Q_\mu{}^\rho, \quad Q_{\mu\nu}P^{\nu\rho}=0 , \\ &P_{\mu\nu}P^{\nu\rho}=R_\mu{}^\rho, \quad P_{\mu\nu}R^{\nu\rho}=P_\mu{}^\rho . \end{aligned} \quad (2.14)$$

Also, from the definition, it follows that

$$\begin{aligned} &R_{\mu\nu}(k)=R_{\nu\mu}(k)=R_{\mu\nu}(-k) , \\ &Q_{\mu\nu}(k)=Q_{\nu\mu}(k)=Q_{\mu\nu}(-k) , \\ &P_{\mu\nu}(k)=-P_{\nu\mu}(k)=-P_{\mu\nu}(-k) . \end{aligned} \quad (2.15)$$

The condition from Bose-Einstein statistics and crossing symmetry is, as before, that $\pi_{\mu\nu}$ be symmetric under the simultaneous interchange of $\mu\leftrightarrow\nu$ and $k\to-k$. This means that the part of $\pi_{\mu\nu}$ that contains even (odd) powers of k must be symmetric (antisymmetric) in μ,ν. In terms of the variables defined in Eq. (2.12), it implies

$$\pi_{\nu\mu}(\kappa,-\omega)=\pi_{\mu\nu}(\kappa,\omega) . \quad (2.16)$$

Using Eq. (2.15), this gives

$$\pi_{T,L,P}(\kappa,-\omega)=\pi_{T,L,P}(\kappa,\omega) . \quad (2.17)$$

The same arguments that lead to Eq. (2.7) can be applied here to show that if the interaction Lagrangian is Hermitian, then

$$\pi^*_{\nu\mu}(\kappa,\omega-i\epsilon)=\pi_{\mu\nu}(\kappa,\omega+i\epsilon) , \quad (2.18)$$

which in turn implies

$$\pi^*_{T,L,P}(\kappa,\omega-i\epsilon)=\pi_{T,L,P}(\kappa,\omega+i\epsilon) . \quad (2.19)$$

Therefore, the imaginary parts of $\pi_{T,L,P}$, if any, must come only from the absorptive part.

Thus, $\pi_{\mu\nu}$ can contain terms that are antisymmetric under $\mu\leftrightarrow\nu$, provided they are odd under $k\to-k$. Such terms are allowed by Bose-Einstein statistics and, in general, the term with the structure tensor $P_{\mu\nu}$ exists in a medium. Our primary interest in what follows is to elucidate the interpretation and physical origin of that term.

III. DISCRETE SYMMETRIES AND THE π_P TERM

The presence of a term proportional to the antisymmetric ϵ tensor is well known and widely discussed in the context of vacuum electrodynamics in three space-time dimensions.[9] It has been pointed out that such a term signifies a violation of discrete symmetries such as parity.

In the four-dimensional case, we similarly ask the question of what symmetries are broken by $P_{\mu\nu}$; e.g., parity

(P), charge conjugation (C), and CP. In other words, we would like to answer under what conditions the $P_{\mu\nu}$ term exists and under what conditions it is zero.

Here again, as in the three-dimensional case, we are tempted to conclude immediately that the presence of the $P_{\mu\nu}$ term must somehow involve the breaking of P and CP. While this is true, its precise implications have to be discussed with great care. The reason is that the background that defines the medium, with respect to which all averages and expectation values are taken, may or may not be symmetric under the symmetries we are concerned with. To make the statements more precise, given a conserved quantity θ, so that

$$[\theta,\mathcal{L}]=0 , \tag{3.1}$$

we will say that the background is θ symmetric if

$$\langle\theta\rangle=0 , \tag{3.2}$$

where $\langle\ \rangle$ stands for the averages in the medium, i.e., the same that is used to define, for example, the propagators. Another way to express Eq. (3.2) is to say that the chemical potentials associated with θ are zero.

The generating function that embodies the effect of the background is

$$Z=\exp\left[-\beta H+\sum_A \alpha_A\theta_A\right] , \tag{3.3}$$

where the quantities θ_A form a complete set of conserved charges.

All our conclusions regarding the implications of the discrete symmetries P, C, and CP on the presence of the $P_{\mu\nu}$ term will be easily understood in terms of the following example. Suppose we are to calculate the expectation value of an operator $\hat{O}$ which is parity odd, i.e.,

$$P^{-1}\hat{O}P=-\hat{O} . \tag{3.4}$$

By definition

$$\langle\hat{O}\rangle=\frac{\mathrm{tr}Z\hat{O}}{\mathrm{tr}Z}=-\frac{\mathrm{tr}(PZP^{-1})\hat{O}}{\mathrm{tr}Z} , \tag{3.5}$$

where we have used the cyclic property of the trace. In this form, any statement now reduces to a statement about the behavior of Z under P. If the Lagrangian (and therefore the Hamiltonian) is invariant under P, and if the chemical potentials of charges that are P odd are zero, then

$$PZP^{-1}=Z \tag{3.6}$$

and so

$$\langle\hat{O}\rangle=0 . \tag{3.7}$$

However if some of the chemical potentials of parity-odd charges are nonzero, then $\langle\hat{O}\rangle$ is not necessarily zero, even if the Lagrangian conserves parity. We will refer to a background for which Eq. (3.6) holds as a parity-symmetric background. Thus, we can summarize these results by saying that $\langle\hat{O}\rangle$ is zero if the Lagrangian is invariant under parity *and* the background is parity symmetric.

We can now return to discuss the implications of the discrete symmetries P, C, and CP on $\pi_{\mu\nu}$. The situation, though not identical, is similar to what was discussed above. If the Lagrangian is invariant under P *and* the background is parity symmetric, then $\pi_{\mu\nu}$ satisfies

$$\pi_{\mu\nu}=\tilde{\pi}_{\mu\nu} , \tag{3.8}$$

where $\tilde{\pi}_{\mu\nu}$ is defined as $\pi_{\mu\nu}$ with the sign of the ϵ term reversed. So, Eq. (3.8) implies

$$\pi_P=0 \tag{3.9}$$

if the two conditions hold. Therefore, π_P can be nonzero only if parity is broken either by the Lagrangian or the background, or both. However, this is not sufficient to guarantee the presence of the π_P term, as we will see in a moment.

The product of two electromagnetic currents is C even and therefore we do not obtain any restriction on $\pi_{\mu\nu}$ even if the Lagrangian preserves C and/or the background is C symmetric. But CP has important implication. As before, if CP is conserved by the Lagrangian and the background is CP symmetric, then again $\pi_{\mu\nu}$ must satisfy Eq. (3.8). Therefore, in order for π_P to be nonzero, not only P has to be broken by either $\mathcal{L}$ or the background, or both, but also CP must be broken by one or the other, or both.

For example, suppose that $\mathcal{L}$ breaks P but CP is conserved, which is a good approximation in practice. If the background consists, for example, of a nonrelativistic, nondegenerate gas of electrons then π_P will receive a nonzero contribution at some order in weak interactions. In this case the background is P symmetric but C and CP asymmetric. The situation resembles the one in the atomic-physics parity-violating experiments. In fact, those experiments can be interpreted and discussed in this formalism.

As a second example, suppose the background is a gas of massless, left-handed neutrinos and their antiparticles. If the lepton number of the gas is zero, then the background is C and P asymmetric but CP symmetric and π_P should be zero if, as before, we assume that CP is conserved by the Lagrangian. However, if the lepton number is not zero, the background becomes CP asymmetric and so π_P would receive contributions at some order in the weak interactions. In the Appendix we give some details of how these conclusions are realized in a specific model.

IV. PHOTON PROPAGATOR AND DISPERSION RELATIONS

In the presence of an external current $j^\mu_{\rm ext}$, the Lagrangian of the electromagnetic field in the vacuum is given by

$$\mathcal{L}_{\rm vacuum}=\tfrac{1}{2}A_\mu(\Delta^{-1})^{\mu\nu}A_\nu-A_\mu j^\mu_{\rm ext} , \tag{4.1}$$

where $\Delta_{\mu\nu}$ is the tree propagator. The matter effects introduce a nontrivial vacuum polarization, so that in the medium we obtain the Lagrangian

$$\mathcal{L}_{\rm medium}=\tfrac{1}{2}A_\mu[(\Delta^{-1})^{\mu\nu}+\pi^{\mu\nu}]A_\nu-A_\mu j^\mu_{\rm ext} . \tag{4.2}$$

The equation of motion of the electromagnetic field is therefore given by

$$[(\Delta^{-1})^{\mu\nu}+\pi^{\mu\nu}]A_\nu=j^\mu_{\rm ext} \ . \qquad (4.3)$$

One now identifies the complete propagator $D_{\mu\nu}$ by the relation

$$\Delta^{-1}_{\mu\nu}+\pi_{\mu\nu}=D^{-1}_{\mu\nu} \qquad (4.4)$$

so that the equation of motion becomes

$$(D^{-1})^{\mu\nu}A_\nu=j^\mu_{\rm ext} \ . \qquad (4.5)$$

Using Eq. (2.13) and the definition in Eq. (4.4), we can write down the inverse propagator of the photon to be

$$D^{-1}_{\mu\nu}=(-k^2+\pi_T)R_{\mu\nu}+(-k^2+\pi_L)Q_{\mu\nu}+\pi_P P_{\mu\nu} \ , \qquad (4.6)$$

where we have discarded possible gauge terms proportional to $k_\mu k_\nu$ since these decouple by gauge invariance. The dispersion relations are found by setting the determinant of $D^{-1}_{\mu\nu}$ to zero, or equivalently, by solving the equations $D^{-1}_{\mu\nu}A^\nu=0$, or still another way by the poles of $D_{\mu\nu}$.

The subject of photon propagation and dispersion relations in absence of the tensor P has been addressed before.[4] There, it was shown that the tensors R and Q govern the propagation of the transverse and longitudinal modes, respectively.

To extend that analysis to include the effects of $P_{\mu\nu}$, we first observe that $P_{\mu\nu}$ relates to the transverse modes. One way to see this is to note, from Eq. (2.14), that P is orthogonal to Q. Alternatively, one can go to the rest frame of the medium where

$$u^\mu=(1,\mathbf{0}) \ . \qquad (4.7)$$

In this frame, k_0 and $|\mathbf{k}|$ reduce to the invariants ω and κ, respectively, as can be seen from Eq. (2.12). We will use the symbols k_0 and $|\mathbf{k}|$ whenever our discussion pertains to the rest frame of the medium. The various components of the tensors R, Q, and P in this frame are as follows:

$$\begin{aligned} &R_{00}=R_{0i}=0, \quad R_{ij}=-\delta_{ij}+\frac{k_ik_j}{|\mathbf{k}|^2} \ , \\ &Q_{00}=-\frac{|\mathbf{k}|^2}{k^2}, \quad Q_{0i}=-\frac{k_0k_i}{k^2}, \quad Q_{ij}=-\frac{k_0^2}{k^2}\frac{k_ik_j}{|\mathbf{k}|^2} \ , \\ &P_{00}=P_{0i}=0, \quad P_{ij}=\frac{i}{|\mathbf{k}|}\epsilon_{ijk}k^k \ . \end{aligned} \qquad (4.8)$$

Obviously, in this frame, the spatial components of the tensors R and P satisfy

$$k^iR_{ij}=k^iP_{ij}=0 \ , \qquad (4.9)$$

which identifies them as describing the transverse degrees of freedom. On the other hand, the spatial components of Q satisfy

$$(\delta_{ij}-k_ik_j/|\mathbf{k}|^2)Q^{jk}=0 \ , \qquad (4.10)$$

so that we identify Q to be the projector of the longitudinal degree of freedom.

To proceed, we introduce two vectors whose components in the rest frame of the medium are $(0,\mathbf{e}_1)$ and $(0,\mathbf{e}_2)$. The spatial components $\mathbf{e}_1$ and $\mathbf{e}_2$ have unit length and are orthogonal to the direction of propagation $\hat{\mathbf{k}}$ (see Fig. 1). They satisfy the relations

$$\mathbf{e}_1=\mathbf{e}_2\times\hat{\mathbf{k}}, \quad \mathbf{e}_2=-\mathbf{e}_1\times\hat{\mathbf{k}}, \quad \hat{\mathbf{k}}\cdot(\mathbf{e}_1\times\mathbf{e}_2)=1 \ . \qquad (4.11)$$

Using the expression of P_{ij} from Eq. (4.8), we can easily generalize Eq. (4.11) to an arbitrary frame:

$$e_{1\mu}=iP_{\mu\nu}e_2^\nu, \quad e_{2\mu}=-iP_{\mu\nu}e_1^\nu, \quad P_{\mu\nu}e_1^\mu e_2^\nu=i \ . \qquad (4.12)$$

It is also useful to define

$$e_3^\mu=\frac{\tilde{u}^\mu}{\sqrt{-\tilde{u}^2}} \ . \qquad (4.13)$$

Note that the square root in the denominator is necessarily real because, by definition, $\tilde{u}^2=1-\omega^2/k^2<0$, since $k^2=\omega^2-\kappa^2<\omega^2$. The set of vectors k^μ, e_1^μ, e_2^μ, and e_3^μ form an orthogonal basis in the four-dimensional space-time and is a convenient one to use. Because of the orthogonality, we can use the vectors e^μ as the polarization vectors. Obviously, e_1^μ and e_2^μ describe transverse-polarization states since their spatial components are transverse to $\hat{\mathbf{k}}$ in the rest frame of the medium, as commented earlier. Similarly, e_3^μ has spatial components parallel to $\hat{\mathbf{k}}$ in the same frame and therefore describes the longitudinal polarization state. Some useful relations are

$$\begin{aligned} &R_{\mu\nu}=-(e_{1\mu}e_{1\nu}+e_{2\mu}e_{2\nu}) \ , \\ &P_{\mu\nu}=i(e_{1\mu}e_{2\nu}-e_{2\mu}e_{1\nu}) \ , \\ &Q_{\mu\nu}=-e_{3\mu}e_{3\nu} \ . \end{aligned} \qquad (4.14)$$

In fact, the second one of this set of equations can be derived from the first one with the use of Eqs. (2.14) and (4.12). The third one follows from Eq. (4.13).

If $\pi_P=0$, one can carry out the analysis in terms of the linear polarization vectors: e_3^μ for the longitudinal mode and $e_{1,2}^\mu$ for the transverse modes. In Ref. 4 it was shown that the longitudinal mode has a dispersion relation

$$k^2-\pi_L=0 \ , \qquad (4.15)$$

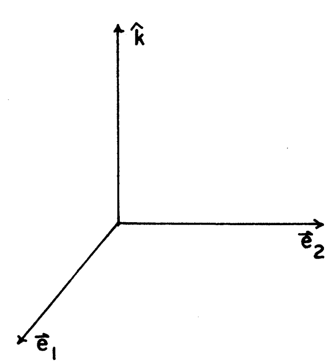

FIG. 1. The mutual orientation of the unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ with respect to the direction of propagation $\hat{\mathbf{k}}$. Equation (4.11) summarizes the relations between the three vectors.

while the transverse modes satisfy

$$k^2-\pi_T=0 \ . \tag{4.16}$$

In the presence of π_P the physics can be best understood by introducing the tensors

$$A_{\mu\nu}=\tfrac{1}{2}(R_{\mu\nu}+P_{\mu\nu}),\quad B_{\mu\nu}=\tfrac{1}{2}(R_{\mu\nu}-P_{\mu\nu}) \ . \tag{4.17}$$

Note that these tensors and $Q_{\mu\nu}$ satisfy the following relations:

$$\begin{aligned} &A_{\mu\nu}A^{\nu\rho}=A_\mu{}^\rho,\quad B_{\mu\nu}Q^{\nu\rho}=0 \ ,\\ &B_{\mu\nu}B^{\nu\rho}=B_\mu{}^\rho,\quad Q_{\mu\nu}A^{\nu\rho}=0 \ ,\\ &Q_{\mu\nu}Q^{\nu\rho}=Q_\mu{}^\rho,\quad A_{\mu\nu}B^{\nu\rho}=0 \ . \end{aligned} \tag{4.18}$$

In other words, A, B, and Q act like projection operators. We will presently see that in fact they project out different polarization states of the photon.

Equation (4.6) can now be written as

$$\begin{aligned} D^{-1}_{\mu\nu}=&(-k^2+\pi_T+\pi_P)A_{\mu\nu}+(-k^2+\pi_T-\pi_P)B_{\mu\nu}\\ &+(-k^2+\pi_L)Q_{\mu\nu} \ . \end{aligned} \tag{4.19}$$

Using Eq. (4.18), the propagator can easily be written as

$$D_{\mu\nu}=-\frac{A_{\mu\nu}}{k^2-\pi_T-\pi_P}-\frac{B_{\mu\nu}}{k^2-\pi_T+\pi_P}-\frac{Q_{\mu\nu}}{k^2-\pi_L} \ . \tag{4.20}$$

Looking at the poles of this expression, we see that the dispersion relation for the longitudinal photon remains unaffected by the π_P term. But the two transverse modes are no more degenerate. Rather, they have the dispersion relation

$$k^2-\pi_T=\pm\pi_P \ . \tag{4.21}$$

In fact, the polarization vectors can no longer be taken to be e_1^μ and e_2^μ since they get mixed up by the $P_{\mu\nu}$ term, as expressed in Eq. (4.12). Rather, the appropriate basis to describe the eigenmodes is provided by the circular polarization vectors

$$e_\mu^{(\pm)}=\frac{1}{\sqrt{2}}(e_{1\mu}\pm ie_{2\mu}) \ . \tag{4.22}$$

Using Eqs. (4.14) and (4.17), it is now easy to see that

$$e_\mu^{(+)*}e_\nu^{(+)}=-B_{\mu\nu},\quad e_\mu^{(-)*}e_\nu^{(-)}=-A_{\mu\nu} \ . \tag{4.23}$$

Therefore,

$$D_{\mu\nu}=\frac{e_\mu^{(-)*}e_\nu^{(-)}}{k^2-\pi_T-\pi_P}+\frac{e_\mu^{(+)*}e_\nu^{(+)}}{k^2-\pi_T+\pi_P}+\frac{e_{3\mu}e_{3\nu}}{k^2-\pi_L} \ . \tag{4.24}$$

This form clearly shows the polarization vectors and the dispersion relations. We now use this form to determine the wave-function renormalization constants that must be used for photons in external fields.

V. PHOTONS IN EXTERNAL LINES

For photons in internal lines of a Feynman diagram, the propagator that should be used in order to take into account the effects of the medium is given by Eq. (4.20). For external photons, the wave functions are proportional to the polarization vectors e_λ^μ (where $\lambda=+,-,3$) with a normalization factor $N_{k\lambda}$. To determine them, we take advantage of the fact that the $N_{k\lambda}$ are Lorentz-invariant quantities, so that we can determine them in the rest frame of the medium. In this frame, we denote by $\omega_{k\lambda}$ the solution to the dispersion relations in Eqs. (4.15) and (4.21). The corresponding polarization vectors are denoted by $e_\mu(k\lambda)$, thus making explicit their momentum dependence.

Writing the wave function in the form

$$\langle 0|A^\mu(x)|k\lambda\rangle=\sqrt{N_{\kappa\lambda}}\,e^\mu(k\lambda)e^{-ik\cdot x} \tag{5.1}$$

we determine $N_{k\lambda}$ by looking at the residue of $D_{\mu\nu}$ at the poles, which are given by the dispersion relations.

Following a standard analysis[11] and using the definition

$$\langle 0|T[A_\mu(x)A_\nu(y)]|0\rangle=i\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-y)}D_{\mu\nu}(k), \tag{5.2}$$

we calculate the one-photon contribution to $D_{\mu\nu}(k)$, with the result

$$D_{\mu\nu}(k)|_{1\text{ photon}}=\sum_{\lambda=+,-,3}N_{k\lambda}e_\mu(k\lambda)e_\nu(k\lambda)\times\frac{1}{k_0^2-\omega_{k\lambda}^2+i\epsilon} \ . \tag{5.3}$$

Therefore, in the limit $k_0\to\omega_{k\lambda}$, the photon propagator behaves like

$$D_{\mu\nu}(k)|_{1\text{ photon}}=\frac{1}{k_0-\omega_{k\lambda}}\frac{N_{k\lambda}}{2\omega_{k\lambda}}e_\mu(k\lambda)e_\nu(k\lambda) \ . \tag{5.4}$$

Starting from Eq. (4.24) and taking the limit $k_0\to\omega_{k\lambda}$, we then obtain

$$\begin{aligned} N_{k\lambda}&=\frac{2\omega_{k\lambda}}{\left[\dfrac{\partial}{\partial k_0}[k_0^2-\pi_T+\lambda\pi_P]\right]_{k_0=\omega_{k\lambda}}}\quad\text{for }\lambda=\pm \ ,\\ N_{k3}&=\frac{2\omega_{k3}}{\left[\dfrac{\partial}{\partial k_0}(k^2-\pi_L)\right]_{k_0=\omega_{k3}}} \ . \end{aligned} \tag{5.5}$$

In calculating cross sections or decay probabilities, a normalization factor $N_{k\lambda}$ must be included for each external photon in the process, in addition to the standard factors. In the limit $\pi_P\to 0$, Eqs. (5.5) reduce to the familiar results,[4] as can be shown easily by using Eq. (6.24) derived later. In the limit $\pi_{T,L,P}\to 0$, all $N_{k\lambda}$ becomes equal to unity, which is the appropriate result in the vacuum.

VI. EQUATIONS OF MACROSCOPIC ELECTRODYNAMICS

The formalism described above is very useful for carrying out practical calculations in the covariant formulation of field theory in a medium, which includes finite-temperature field theory and the relativistic plasma. While, as we have seen, the effect of the π_P term on the propagation of photons can be simply deduced within that formalism also, a complete picture of the physical situation is obtained by making contact with the macroscopic formulation of electrodynamics. Notice that maintaining explicit covariance is very useful in the field-theoretic calculation of $\pi_{\mu\nu}$; but it is not needed for making contact with the macroscopic formulation after $\pi_{T,L,P}$ are calculated. Therefore, the entire discussion of this section is carried out in the rest frame of the medium defined in Eq. (4.7).

Maxwell's equations in momentum space are

$$\begin{aligned} &i\mathbf{k}\cdot\mathbf{E}=\rho, \quad i\mathbf{k}\times\mathbf{B}=\mathbf{j}-ik_0\mathbf{E} , \\ &\mathbf{k}\cdot\mathbf{B}=0, \quad \mathbf{k}\times\mathbf{E}=k_0\mathbf{B} . \end{aligned} \tag{6.1}$$

Here, ρ and $\mathbf{j}$ contain, in addition to any external sources, the induced sources of the medium, which are the ones calculated in field theory through $\pi_{\mu\nu}$ [see Eq. (6.22) below]. The standard procedure is to make a separation between the two kinds of sources by writing

$$\rho=\rho_{\rm ext}+\rho_{\rm ind}, \quad \mathbf{j}=\mathbf{j}_{\rm ext}+\mathbf{j}_{\rm ind} , \tag{6.2}$$

and then to parametrize the induced sources in the form

$$\rho_{\rm ind}=-i\mathbf{k}\cdot\mathbf{P}, \quad \mathbf{j}_{\rm ind}=-ik_0\mathbf{P}+i\mathbf{k}\times\mathbf{M} . \tag{6.3}$$

Now, expressing $\mathbf{P}$ and $\mathbf{M}$ as

$$\mathbf{P}=(\varepsilon-1)\mathbf{E}, \quad \mathbf{M}=\left[1-\frac{1}{\mu}\right]\mathbf{B} , \tag{6.4}$$

the equations become

$$\begin{aligned} &i\mathbf{k}\cdot\mathbf{D}=\rho_{\rm ext}, \quad i\mathbf{k}\times\mathbf{H}=\mathbf{j}_{\rm ext}-ik_0\mathbf{D} , \\ &\mathbf{k}\cdot\mathbf{B}=0, \quad \mathbf{k}\times\mathbf{E}=k_0\mathbf{B} , \end{aligned} \tag{6.5}$$

in terms of the vectors

$$\mathbf{D}\equiv\mathbf{E}+\mathbf{P}=\varepsilon\mathbf{E}, \quad \mathbf{H}\equiv\mathbf{B}-\mathbf{M}=\frac{1}{\mu}\mathbf{B} . \tag{6.6}$$

Thus, the effects of the medium of the propagation of photons are parametrized by ε and μ. We can calculate them if we know how to calculate $\mathbf{P}$ and $\mathbf{M}$, or equivalently $\mathbf{j}_{\rm ind}$, from first principles.

A better way to proceed is as follows. We can eliminate $\mathbf{B}$ from the equations of motion from the beginning. The two homogeneous equations in Eq. (6.1) give

$$\mathbf{B}=\frac{1}{k_0}\mathbf{k}\times\mathbf{E} . \tag{6.7}$$

Thus, $\mathbf{B}$ is already solved for. We therefore need to consider only the inhomogeneous equations in Eq. (6.1).

Instead of writing $\rho_{\rm ind}$ and $\mathbf{j}_{\rm ind}$ as in Eq. (6.3) with Eq. (6.4), we can use Eq. (6.7) to eliminate $\mathbf{B}$, obtaining

$$i\mathbf{k}\cdot\mathbf{E}=\rho, \quad \frac{i}{k_0}\mathbf{k}\times(\mathbf{k}\times\mathbf{E})=\mathbf{j}-ik_0\mathbf{E} . \tag{6.8}$$

Since $\mathbf{k}\times(\mathbf{k}\times\mathbf{E})=(\mathbf{k}\cdot\mathbf{E})\mathbf{k}-|\mathbf{k}|^2\mathbf{E}$, the upshot of the second equation in Eq. (6.8) is that $\mathbf{j}$ contains a term proportional to $\mathbf{E}$ and another component in the direction of $\mathbf{k}$. Introducing the notations

$$\mathbf{E}_l=\hat{\mathbf{k}}(\hat{\mathbf{k}}\cdot\mathbf{E}), \quad \mathbf{E}_t=\mathbf{E}-\mathbf{E}_l \tag{6.9}$$

for the components of $\mathbf{E}$ parallel and perpendicular to $\mathbf{k}$ in the plane containing the two vectors, we can rewrite Eq. (6.8) as

$$i|\mathbf{k}||\mathbf{E}_l|=\rho, \quad ik_0\left[\mathbf{E}_l+\left[1-\frac{|\mathbf{k}|^2}{k_0^2}\right]\mathbf{E}_t\right]=\mathbf{j} . \tag{6.10}$$

Needless to say, this equation gives the total charge and current densities. As mentioned in Eq. (6.2), some parts of them come from external sources and some parts are induced by the medium. We parametrize the induced charge and current densities by

$$\begin{aligned} &\rho_{\rm ind}=i(1-\varepsilon_l)|\mathbf{k}||\mathbf{E}_l| , \\ &\mathbf{j}_{\rm ind}=ik_0[(1-\varepsilon_l)\mathbf{E}_l+(1-\varepsilon_t)\mathbf{E}_t] . \end{aligned} \tag{6.11}$$

Putting these expressions back in Eq. (6.8), we obtain

$$i\mathbf{k}\cdot\mathbf{D}=\rho_{\rm ext}, \quad \frac{i}{k_0}\mathbf{k}\times(\mathbf{k}\times\mathbf{E})=\mathbf{j}_{\rm ext}-ik_0\mathbf{D} , \tag{6.12}$$

where

$$D_i=\varepsilon_{ij}E_j \tag{6.13}$$

with

$$\varepsilon_{ij}=\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j\varepsilon_l+(\delta_{ij}-\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j)\varepsilon_t . \tag{6.14}$$

Thus, the independent dynamical variable is taken to be $\mathbf{E}$, whereas $\varepsilon_{t,l}$ merely parametrize the effects of the medium. Explicitly, the equation for $\mathbf{E}$ is

$$ik_0\left[\varepsilon_l\mathbf{E}_l+\left[\varepsilon_t-\frac{\mathbf{k}^2}{k_0^2}\right]\mathbf{E}_t\right]=\mathbf{j}_{\rm ext} , \tag{6.15}$$

from which the dispersion relations are determined.

In this way of formulating the problem, it is almost obvious that Eq. (6.14) is not the most general form of ε_{ij}. In general, one can write down the following general form consistent with the rotational symmetry of space in the rest frame of the medium:

$$\varepsilon_{ij}=\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j\varepsilon_l+(\delta_{ij}-\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j)\varepsilon_t+i\epsilon_{ijk}\hat{\mathbf{k}}^k\varepsilon_p . \tag{6.16}$$

Equivalently, one can say that the expression for $\mathbf{j}_{\rm ind}$ in Eq. (6.11) is not general, since a three-vector cannot in general be written as a superposition of two basis vectors $\mathbf{E}_l$ and $\mathbf{E}_t$. There might be a component of $\mathbf{j}_{\rm ind}$ perpendicular to the plane of $\mathbf{k}$ and $\mathbf{E}$, so that in general we should write

$$\mathbf{j}_{\rm ind}=ik_0[(1-\varepsilon_l)\mathbf{E}_l+(1-\varepsilon_t)\mathbf{E}_t-i\varepsilon_p\hat{\mathbf{k}}\times\mathbf{E}] . \tag{6.17}$$

There are two things to notice at this point. First, one cannot add any more terms, since we have already spanned the three-dimensional space. Second, the P and CP properties of the new term in $\mathbf{j}_{\rm ind}$ is different from that of the other terms on the right-hand side of Eq. (6.17). Thus, the presence of this term requires P- or CP-odd effects in the Lagrangian or in the system, or maybe in both. As we will now demonstrate, it is connected to the π_P term in $\pi_{\mu\nu}$.

The dispersion relations are obtained as the poles of $D_{\mu\nu}$. Through Eq. (4.5), they should coincide with the zeros of $j^{\mu}_{\rm ext}$. Notice that Eqs. (6.10) and (6.17) imply

$$\mathbf{j}_{\rm ext}=ik_0\left[\varepsilon_l\mathbf{E}_l+\left[\varepsilon_t-\frac{|\mathbf{k}|^2}{k_0^2}\right]\mathbf{E}_t+i\varepsilon_p\hat{\mathbf{k}}\times\mathbf{E}\right] . \quad (6.18)$$

Existence of solutions of the homogeneous equation obtained by putting $\mathbf{j}_{\rm ext}=0$ requires the vanishing of the determinant of the matrix containing the coefficients of different components of E. In order to simply read the solutions directly, we introduce

$$\mathbf{E}_{\pm}=\tfrac{1}{2}(\mathbf{E}_t\pm i\hat{\mathbf{k}}\times\mathbf{E}) \quad (6.19)$$

to rewrite Eq. (6.18) as

$$\mathbf{j}_{\rm ext}=ik_0\left[\varepsilon_l\mathbf{E}_l+\left[\varepsilon_t+\varepsilon_p-\frac{|\mathbf{k}|^2}{k_0^2}\right]\mathbf{E}_+ +\left[\varepsilon_t-\varepsilon_p-\frac{|\mathbf{k}|^2}{k_0^2}\right]\mathbf{E}_-\right] . \quad (6.20)$$

Thus, the dispersion relations are obtained as the solutions of the equations

$$\begin{aligned}&\varepsilon_l=0, \quad \text{longitudinal mode} ,\\ &\frac{|\mathbf{k}|^2}{k_0^2}=\varepsilon_t\pm\varepsilon_p, \quad \text{transverse modes} .\end{aligned} \quad (6.21)$$

To show that these dispersion relations are the same as the ones obtained in Sec. IV, we recall Eq. (4.3) once again. An alternative meaning of this equation other than the one given in Eq. (4.5) is obtained by identifying the vacuum-polarization term as the source term due to induced currents:

$$j^{\mu}_{\rm ind}=-\pi^{\mu\nu}A_\nu , \quad (6.22)$$

so that the equation of motion reads

$$[(\Delta^{-1})^{\mu\nu}]A_\nu=j^{\mu}_{\rm ext}+j^{\mu}_{\rm ind} . \quad (6.23)$$

Using Eq. (2.13) and comparing Eqs. (6.17) and (6.22), one can obtain the following relations written in a Lorentz-invariant way:

$$1-\varepsilon_t=\pi_T/\omega^2, \quad 1-\varepsilon_l=\pi_L/k^2, \quad \varepsilon_p=\pi_P/\omega^2 . \quad (6.24)$$

As an example of how these relations are derived, consider the equation for the time component of $j^{\mu}_{\rm ind}$ from Eq. (6.22). Using Eq. (4.8), we see that only the components of $Q_{\mu\nu}$ contributes to this equation. Thus, we obtain

$$\rho_{\rm ind}=\frac{\pi_L}{k^2}(|\mathbf{k}|^2A^0-k_0\mathbf{k}\cdot\mathbf{A})=\frac{\pi_L}{k^2}i\mathbf{k}\cdot\mathbf{E} , \quad (6.25)$$

using, in the last step, the momentum-space relation

$$\mathbf{E}=-i(A^0\mathbf{k}-k^0\mathbf{A}) . \quad (6.26)$$

Comparing Eq. (6.25) with Eq. (6.11), the second equation in Eq. (6.24) follows. The other two equations are obtained similarly by considering $\mathbf{j}_{\rm ind}$ in Eq. (6.22). Using Eq. (6.24), we can see that the dispersion relations obtained in Eq. (6.21) are the same as the ones given in Eq. (4.15) and Eq. (4.21), as they should be.

VII. CONCLUSIONS

In summary, we have discussed in detail the general formulas for the propagation of electromagnetic field within a medium. We have also discussed the relation between the microscopically calculated quantity $\pi_{\mu\nu}$ and the macroscopic equations of electrodynamics. The connection between these two approaches is very useful, because while the calculation of π_P is most conveniently and consistently done in quantum field theory, the macroscopic consequences are best elucidated in the more intuitive and familiar setting of macroscopic electrodynamics.

In particular, our attention was directed to the effects of a possible term in the vacuum-polarization tensor proportional to the antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$. The presence of the term requires *P* or *CP*-odd effects in the Lagrangian or in the medium. The effect of this term is to split the otherwise degenerate right- and left-handed circular polarization modes of the photon. Thus, the two modes propagate with different speeds within the medium. This phenomenon, known as *optical activity,* is known to occur in anisotropic systems. Our arguments in this paper show that it can take place in isotropic systems as well if *P*- or *CP*-odd effects are present.

ACKNOWLEDGMENTS

We thank G. Bhattacharya for discussions. The research has been partially supported by the National Science Foundation.

APPENDIX: AN ILLUSTRATIVE EXAMPLE

In this appendix, we present an example of how the π_P term of Eq. (2.13) actually arises in a specific model and thereby corroborate the statements, made at the end of Sec. III, about the necessity of both *P* and *CP* asymmetry in the Lagrangian and/or in the background for the appearance of such a term.

We consider the standard model in a background of neutrinos and antineutrinos. For simplicity, we consider the Lagrangian with just one fermion family, so that it automatically conserves *CP* while violating *P*. Also, we choose to work in the rest frame of the neutrino bath, defined in Eq. (4.7). The neutrino propagator should now be written as[5]

$$\begin{aligned}iS_F(p)&=\not{p}\left[\frac{i}{p^2+i\epsilon}-2\pi\delta(p^2)n_F(p^0)\right]\\ &\equiv iS_F^0(p)+S_F'(p) .\end{aligned} \quad (A1)$$

Here, we have used the real-time formalism which is convenient for us. The symbol $n_F(p^0)$ denotes the Fermi-Dirac distribution function

$$n_F(p^0)=\frac{\theta(p^0)}{e^{\beta(p^0-\mu)}+1}+\frac{\theta(-p^0)}{e^{-\beta(p^0-\mu)}+1} . \quad (A2)$$

The two-loop diagrams which involve the internal neutrino lines and contribute to $\pi_{\mu\nu}$ have been given in Ref. 7. There, it has been pointed out that the background contribution to the vacuum-polarization tensor $\pi'_{\mu\nu}$ is given by

$$\pi'_{\mu\nu}(k) = -i\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}[S'_F(p)T_{\mu\nu}(p,k)] \ , \tag{A3}$$

where $\bar{u}(p)T_{\mu\nu}(p,k)u(p)$ is the neutrino-photon forward-scattering amplitude, $u(p)$ being the neutrino spinor at momentum p. To calculate $T_{\mu\nu}$, we can use all zero-temperature propagators since at the lowest loop level, there are no internal neutrino lines for this process. Writing $T_{\mu\nu}(p,k)=\sum_n T^{(n)}_{\mu\nu}(p,k)$ where the index n refers to the power of p in the terms, we can show that only $T^{(0)}_{\mu\nu}$ is relevant for calculating $\pi'_{\mu\nu}$ to $O(1/M_W^2)$. Bose symmetry for the photons and the chirality of neutrino interactions then dictate the form

$$T^{(0)}_{\mu\nu} = T\epsilon_{\mu\nu\alpha\beta}k^\alpha\gamma^\beta(1-\gamma_5) \ . \tag{A4}$$

Turning back to Eq. (A3), we see that the only contribution to $\pi'_{\mu\nu}$ to this order is

$$\pi'_{\mu\nu} = -i\epsilon_{\mu\nu\alpha\beta}k^\alpha T\int \frac{d^4p}{(2\pi)^4}\delta(p^2)n_F(p^0)\mathrm{tr}(\not{p}\gamma^\beta) \ . \tag{A5}$$

To perform the remaining p integration, note that

$$\int \frac{d^4p}{(2\pi)^4}\delta(p^2)n_F(p^0)p^\beta = \tfrac{1}{2}u^\beta\int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{e^{\beta(|\mathbf{p}|-\mu)}+1} - \frac{1}{e^{\beta(|\mathbf{p}|+\mu)}+1}\right] = \tfrac{1}{2}(n_\nu - n_{\bar{\nu}})u^\beta \ , \tag{A6}$$

where n_ν and $n_{\bar{\nu}}$ are the number density of neutrinos and antineutrinos. Combining Eqs. (A5) and (A6), we see that π_P is proportional to $n_\nu - n_{\bar{\nu}}$, or the lepton number, as indicated in the discussion of Sec. III. Thus, since we have P violation in the Lagrangian but no CP violation, we need a net lepton number of the medium so that the background can provide the CP asymmetry necessary for the π_P term in Eq. (2.14). The details of the evaluation of π_P in this and other cases will be presented in a future publication.

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[9]In three space-time dimensions, the most general expression of $\pi_{\mu\nu}$ in the vacuum contains a term proportional to $\tilde{g}_{\mu\nu}$ and another term proportional to $\epsilon_{\mu\nu\rho}k^\rho$. The effect of the latter term is to contribute a piece proportional to $\epsilon_{\mu\nu\rho}A^\mu F^{\nu\rho}$ to the Lagrangian. See, e.g., S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) **140**, 372 (1982), and references therein.

[10]The components of the antisymmetric tensor are defined by $\epsilon^{0123} = -\epsilon_{0123} = 1$. However, for the spatial antisymmetric tensor used in defining the components in Eq. (4.8) and in Sec. VI, we use $\epsilon_{123} = 1$.

[11]See, e.g., J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).