Geometrical origin of supersymmetric gauge theories

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We show that the kinematical properties of any supersymmetric gauge theory may be obtained by mapping a geometric group structure of loops in superspace into some particular Lie group. The underlying group structure of the usual constrained supergauge theories turns out to be the group of even (bosonic) loops.

I. INTRODUCTION

The relevance of the group of loops as the basic underlying geometric structure of gauge theories has been emphasized by many authors.[1-6] It has been explicitly shown that the kinetic content of any Yang-Mills theory is a direct consequence of the geometric structure of the group of loops.[4] A conventional Yang-Mills theory may be obtained by mapping this structure into a specific compact Lie group. Following this view, it was shown[7] that the gauge theory of the Lorentz group provides in a unique way all the necessary kinematical elements for a local description of a general four-dimensional manifold with torsion.

Loops have been used at the quantum level, typically in nonperturbative work. The investigation of the equations of motion for the loop functionals was initiated by Polyakov,[8,9] Nambu,[10] Neveu and Gervais,[11] and further developed by many others.[12-16] During the last year, loops are becoming popular as the more natural gauge-invariant description of the Hilbert space in Hamiltonian lattice gauge theories.[17-19]

The extension of these ideas to supersymmetric gauge theories has little been explored.[20,21] Any program of application of the loop-space techniques should start by identifying the basic underlying geometric structure of the supersymmetric gauge theories. In this paper we shall consider the extended group of loops in superspace and show that it leads naturally to the maximal[22] (not constrained) supersymmetric gauge theories. This extension is rather obvious because the maximal approach to supersymmetric gauge theories is, up to torsion, formally identical to the usual gauge theories.

It seems more interesting to recover the usual minimal (constrained) formulation of the super-gauge-invariant Yang-Mills theories[23,24] which involves only a single real superfield.

We shall see that it is possible to define in superspace a more restricted geometrical structure, the group of even loops. Minimal supersymmetric Yang-Mills theories will be obtained as representations of this group.

This paper is organized as follows. In Sec. II we introduce the group of loops in superspace. We study its local structure and show the relation between the infinitesimal generators of the group and the field strengths and potentials of the super-gauge-invariant theories. In Sec. III we analyze the usual minimal formulation and show that it arises as a real representation of the group of even loops. Conclusions and some further perspectives are left for Sec. IV.

II. THE GROUP OF LOOPS IN SUPERSPACE

Flat superspace is a vanishing curvature manifold with torsion. A closed polygonal path in superspace may be described in an intrinsic way following a procedure first introduced by Mandelstam.[25] One starts by considering a frame of reference at some point 0, taken as the origin of the paths. The first step of the path is given by a vector of components u_1^A in that frame. Next, one considers the parallel-transported frame at the point $0+u_1^A$. The second step of the path is described by its components in the new frame. One proceeds until the last vector of the chain is introduced.

In flat superspace, the frames of reference obtained by parallel transport are path independent. A natural basis at the point $x(x^\mu,\theta^\alpha,\theta^{\dot\alpha})$ is defined by the covariant derivatives

$$\begin{aligned} e_\mu(x)&=D_\mu=\frac{\partial}{\partial x^\mu}\ ,\\ e_\alpha(x)&=D_\alpha=\frac{\partial}{\partial\theta^\alpha}+i\bar\theta^{\dot\alpha}\sigma_{\alpha\dot\alpha}{}^{\mu}\frac{\partial}{\partial x^\mu}\ ,\\ e_{\alpha}(x)&=D_{\dot\alpha}=\frac{\partial}{\partial\theta^{\dot\alpha}}+i\theta^{\alpha}\sigma_{\alpha\dot\alpha}{}^{\mu}\frac{\partial}{\partial x^\mu}\ , \end{aligned} \tag{2.1}$$

which satisfy

$$[D_A,D_B\}=iT_{AB}{}^{C}D_C\ . \tag{2.2}$$

Capital indices denote any element of the set $(\mu,\alpha,\dot\alpha)$ and the nontrivial components of the torsion are

$$T^\mu{}_{\alpha\dot\beta}=2\sigma^\mu{}_{\alpha\dot\beta}=T^\mu{}_{\dot\beta\alpha}\ . \tag{2.3}$$

We shall describe an open path with origin at some point 0 with coordinates $(x_0{}^{\mu},\theta_0{}^{\alpha},\bar{\theta}_0{}^{\dot{\alpha}})$ by the chain

$$(u_1{}^A,u_2{}^A,\ldots,u_N{}^A)\ . \tag{2.4}$$

We suppose that each vector has a definite parity in the Grassmann algebra. The end point of the open path will be at the point X with coordinates

$$\begin{aligned} x^{\mu}&=x_0{}^{\mu}+\sum_{j=1}^{N}u_j{}^{\mu}+i\sigma^{\mu}{}_{\alpha\dot{\alpha}}\sum_{j=1}^{N}(\bar{\theta}_j{}^{\dot{\alpha}}u_j{}^{\alpha}+\theta_j{}^{\alpha}u_j{}^{\dot{\alpha}})\ ,\\ \theta^{\alpha}&=\theta_0{}^{\alpha}+\sum_{j=1}^{N}u_j{}^{\alpha}\ ,\\ \bar{\theta}^{\dot{\alpha}}&=\bar{\theta}_0{}^{\dot{\alpha}}+\sum_{j=1}^{N}u_j{}^{\dot{\alpha}}\ , \end{aligned} \tag{2.5}$$

where

$$\theta_j{}^{\alpha}=\theta_0{}^{\alpha}+\sum_{i=1}^{j-1}u_i{}^{\alpha}\ . \tag{2.6}$$

A closed polygonal path with origin 0 is given by a chain of vectors such that

$$\begin{aligned} &\sum_{j=1}^{N}[u_j{}^{\mu}+i\sigma^{\mu}_{\alpha\dot{\alpha}}(\bar{\theta}_j{}^{\dot{\alpha}}u_j{}^{\alpha}+\theta_j{}^{\alpha}u_j{}^{\dot{\alpha}})]=0\ ,\\ &\sum_{j=1}^{N}u_j{}^{\alpha}=0,\quad \sum_{j=1}^{N}u_j{}^{\dot{\alpha}}=0\ . \end{aligned} \tag{2.7}$$

The product of two closed paths may be defined as

$$(u_1,\ldots,u_N)(v_1,\ldots,v_M)=(u_1,\ldots,u_N,v_1,\ldots,v_M)\ . \tag{2.8}$$

This operation is associative and has an identity element, the empty chain. However, there is no inverse. In order to give a group structure to the set of closed paths, some identification between paths is required.

We define the contraction of a chain

$$(u_1,\ldots,u_N)_c \tag{2.9}$$

as the operation which replaces any two collinear adjacent vector in the chain by its resulting displacement. Thus, if $u_2=-u_3$ and $u_4=\lambda u_1$, one has

$$(u_1,u_2,u_3,u_4,u_5,u_6)_c=(u_1+u_4,u_5,u_6)\ . \tag{2.10}$$

Let us now define a loop as any chain that is equal to its contraction. The product between loops will be given by

$$(u_1,\ldots,u_N)(v_1,\ldots,v_M)=(u_1,\ldots,u_N,v_1,\ldots,v_M)_c\ . \tag{2.11}$$

The inverse of the loop $(u_1,\ldots,u_N)$ is now given by $(\bar{u}_N,\ldots,\bar{u}_1)$ where $\bar{u}_i=-u_i$. It is immediately seen that the set of loops forms a group under this operation. We shall denote by L_0 the group of loops with origin 0. One can easily prove that groups corresponding to different reference points are isomorphic and then one may fix the origin at will. For convenience, we shall fix the even coordinates of the origin at the spatial infinite.

By following the same procedure, open paths may be defined as contracted chains of vectors.

The local structure of the group of loops may be obtained as usual by considering its infinitesimal generators. As we shall prove they are simply related with the physical quantities of the supersymmetric gauge theories.

Let us start by considering the infinitesimal loop

$$\delta_1L(P,u,v)=Puv\bar{u}\bar{v}\bar{w}\bar{P}\ , \tag{2.12}$$

where

$$w^A=iT^A{}_{BC}u^Bv^C\ . \tag{2.13}$$

This loop is obtained by following some path P starting at the origin 0, then an infinitesimal loop $uv\bar{u}\bar{v}\bar{w}$, and finally going back to the origin along $\bar{P}$. Because of the torsion, the last step $\bar{w}$ is required in order to close the path. When u or v vanish, or when $u=\lambda v$, δ_1L reduces to the null loop $P\bar{P}$. It may be immediately seen, by using the defining property of the group of loops, that there is a differential operator which satisfies

$$\psi(\delta L_1L)=[1+u^Av^B\Delta_{AB}(P)]\psi(L) \tag{2.14}$$

with

$$\Delta_{AB}(P)=(-1)^{\epsilon_A\epsilon_B+1}\Delta_{BA}(P)\ . \tag{2.15}$$

The path-independent differential operators $\Delta_{\mu\nu}(P)$ are noncommutative and generalize the "area derivative" first considered by Mandelstam.[25] The quantities ϵ_A are such that $\epsilon_{\mu}=0$, $\epsilon_{\alpha}=\epsilon_{\dot{\alpha}}=1$.

A second class of infinitesimal loops will be considered:

$$\delta_2L=Pu\bar{P}{:}u\ , \tag{2.16}$$

where P is an open path with origin at spatial infinite and $\bar{P}{:}u$ is the chain P followed in the opposite sense after the step u. The differential operator associated with this element satisfies

$$\psi(\delta_2LL)=[1+u^A\delta_A(P)]\psi(L)\ . \tag{2.17}$$

Both operators Δ_{AB} and δ_A may be considered as deformation generators of a path. When acting on open paths they do not move their end points. In order to induce end-point motions we are going to introduce two differential operators.

The first one is the Mandelstam[25] derivative, which measures the effect on any path-dependent object of a linear extension of the path from the end point x to $x+u$:

$$\phi(Pu)=(1+u^A\nabla_A)\phi(P)\ . \tag{2.18}$$

The second differential operator measures the effect on a path P with end points $\mathbf{r}$ of a parallel translation of the path $P{:}u$ through the end point:

$$\mathbf{r}+u^A\mathbf{e}_A(\mathbf{r})\ . \tag{2.19}$$

It satisfies

$$\phi(P{:}u)=(1+u^AD_A)\phi(P)\ . \tag{2.20}$$

We shall call this operator the parallel derivative.

These differential operators obey a set of restrictions which arise from the geometric properties of the group of loops and reproduce the kinematical relation of the supersymmetric gauge theories.

The first equation results from the following identity:

$$Pu = Pu\bar{P}{:}uP{:}u \tag{2.21}$$

which may be written in terms of the generators

$$(1+u^A\nabla_A)\phi(P)=[1+u^A\delta_A(P)](1+u^AD_A)\phi(P)\ . \tag{2.22}$$

Therefore,

$$\nabla_A = D_A + \delta_A(P)\ . \tag{2.23}$$

Thus, Mandelstam's derivative which plays the role of a covariant derivative in any gauge theory may be written as the sum of the parallel derivative and the deformation derivative. A relation between the "area derivative" Δ_{AB} and the deformation derivative δ_A may be derived by considering the somewhat elaborated form of the identity loop

$$I=P_1uv\bar{u}\bar{v}\bar{w}\bar{P}_1P_1w\bar{P}_5P_5v\bar{P}_4P_4u\bar{P}_3P_3\bar{v}\bar{P}_2P_2u\bar{P}_1\ , \tag{2.24}$$

where u and v are infinitesimal vectors, w is given by (2.14), and $P_2,\ldots,P_5$ are paths obtained by parallel transport of P_1.

When written in terms of the differential operators this relation takes the form

$$\begin{aligned}\phi(c)=&[1+u^Av^B\Delta_{AB}(P)][1+w^C\delta_C(P)]\\&\times[1+v^B\delta_B(P{:}w)][1+u^A\delta_A(P{:}wv)]\\&\times[1-v^B\delta_B(P{:}uv)][1-u^A\delta_A(P{:}u)]\phi(c)\end{aligned} \tag{2.25}$$

that leads to the identity

$$\begin{aligned}\Delta_{AB}(P)=(-1)^{\epsilon_A\epsilon_B}(&D_A\delta_B(P)-(-1)^{\epsilon_A\epsilon_B}D_B\delta_A(P)\\&+[\delta_A(P),\delta_B(P)\}\\&-i(-1)^{\epsilon_A\epsilon_B}T^C{}_{AB}\delta_C(P))\end{aligned} \tag{2.26}$$

which reproduce the relation between field strengths and potentials of the maximal (nonconstrained) supersymmetric gauge theory.

It is important to remark that this equation arises from geometric considerations, without any mention to a particular gauge group or a specific gauge theory.

Finally, the path-dependent version of the Ricci identities may be obtained from Eqs. (2.23) and (2.26):

$$[\nabla_A,\nabla_B\}\phi(P)=(-1)^{\epsilon_A\epsilon_B}[\Delta_{AB}(P)+iT_{AB}{}^C\nabla_C]\phi(P)\ . \tag{2.27}$$

The differential operators Δ_{AB} and δ_A are path-dependent objects. We shall derive the restrictions on the path dependence of these operators. Let us consider a general deformation of the path going from P to LP where L is an arbitrary loop.

For convenience, we introduce the operator $U(L)$ defined by its action on loop-dependent functions

$$\hat{U}(L)\psi(C)=\psi(LC)\ . \tag{2.28}$$

When $L=\delta_1L=Puv\bar{u}\bar{v}\bar{w}\bar{P}$, $U(\delta_1L)$ may be written in terms of the differential operator Δ_{AB}:

$$\hat{U}(\delta_1L)=1+u^Av^B\Delta_{AB}(P)\ . \tag{2.29}$$

The group structure of the loop space ensures that

$$\hat{U}(L_1)\hat{U}(L_2)=\hat{U}(L_1L_2) \tag{2.30}$$

and

$$\hat{U}^{-1}(L)=\hat{U}(\bar{L})\ . \tag{2.31}$$

The path dependence of the operator Δ_{AB} results from

$$\begin{aligned}1+u^Av^B\Delta_{AB}(CP)&=\hat{U}(CPuv\bar{u}\bar{v}\bar{w}\bar{P}\bar{C})\\&=\hat{U}(C)\hat{U}(Puv\bar{u}\bar{v}\bar{w}\bar{P})\hat{U}(\bar{C})\ ;\end{aligned} \tag{2.32}$$

that is,

$$\Delta_{AB}(P)=\hat{U}(C)\Delta_{AB}(P)\hat{U}^{-1}(C)\ . \tag{2.33}$$

An analogous procedure allows us to obtain

$$\delta_A(CP)=\hat{U}(C)\delta_A(P)\hat{U}^{-1}(C)+\hat{U}(C)D_A\hat{U}^{-1}(C)\ . \tag{2.34}$$

Once more, these geometrical relations recall the gauge transformation laws of the field strength and the potential.

Let us now show how the conventional gauge-dependent fields arise from the path-dependent differential operators.

As in any gauge theory, the point-dependent formulation must be obtained by "freezing" the path dependence of the generators. A convenient way to achieve this is to choose some arbitrary reference path R from infinity to some fixed point, say the origin of coordinates. Let us denote R_x as the path parallel to R with end at x. Point-dependent operators may be obtained from the corresponding path-dependent objects by

$$\Delta_{AB}(x)=\Delta_{AB}(R_x),\quad \delta_A(x)=\delta_A(R_x)\ .$$

It is evident that the above path fixing process maps the parallel derivative into ordinary derivatives and therefore it holds that

$$\begin{aligned}\Delta_{AB}(x)=(-1)^{\epsilon_A\epsilon_B}(&D_A\delta_B(x)-(-1)^{\epsilon_A\epsilon_B}D_B\delta_A(x)\\&+[\delta_A(x),\delta_B(x)\}\\&-i(-1)^{\epsilon_A\epsilon_B}T^C{}_{AB}\delta_C(x))\ .\end{aligned} \tag{2.35}$$

Once a reference path is fixed, the differential operators are uniquely defined, but a change of reference from R to R' produces a change in the generators given by

$$\begin{aligned}\delta'_A(x)=\delta_A(R'_x)&=\delta_A(R'_x\bar{R}_x\cdot R_x)\\&=\hat{U}(x)\delta_A(R_x)\hat{U}^{-1}(x)\\&\quad+\hat{U}(x)D_A\hat{U}^{-1}(x)\end{aligned} \tag{2.36}$$

with

$$\hat{U}(x)=\hat{U}(R'_x\bar{R}_x) \; . \tag{2.37}$$

Therefore, the point-dependent generators of the group of loops transform as the potentials under a gauge transformation.

The maximal approach to the supersymmetric gauge theories now arises as representations of the group of loops. Let us consider the loop functional $U(L)$ given by

$$U(L)=U^0(L)I+U^a(L)X^a \; , \tag{2.38}$$

where U^0, U^a are elements of a Grassmann algebra and X^a are the generators of certain Lie group G, and let us suppose that they are a representation of the group of loops

$$U(L_1)U(L_2)=U(L_1L_2), \quad U(L)U(\bar{L})=I \; . \tag{2.39}$$

Then it holds that

$$\begin{aligned} U(R_x u \mathbf{R}_{x+u} L) &= [1+u^A\delta_A(x)]U(L) \\ &= U(R_x u\bar{R}_{x+u})U(L) \\ &= [1+u^B A_B(x)]U(L) \end{aligned} \tag{2.40}$$

because $U(R_x u\bar{R}_{x+u})$ is near to the identity. Therefore

$$\delta_B(x)U(L)=iA_B(x)U(L) \; . \tag{2.41}$$

An analogous procedure leads to

$$\Delta_{AB}(x)U(L)=i(-1)^{\epsilon_A\epsilon_B}F_{AB}(x)U(L) \; , \tag{2.42}$$

where the factor $(-1)^{\epsilon_A\epsilon_B}$ has been introduced for convenience in order to recover the usual relation between the field strength and the potential. In fact, Eq. (2.35) can be brought to the well-known relation

$$\begin{aligned} F_{AB}(x) &= D_A A_B(x)-(-1)^{\epsilon_A\epsilon_B}D_B A_A(x) \\ &\quad +i[A_A, A_B\}+i(-1)^{\epsilon_A\epsilon_B}T^C{}_{AB}A_C(x) \; . \end{aligned} \tag{2.43}$$

Finally, a change of the reference path from R_x to R'_x induces via Eqs. (2.36) and (2.41) a gauge transformation of the potential. Thus, we have proved that the kinematical properties of any maximal supersymmetric gauge theory may be obtained by mapping the group of loops into the particular group G being gauged. With this representation we have associated a matrix generator to every loop differential operator, and the kinematics of the supergauge theory associated with G was recovered as the induced image of the kinematics of the group of loops.

III. THE MINIMAL APPROACH

In order to get us back to the minimal scheme we need to impose super- and gauge-covariant constraints on the theory. We are going to show that they have a simple geometrical meaning. In fact, as a consequence of these constraints the group of loops turns out to be restricted to its even (bosonic) part.

The geometrical form of the conventional and representation preserving constraints is

$$\Delta_{\alpha\beta}\phi(C)=\Delta_{\dot{\alpha}\dot{\beta}}\phi(C)=\Delta_{\alpha\dot{\beta}}\phi(C)=0 \; . \tag{3.1}$$

Because of this set of equations, functions $\phi(C)$ depend on certain equivalence class of loops. In fact, two loops that differ by a chain of odd vectors are equivalent.

In order to define the factor group associated with this equivalence relation we are going to modify the notion of contraction of a polygonal path. We define

$$(u_1, \ldots, u_N)_{c'}$$

as the operation which replaces (i) two adjacent odd vectors of a chain

$$u^A, v^A \;\text{ by }\; \frac{i}{2}T^A{}_{BC}u^Bv^C, u^A+v^A \; ,$$

and (ii) two adjacent collinear even vectors

$$u^A, \lambda u^A \;\text{ by }\; (1+\lambda)u^A \; .$$

We define an even loop as a closed polygonal chain that is equal to its contraction. By definition, the odd parts of an even loop are always substituted by a single vector. Thus an even loop is uniquely determined by the position of its even parts.

The product between loops will be given by juxtaposition and contraction of their chains and once again it may be easily seen that they form a group.

Let us study the local structure of this group. Infinitesimal loops

$$Puv\bar{u}\bar{v}\bar{w}P$$

with u,v, odd, are equivalent to the identity loop. Therefore the nontrivial components of the "area derivative" are

$$\Delta_{\alpha\mu}(P), \quad \Delta_{\dot{\alpha}\mu}(P), \quad \Delta_{\mu\nu}(P) \; .$$

The differential operators $\delta_A(P)$ associated with the loops $Pu\bar{P}{:}u$ are not independent. In fact, one may obtain from the identity (2.24) the relations

$$\begin{aligned} & D_\alpha\delta_\beta+D_\beta\delta_\alpha+\{\delta_\alpha,\delta_\beta\}=0 \; , \\ & D_{\dot{\alpha}}\delta_{\dot{\beta}}+D_{\dot{\beta}}\delta_{\dot{\alpha}}+\{\delta_{\dot{\alpha}},\delta_{\dot{\beta}}\}=0 \; , \\ & \delta_\mu(P)=-\frac{i}{4}\sigma_\mu{}^{\alpha\dot{\beta}}[D_\alpha\delta_{\dot{\beta}}+D_{\dot{\beta}}\delta_\alpha+\{\delta_\alpha,\delta_{\dot{\beta}}\}] \; . \end{aligned} \tag{3.2}$$

The general solution of the first two equations may be written in terms of the deformation operator $\hat{U}(c)$, in fact taking into account (2.34) one gets

$$\delta_\alpha(P)=U(P\bar{P}_0^x)D_\alpha\hat{U}(P_0^x\bar{P})$$

and

$$\delta_{\dot{\alpha}}(P)=U(P\bar{P}_0'^x)D_{\dot{\alpha}}U(P_0'^x\bar{P}) \; . \tag{3.3}$$

Paths $P_0^x, P_0'^x$ going from infinity through the point x are, respectively, parallel to the reference paths P_0, P'_0.

The conventional gauge-dependent fields arise from the path-dependent operators in the following way. We start by "freezing" the path dependence in order to obtain point-dependent operators. A natural choice is fixing $P=P_0^x$ or $P=P_0'^x$. The usual supersymmetric choice of gauge corresponds to $P=P_0'^x$, then deformation derivatives take the form

$$\delta_\alpha(x)=\hat{U}(P_0'^x\bar{P}_0^x)D_\alpha\hat{U}(P_0\bar{P}_0'^x)\;,$$
$$\delta_{\dot\alpha}(x)=0,\quad \delta_\mu(x)=-\frac{i}{4}\sigma_\mu{}^{\dot\beta\alpha}D_{\dot\beta}\delta_\alpha(x)\;. \tag{3.4}$$

The usual superfields arise from a representation of the group of even loops into a supersymmetric extension of some Lie group

$$U(L)=U^0(L)I+U^a(L)X^a\;.$$

As was shown in Sec. II, to each relation between differential operators corresponds a relation between superfields. Then, there exist reference paths P_0^x and $P_0'^x$ such that the gauge potentials take the form

$$A_\alpha(x)=-iU(P_0'^x\bar{P}_0^x)D_\alpha U(P_0^x\bar{P}_0'^x)\;,$$
$$A_{\dot\alpha}(x)=0,\quad A_\mu(x)=-\tfrac{1}{4}\sigma_\mu{}^{\dot\beta\alpha}D_{\dot\beta}A_\alpha(x)\;. \tag{3.5}$$

Reference paths will, in general, not be unique. If R_0^x and $R_0'^x$ are a new set of reference paths, then

$$U(R_0^x\bar{R}_0'^x)=e^{-i\bar\Lambda'}U(P_0\bar{P}_0'^x)e^{i\Lambda}\;, \tag{3.6}$$

where Λ is a chiral superfield and $\bar\Lambda'$ antichiral.

Therefore, any representation of the group of even loops may be written in terms of a single point-dependent superfield

$$U(x)=U(P_0^x\bar{P}_0'^x)\;.$$

The reality constraint remains to be imposed. To do that we are going to define a real representation of the group of even loops. A representation of the group of even loops will be real if there exist reference paths $P_0^x, P_0'^x$ such that $U(P_0^x\bar{P}_0'^x)$ is real for all x. That is

$$U(x)=U^\dagger(x)$$

and, consequently, the usual minimal supersymmetric gauge theories arise as real representations of the group of even loops.

IV. CONCLUSIONS

In this paper we have shown that the kinematical properties of any supersymmetric gauge theory may be obtained by mapping a group of loops into some particular Lie group. While the maximal formulations are related with the general group of loops in superspace, the usual, minimal, constrained formulations, result from the group of even loops. In a recent paper, Sazdovic[26] observed the relevance of the even paths in superspace for the study of a supergauge theory of electric and magnetic charges.

Our analysis suggests that even loops should be considered as a key geometrical structure for the study of any minimal gauge theory in superspace. In particular, it seems that the extension of the loop space methods to the supersymmetric case will be simple and suitable for nonperturbative work.

Here, we have restricted ourselves to the $N=1$ supersymmetries. It is reasonable to expect that the well-known difficulties[27,28] to have a fully supersymmetric (off-shell) gauge theory for more than two supersymmetric charges,[29] $N>2$, will manifest in some geometric obstruction to define the underlying group of loops.

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