Dynamical supersymmetry breaking due to vacuum tunneling

A. Casher and Y. Shamir
School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel
(Received 15 December 1987)

We investigate massive supersymmetric (SUSY) Higgs theories with real matter fields in two and three space-time dimensions. An exact expression for the nonperturbative correction to the vacuum energy density due to a dilute instanton gas is obtained. The first quantum correction vanishes identically, whereas the second correction is found to be negative definite for a specific two-dimensional theory. Consequently in general dynamical breakdown of the SUSY algebra due to vacuum tunneling takes place in two and three space-time dimensions. It is shown that there exists a nonzero leakage of SUSY flux which is proportional to the volume of space-time. The resulting boson-fermion mass splitting is calculated.

I. INTRODUCTION

Dynamical supersymmetry (SUSY) breakdown has been extensively studied in the past few years.[1-8] Whereas perturbative effects cannot induce SUSY breaking because of the nonrenormalization theorems, it may happen that nonperturbative effects could induce it. This possibility was originally suggested by Abbott, Grisaru, and Schnitzer,[1] and later it was pointed out by Witten that nonperturbative SUSY breakdown could provide a dynamical mechanism for resolution of the hierarchy problem in grand unified theories.[2]

To investigate whether SUSY is indeed broken nonperturbatively one has to study the role of instantons in SUSY theories. (We use the term instanton to describe any topologically nontrivial solution of the Euclidean field equations. We therefore refer to fluxons, monopoles, and Yang-Mills instantons in two, three, and four space-time dimensions, respectively.) In the semiclassical approximation instantons do not contribute to the SUSY partition function due to the presence of the fermionic zero modes. In view of this fact, several mechanisms for dynamical SUSY breaking have been proposed. The first is that under certain circumstances instantons may induce the formation of a fermionic condensate.[2-4] The vacuum energy density is then said to become positive definite, thus indicating spontaneous SUSY breaking. Another approach is to study the contribution of instanton–anti-instanton correlations to the partition function.[6,7] If this contribution is dominant, the vacuum energy density will again be nonzero. However, one cannot determine *a priori* whether it will become positive or negative. A crucial ingredient of both approaches is the assumption that the quantum corrections to the fermionic determinant in the instanton background vanish to all orders in perturbation theory.[9]

It has been conjectured by one of us that the latter assumption may be incorrect.[8] Indeed, the existence of fermionic zero modes implies that, to leading order, tunneling from the perturbative vacuum of a given topological sector ends up in an orthogonal state of another topological sector. Therefore the amplitude to tunnel from one perturbative vacuum into another is zero in this approximation, and so the vacuum energy density remains zero. However, in principle, small fluctuations could mix the above state with the vacuum state of the same topological sector. Under these circumstances the amplitude to tunnel from one perturbative vacuum into another would no longer be zero, and consequently the true vacuum energy density would become negative definite.

Since a negative vacuum energy density signals a *dynamical breakdown of the SUSY algebra,* the physical motivation for such a conjecture should be explained. The SUSY generators satisfy an algebra which contains the Hamiltonian, and consequently the SUSY transformations mix the canonical fields with their conjugate momenta. As a result one cannot define a supersymmetric system on a finite domain with boundaries, since no boundary conditions are compatible with the vanishing outflow of SUSY current. In a topologically trivial background the amount of SUSY flux that "leaks" from the boundaries depends on the size of the system, and exact SUSY is restored when the size of the system tends to infinity. However, when the background topology is nontrivial one may suspect that a finite leakage of SUSY flux will survive even in the infinite-volume limit. If moreover the contributions of different classical lumps to the total flux are additive, an explicit dynamical breakdown of the SUSY algebra will be induced.

This scenario is supported by an examination of the vector potentials of multi-instanton configurations. In a regular gauge the vector field of a single instanton falls as r^{-1} at large distances. As a result, multi-instanton configurations are well defined only in a singular gauge. The existence of singular field configurations in the exact path integral means that the true phase space of a gauge theory is not the manifold of the canonical fields, namely, the Lie-algebra-valued vector potentials. Rather, the correct variables span the gauge group manifold. In particular, the action is rendered finite in a singular gauge by defining $F_{\mu\nu}^2$ as the limit of a small Wilson loop and not as the square of a (singular) curl of A_μ. We may thus anticipate that symmetry relations and operator algebras might change due to the change in the nature of the variables of the theory, and should be reexamined. A strong indication that SUSY may be broken by such effects is the lack of success in constructing a lattice version of SUSY gauge theories. We conjecture that this is due to the necessity of using group manifolds which do not have the required Kähler structure.[10] The extent to which the "group" effects persist in the exact spectrum is precisely quantified by the measure of topologically nontrivial configurations in the Euclidean partition function.

Finally, we remark that SUSY violation due to topological effects of gauge fields do occur in quantum mechanics and follow the pattern described above—the classical equations of motion are supersymmetric, but the operator algebra is violated by singular gauge topologies.[11] Moreover, the SUSY violation is not due to the global topology at infinity, but rather depends on the distribution of "holes" in configuration space. In particular, the violation due to a well-separated fluxon-antifluxon pair is not zero, but twice the violation due to a single fluxon.

In this paper we prove that dynamical breakdown of the SUSY algebra takes place in two and three space-time dimensions by performing a direct calculation of the vacuum energy density. We consider SUSY theories with real matter fields which are obtained from $N=1$ SUSY in four dimensions through dimensional reduction. It is assumed that all fields are massive. In particular, the gauge symmetry is completely broken through the Higgs mechanism. As a result, all classical field configurations and all propagators decrease exponentially, thus rendering the dilute-gas approximation well defined and reliable in the weak-coupling limit.

In Sec. II we find an exact expression for the one-instanton contribution to the SUSY partition function. We show that the contributions of multi-instanton configurations factorize and obtain an expression for the vacuum energy density. In Sec. III we define the Euclidean SUSY Lagrangian and discuss several algebraic relations among the differential operators that define the quantum fluctuations of bosons, fermions, and ghosts. Of special importance is Eq. (3.43) which relates the eigenvalue equations for bosonic and fermionic fluctuations.

In Sec. IV we find that the first quantum correction to the fermionic determinant is a tree diagram. However, using the identities of Sec. III we show that the tree correction vanishes identically.

In Sec. V we identify all the one-loop corrections to the fermionic determinant. Because of the complexity of the involved expressions we have not calculated the full one-loop correction. Instead we present a specific massive SUSY Higgs model in two space-time dimensions, and prove that the one-loop correction is nonzero for this model. We then argue that our model is by no means special, and that explicit SUSY breaking will, in general, take place in any massive SUSY Higgs theory in two and three space-time dimensions.

In Sec. VI we show that the SUSY flux leakage in field theory follows the same pattern as SUSY violation in quantum mechanics with topological singularities, namely, a nonzero amount of flux emerges from the core of every classical lump, the contributions of many well-separated lumps to the total flux are additive, and there is no cancellation between the contributions of lumps with opposite topological charges. In particular we find that the SUSY current continuity equation is violated by nonperturbative effects, and show that the extra term is not a total divergence. Using the above properties of the SUSY flux we then obtain a formula for the mass splitting between bosons and fermions.

In Sec. VII we summarize our results and conclude with a discussion of some open questions, among which is the possibility of dynamical breakdown of the SUSY algebra in four dimensions. Appendixes A–E are devoted to the elaboration of several technical points.

II. THE NONPERTURBATIVE EUCLIDEAN PATH INTEGRAL

The Euclidean SUSY partition function is defined through

$$Z=\mathrm{Tr}\{\exp(-\hat{\mathcal{H}}t)\}$$
$$=\int[dA][d\Phi][d\Phi^*]\delta[\mathcal{G}^a(A,\Phi,\Phi^*)]\,\mathrm{Det}\left[\frac{\delta\mathcal{G}^a}{\delta\omega^c}(A,\Phi,\Phi^*)\right]\mathrm{Det}^{1/2}[D_F(A,\Phi,\Phi^*)]\exp[-S_B(A,\Phi,\Phi^*)] \tag{2.1}$$

with periodic boundary conditions for bosons and antiperiodic boundary conditions for fermions. $S_B(A,\Phi,\Phi^*)$ is the action functional of all Bose fields, $A^c_\mu,\mu=1,\ldots,4$, is the gauge field, and Φ_α stands for all matter fields. $\mathcal{G}^a(A,\Phi,\Phi^*)$ is the gauge condition and $\mathrm{Det}[\delta\mathcal{G}^a/\delta\omega^c(A,\Phi,\Phi^*)]$ is the Faddeev-Popov determinant. All the necessary counterterms are lumped into $S_B(A,\Phi,\Phi^*)$ for notational simplicity. In Eq. (1) the fermions have been integrated out. $D_F(A,\Phi,\Phi^*)$ is the full Dirac operator in the background of the bosonic fields. The $\frac{1}{2}$ factor is due to the Majorana representation we employ for the Weyl fermions used in SUSY. The partition function Z is normalized by the requirement that it be equal to one for free fields—a normalization which is preserved to all orders in perturbation theory due to the nonrenormalization theorems.

We assume the SUSY theory admits instanton configurations in d space-time dimensions, where d is either two or three. The instanton field $(a^c_\mu(x),\varphi_\alpha(x))$ is therefore a solution of the Euclidean field equations. [It is understood that $a^c_\sigma(x)=0$ for $d+1\le\sigma\le 4$.] We label different classical solutions by collective coordinates $x^0_k,k=1,\ldots,d$, which describe the instanton's center-of-mass position. To quantize the one-instanton sector we expand the bosonic fields around the classical solution in a gauge-invariant way:[12]

$$\Phi(x)=U(x,x^0)[\varphi(x-x^0)+\phi'(x-x^0)] , \quad (2.2a)$$

$$A_\mu(x)=U(x,x^0)\left[-\frac{i}{e}\partial_\mu+a_\mu(x-x^0) + \alpha'_\mu(x,x^0)\right]U^\dagger(x,x^0) ,$$

$$U(x,x^0)=P\left[\exp\left[-ie\int_{\Upsilon(x^0)}dy_j a_j(x-y)\right]\right] , \quad (2.3)$$

$U(x,x^0)$ is the (path-ordered) parallel transport along a path $\Upsilon(x^0)$ from the origin to x^0. The precise definition of $\Upsilon(x^0)$ is discussed in Appendix A. We recall that on the quantum level Φ_α and Φ_α^* are treated as independent variables. Under such circumstances we make the replacement

$$\Phi_\alpha^* \to (\Phi_*)_\alpha \equiv \Phi_{\alpha^*} . \quad (2.4)$$

To allow a more compact formulation we now introduce B_I as a generic name for all Bose fields. Thus the capital index I stands for the indices α,α^*,μ,c and explicitly

$$B_\alpha \equiv B^{\alpha^*} \equiv (B_*)^\alpha \equiv \Phi_\alpha ,$$
$$B^\alpha \equiv B_{\alpha^*} \equiv (B_*)_\alpha \equiv \Phi_{\alpha^*} , \quad (2.5)$$
$$B_{(\mu,c)} \equiv B^{(\mu,c)} \equiv (B_*)_{(\mu,c)} \equiv A_\mu^c .$$

Equation (2.2a) can therefore be rewritten as

$$B_I(x)={}^U b_I(x-x^0)+{}^U\beta'_I(x-x^0) , \quad (2.2b)$$

where $b_I(x)$ is the classical instanton field and $\beta'_I(x)$ are the quantum fluctuations. Upon substituting the expansion (2.2) in $S_B(B)$ we obtain

$$S_B(B)=S_c+\tfrac{1}{2}(\beta'|H_B(b)|\beta')+\int d^dx\,\mathcal{L}_B^{\rm int}(b,\beta') , \quad (2.6)$$

$$H_B(b)_I{}^J=\left.\frac{\delta\mathcal{L}_B}{\delta B^I\delta B_J}\right|_{B=b} , \quad (2.7)$$

$$S_c\equiv S_B(b) . \quad (2.8)$$

The inner product on the right-hand side (RHS) of Eq. (2.6) is defined through

$$(\beta^{(1)}|\beta^{(2)})\equiv\int d^dx\,\beta^{(1)I}\beta_I^{(2)}$$
$$=\int d^dx(\alpha_\mu^{(1)c}\alpha_\mu^{(2)c}+\phi_{\alpha^*}^{(1)}\phi_\alpha^{(2)}+\phi_\alpha^{(1)}\phi_{\alpha^*}^{(2)}) . \quad (2.9)$$

All indices will usually be omitted from the inner product (2.9). However, they will be reintroduced if, otherwise, an ambiguity may arise.

The operator $H_B(b)$ defines the eigenvalue equation for bosonic fluctuations in the presence of the background field $b(x)$:

$$H_B(b)_I{}^J\beta'_J=\lambda^2\beta'_I . \quad (2.10)$$

We comment that $H_B(b)$ can be thought of as a quantum-mechanical Hamiltonian in $(d+1)$-dimensional Minkowskian space-time. All eigenvalues of $H_B(b)$ are non-negative because the instanton minimizes the (positive-definite) action functional in the relevant topological sector. We now choose the following gauge condition [defined directly in terms of $\beta'_I(x)$]:

$$\mathcal{G}^c(\beta'_I)=-(\nabla_j\alpha'_j)^c+ie(\varphi^\dagger T^c\phi'-\phi'^T_* T^c\varphi) , \quad (2.11)$$

$$\nabla_j=\partial_j+iea_j . \quad (2.12)$$

After imposing the gauge (2.11) the only remaining bosonic zero modes correspond to infinitesimal translations of the instanton. For gauge theories the appropriate (i.e., normalizable) translational zero modes are the covariant derivatives of the instanton field.[13] The covariant derivatives are orthogonal relative to the inner product (2.9) and their normalized form is

$$b_{I;l}=\begin{cases} S_c^{-1/2}(\nabla_l\varphi)_\alpha, & I=\alpha , \\ S_c^{-1/2}(\nabla_l\varphi)_\alpha^*, & I=\alpha^* , \\ S_c^{-1/2}f_{lj}^c, & I=(j,c) , \end{cases} \quad (2.13)$$

$$f_{lj}=-\frac{i}{e}[\nabla_l,\nabla_j] . \quad (2.14)$$

We now expand $\beta'_I(x)$ as

$$\beta'_I(x)=b_{I;l}(x)\xi_l+\beta_I(x) , \quad (2.15)$$

where ξ_l are the zero-mode amplitudes and $\beta_I(x)$ is orthogonal to the zero modes and satisfies the gauge condition (2.11). After a unitary change of variables the functional measure becomes

$$[dB]\delta[\mathcal{G}^a({}^{U^\dagger}B_I(x+x^0)-b_I(x))]=[d\beta]d\xi_l . \quad (2.16)$$

To construct a well-defined perturbative expansion of the one-instanton sector we replace the zero-mode amplitudes ξ_l by the collective coordinates x_k^0 using the Faddeev-Popov procedure. To this end we introduce the following identity into the path integral:

$$1=\int dx_k^0\left|\det\left[\frac{\partial\mathcal{C}_l}{\partial x_k^0}\right]\right|\delta(\mathcal{C}_l) , \quad (2.17)$$

$$\mathcal{C}_l(x,x^0)=(B(x)-{}^Ub(x-x^0)|{}^Ub_{;l}(x-x^0)) . \quad (2.18)$$

Equations (2.2) and (2.15) imply that

$$\delta(\mathcal{C}_l)=\delta(\xi_l) . \quad (2.19)$$

The Jacobian factor is calculated in Appendix A. The result is

$$\frac{\partial\mathcal{C}_l}{\partial x_k^0}=S_c^{1/2}I_{kl}(\beta)+\text{nonlocal terms} , \quad (2.20)$$

$$I_{kl}(\beta)=\delta_{kl}+S_c^{-1/2}(\beta_{;k}|b_{;l}) . \quad (2.21)$$

The nonlocal terms arise because the constraints $\mathcal{C}_l$ depend on the parallel transport $U(x,x^0)$. Nevertheless we prove in Appendix A that the nonlocal terms do not contribute to the partition function. Substituting Eqs. (2.15)–(2.21) in Eq. (2.1) and integrating over ξ_l and x_k^0 we find

$$Z_1 = \mathcal{V} S_c^{d/2} \int [d\beta] \det[I_{kl}(\beta)] \,\mathrm{Det}\left[\frac{\delta \mathcal{G}^a}{\delta\omega^c}(b+\beta)\right] \times \mathrm{Det}^{1/2}[D_F(b+\beta)] \exp[-S_B(b+\beta)] , \quad (2.22)$$

where $\mathcal{V}$ is the volume of space-time.

We now wish to perform the functional integration in Eq. (2.22) and obtain an exact diagrammatic expansion of the one-instanton sector in terms of interaction vertices and boson, fermion, and ghost propagators. Having eliminated the bosonic zero modes we already have a well-defined bosonic propagator:

$$G_B(x,y)_I{}^J = \sum_{\lambda^2>0} \frac{1}{\lambda^2} \beta_I(x,\lambda)[\beta_J(y,\lambda)]^* \quad (2.23)$$

which satisfies

$$H_B(b)_I{}^K G_B(x,y)_K{}^J = \delta_I{}^J(x-y) - b_{I;l}(x) b^J{}_{;l}(y) - \sum_{\lambda_1} \check{b}_I(x,\lambda_1)\check{b}^J(y,\lambda_1) . \quad (2.24)$$

In the last term of Eq. (2.24) we sum over all eigenstates which are orthogonal to the gauge condition (2.11). We next expand the full ghost operator as

$$\frac{\delta\mathcal{G}^a}{\delta\omega^c}(b+\beta) = H_{\mathrm{gh}}(b)_{ac} + V_{\mathrm{gh}}(\beta)_{ac} , \quad (2.25)$$

$$H_{\mathrm{gh}}(b)_{ac} \equiv \frac{\delta\mathcal{G}^a}{\delta\omega^c}(b) . \quad (2.26)$$

The operator $H_{\mathrm{gh}}(b)$ is always positive definite and gives rise to a well-defined ghost propagator:

$$H_{\mathrm{gh}}(b)_{ad} G_{\mathrm{gh}}(x,y)_{dc} = \delta_{ac}(x-y) . \quad (2.27)$$

Finally we expand the full Dirac operator in a similar way:

$$D_F(b+\beta)_{I'}{}^{J'} = H_F(b)_{I'}{}^{J'} + V(\beta)_{I'}{}^{J'} , \quad (2.28)$$

$$H_F(b)_{I'}{}^{J'} \equiv D_F(b)_{I'}{}^{J'} . \quad (2.29)$$

In what follows we assume that $H_F(b)$ is a Hermitian operator. The definition of the primed capital indices I', J' is given in the next section. Because of SUSY $H_F(b)$ always admits fermionic zero modes $\chi_n^0, n=1,\ldots,N$, which are responsible for the suppression of tunneling in the semiclassical approximation. (To keep the derivation general we do not specify their number at this stage.) As a result $H_F(b)$ is not invertible and a more careful treatment is needed. We first construct the fermionic propagator only from the nonzero eigenstates of $H_F(b)$:

$$G_F(x,y)_{I'}{}^{J'} = \sum_{\lambda\neq 0} \frac{1}{\lambda} \chi_{I'}(x,\lambda)\chi_{J'}^*(y,\lambda) , \quad (2.30)$$

where $\chi_{I'}(x,\lambda)$ are c-number spinor eigenfunctions of $H_F(b)$. Therefore,

$$H_F(b)_{I'}{}^{K'} G_F(x,y)_{K'}{}^{J'} = \delta_{I'}{}^{J'}(x-y) - \sum_{n=1}^{N} \chi_{nI'}^0(x)\chi_{nJ'}^{0*}(y) . \quad (2.31)$$

We next have to extract the zero modes dependence of the full fermionic determinant. To this end we decompose the full fermionic determinant as

$$\mathrm{Det}^{1/2}[D_F(b+\beta)] = \underline{\mathrm{Det}}^{1/2}[H_F(b)+V(\beta)] \det{}^{1/2}[E_{mn}(\beta)] = \underline{\mathrm{Det}}^{1/2}[H_F(b)]\, \underline{\mathrm{Det}}^{1/2}[1+G_F V(\beta)] \det{}^{1/2}[E_{mn}(\beta)] , \quad (2.32)$$

$$E_{mn}(\beta) = \left(\chi_m^{0\dagger} \middle| V(\beta) \sum_{p=0}^{\infty} [-G_F V(\beta)]^p \middle| \chi_n^0 \right) . \quad (2.33)$$

The proof of Eqs. (2.32) and (2.33) is given in Appendix B. The underlines indicate that the functional determinants in Eq. (2.32) are constructed only from the nonzero eigenstates of $H_F(b)$. Notice that $\ln \underline{\mathrm{Det}}^{1/2}[1+G_F V(\beta)]$ is the sum of all connected closed fermionic loops with bosons on the external legs (see Fig. 1), and that upon diagonalizing $E_{mn}(\beta)$ we obtain the standard expression of degenerate perturbation theory for the shift of zero eigenvalues[14] (see Fig. 2).

Using Eqs. (2.23)–(2.33) the bosonic integration in Eq. (2.22) can be carried out. The final expression for the one-instanton sector of the partition function is

$$Z_1 = \mathcal{V}\Omega , \quad (2.34)$$

$$\Omega = S_c^{d/2} \exp(-S_c)\, \underline{\mathrm{Det}}^{-1/2}[H_B(b)]\, \underline{\mathrm{Det}}^{1/2}[H_F(b)]\, \mathrm{Det}[H_{\mathrm{gh}}(b)] \langle \det[I_{kl}(\beta)] \det{}^{1/2}[E_{mm}(\beta)] \rangle \exp[-\mathcal{W}(b)] . \quad (2.35)$$

Here the angular brackets stand for the expectation value of the enclosed operators and $\mathcal{W}(b)$ is the sum of all connected bubble diagrams. Both are constructed using the propagators defined in Eqs. (2.23), (2.27), and (2.30) and the interaction vertices defined in Eqs. (2.6), (2.25), and (2.28). We recall that Ω is essentially the space-time instanton density.

So far we have been considering configurations that in-

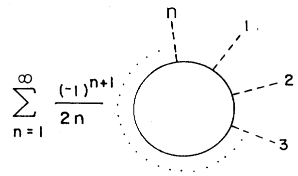

FIG. 1. Diagrammatic representation of $\ln \underline{\det}^{1/2}[1+G_F V(\beta)]$ [see Eq. (2.32)].

volved a single instanton. However, if the exact nonperturbative measure of any topologically nontrivial classical lump is nonzero, the vacuum is dominated by a finite *density* of such lumps. We therefore have to calculate the contribution of multilump configurations to the partition function. To this end we group all configurations into classes which are characterized by the number of lumps of each variety. (For example, in two space-time dimensions there exist fluxon and antifluxon solutions of any integer topological charge. We shall return to this point in Sec. V.) Notice that infinitely many classes correspond to a single topological sector, since different lumps may carry opposite topological charges. Nevertheless, the lump's size is m_0^{-1}, where m_0 is the smallest mass parameter in the theory, whereas the mean interlump distance is

$$l \approx \Omega^{-1/d} . \tag{2.36}$$

As a result we can unambiguously identify the class to which every configuration belongs. In fact, because of the boundary conditions implied by Eq. (2.1), we have to sum only over classes that belong to the trivial topological sector.

We will now prove that the multilump contributions factorize. We first consider a background field that consists of n identical lumps. Within the dilute-gas approximation we now have dn (Nn) bosonic (fermionic) zero modes. Clearly, zero modes that belong to different lumps have no overlap. Moreover, any diagram that involves zero modes that belong to more that one lump vanishes due to the exponential decrease of both zero modes and propagators. Therefore the term in angular brackets in Eq. (2.35) factorizes. Similar arguments imply that the contributions of different lumps to $\mathcal{W}(b)$ and to the logarithm of the functional determinants are additive. In fact, due to SUSY the contribution of interlump space-time to the above terms vanishes identically in the dilute-gas approximation. Consequently all terms in Eq. (2.35) factorize. Introducing dn collective coordinates and repeating the Faddeev-Popov procedure (2.15)–(2.21) with dn constraints we obtain

$$Z_n = \frac{1}{n!}(\mathcal{V}\Omega)^n . \tag{2.37}$$

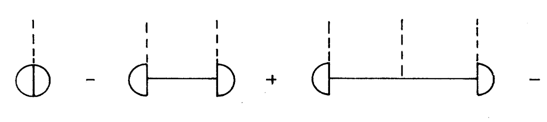

FIG. 2. Diagrammatic representation of $E_{mn}(\beta)$ [see Eq. (2.33)]. The right semicircle represents χ_n^0 and the left semicircle represents $\chi_m^{0\dagger}$.

The $(n!)^{-1}$ factor is due to Bose statistics of the lumps.

Now let τ label the different types of lumps, and assume that there are n_τ lumps of type τ in a given configuration. Equation (2.37) readily generalizes to

$$Z_{n_1,n_2,\ldots,n_\tau,\ldots} = \prod_\tau \frac{1}{n_\tau!}(\mathcal{V}\Omega_\tau)^{n_\tau} . \tag{2.38}$$

To calculate the partition function (2.1) we impose the condition of a globally trivial topology by introducing a Kronecker δ of the total topological charge:

$$Z = \sum_{n_\tau} \delta_{\sum_\rho n_\rho q_\rho, 0} Z_{n_1,n_2,\ldots,n_\tau,\ldots} . \tag{2.39}$$

Here q_ρ is the ρth lump topological charge. Using a representation of the Kronecker δ we finally obtain

$$Z = \int_0^{2\pi} d\theta \exp\left[\mathcal{V}\sum_\tau e^{i\theta q_\tau}\Omega_\tau\right] . \tag{2.40}$$

Equation (40) exhibits the formation of "θ vacua" out of the degenerate perturbative vacua, provided the expression in large parentheses is not identically zero. The true vacuum energy density is therefore

$$\epsilon_0 = -\sum_\tau e^{i\theta_0 q_\tau}\Omega_\tau , \tag{2.41}$$

where

$$\sum_\tau e^{i\theta_0 q_\tau}\Omega_\tau = \max\left[\sum_\tau e^{i\theta q_\tau}\Omega_\tau\right]_{\theta=0}^{2\pi} . \tag{2.42}$$

In Sec. V we will employ Eqs. (2.35) and (2.41) to show that the vacuum energy density is negative definite for a specific two-dimensional SUSY theory.

III. THE EUCLIDEAN SUSY ACTION

In this paper we investigate SUSY Higgs theories in two or three space-time dimensions, which are obtained from renormalizable $N=1$ SUSY theories in four dimensions through dimensional reduction. We assume the following.

(a) All fields are massive. In particular, the gauge symmetry is spontaneously broken through the Higgs mechanism.

(b) There exists an R symmetry in the theory. The reader who is not familiar with the properties of R symmetry is referred to Appendix C.

(c) The matter fields belong to a real (in general reducible) representation $\mathcal{R}(G)$ of the gauge group. $\mathcal{R}(G)$ can therefore be decomposed as

$$\mathcal{R}(G) = \mathcal{R}_0 \oplus \mathcal{C} \oplus \overline{\mathcal{C}} . \tag{3.1}$$

The simple components of $\mathcal{R}_0$ are real, and the simple components of $\mathcal{C}$ are complex or pseudoreal. Let T^a be the Hermitian group generators in the $\mathcal{R}(G)$ representation. Since $\mathcal{R}(G)$ is real there exists a matrix L such that

$$LT^aL^{-1} = (-T^a)^* = (-T^a)^T , \tag{3.2}$$

$$L^T = L, \quad L^+ = L^{-1} . \tag{3.3}$$

In every simple component of $\mathcal{R}_0$ we can choose a basis such that the restriction of L to $\mathcal{R}_0$ is the identity matrix. Relative to this basis L is given explicitly by

$$L\begin{bmatrix}\mathcal{R}_0\\ \mathcal{C}\\ \overline{\mathcal{C}}\end{bmatrix}=\begin{bmatrix}I&0&0\\0&0&I\\0&I&0\end{bmatrix}\begin{bmatrix}\mathcal{R}_0\\ \mathcal{C}\\ \overline{\mathcal{C}}\end{bmatrix}. \tag{3.4}$$

We further assume that the superpotential $W(\Phi)$ is invariant under the action of L:

$$W(L\Phi)=W(\Phi)\ . \tag{3.5}$$

The bosonic Lagrangian is given by

$$\mathcal{L}_B=\tfrac{1}{4}F^c_{\mu\nu}F^c_{\mu\nu}+|D_\mu\Phi|^2+|W_\alpha(\Phi)|^2+\frac{e^2}{2}[D^c(\Phi)]^2\ ,$$
$$W_\alpha(\Phi)\equiv\frac{\partial W}{\partial\Phi_\alpha}\ , \tag{3.6}$$
$$D^c(\Phi)=\Phi^\dagger T^c\Phi\ .$$

In the bosonic Lagrangian (3.6) and in the fermionic Lagrangian [to be defined in Eq. (3.20)] it is understood that $\partial_\sigma=0$ for $d+1\le\sigma\le4$. We recall that the superpotential $W(\Phi)$ is an analytic polynomial whose degree is less than or equal to three.

We now consider the instanton sector of the theory. We assume that the instanton field satisfies the reality condition

$$L\varphi=\varphi^* \tag{3.7}$$

and respects R symmetry:

$$\varphi_\alpha(x)=0\quad\text{for } Q_R(\Phi_\alpha)\neq0\ , \tag{3.8}$$

where Q_R is the R symmetry charge. The instanton field is therefore a solution of the following field equations:

$$-(\nabla^2\varphi)_\alpha+W^*_{\alpha\beta}(\varphi)W_\beta(\varphi)=0\ , \tag{3.9a}$$

$$-(\nabla_l f_{lk})^c+ie[(\nabla_k\varphi)^\dagger T^c\varphi-\varphi^\dagger T^c(\nabla_k\varphi)]=0\ . \tag{3.9b}$$

The small fluctuations operator $H_B(b)$ is defined in Eq. (2.7). After substituting the gauge condition (2.11) we obtain an alternative form for $H_B(b)$ which is explicitly given by

$$-(\nabla^2\phi)_\alpha+W^*_{\alpha\beta}(\varphi)W_{\beta\gamma}(\varphi)\phi_\gamma+W^*_{\alpha\beta\gamma}W_\beta(\varphi)\phi_{\gamma*}+2e^2(T^c\varphi)_\alpha(\varphi^\dagger T^c\phi)-2ie(T^c\nabla_k\varphi)_\alpha\alpha^c_k=\lambda^2\phi_\alpha\ , \tag{3.10a}$$

$$-(\nabla^2\alpha_k)^c+2ie[f_{lk},\alpha_l]^c+e^2\varphi^\dagger\{T^c,\alpha_k\}\varphi+2ie[(\nabla_k\varphi)^\dagger T^c\phi-(\nabla_k\varphi)^T(T^c)^T\phi_*]=\lambda^2\alpha^c_k,\quad k=1,\dots,d \tag{3.10b}$$

$$-(\nabla^2\alpha_\sigma)^c+e^2\varphi^\dagger\{T^c,\alpha_\sigma\}\varphi=\lambda^2\alpha^c_\sigma,\quad d+1\le\sigma\le4\ . \tag{3.10c}$$

Throughout this paper we always use the form (3.10) for $H_B(b)$.

The ghost operator $H_{\rm gh}(b)$ is defined in Eq. (2.26). For the gauge condition (2.11) we find

$$H_{\rm gh}(b)_{ac}=-\nabla^2_{ac}+e^2\varphi^\dagger\{T^a,T^c\}\varphi\ . \tag{3.11}$$

Notice that the differential operators (3.11) and (3.10c) are identical. The ghost propagator is closely related to the projection over all states which are orthogonal to the gauge condition [the last term in Eq. (2.24)]. Since these states correspond to infinitesimal local gauge transformations of the classical field, they admit the following parametrization:

$$\check{b}_I(x)=\delta_c b_I(x)\eta^c(x)\ , \tag{3.12a}$$

$$\delta_c b_I=\begin{cases}-ie(T^c\varphi)_\alpha, & I=\alpha\ ,\\ ie(T^c\varphi)^*_\alpha, & I=\alpha^*\ ,\\ (\nabla_k)_{ac}, & I=(k,a)\ .\end{cases} \tag{3.12b}$$

The normalization integral for $\check{b}_I(x)$ is, therefore,

$$(\check{b}|\check{b})=(\eta^a|-\nabla^2_{ac}+e^2\varphi^\dagger\{T^a,T^c\}\varphi|\eta^c)\ . \tag{3.13}$$

In Eq. (3.13) we have dropped a surface term, which vanishes due to the boundary conditions imposed on local gauge transformations. Consequently, taking $\eta^c(x)$ to be eigenstates of $H_{\rm gh}(x)$ we find

$$\sum_{\lambda_1}\check{b}_I(x,\lambda_1)\check{b}^J(y,\lambda_1)$$
$$=\sum_\lambda\frac{1}{\lambda^2}[\delta_a b_I(x)\eta^a(x,\lambda)][\eta^c(y,\lambda)\overleftarrow{\delta}_c b^J(y)]$$
$$=\delta_a b_I(x)G_{\rm gh}(x,y)_{ac}\overleftarrow{\delta}_c b^J(y)\ . \tag{3.14}$$

Consider next the fermionic sector of the Lagrangian. In this paper we use anti-Hermitian γ_μ matrices which satisfy the Euclidean Dirac algebra:

$$\{\gamma_\mu,\gamma_\nu\}=-2\delta_{\mu\nu},\quad \mu,\nu=1,\dots,4\ . \tag{3.15}$$

We define the charge-conjugation matrix through

$$C\gamma_\mu C^{-1}=-\gamma^*_\mu=\gamma^T_\mu\ . \tag{3.16}$$

Choosing chiral representation for the γ_μ matrices, we have

$$C=\gamma_1\gamma_3,\quad C^\dagger=C^{-1}=C^T=-C\ . \tag{3.17}$$

In this paper we also use the notation

$$[\pm]=\tfrac{1}{2}(1\pm\gamma_5)\ . \tag{3.18}$$

Let Ψ_α be the fermionic matter fields and Λ_c the gaugino. Similarly to the bosonic case we let $\chi_{I'}$ be a generic name for all Fermi fields, where now the primed capital index I' stands for α and c. Explicitly,

$$\chi_\alpha\equiv\Psi_\alpha,\quad \chi_c\equiv\Lambda_c\ . \tag{3.19}$$

The fermionic Lagrangian is

$$\mathcal{L}_F=\tfrac{1}{2}\bar{\chi}_{I'}D_F(B)_{I'}{}^{J'}\chi_{J'}=\frac{i}{2}\bar{\Psi}\gamma_5 \not{D}\Psi+\frac{i}{2}\bar{\Lambda}\gamma_5\not{D}\Lambda+\frac{ie}{\sqrt{2}}(\bar{\Lambda}_c\gamma_5\tilde{\Phi}^\dagger T^c\Psi-\bar{\Psi}\gamma_5 T^c L\tilde{\Phi}^*\Lambda_c)-\tfrac{1}{2}\bar{\Psi}_\alpha\gamma_5 L_{\alpha\beta}W_{\beta\gamma}(\tilde{\Phi})\Psi_\gamma\ , \tag{3.20}$$

$$\tilde{\Phi}\equiv[+]\Phi+[-]L\Phi^*\ . \tag{3.21}$$

The reader is referred to Appendix D for the derivation of Eq. (3.20). Notice that $\chi_{I'}$ are Majorana fermions with transformation properties of Dirac spinors. The Majorana condition is

$$\bar{\chi}\equiv\chi^T CL\ . \tag{3.22}$$

The structure of the fermionic field equations and of the boson-fermion interaction can now be read off Eqs. (3.20) and (2.28). The interaction matrix $V(\beta)$ has the following symmetries:

$$V^T(\beta)=-CLV(\beta)CL\ , \tag{3.23}$$

$$V^\dagger(\beta)=V(L\beta^*)\ . \tag{3.24}$$

Equation (3.23) is a manifestation of the built-in Majorana symmetry. Equation (3.24) follows from the use of real matter fields and in particular Eq. (3.5). The Dirac operator $H_F(b)$ is explicitly given by

$$H_F(b)=\gamma_5[i\not{\nabla}-M(\varphi)]\ , \tag{3.25a}$$

$$M(\varphi)_{I'}{}^{J'}\chi_{J'}=\begin{bmatrix} L_{\alpha\beta}W_{\beta\gamma}(\varphi) & ie\sqrt{2}(T^c\varphi)_\alpha \\ -ie\sqrt{2}(\varphi^+T^a)_\gamma & \end{bmatrix}\begin{bmatrix}\Psi_\gamma\\ \Lambda_c\end{bmatrix}\ . \tag{3.25b}$$

Equation (3.24) together with the reality condition (3.7) imply that $H_F(b)$ is a Hermitian operator. (As in the bosonic case, it can be thought of as the fermionic Hamiltonian in $d+1$ dimensions.) The hermiticity of $H_F(b)$ and Eq. (3.23) imply the existence of charge-conjugation symmetry:

$$H_F(b)CL=CLH_F^*(b)\ . \tag{3.26}$$

Thus, if χ is an eigenstate of $H_F(b)$, $CL\chi^*$ is another eigenstate with the same eigenvalue. We comment that χ and $CL\chi^*$ are always different states since charge-conjugation reverses the sign of the total angular momentum [see, for example, Eq. (5.9)].

Finally, because of Eq. (3.8), $M(\varphi)$ has nonvanishing entries only between fermions with opposite R symmetry charges: i.e.,

$$\{M(\varphi),Q_R\}=0\ . \tag{3.27}$$

Let

$$Q_5=Q_R\gamma_5\ . \tag{3.28}$$

We readily see that $H_F(b)$ anticommutes with generalized chirality defined relative to Q_5:

$$\{H_F(b),Q_5\}=0\ . \tag{3.29}$$

Q_5 chirality commutes with the charge-conjugation transformation (3.26), since independently of the particular representation used for the γ_μ matrices, we have

$$Q_5CL=CLQ_5^*\ . \tag{3.30}$$

As a result, nonzero eigenstates of $H_F(b)$ appear in quartets:

$$\begin{aligned}&\chi, CL\chi^* \text{ with eigenvalue } \lambda\ ,\\ &Q_5\chi, Q_5CL\chi^* \text{ with eigenvalue } -\lambda\ .\end{aligned} \tag{3.31}$$

Equation (3.29) also implies that we can define the index of $H_F(b)$ relative to Q_5 in a natural way. In four-dimensional theories this index is of crucial importance. However, for the theories discussed in this paper the index is trivially zero due to the existences of the following discrete symmetry:

$$\begin{aligned}&\chi\to\gamma_4\gamma_5\chi,\quad \bar{\chi}\to-\bar{\chi}\gamma_4\gamma_5\ ,\\ &\Phi\to L\Phi^*\ ,\\ &\alpha_4\to-\alpha_4,\quad \alpha_m\to\alpha_m, m=1,2,3\ .\end{aligned} \tag{3.32}$$

The invariance of the action under the transformation (3.32) implies

$$\{H_F(b),\gamma_4\gamma_5\}=0 \tag{3.33}$$

and

$$[H_F(b),\gamma_4Q_R]=0\ . \tag{3.34}$$

Therefore if χ^0 is a zero mode of $H_F(b)$ with definite Q_5 chirality, then $\gamma_4\gamma_5\chi^0$ is another zero mode with opposite Q_5 chirality.

The global SUSY transformation that corresponds to the Lagrangians (3.6) and (3.20) is

$$\delta\tilde{\Phi}_\alpha=C\Psi_\alpha,\quad \delta A_\mu^c=\frac{1}{i\sqrt{2}}C\gamma_\mu\Lambda_c\ , \tag{3.35a}$$

$$\begin{aligned}&\delta\Psi_\alpha=\gamma_5[i\not{D}_{\alpha\beta}L_{\beta\gamma}\tilde{\Phi}_\gamma^*-W_\alpha^*(\tilde{\Phi})]\ ,\\ &\delta\Lambda_c=\frac{1}{2i\sqrt{2}}\gamma_5[\sigma_{\mu\nu}F_{\mu\nu}^c+2eD^c(\tilde{\Phi})]\ .\end{aligned} \tag{3.35b}$$

Several relations which are consequences of SUSY are more easily formulated in terms of a matrix Γ that we define below; besides the capital fermionic and bosonic indices (I' and I), Γ has a (suppressed) Dirac index, and its nonzero entries are

$$\begin{aligned}&\Gamma_\alpha{}^\beta=[+]\delta_\alpha{}^\beta\ ,\\ &\Gamma_\alpha{}^{\beta^*}=[-]L_\alpha{}^{\beta^*}\ ,\\ &\Gamma_\alpha^{(\mu,c)}=\frac{1}{i\sqrt{2}}\gamma_\mu\delta_{ac}\ .\end{aligned} \tag{3.36}$$

For example,

$$\Gamma_{I'}{}^I B_I=\begin{bmatrix}\tilde{\Phi}_\alpha\\ \dfrac{1}{i\sqrt{2}}A^a\end{bmatrix}\ . \tag{3.37}$$

Using the above definition one can verify that on the full field equations SUSY is realized through the identity

$$\Gamma_{I'}{}^{I}\left[-\partial_k \frac{\delta\mathcal{L}_B}{\delta(\partial_k L^{IJ}B_J)}+\frac{\delta\mathcal{L}_B}{\delta(L^{IK}B_K)}\right]=D_F(B)_{I'}{}^{J'}\delta\chi_{J'}(B)\ . \quad (3.38)$$

By expanding the bosonic fields in Eq. (3.38) one obtains four interesting relations. To this end we first expand $\delta\chi_{I'}(B)$ as

$$\delta\chi_{I'}(b+\beta)=S_c^{1/2}\delta\chi^0_{I'}+H_F(b)_{I'}{}^{J'}\Gamma_{J'}{}^{J}L_{JI}\beta^I+\tfrac{1}{2}(V_{I'}{}^{LK'}L_{LN}\beta^N)\Gamma_{K'}{}^{K}L_{KM}\beta^M\ , \quad (3.39)$$

where

$$\delta\chi^0_{I'}\equiv S_c^{-1/2}\delta\chi_{I'}(b) \quad (3.40)$$

and $V_{I'}{}^{LK'}$ is defined through [see Eq. (2.28)]

$$V_{I'}{}^{LK'}\beta_L\equiv V(\beta)_{I'}{}^{K'}\ . \quad (3.41)$$

We have applied the gauge condition (2.11) to obtain the second term on the RHS of Eq. (3.39). We now find that Eq. (3.38) gives rise to the identities

$$H_F(b)\delta\chi^0=0\ , \quad (3.42)$$

$$\Gamma_{I'}{}^{I}H_B(b)_I{}^{J}L_{JK}\beta^K=H_F^2(b)_{I'}{}^{M'}\Gamma_{M'}{}^{M}L_{MN}\beta^N+S_c^{1/2}(V_{I'}{}^{LK'}\beta_L)\delta\chi^0_{K'}\ , \quad (3.43)$$

$$\tfrac{1}{2}\Gamma_{I'}{}^{I}\mathcal{L}^{(3)}_{IJK}\beta^J\beta^K=\tfrac{1}{2}H_F(b)_{I'}{}^{J'}(V_{J'}{}^{NL'}\beta_N)\Gamma_{L'}{}^{L}\beta_L+(V_{I'}{}^{PK'}L_{PQ}\beta^Q)H_F(b)_{K'}{}^{M'}\Gamma_{M'}{}^{M}\beta_M\ , \quad (3.44)$$

$$\tfrac{1}{3}\Gamma_{I'}{}^{I}\mathcal{L}^{(4)}_{IJKL}\beta^J\beta^K\beta^L=(V_{I'}{}^{NJ'}L_{NP}\beta^P)(V_{J'}{}^{QM'}\beta_Q)\Gamma_{M'}{}^{M}\beta_M\ , \quad (3.45)$$

where

$$\mathcal{L}^{(3)}_{IJK}=\left.\frac{\delta\mathcal{L}_B}{\delta B^I\delta B^J\delta B^K}\right|_{B=b}\ , \quad (3.46)$$

$$\mathcal{L}^{(4)}_{IJKL}=\frac{\delta\mathcal{L}_B}{\delta B^I\delta B^J\delta B^K\delta B^L}\ . \quad (3.47)$$

Equation (3.42) implies that the matrix $\delta\chi^0$ is composed of four fermionic zero modes [actually Eq. (3.40) gives the normalized form of the zero modes]. Equation (3.43) is of crucial importance, since it implies that the *Schrödinger-type operators $H_B(b)$ and $H_F^2(b)$ involve different scattering potentials.* [The only exception arises when $\delta\chi^0$ contains a Dirac projection operator. This is the case of the exact (massless) Yang-Mills instanton in four dimensions, and of certain SUSY quantum mechanics theories. Notice that in these cases there exist only two fermionic zero modes.] We will employ Eq. (3.43) extensively in the following sections. Finally, Eqs. (3.44) and (3.45) provide relations between the bosonic and fermionic vertices.

IV. FIRST QUANTUM CORRECTION

Having defined the Euclidean SUSY Lagrangian we are now able to calculate quantum corrections to the zero fermionic eigenvalues. The theories discussed in Sec. III always admit four zero modes which are given in a matrix form in Eq. (3.40). Taking into account the properties of the background field (3.7) and (3.8), it is useful to write down the zero modes explicitly:

$$\delta\Psi^0_\alpha=\begin{cases}S^{-1/2}i\gamma_5(\nabla\varphi)_\alpha, & Q_R(\Phi_\alpha)=0\ ,\\ -S_c^{-1/2}\gamma_5W^*_\alpha(\varphi), & Q_R(\Phi_\alpha)=1\ ,\\ 0 \ \text{otherwise}\ ,\end{cases} \quad (4.1)$$

$$\delta\Lambda^0_c=\frac{1}{i2\sqrt{2}}\gamma_5\sigma_{kl}f^c_{kl}\ .$$

The matrix $E(\beta)$ in (2.33) is

$$E(\beta)=\left(\delta\chi^{0\dagger}\left|V(\beta)\sum_{p=0}^{\infty}[-G_FV(\beta)]^p\right|\delta\chi^0\right)\ . \quad (4.2)$$

Majorana symmetry [Eqs. (3.23) and (3.26)] implies that $CE(\beta)$ is antisymmetric. Using Eq. (3.17) we find that $E(\beta)$ has the following structure:

$$E(\beta)=\begin{bmatrix}E_1(\beta) & 0 & E_6(\beta) & -E_4(\beta)\\ 0 & E_1(\beta) & -E_5(\beta) & E_3(\beta)\\ E_3(\beta) & E_4(\beta) & E_2(\beta) & 0\\ E_5(\beta) & E_6(\beta) & 0 & E_2(\beta)\end{bmatrix} \quad (4.3)$$

and so

$$\det{}^{1/2}[E(\beta)]=E_1(\beta)E_2(\beta)+E_4(\beta)E_5(\beta)-E_3(\beta)E_6(\beta)\ . \quad (4.4)$$

This result can also be written as

$$\det{}^{1/2}[E(\beta)]=\tfrac{1}{4}\,\mathrm{tr}\{E(\beta)[+]\}\,\mathrm{tr}\{E(\beta)[-]\}+\tfrac{1}{16}\,\mathrm{tr}\{E(\beta)\gamma_\mu\}\,\mathrm{tr}\{E(\beta)\gamma_\mu\}\ . \quad (4.5)$$

The last expression is manifestly independent of the particular representation chosen for the γ_μ matrices. We comment that Eq. (2.41) for the vacuum energy density implies that the sign ambiguity in Eq. (4.4) has no physical consequences.

The leading approximation to the expectation value of $\det^{1/2}[E(\beta)]$ is obtained as follows [see Eq. (2.35)]: on the RHS of Eq. (4.5) we replace $E(\beta)$ by $E^{(1)}(\beta)$ where

$$E^{(1)}(\beta)=(\delta\chi^{0\dagger}|V(\beta)|\delta\chi^0) \quad (4.6)$$

and contract the two bosonic fields in each term. The first quantum correction $\varepsilon^{(1)}$ to the fermionic determinant is therefore a tree diagram (see Fig. 3).

We will now show that the tree correction vanishes identically. First, applying Eq. (3.43) to $E^{(1)}(\beta)$ we find

$$E^{(1)}(\beta)=S_c^{-1/2}(\delta\chi^{0\dagger}_{I'}\Gamma_{I'}{}^{I}|H_B(b)_I{}^{J}|L_{JK}\beta^K)\ . \quad (4.7)$$

Therefore,

$$\varepsilon^{(1)}=S_c^{-1/2}\sum_{\lambda^2>0}\frac{1}{\lambda^2}\{\tfrac{1}{8}\,\mathrm{tr}(\delta\chi_{J'}^{0\dagger}|V_{J'}{}^{LK'}[\beta^L(\lambda)]^*|\delta\chi^0_{K'}[\pm])\,\mathrm{tr}(\delta\chi_{I'}^{0\dagger}\Gamma_{I'}{}^{I}|H_B(b)_I{}^J|L_{JK}\beta^K(\lambda)[\mp])$$
$$+\tfrac{1}{16}\,\mathrm{tr}(\delta\chi_{J'}^{0\dagger}|V_{J'}{}^{LK'}[\beta^L(\lambda)]^*|\delta\chi^0_{K'}\gamma_\mu)\,\mathrm{tr}(\delta\chi_{I'}^{0\dagger}\Gamma_{I'}{}^{I}|H_B(b)_I{}^J|L_{JK}\beta^K(\lambda)\gamma_\mu\})\ . \qquad (4.8)$$

Using Eq. (2.24) the bosonic propagator can be eliminated from Eq. (4.8). If the bosonic partner of $\delta\chi_{I'}^{0\dagger}$ is a scalar field with one unit of R symmetry charge, it is obvious that a nonzero contribution to $\varepsilon^{(1)}$ is obtained only by applying the $\delta_I{}^J(x-y)$ term of Eq. (2.24) to the first row of Eq. (4.8). We next consider all $\delta\chi_{I'}^{0\dagger}$ whose bosonic partners have zero R symmetry charge. (Notice that this always includes $\delta\Lambda_c^0$.) Using Eq. (4.1) we find

$$\delta\chi_{I'}^{0\dagger}\Gamma_{I'}{}^{I}=-\frac{i}{2}\gamma_5\gamma_k b^I{}_{;k}+i\gamma_\mu d^I{}_\mu\ ,\quad Q_R(B_I)=0\ , \qquad (4.9)$$

where

$$d_{I\mu}=\begin{cases}\tfrac{1}{2}S_c^{-1/2}(\nabla_\mu\varphi)_\alpha, & I=\alpha\ ,\\ -\tfrac{1}{2}S_c^{-1/2}(\nabla_\mu\varphi)^*_\alpha, & I=\alpha^*\ ,\\ \tfrac{1}{4}S_c^{-1/2}\epsilon_{\mu k l \nu}f^c_{kl}, & I=(\nu,c)\ .\end{cases} \qquad (4.10)$$

Now a nonzero contribution to $\varepsilon^{(1)}$ arises only upon substituting the second term of Eq. (4.9) in the second row of Eq. (4.8). As in the previous case, only the $\delta_I{}^J(x-y)$ term of Eq. (2.24) contributes to $\varepsilon^{(1)}$. [In three space-time dimensions one has to apply Bianchi identity in order to prove that the projection on infinitesimal gauge transformations does not contribute to $\varepsilon^{(1)}$, and to use the tensorial structure of $f^c_{kl}(x)$ in order to prove that the projection on the translational zero modes does not contribute to $\varepsilon^{(1)}$ as well.]

A straightforward calculation along the above lines reveals that up to numerical factors all contributions to $\varepsilon^{(1)}$ have the general form

$$\int d^dx\ \mathrm{tr}(\gamma_5\delta\chi^0_{I'}\delta\chi^0_{J'}\delta\chi^0_{K'})\ , \qquad (4.11)$$

and upon summing all these contributions we find that the integrand in Eq. (4.8) vanishes identically.

In order to establish an identity that will prove useful in the next section we will now give an alternative proof that $\varepsilon^{(1)}=0$ for a somewhat more restricted class of theories. To this end we observe that a term of the general form (4.11) does not vanish only if all three bosonic partners have either zero or one unit of R symmetry charge. As a result, a third order term of the superpotential (call it $\Phi_1\Phi_2\Phi_3$) may contribute only if say, Φ_1 and Φ_2 have zero charge and Φ_3 has one unit of charge. We now assume that all third-order terms of the superpotential have the above structure, and moreover that in every such term the field with one unit of R symmetry charge is a singlet of the gauge group. Under these circumstances it is a straightforward exercise to verify the following identity:

$$\sum_{\lambda^2>0}\frac{1}{\lambda^2}\{V_{J'}{}^{LK'}[\beta^L(\lambda)]^*\delta\chi^0_{K'}[\pm]\,\mathrm{tr}(\delta\chi_{I'}^{0\dagger}\Gamma_{I'}{}^{I}|H_B(b)_I{}^J|L_{JK}\beta^K(\lambda)[\mp])$$
$$+\tfrac{1}{2}V_{J'}{}^{LK'}[\beta^L(\lambda)]^*\delta\chi^0_{K'}\gamma_\mu\,\mathrm{tr}(\delta\chi_{I'}^{0\dagger}\Gamma_{I'}{}^{I}|H_B(b)_I{}^J|L_{JK}\beta^K(\lambda)\gamma_\mu)\}=-S_c^{-1/2}H_F(b)_{J'}{}^{L'}(i\nabla\!\!\!/\,\delta\chi^0)_{L'}\gamma_5\ . \qquad (4.12)$$

Substituting Eq. (4.12) in Eq. (4.8) we find

$$\varepsilon^{(1)}=-\frac{1}{8S_c}\,\mathrm{tr}(\delta\chi^{0\dagger}|H_F(b)|i\nabla\!\!\!/\,\delta\chi^0\gamma_5)=0\ . \qquad (4.13)$$

V. SECOND QUANTUM CORRECTION

The structure of the second quantum correction to the vacuum energy density $\varepsilon^{(2)}$ is much more complicated. It consists of 15 types of one-loop contributions to $\langle\det^{1/2}[E(\beta)]\rangle$ (see Fig. 4) and one tree diagram that arises from expanding $\det[I_{kl}(\beta)]$ in Eq. (2.35) (see Fig. 5). We point out that the diagrams in Fig. 6 vanish due to various conservation laws. Because of the complexity of the involved expressions we have not calculated the full expression for $\varepsilon^{(2)}$. Instead we will present a specific SUSY Higgs model in two space-time dimensions and prove that $\varepsilon^{(2)}$ is nonzero for that theory.

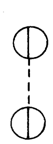

FIG. 3. A tree correction to the fermionic determinant.

The basic ingredient of our model is a supersymmetric version of the Abelian Higgs model (in fact it is the Euclidean analog of the model discussed in detail in Ref. 13). It consists of a U(1) gauge supermultiplet $\{A_\mu,\Lambda\}$ and three scalar supermultiplets $\{\Phi_q,\Psi_q\}$ with U(1) charges $q=+,-,0$. The superpotential is

$$W(\Phi)=y\Phi_0(\Phi_+\Phi_- - v^2)\ . \qquad (5.1)$$

Therefore

$$\sum_q|W_q(\Phi)|^2+\frac{e^2}{2}D^2(\Phi)=y^2|\Phi_+\Phi_- - v^2|^2+y^2|\Phi_\pm\Phi_0|^2$$
$$+\frac{e^2}{2}(|\Phi_+|^2-|\Phi_-|^2)^2\ . \qquad (5.2)$$

The bosonic potential (5.2) admits a supersymmetric minimum which breaks the gauge symmetry spontaneously:

$$\langle \Phi_+ \rangle = \langle \Phi_- \rangle = v, \quad \langle \Phi_0 \rangle = 0 . \tag{5.3}$$

The classical mass spectrum consists of a scalar supermultiplet of mass $\sqrt{2}\,m$ where $m = yv$ and a broken gauge multiplet of mass 2μ where $\mu = ev$. The R symmetry charges of the scalar fields are

$$Q_R(\Phi_0) = 1, \quad Q_R(\Phi_\pm) = 0 . \tag{5.4}$$

The topological Euclidean configuration of our model

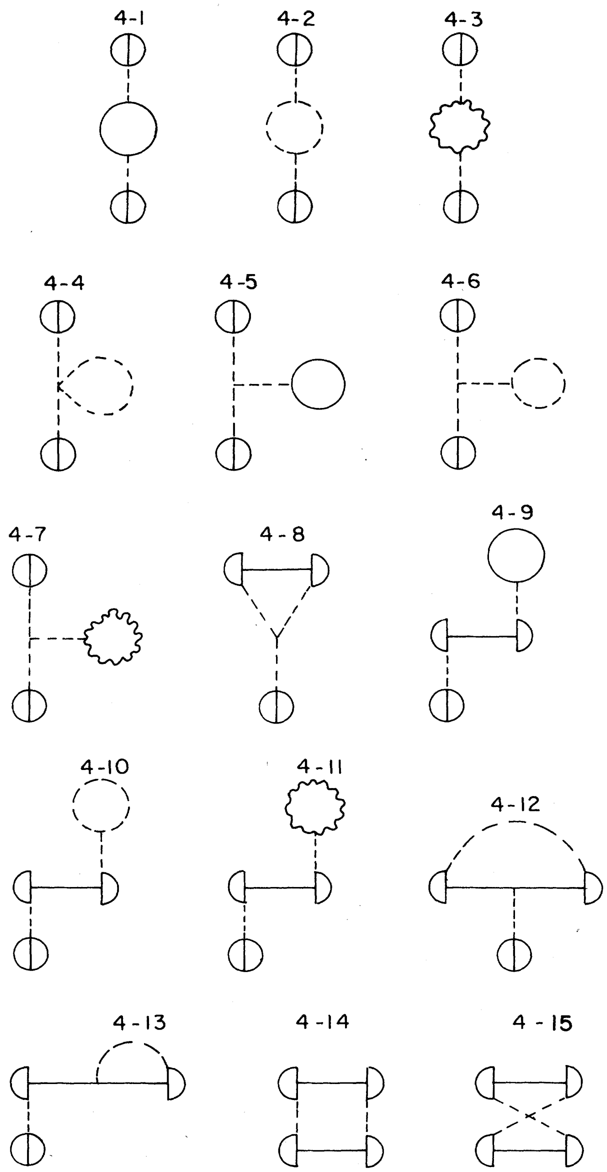

FIG. 4. One-loop corrections to the fermionic determinant. Wavy lines represent ghosts' propagators. A convenient procedure to identify all possible diagrams is the following. First find all the one-loop contributions to the fermionic four-point function, and then replace the external legs by fermionic zero modes.

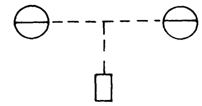

FIG. 5. A tree diagram which arises upon expanding $\det[I_{kl}(\beta)]$ in Eq. (2.35). The rectangle source is $-S_c^{-1/2} b_{I;l;l}$.

is the Nielsen-Olesen fluxon.[15] In a regular gauge[16] the background fields of the fluxon satisfy the ansatz

$$\langle \Phi_\pm(x) \rangle \equiv v\varphi_\pm(x) = v e^{\mp i\theta} \varphi(r) , \tag{5.5a}$$

$$\langle A_\theta(x) \rangle \equiv a_\theta(x) = \frac{a(r)}{er} , \tag{5.5b}$$

$$\langle \Phi_0(x) \rangle = \langle A_r(x) \rangle = \langle A_3(x) \rangle = \langle A_4(x) \rangle = 0 . \tag{5.5c}$$

For later convenience we have normalized the classical scalar fields in Eq. (5.5a) differently from Eq. (2.2). The asymptotic behavior of the fields is

$$\varphi(r) \xrightarrow[r\to\infty]{} 1, \quad a(r) \xrightarrow[r\to\infty]{} 1 . \tag{5.6}$$

The functions $\varphi(r)$ and $a(r)$ are solutions of the following nonlinear equations:

$$\left[-\partial_r^2 - \frac{1}{r}\partial_r + \left[\frac{1-a}{r} \right]^2 \right] \varphi + m^2(\varphi^2 - 1)\varphi = 0 , \tag{5.7a}$$

$$\left[-\partial_r^2 + \frac{1}{r}\partial_r \right] a + 4\mu^2 \varphi^2 (a-1) = 0 . \tag{5.7b}$$

An approximate solution of Eqs. (5.7) can be found in Ref. 13. The fermionic zero modes of our model are

$$\delta\Psi_0 = -S_c^{-1/2} v \gamma_5 m (\varphi^2 - 1) , \tag{5.8a}$$

$$\delta\Psi_\pm = S_c^{-1/2} i v \gamma_5 \nabla\!\!\!/\, \varphi_\pm = S_c^{-1/2} v \gamma_5 e^{\mp i\theta} \left[\gamma_r (i\partial_r \varphi) \pm \gamma_\theta \left[\frac{1-a}{r} \right] \varphi \right] , \tag{5.8b}$$

$$\delta\Lambda = S_c^{-1/2} v \frac{1}{i\mu\sqrt{2}} \gamma_5 \sigma_3 b , \tag{5.8c}$$

where

$$b(r) = \frac{\partial_r a(r)}{r}$$

is proportional to the magnetic field. We also record the structure of the total angular momentum in the fluxon sector:

$$J_3 = L_3 + S_3 + Q . \tag{5.9}$$

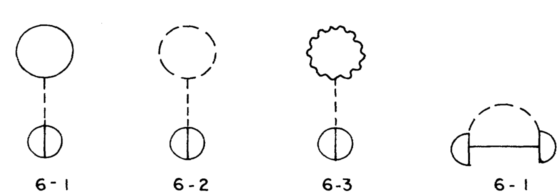

FIG. 6. Diagrams that vanish identically due to various conservation laws.

Here Q is the U(1) charge.

We now introduce an additional neutral sector which consists of two neutral-scalar supermultiplets $\{\Phi_1,\Psi_1\}$ and $\{\Phi_2,\Psi_2\}$. Their R symmetry charges are

$$Q_R(\Phi_1)=0,\quad Q_R(\Phi_2)=1\ . \tag{5.10}$$

The new superpotential is

$$W(\Phi)=y\Phi_0(\Phi_+\Phi_- - v^2)+z\Phi_0\Phi_1^2+m_1\Phi_1\Phi_2\ . \tag{5.11}$$

Consequently all fields in the new sector have mass m_1.

One can easily verify that the classical solution (5.5)–(5.7) is left unchanged and, in addition,

$$\langle \Phi_1(x)\rangle=\langle\Phi_2(x)\rangle=0\ . \tag{5.12}$$

Thus S_c and the zero modes (5.8) remain the same as before and

$$\delta\Psi_1=\delta\Psi_2=0\ . \tag{5.13}$$

The original sectors of $H_B(b)$ and $H_F(b)$ are left unchanged as well. Ψ_1 and Ψ_2 form a massive Dirac field which satisfies the free Dirac equation:[19]

$$\gamma_5\left[i\partial\!\!\!/-\begin{bmatrix}0 & m_1\\ m_1 & 0\end{bmatrix}\right]\begin{bmatrix}\Psi_1\\ \Psi_2\end{bmatrix}=\lambda\begin{bmatrix}\Psi_1\\ \Psi_2\end{bmatrix}\ . \tag{5.14}$$

The ϕ_2 field is a free massive bosonic field, whereas ϕ_1 and ϕ_{1*} satisfy the field equations

$$\begin{aligned}\left[-\partial_r^2-\frac{1}{r}\partial_r+\frac{l^2}{r^2}\right]\phi_1+m_1^2\phi_1+2mm_z(\varphi^2-1)\phi_{1*}&=\lambda^2\phi_1\ ,\\ \left[-\partial_r^2-\frac{1}{r}\partial_r+\frac{l^2}{r^2}\right]\phi_{1*}+m_1^2\phi_{1*}+2mm_z(\varphi^2-1)\phi_1&=\lambda^2\phi_{1*}\ ,\end{aligned} \tag{5.15}$$

where

$$m_z=zv\ . \tag{5.16}$$

In what follows we assume that the orders of magnitude of the mass parameters are related through

$$m\gg\mu,\quad m_1^2\gg mm_z\ . \tag{5.17}$$

Our purpose is to prove that under the assumptions (5.17) the m_z dependence of $\varepsilon^{(2)}$ is nontrivial. This dependence arises from two sources—interaction vertices and the ϕ_1 propagator. The bosonic vertices that depend on z are

$$\begin{aligned}&zm\phi_1^2\phi_{\pm*}\varphi_\pm+\text{H.c.},\quad 2zm_1\phi_0\phi_1\phi_{2*}+\text{H.c.}\ ,\\ &yz\phi_1^2\phi_{+*}\phi_{-*}+\text{H.c.},\quad 4z^2\phi_0\phi_1\phi_{0*}\phi_{1*},\quad z^2\phi_1^2\phi_{1*}^2\ .\end{aligned} \tag{5.18}$$

The (symmetrized) fermionic vertices are

$$-2z(\bar\Psi_1\gamma_5\tilde\Phi_1\Psi_0+\bar\Psi_1\gamma_5\tilde\Phi_0\Psi_1+\bar\Psi_0\gamma_5\Phi_1\Psi_1)\ . \tag{5.19}$$

The m_z dependence of the ϕ_1 propagator can be extracted by expanding in Born series:

$$\begin{aligned}G_{1^*,1}&=G_{1,1^*}\\ &=G_{m_1}\sum_{p=0}^{\infty}[4m^2m_z^2(\varphi^2-1)G_{m_1}(\varphi^2-1)G_{m_1}]^p\ ,\\ G_{1,1}&=G_{1^*,1^*}=-2mm_zG_{m_1}(\varphi^2-1)G_{1^*,1}\ .\end{aligned} \tag{5.20}$$

Here G_{m_1} is a free (two-dimensional) bosonic propagator of mass m_1:

$$(-\partial_x^2+m_1^2)G_{m_1}(x,y)=\delta^2(x-y)\ . \tag{5.21}$$

Our next task is to identify all diagrams that have some m_z dependence. Taking into account the preceding discussion and in particular Eqs. (5.12) and (4.7), we find that all these diagrams have a common property (see Fig. 7); namely, the fields that run through the loop always belong to the new neutral sector, whereas the fields on the external "tails" always belong to the original sector. This suggests the following strategy for extracting the m_z dependence of $\varepsilon^{(2)}$: by using various identities we will try to eliminate the external tails as much as possible, and replace them by local sources. Using Eq. (5.20), different contributions may then cancel each other, and we will end up with a relatively simple result.

The first step toward accomplishing the above program is to apply Eq. (4.7) and the following discussion. After that there remain tails only in the diagrams of Figs. 7-6, 7-10, and 7-13. We next observe that Eq. (4.12) applies to our model. Consequently the tail in Fig. 7-13 can be completely eliminated (see Fig. 8), as well as the fermionic propagator of the tail in Fig. 7-10. The sum of diagrams 7-6 and 7-10 now has the structure shown in Fig. 9, where the square source is given by

$$H_B(b)d'+d''\ , \tag{5.22}$$

where

$$d_I'=\begin{cases}\dfrac{v}{4S_c^2}(\nabla^2\varphi)_\alpha, & I=\alpha=+,-,0,1,2\ ,\\[2mm] \dfrac{v}{4S_c^2}(\nabla^2\varphi)_\alpha^*, & I=\alpha^*\ ,\\[2mm] -\dfrac{1}{4S_c^2}\partial_l f_{lk}, & I=k\ ,\end{cases} \tag{5.23a}$$

$$d''=\begin{cases}-\dfrac{2\mu^2v}{S_c^2}\left[\dfrac{1-a}{r}\right]^2\varphi^2\varphi_\alpha, & I=\alpha\ ,\\[2mm] -\dfrac{2\mu^2v}{S_c^2}\left[\dfrac{1-a}{r}\right]^2\varphi^2\varphi_\alpha^*, & I=\alpha^*\ ,\\[2mm] 0, & I=k\ .\end{cases} \tag{5.23b}$$

Another application of Eq. (2.24) gives rise to (see Appendix E for our notation conventions)

$$\varepsilon_6^{(2)}+\varepsilon_{10}^{(2)}=\varepsilon'+\varepsilon''\ , \tag{5.24}$$

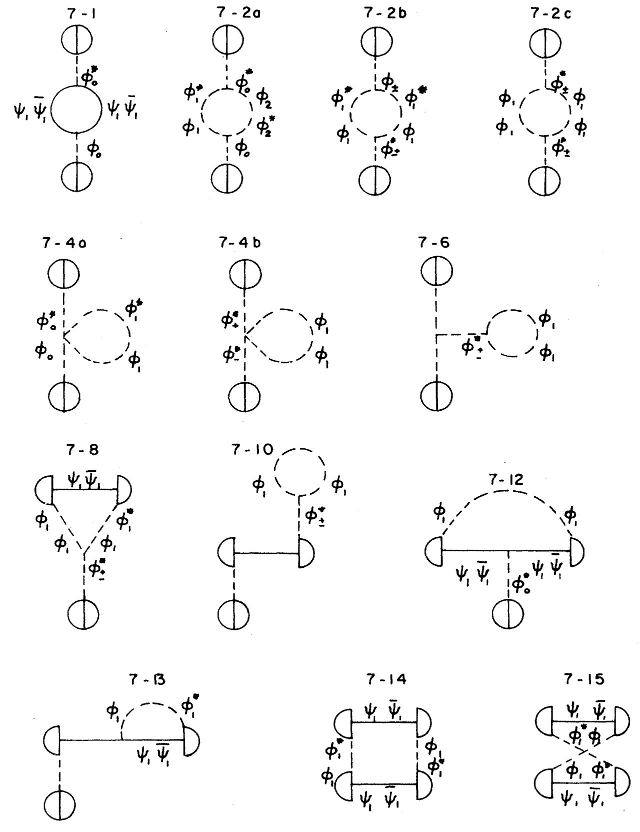

FIG. 7. Diagrams that have some m_z dependence. The various diagrams are labeled in agreement with Fig. 4.

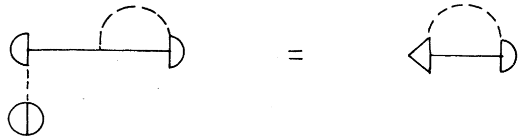

FIG. 8. Elimination of the "tail" in diagram 7-13 using Eq. (4.12). The triangle source is $i[(8S_c)^{-1}\not{\partial}\delta\Psi_0\gamma_5]^\dagger$.

where

$$\varepsilon'=\frac{mm_z}{2S_c^2}\,\mathrm{Tr}(G_{1,1}\varphi_\pm\nabla^2\varphi_\mp)\ , \tag{5.25}$$

and (see Fig. 10)

$$\varepsilon''=-8\frac{mm_z\mu^2}{S_c^2}\sum_{\lambda^2>0}\frac{1}{\lambda^2}\,\mathrm{Tr}[G_{1,1}\varphi\phi_R(\lambda)]\times\left[[\phi_R(\lambda)]^*\middle|\left[\frac{1-a}{r}\right]^2\varphi^3\right]\ , \tag{5.26}$$

$$\phi_R=\tfrac{1}{2}(e^{i\theta}\phi_+ + e^{-i\theta}\phi_- + e^{-i\theta}\phi_{+*} + e^{i\theta}\phi_{-*})\ . \tag{5.27}$$

A detailed list of the final expressions for all diagrams that have some m_z dependence is given in Appendix E. Applying Eqs. (5.20) and (5.21) to the results given in Appendix E we are now able to calculate the leading m_z dependence of $\varepsilon^{(2)}$. It is found that the contributions of all diagrams except ε'' cancel each other. Consequently,

$$\varepsilon^{(2)}_{m_z^2}=\varepsilon''_{m_z^2}=16\left[\frac{mm_z\mu}{S_c}\right]^2\int d^2x\,d^2y[\varphi^2(|x|)-1]G^2_{m_1}(x,y)h(|y|)\ , \tag{5.28}$$

$$h(r)=2\pi\sum_{\substack{\lambda^2>0\\ j_3=0}}\frac{1}{\lambda^2}\phi_R(r,\lambda)\int dr'r'[\phi_R(r',\lambda)]^*\left[\frac{1-a(r')}{r'}\right]^2\varphi^3(r')\ . \tag{5.29}$$

For zero total angular momentum, ϕ_R mixes only with the tangential component of the vector field a_θ. We thus define

$$h_\theta(r)=2\pi\sum_{\substack{\lambda^2>0\\ j_3=0}}\frac{1}{\lambda^2}a_\theta(r,\lambda)\int dr'r'[\phi_R(r',\lambda)]^*\left[\frac{1-a(r')}{r'}\right]^2\varphi^3(r')\ . \tag{5.30}$$

Using the explicit structure of $H_B(b)$ for zero total angular momentum, Eqs. (5.29) and (5.30) can be inverted as

$$\begin{bmatrix}\left[\frac{1-a}{r}\right]^2\varphi^3\\ 0\end{bmatrix}=\begin{bmatrix}-\partial_r^2-\frac{1}{r}\partial_r+\left[\frac{1-a}{r}\right]^2+2m^2\varphi^2+m^2(\varphi^2-1) & -4\mu\left[\frac{1-a}{r}\right]\varphi\\ -4\mu\left[\frac{1-a}{r}\right]\varphi & -\partial_r^2-\frac{1}{r}\partial_r+\frac{1}{r^2}+4\mu^2\varphi^2\end{bmatrix}\begin{bmatrix}h\\ h_\theta\end{bmatrix}\ . \tag{5.31}$$

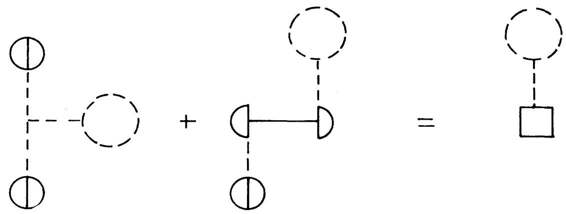

FIG. 9. Calculation of the sum of diagrams 7-6 and 7-10. The square source is given in Eq. (5.22).

Since the $(\alpha_\theta,\alpha_\theta)$ component of Eq. (5.31) is a positive-definite operator, it follows that $h(r)$ cannot vanish on any open neighborhood. Taking into account its asymptotic behavior we conclude that $h(r)$ has (at most) a finite number of zeros. Similar arguments imply that the convolution

$$g(r)=g(|x|)=\int d^2y[\varphi^2(|y|)-1]h(|x-y|) \qquad (5.32)$$

also has a finite number of zeros. Notice that $g(r)$ is determined only by properties of the original sector (5.1)–(5.9), and in particular is completely independent of m_1. We may now rewrite Eq. (5.28) as

$$\varepsilon^{(2)}_{m_z^2}=32\pi\left[\frac{mm_z\mu}{S_c}\right]^2\int dr\, rG^2_{m_1}(0,r)g(r)=32\pi\left[\frac{mm_z\mu}{S_c}\right]^2\int dr\, rK_0^2(m_1r)g(r)\ . \qquad (5.33)$$

Taken as a function of m_1, the last expression can be thought of as a generalized Laplace transform of $g(r)[K_0(m_1r)\sim\exp(-m_1r)]$. Hence Eq. (5.33) vanishes at most for a finite number of values of m_1. [As an example, choosing $m_1\gg m$ we can replace $g(r)$ by its leading power behavior near the origin, and so the RHS of Eq. (5.33) will necessarily be nonzero.] Thus $\varepsilon^{(2)}_{m_z^2}$ as well as $\varepsilon^{(2)}$ itself are in general nonzero.

Consider now Eq. (2.41) for the vacuum energy density. Because of the symmetries of our model it is clear that Ω_{fluxon} and $\Omega_{\text{antifluxon}}$ are equal (and nonzero). We now have to estimate the contributions of fluxons with higher topological charges. One can easily extend the results of Ref. 13 to obtain

$$S_c^{q_\tau}\approx 4\pi v^2q_\tau^2\ln\left[\frac{m}{\mu}\right]\approx q_\tau^2S_c\ . \qquad (5.34)$$

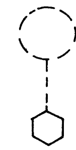

FIG. 10. Diagrammatic representation of Eq. (5.26).

Therefore all fluxons with higher topological charges do not contribute in the dilute-gas approximation. We finally obtain, for our model,

$$\epsilon_0=-2|\Omega_{\text{fluxon}}|<0\ . \qquad (5.35)$$

We now briefly comment on the generality of our result. First, since there is no generic difference between the original model (5.1)–(5.9) and the extended one, it should be concluded that the vacuum energy density of the original model is negative definite too. Second, since the derivation of Sec. II depends crucially on the assumption that all fields are massive, it is expected that relaxing assumptions (b) and (c) of Sec. III or considering three-dimensional theories, will not affect our result as well. We therefore conclude that dynamical breakdown of the SUSY algebra due to vacuum tunneling will take place in every massive SUSY Higgs theory in two and three space-time dimensions.

VI. SUSY FLUX LEAKAGE

We have found in Sec. V that tunneling lowers the vacuum energy density of SUSY theories in two and three space-time dimensions. As a result, the nonperturbative effects *necessarily violate the SUSY algebra.* Such a violation can be induced only if there exists a strong enough leakage of SUSY flux at the boundaries of space-time and in particular it implies that the SUSY current continuity equation is violated as well.

In this section we investigate the properties of the SUSY flux in detail, and obtain a quantitative description of the above statements. As in Sec. II, we assume a singular gauge for the background fields. The properties of the SUSY flux are best illustrated by looking at the nonperturbative corrections to physical quantities such as boson-fermion mass splitting. We work in a finite domain of space-time of volume $\mathcal{V}$. The finite-volume normalization of the quantum eigenstates is, therefore,

$$\chi(\lambda_F)\to\mathcal{V}^{-1/2}\chi(\lambda_F),\quad \beta(\lambda_B)\to\mathcal{V}^{-1/2}\beta(\lambda_B)\ . \qquad (6.1)$$

Using Eqs. (3.35) and (3.39) we obtain the (linearized) SUSY Ward identity:

$$\left\langle\chi(\mathbf{x})\left[\oint d\sigma_kS_k\right]\beta^T(\mathbf{y})L\right\rangle=H_F(\langle B\rangle)\Gamma\langle L\beta^*(\mathbf{x})\beta^T(\mathbf{y})L\rangle-\langle\chi(\mathbf{x})\bar\chi(\mathbf{y})\rangle\Gamma=\mathcal{V}^{-1}[H_F(\langle B\rangle)\Gamma\tilde G_B(\mathbf{x},\mathbf{y})-\tilde G_F(\mathbf{x},\mathbf{y})\Gamma]\ . \qquad (6.2)$$

The linearized SUSY current is

$$S_k=i\bar\chi\gamma_5\gamma_kH_F(b)\Gamma L\beta^*\ . \qquad (6.3)$$

In Eq. (6.2) $\langle B\rangle$ stands for the full vacuum expectation values of the bosonic fields, to be evaluated after having calculated the nonperturbative corrections to the bosonic potential. Similarly, the full bosonic (fermionic) propagator $\tilde G_B$ $(\tilde G_F)$ should be considered as a finite-volume propagator whose mass matrix M_B (M_F) has to be determined. Obviously,

$$M_B^2=(M_0+\delta M_B)^2, \quad M_F=M_0+\delta M_F \ , \tag{6.4}$$

where M_0 is the classical mass matrix and δM_B (δM_F) is a small nonperturbative correction.

As was explained in Sec. II, the full vacuum is dominated by finite density of topologically nontrivial configurations. We thus begin with a calculation of matrix elements of the SUSY flux in the background field of N classical lumps (which we take to be identical for notational simplicity). A general eigenstate which consists of an incoming plane wave that scatters off N identical (well separated) centers has the structure

$$\psi(\mathbf{x})=e^{-i\mathbf{p}\cdot\mathbf{x}}+\sum_{n=1}^{N} e^{-i\mathbf{p}\cdot\mathbf{x}^{0,n}}\frac{e^{ipr'_n}}{(pr'_n)^{(d-1)/2}}f_{\mathbf{p}}(\hat{\mathbf{r}}'_n) \ . \tag{6.5}$$

Here $\mathbf{x}^{0,n}$ are the collective coordinates of the nth lumps and

$$r'_n=|\mathbf{x}-\mathbf{x}^{0,n}|, \quad \hat{\mathbf{r}}'_n=\frac{\mathbf{x}-\mathbf{x}^{0,n}}{r'_n} \ .$$

The additional phase $\exp(-i\mathbf{p}\cdot\mathbf{x}^{0,n})$ is introduced to compensate for the position of the nth lump. Equation (6.5) exhibits the fact, valid within the dilute-gas approximation, that the scattered waves of different lumps are uncorrelated. Equation (6.5) applies everywhere, except inside the cores of each lump.

Matrix elements of the SUSY flux are constructed from bosonic and fermionic eigenstates whose generic structure is similar to Eq. (6.5). Because of translational invariance only flux portions which are independent of $\mathbf{x}^{0,n}$ may contribute to the LHS of Eq. (6.2). A glance at Eq. (6.5) reveals two necessary conditions.

(a) The incoming bosonic and fermionic momenta are equal:

$$\mathbf{p}_B=\mathbf{p}_F \ . \tag{6.6}$$

(b) The flux is due to bosonic and fermionic waves that scatter off the same lump.

An immediate consequence of property (b) is that the contributions of different lumps to the total flux are additive. Using the above properties we are able to calculate the LHS of Eq. (6.2):

$$\begin{aligned}\langle \chi(\mathbf{x})\left[\oint d\sigma_k S_k\right]\beta^T(\mathbf{y})L\rangle &=\frac{1}{\mathcal{V}^2 Z}\sum_{N_\tau=1}^{\infty}\delta_{\sum_{\rho'} N_{\rho'}q_{\rho'},0}\prod_{\sigma}\frac{(\mathcal{V}\Omega_\sigma)^{N_\sigma}}{N_\sigma!}\sum_{\mathbf{p}}\frac{1}{\lambda_F\lambda_B^2}\chi(\mathbf{x},\mathbf{p})\left[\oint d\sigma_k S_k(b^{(N_\tau)};\mathbf{p})\right]\beta^T(\mathbf{y},\mathbf{p})L\\ &=\frac{1}{\mathcal{V}^2 Z}\sum_{N_\tau=1}^{\infty}\delta_{\sum_{\rho'} N_{\rho'}q_{\rho'},0}\left]\prod_{\sigma}\frac{(\mathcal{V}\Omega_\sigma)^{N_\sigma}}{N_\sigma!}\sum_{\mathbf{p}}\frac{1}{\lambda_F\lambda_B^2}\chi(\mathbf{x},\mathbf{p})\sum_\rho N_\rho\left[\oint d\sigma_k S_k(b^{(\rho)};\mathbf{p})\right]\beta^T(\mathbf{y},\mathbf{p})L\right.\\ &=\frac{1}{\mathcal{V}}\sum_{\tau}e^{i\theta_0 q_\tau}\Omega_\tau\sum_{\mathbf{p}}\frac{1}{\lambda_F\lambda_B^2}\chi(\mathbf{x},\mathbf{p})\left[\oint d\sigma_k S_k(b^{(\tau)};\mathbf{p})\right]\beta^T(\mathbf{y},\mathbf{p})L \ .\end{aligned} \tag{6.7}$$

The θ_0 phase is defined in Eq. (2.42), $b^{(N_\tau)}$ is the background field of N_τ lumps of type τ and $b^{(\rho)}$ is the background field of a single lump of type ρ. The transition from the third row to the last row is justified, since in the infinite-volume limit the partition function is dominated by the true vacuum.

Comparing Eqs. (6.7) and (6.2) we find that the $\mathcal{V}$ dependence cancels. We thus see that additivity of the contributions of different lumps renders the total flux proportional to the volume of space-time, and that this phenomenon is indeed necessary in order to obtain any physical consequences.

So far we have used only the kinematical structure of the SUSY flux. (In fact, the preceding discussion applies to any other symmetry.) What we have learned is the following: the dynamical question is whether there exists a nonzero leakage of SUSY flux due to the presence of a single instanton.

Since we now have to consider a single scattering center, it is possible (and more convenient) to expand in partial waves. To simplify the kinematical structure we limit our discussion to the case of elastic scattering only. (Consequently our discussion applies without modifications when one is below the second Euclidean mass threshold.) A bosonic eigenstate with a definite total angular momentum has the general structure

$$\beta_{I(J)}=[\delta_{IJ}e^{-ipr}+(\mathcal{S}_B)_{IJ}e^{ipr}](pr)^{(1-d)/2} \ , \tag{6.8}$$

where $\mathcal{S}_B$ is the quantum-mechanical scattering matrix. Here I labels the various channels of a given eigenstate and J labels different eigenstates which involve an incoming wave in channel J.

To construct a similar decomposition of fermionic eigenstates we first notice that, for large r,

$$H_F(b)\approx\gamma_5(i\gamma_r\partial_r-M_0), \quad \gamma_r=\frac{1}{r}\gamma_k x_k \ . \tag{6.9}$$

Consequently one can construct a fermionic eigenstate from pairs of radial channels (γ_5 is diagonal)

$$\chi_{I'}=[g_{I'}(r)+i\gamma_5\gamma_r h_{I'}(r)]\begin{bmatrix}Y_{I'}(\hat{r})\\0\end{bmatrix} \ . \tag{6.10}$$

The two-component spinors $Y_{I'}(\hat{\mathbf{r}})$ are eigenstates of the total angular momentum. [Strictly speaking, here the capital indices I (I') label all radial channels of a given bosonic (fermionic) eigenstate, and so their meaning is somewhat different from previous sections.] The $g_{I'}(r)$ channels can now be expanded similarly to Eq. (6.8), whereas the $h_{I'}(r)$ channels are related to the $g_{I'}(r)$ channels through Eqs. (6.9) and (6.10). Using Eqs. (6.3) and (6.8)–(6.10) we find

$$\oint d\sigma_k S_k(b;p,j)=\oint d\sigma_k i\chi^\dagger(p,j)\gamma_5\gamma_k H_F(b)\Gamma L\beta_*(p,j) \propto p^{2-d}(\Gamma-\mathcal{S}_F^\dagger\Gamma\mathcal{S}_B)\ . \tag{6.11}$$

The proportionality constant in Eq. (6.11) depends on the particle's mass and on the total angular momentum j. Note that the dependence on the surface radius $(r \gg \|M_0^{-1}\|)$ cancels out and the RHS of Eq. (6.11) is finite. The Γ matrix merely serves to match the bosonic and fermionic indices. Hence what we have on the RHS of Eq. (6.11) is, generically,

$$\text{flux} \propto I-\mathcal{S}_F^\dagger\mathcal{S}_B\ . \tag{6.12}$$

We now recall that, because of Eq. (3.43),

$$\mathcal{S}_F\neq\mathcal{S}_B \tag{6.13}$$

for every massive field theory, and so the *SUSY flux (6.11) is in general nonzero.* Conversely, were $\mathcal{S}_B$ and $\mathcal{S}_F$ equal, unitarity would have ensured the vanishing of Eq. (6.12).

To obtain an explicit expression for the mass splitting we take the Fourier transform of Eqs. (6.2) and (6.7) and extract the coefficient of $\gamma_5 \not{p}$. The result is

$$M_F^2(p^2)\Gamma-\Gamma M_B^2(p^2)=\sum_\tau e^{i\theta_0 q_\tau}\Omega_\tau\oint d\sigma_k S_k(b^{(\tau)};\mathbf{p})\ . \tag{6.14}$$

The analytic continuation of Eq. (6.14) to $p^2=-M^2$ provides the general expression for the boson-fermion mass splitting which is generated due to the nonperturbative breakdown of the SUSY algebra.

It is interesting to investigate the possible phenomenological implications of Eq. (6.14). Clearly, before such an investigation can be carried out in detail, there are several conjectures which have to be confirmed: first, that explicit SUSY breakdown due to vacuum tunneling occurs in four dimensions as well and, second, that the results of this paper can be extended to partially massless chiral theories as well. The possibility of explicit SUSY breaking in four dimensions is discussed in the last section. As an illustrative example let us assume that the results of Sec. V remain valid if we add a massless sector to the model discussed there. The particularly simple kinematics of a massless sector allows further simplification of the RHS of Eq. (6.14). To this end we first apply Eq. (3.43) to obtain an expression for $\partial_k S_k$:

$$\begin{aligned}\partial_k S_k(b;\lambda_F,\lambda_B)&=\chi^\dagger(\lambda_F)[-\overleftarrow{H}_F(b)+\overrightarrow{H}_F(b)]\overrightarrow{H}_F(b)\Gamma L\beta_*(\lambda_B)\\&=(\lambda_B^2-\lambda_F^2)\chi^\dagger(\lambda_F)\Gamma L\beta_*(\lambda_B)-i\lambda_F\partial_k[\chi^\dagger(\lambda_F)\gamma_5\gamma_k\Gamma L\beta_*(\lambda_B)]-S_c^{1/2}\chi^\dagger(\lambda_F)V(\beta(\lambda_B))\delta\chi^0\ .\end{aligned} \tag{6.15}$$

The first term in the last row has no physical consequences since it vanishes for $\lambda_F=\lambda_B$ [see Eq. (6.6)]. Proceeding along the lines of Eqs. (6.8)–(6.11) we find that the flux leakage due to the total divergence term in the last row is precisely opposite to the flux leakage due to $\partial_k S_k$. Consequently,

$$\begin{aligned}\oint d\sigma_k S_k(b;\lambda)&=\tfrac{1}{2}\oint d\sigma_k[S_k(b;\lambda)-i\lambda\chi^\dagger(\lambda)\gamma_5\gamma_k\Gamma L\beta_*(\lambda)]-\tfrac{1}{2}S_c^{1/2}(\chi^\dagger(\lambda)|V(\beta(\lambda))|\delta\chi^0)\\&=-\tfrac{1}{2}S_c^{1/2}(\chi^\dagger(\lambda)|V(\beta(\lambda))|\delta\chi^0)\ .\end{aligned} \tag{6.16}$$

Substituting Eq. (6.16) into Eq. (6.14) we find, for the massless sector,

$$M_F^2\Gamma-\Gamma M_B^2=-\tfrac{1}{2}\sum_\tau (S_c^\tau)^{1/2}e^{i\theta_0 q_\tau}\Omega_\tau(\chi_\tau^\dagger(p=0)|V(\beta_\tau(p=0))|\delta\chi_\tau^0)\ . \tag{6.17}$$

To leading approximation we can assume that the $p=0$ eigenstates are equal to one over the core of the instanton. Hence,

$$(M_F^2\Gamma-\Gamma M_B^2)_{I'}{}^K=-\tfrac{1}{2}\sum_\tau (S_c^\tau)^{1/2}e^{i\theta_0 q_\tau}\Omega_\tau V_{I'}{}^{KJ'}\int d^dx\ \delta\chi_{\tau J'}^0(x)\ . \tag{6.18}$$

We now observe that $\delta\Psi_\pm$ [see Eq. (5.8b)] does not contribute to the RHS of Eq. (6.18) because $\nabla_k\varphi_\pm$ are vectorial quantities. [Since we use a singular gauge in this section, the $\exp(\mp i\theta)$ factor of Eq. (5.8b) is absent from Eq. (6.18).] A similar conclusion applies to $\delta\Lambda$ (5.8c) after summing the contributions of fluxon and antifluxon. (We recall that these are the only important contributions in the dilute-gas approximation.) Therefore only $\delta\Psi_0$ (5.8a) contributes to the RHS of Eq. (6.18), thus giving rise to

$$(M_F^2\Gamma-\Gamma M_B^2)_{I'}{}^K=-S_c^{1/2}|\Omega_{\text{fluxon}}|V_{I'}{}^{K0}\int d^2x\ \delta\Psi_0(x)\ .$$

Hence under our assumption the particles of the massless sector have indeed acquired a nontrivial mass matrix which violates the SUSY algebra.

VII. DISCUSSION AND CONCLUSIONS

In this paper we proved that dynamical breakdown of the SUSY algebra due to vacuum tunneling takes place in two and three space-time dimensions. We also identified the physical mechanism which generates SUSY violation, namely, the discrepancy between the bosonic and fermionic scattering matrices in an instanton background induces a nonzero and, in fact, extensive leakage of SUSY flux. Indeed, the expression for boson-fermion mass splitting derived in Sec. VI is directly related to the SUSY flux leakage. We conjecture that the full expression for the one-loop correction to the vacuum energy density, when calculated, will also reveal an inherent dependence on the SUSY flux leakage.

It should be emphasized that what we actually proved in Sec. VI is the following: a nonzero measure for classical configurations implies that matrix elements of the SUSY flux are, in general, nonzero as well. Only because of technical difficulties have we not established the opposite assertion: namely, that the measure of classical configurations is nonzero due to the existence of a nonzero SUSY flux leakage. A more sophisticated calculation of the one-loop correction to the fermionic determinant should make use of all four identities (3.42)–(3.45), and would necessarily involve integrations by parts. These integrations by parts would give rise to surface contributions proportional to the flux of the full (nonlinear) SUSY current. We conjecture that it is precisely those terms that would survive after all algebraic cancellations have been taken into account, and which are therefore responsible for the nonvanishing of the one-loop correction to the vacuum energy density.

In fact, the master identity (3.38), which contains all the SUSY information and governs all the cancellations between differential operators and vertices can be rewritten as

$$\chi^\dagger\Gamma\left[\overleftarrow{H}_B(b)L\beta_* + \frac{\delta\mathcal{L}_B^{\rm int}}{\delta(L\beta_*)}\right] - \chi^\dagger[\overleftarrow{H}_F(b)+V(\beta)]\delta\chi(b+\beta)=\partial_k S_k(\chi,b+\beta)\,, \tag{7.1}$$

where χ is an arbitrary c-number spinor and $S_k(\chi,b+\beta)$ is the full SUSY current. Equation (7.1) is then used to express the interaction vertex $(\chi^+|V(\beta)|\delta\chi^0)$ in terms of diagrams in which the adjacent bosonic and fermionic propagators have been eliminated in turn, higher loop graphs, and insertions of the SUSY flux. We thus expect that due to the existence of a flux leakage, the application of the identities (3.38) and (3.42)–(3.45) which amounts to using *all* the SUSY information in the theory, would fail to ensure the vanishing of the quantum corrections to the fermionic determinant (see Fig. 11).

It is interesting to investigate whether solitons could also induce explicit SUSY breaking, being another topological effect of gauge theories. This possibility was elaborated in Ref. 13, where it was found that the one-loop correction to the soliton-solitino mass splitting vanishes identically. However, a second look reveals that the one-loop contribution to the soliton-solitino mass splitting is analogous to the tree correction for the zero fermionic eigenvalues in the instanton background. We therefore expect that a two-loop calculation will give rise to a nonzero soliton-solitino mass splitting.

The most interesting question concerning possible application of the above phenomenon to the real world is, of course, whether dynamical breakdown of the SUSY algebra takes place in four dimensions. When dealing with massive theories there is a qualitative difference between four and lower dimensions. The reason is that due to the scale invariance of the underlying massless theory, massive four-dimensional theories do not admit exact instanton solutions. This has two major consequences: the first is technical in nature, namely, the integration over scales, which is a classical symmetry of the massless theory, has to be replaced by integration over a constraint parameter;[17] the second and more crucial consequence is that the SUSY variation of the classical fields (3.40) no longer generates fermionic zero modes. Instead, fermionic zero modes exist in four dimensions due to index theorems. (For massive theories only recently proved open space index theorems are applicable.)[18] A strong indication that the four-dimensional zero modes are not related to SUSY in some indirect way is that their number is not equal to the number of supersymmetries. The number of zero modes is model dependent, and, in fact, one can construct explicit examples for which the index is zero.

Obviously, the construction of a massive SUSY theory with no zero modes at all (and not only zero index) would prove that explicit SUSY breaking takes place in four dimensions. However, even if the index is nonzero (and this is the generic case) it is still most likely that explicit SUSY breaking will occur for the following reason: massive theories admit nontrivial index structure only in the presence of an R symmetry (see Sec. III). An exact R symmetry would imply that the index is conserved not only under classical deformations which respect the R symmetry, but also under quantum fluctuations. However, in four dimensions the R symmetry is always anomalous. In particular, if due to the anomaly, a field with nonzero R symmetry charge acquires a nonzero expectation value in the constrained instanton background, the index will no longer be conserved. Under these circumstances a nonzero quantum correction to the fermionic determinant will be generated, and consequently explicit SUSY breaking will be induced. A more detailed investigation along the lines described above will be presented elsewhere. We hope that it may also clarify certain subtleties concerning the SUSY anomaly in four dimensions.[1]

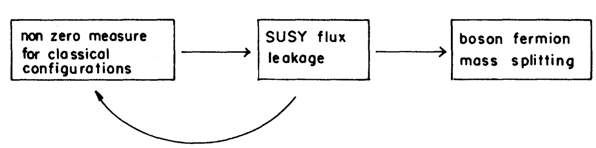

FIG. 11. The physical mechanism which induces the nonperturbative breaking of the SUSY algebra.

Finally, we comment that if it is proved that explicit SUSY breaking is generated in partially massless chiral theories as well, the induced mass matrix for classically massless particles [see Eqs. (6.17)–(6.19)] may play some role in resolution of the hierarchy problem in GUT's.

ACKNOWLEDGMENTS

This work was supported in part by the U.S.-Israel Binational Science Foundation and the Israel Academy of Sciences.

APPENDIX A

In this appendix we calculate the Jacobian that arises in replacing the bosonic zero modes amplitudes by collective coordinates, and prove Eq. (2.22). To this end we show that the local part of $\partial\mathcal{C}_l/\partial x_k^0$ is

$$\left[\frac{\partial\mathcal{C}_l}{\partial x_k^0}\right]_{\rm local}=S_c^{1/2}(b_{;k}|b_{;l})+(\beta_{;k}|b_{;l})=S_c^{1/2}\delta_{kl}+(\beta_{;k}|b_{;l}) \tag{A1}$$

and prove that the nonlocal part does not contribute to the partition function.

Before going into details of the proof, it is useful to get a qualitative understanding of the nonlocal terms by considering a specific family of paths $\{\Upsilon(x^0)\}$. We define $\Upsilon(x^0)$ as follows: we first go from the origin along a straight line to the point $(x_1^0,0,\ldots,0)$, then from $(x_1^0,0,\ldots,0)$ to $(x_1^0,x_2^0,0,\ldots,0)$ and so on, until we reach $x^0=(x_1^0,x_2^0,\ldots,x_d^0)$. Calculating the derivative of the parallel transport

$$U(x,x^0)=P\left[\exp\left[-ie\int_{\Upsilon(x^0)}dy_j a_j(x-y)\right]\right], \tag{A2}$$

we find (see Fig. 12)

$$\frac{\partial U(x,x^0)}{\partial x_k^0}=-ieU(x,x^0)a_k(x-x^0)-ie\int_{\Upsilon(x^0)-\Upsilon_{(k)}(x^0)}dy_j\,U(x,y)f_{jk}(x-y)U^\dagger(x,y)U(x,x^0), \tag{A3}$$

where $\Upsilon_{(k)}(x^0)$ is the path from the origin to the point $(x_1^0,x_2^0,\ldots,x_k^0,0,\ldots,0)$. The first term on the RHS of Eq. (A3) together with the simple fields derivative give rise to the local part of the Jacobian (A1). The second term gives rise to a nonlocal contribution.

We will now prove that the nonlocal part of the Jacobian does not contribute to the partition function by showing that its contribution can be made arbitrarily small. To this end we divide space-time into cubes $\{\Box_p\}$ of size Δ. For a point x^0 that belongs to a given cube $\Box_{p_0}$ we define a new path $\tilde{\Upsilon}(x^0)$ as follows: we first go from the origin to the center of $\Box_{p_0}$ along an arbitrary fixed path. We then define the path from the center of $\Box_{p_0}$ to x^0 analogous to the definition of the path $\Upsilon(x^0)$ from the origin to x^0 (see Fig. 13).

For all inner points of the cubes the nonlocal part of the Jacobian is clearly $O(\Delta)$ and hence can be made arbitrarily small by choosing a sufficiently small Δ. However, the nonlocal part also receives δ function contributions from the boundaries of the cubes. Since the boundary area is $O(1/\Delta)$, the total boundary contribution to the RHS of Eq. (2.17) is $O1$.

To show that the boundary term is not physical and does not contribute to the partition function we will now somewhat modify the Faddeev-Popov procedure: we enlarge each cube to an open cube $\Box'_p$ of size $\Delta+2\epsilon$ (see Fig. 13), and replace Eq. (2.17) by

$$n(B)=\sum_p\int_{\Box'_p}dx_k^0\left|\det\left[\frac{\partial\mathcal{C}_l}{\partial x_k^0}\right]\right|\delta(\mathcal{C}_l)\,. \tag{A4}$$

A configuration $B(x)$ that belongs to the one-instanton sector can always be constructed from an instanton located at a given point plus quantum fluctuations which are orthogonal to the bosonic zero modes. (Since the bosonic zero modes correspond to infinitesimal translations of the instanton, including them in the allowed quantum fluctuations would amount to overcounting of the actual configurations.) The function $n(B)$ is therefore equal to the number of open cubes to which the instanton's center of mass belong.

Using paths defined analogous to $\tilde{\Upsilon}(x^0)$, we now observe that due to the absence of boundary surfaces there is no singular contribution to the nonlocal part of the Jacobian. Consequently the nonlocal part is always $O(\Delta+2\epsilon)$. Moreover, when substituting Eq. (A4) in the partition function, the deviation of the LHS from one

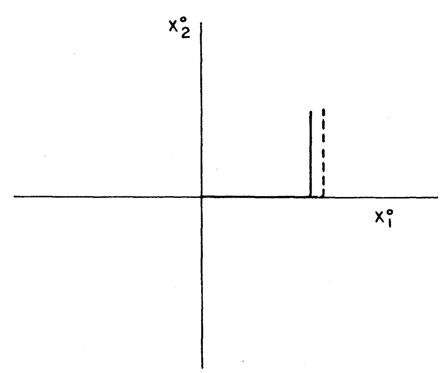

FIG. 12. A typical path $\Upsilon(x^0)$ in two-dimensional spacetime. The dashed line shows the varies path $\Upsilon(x^0+\delta x_1^0,0)$ where $\delta x^0=(\delta x_1^0,0)$.

leads to violation of Eq. (2.22) which is $O(\epsilon/\Delta)$. Therefore the total violation of Eq. (2.22) is $O(\Delta+2\epsilon+\epsilon/\Delta)$, and by appropriately choosing Δ and ϵ it can be made arbitrarily small. This completes the proof of Eq. (2.22).

APPENDIX B

In order to prove Eqs. (2.32) and (2.33) we reintroduce the fermionic path integral. Let $\chi_n^0, n=1,\ldots,N$, be the fermionic zero modes, ζ_n the zero-mode amplitudes, and let χ be orthogonal to all zero modes. Taking into account that the fermionic variables are Majorana spinors we have

$$\mathrm{Det}^{1/2}[D_F(B)] = \int [d\chi] d\zeta \exp[\tfrac{1}{2}(\bar\chi+\bar\chi_m^0\zeta_m | D_F(B) | \chi+\chi_n^0\zeta_n)] , \tag{B1}$$

$\bar\chi(\bar\chi_n^0)$ is related to $\chi(\chi_n^0)$ through Eq. (3.22). We construct the fermionic propagator G_F only from the nonzero eigenstates of $H_F(b)$ [see Eqs. (2.28)–(2.31)]. Integration over $[d\chi]$ gives rise to

$$\mathrm{Det}^{1/2}[D_F(B)] = \underline{\mathrm{Det}}^{1/2}[D_F(B)] \times \int d\zeta \exp[\tfrac{1}{2}\zeta_m E'_{mn}(\beta)\zeta_n] , \tag{B2}$$

$$E'_{mn}(\beta) = \left[\bar\chi_m^0 \left| V(\beta) \sum_{p=0}^{\infty} [-G_F V(\beta)]^p \right| \chi_n^0 \right] . \tag{B3}$$

Similarly to the fermionic propagator, the functional determinant in Eq. (B2) is constructed only from the nonzero eigenstates of $H_F(b)$. The $p=0$ term in the expansion of $E'_{mn}(\beta)$ comes from substituting the zero modes in the fermionic Lagrangian. All higher terms arise because performing the functional integration only over the nonzero modes is equivalent to treating both bosonic fields and fermionic zero modes as external sources.

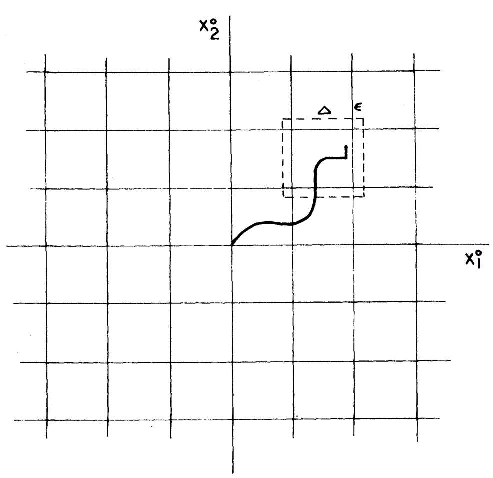

FIG. 13. Dividing space-time into cubes $\{\Box_p\}$ of size Δ. The thick line shows a typical path $\tilde\Upsilon(x^0)$. The dashed line corresponds to one enlarged open cube $\Box'_p$.

Consequently besides closed fermionic loops one also obtains open fermionic lines with zero modes at the ends and any finite number of $V(\beta)$ insertions along the line.

We now observe that due to Hermiticity and Majorana symmetries of $H_F(b)$, $\bar\chi_n^0$ is a permutation of $\chi_n^{0\dagger}$ [see in particular Eq. (3.26)]. Consequently performing the integration over $d\zeta$ we find

$$\mathrm{Det}^{1/2}[D_F(B)] = \underline{\mathrm{Det}}^{1/2}[D_F(B)] \det{}^{1/2}[E_{mn}(\beta)] , \tag{B4}$$

where

$$E_{mn}(\beta) = \left[\chi_m^{0\dagger} \left| (\beta) \sum_{p=0}^{\infty} [-G_F V(\beta)]^p \right| \chi_n^0 \right] . \tag{B5}$$

APPENDIX C

A SUSY theory admits an R symmetry provided one can assign charges to the scalar fields, such that the superpotential transforms with charge $Q_R[W(\Phi)]=1$. In this case the various fields have the following charges:

$$Q_R(A_\mu^c)=0 , \tag{C1}$$

$$Q_R(\Lambda_c)=\tfrac{1}{2} , \tag{C2}$$

$$Q_R(\Psi_\alpha)=Q_R(\bar\Psi_\alpha)=Q_R(\Phi_\alpha)-\tfrac{1}{2} . \tag{C3}$$

Using the $\tilde\Phi$ notation for the scalar fields [see Eq. (3.21)], the generator of R transformations is

$$Q_5=Q_R\gamma_5 . \tag{C4}$$

APPENDIX D

The fermionic SUSY Lagrangian in Minkowski space is defined through

$$\mathcal{L}_F^M = \frac{i}{2}\bar\Psi^M \not{D}_5 \Psi^M + \frac{i}{2}\bar\Lambda^M \not{D}_5 \Lambda + ie\sqrt{2}\,\bar\Lambda_c^M \Phi_5^\dagger T_5^c \Psi^M - \tfrac{1}{2}\bar\Psi_\alpha^M W_{\alpha\beta}(\Phi_5)\Psi_\beta^M ,$$

$$W_\alpha = \frac{\partial W}{\partial \Phi_\alpha}, \quad \text{etc.} ,$$

$$\Phi_5 = [+]\Phi + [-]\Phi^* = \tfrac{1}{2}(1+\gamma_5)\Phi + \tfrac{1}{2}(1-\gamma_5)\Phi^* , \tag{D1}$$

$$T_5^c = [+]T^c + [-](-T^c)^* ,$$

$$D_{\mu 5} = \partial_\mu + ieA_\mu^c T_5^c .$$

The Majorana fermions χ^M are constructed from Weyl fermions as

$$\chi^M \equiv \begin{bmatrix} \chi_W \\ i\sigma_2\chi_W^* \end{bmatrix}, \quad \bar\chi^M = i(\chi^M)^T\gamma_0\gamma_2 . \tag{D2}$$

In going to Euclidean space we perform the standard continuation to imaginary time and make the replacement

$$\chi^M \to \chi_5 \equiv \begin{bmatrix} \chi_R \\ i\sigma_2\chi_L^* \end{bmatrix} . \tag{D3}$$

We recall that in Euclidean space χ_L and χ_R are not conjugate. The resulting fermionic Lagrangian has the general form

$$\mathcal{L}'_F=\tfrac{1}{2}\bar{\chi}_5\hat{D}(B)\chi_5 \; . \tag{D4}$$

We now observe that the eigenvalue equation

$$\hat{D}(B)\chi_5=\lambda\chi_5 \tag{D5}$$

is inconsistent with gauge symmetry, since $\hat{D}(B)\chi_5$ has the transformation properties of $i\sigma_2\chi_5^*$. In general the only way out is to consider the eigenvalue equation (here χ_5 and $i\sigma_2\chi_5^*$ are treated as independent variables)

$$\begin{bmatrix} 0 & \hat{D}^\dagger(B) \\ \hat{D}(B) & 0 \end{bmatrix}\begin{bmatrix} \chi_5 \\ i\sigma_2\chi_5^* \end{bmatrix}=\lambda\begin{bmatrix} \chi_5 \\ i\sigma_2\chi_5^* \end{bmatrix} . \tag{D6}$$

Equation (D6) involves redoubling of the number of fermionic degrees of freedom relative to the original Weyl fermions.

However, when the matter fields belong to a real representation of the gauge group, χ_5 and $i\sigma_2\chi_5^*$ have fermionic components with identical transformation properties, only in different order. Consequently the operator [see Eq. (3.2)]

$$D_F(B)=i\gamma_5([+]L+[-]I)\hat{D}(B)([+]I+[-]L) \tag{D7}$$

gives rise to an eigenvalue equation which respects all the relevant symmetries. Clearly,

$$\mathrm{Det}^{1/2}[D_F(B)]=\det{}^{1/2}[\hat{D}(B)] \; . \tag{D8}$$

We may therefore define the fermionic sector of the theory using $D_F(B)$. The explicit structure of $D_F(B)$ is given in Eq. (3.20).

APPENDIX E

We give below a detailed list of the final expressions for all diagrams that have some m_z dependence (see Fig. 7). Our notation conventions can be read off Eqs. (E1) and (E4). The following diagrams contribute to order m_z^2 [note that due to Eq. (5.20) $G_{1,1}=O(m_z)$]:

$$\begin{aligned}\varepsilon_1^{(2)}&=4S_c^{-2}(mm_z)^2\,\mathrm{Tr}[(\varphi^2-1)(\partial_k G_{m_1})(\varphi^2-1)(\partial_k G_{m_1})]\\&=4S_c^{-2}(mm_z)^2\int d^2x\, d^2y[\varphi^2(|x|)-1]\left[\frac{\partial}{\partial x_k}G_{m_1}(x,y)\right][\varphi^2(|y|)-1]\left[\frac{\partial}{\partial y_k}G_{m_1}(y,x)\right] ,\end{aligned} \tag{E1}$$

$$\varepsilon_{2a}^{(2)}=-4S_c^{-2}(mm_1m_z)^2\,\mathrm{Tr}[G_{m_1}(\varphi^2-1)G_{1*,1}(\varphi^2-1)] , \tag{E2}$$

$$\varepsilon_{2b}^{(2)}=S_c^{-2}(mm_z)^2\,\mathrm{Tr}[(\partial_k\varphi^2)G_{1*,1}(\partial_k\varphi^2)G_{1*,1}] , \tag{E3}$$

$$\varepsilon_{4a}^{(2)}=4S_c^{-2}(mm_z)^2\,\mathrm{Tr}[(\varphi^2-1)^2G_{1*,1}]=4S_c^{-2}(mm_z)^2\int d^2x(\varphi^2(|x|)-1)^2G_{1*,1}(x,x) , \tag{E4}$$

$$\varepsilon_{4b}^{(2)}=S_c^{-2}mm_z\,\mathrm{Tr}[(\nabla_k\varphi_+)(\nabla_k\varphi_-)G_{1,1}] , \tag{E5}$$

$$\varepsilon_{13}^{(2)}=8S_c^{-2}(mm_z)^2\,\mathrm{Tr}[(\varphi^2-1)(\partial_k G_{m_1})(\partial_k\varphi^2)G_{1*,1}] , \tag{E6}$$

$$\varepsilon_6^{(2)}+\varepsilon_{10}^{(2)}=\varepsilon'+\varepsilon'' , \tag{E7}$$

$$\varepsilon'=\tfrac{1}{2}S_c^{-2}mm_z\,\mathrm{Tr}(G_{1,1}\varphi_\pm\nabla^2\varphi_\mp) , \tag{E7a}$$

$$\varepsilon''=-8S_c^{-2}mm_z\mu^2\sum_{\lambda^2>0}\frac{1}{\lambda^2}\,\mathrm{Tr}[G_{1,1}\varphi\phi_R(\lambda)]\left[[\phi_R(\lambda)]^*\left|\left[\frac{1-a}{r}\right]^2\varphi^3\right.\right] . \tag{E7b}$$

The following diagrams contribute only to higher orders:

$$\varepsilon_{2c}^{(2)}=-S_c^{-2}(mm_z)^2\,\mathrm{Tr}[(\partial_k\varphi^2)G_{1,1}(\partial_k\varphi^2)G_{1,1}] , \tag{E8}$$

$$\varepsilon_8^{(2)}=-16S_c^{-2}(mm_z)^3\,\mathrm{Tr}[G_{1*,1}(\varphi^2-1)(\partial_k G_{m_1})(\varphi^2-1)(\partial_k\varphi^2)] , \tag{E9}$$

$$\varepsilon_{12}^{(2)}=-16S_c^{-2}(mm_z)^3\,\mathrm{Tr}[G_{1,1}(\varphi^2-1)(\partial_k G_{m_1})(\varphi^2-1)(\partial_k G_{m_1})(\varphi^2-1)] , \tag{E10}$$

$$\begin{aligned}\varepsilon_{14}^{(2)}+\varepsilon_{15}^{(2)}=&-16S_c^{-2}(mm_z)^4\,\mathrm{Tr}[G_{1*,1}(\varphi^2-1)(\partial_k G_{m_1})(\varphi^2-1)G_{1*,1}(\varphi^2-1)(\partial_k G_{m_1})(\varphi^2-1)]\\&+16S_c^{-2}(mm_z)^4\,\mathrm{Tr}[G_{1,1}(\varphi^2-1)(\partial_k G_{m_1})(\varphi^2-1)G_{1,1}(\varphi^2-1)(\partial_k G_{m_1})(\varphi^2-1)] .\end{aligned} \tag{E11}$$

[1] L. F. Abbott, M. T. Grisaru, and H. J. Schnitzer, Phys. Rev. D **16**, 2995 (1977); **16**, 3002 (1977).
[2] E. Witten, Nucl. Phys. **B188**, 513 (1981); **B202**, 253 (1982).
[3] G. Veneziano and S. Yankielowicz, Phys. Lett. **113B**, 231 (1982); T. Taylor, G. Veneziano, and S. Yankielowicz, Nucl. Phys. **B218**, 493 (1983); I. Affleck, J. Harvey, and E. Witten, *ibid.* **B206**, 413 (1982); I. Affleck, M. Dine, and N. Seiberg, *ibid.* **B241**, 493 (1984).

[4]D. Amati, C. G. Rossi, and G. Veneziano, Nucl. Phys. **B249**, 1 (1985).

[5]A. I. Vainshtein and V. I. Zakharov, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 258 (1982) [JETP Lett. **35**, 323 (1982)]; V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B223**, 445 (1983); **B229**, 381 (1983); **B229**, 407 (1983); **B260**, 157 (1985).

[6]L. Mizrachi, Phys. Lett. B **175**, 325 (1986).

[7]R. K. Kaul and L. Mizrachi, CERN Report No. TH-3816, 1984 (unpublished); L. Mizrachi, Phys. Rev. D **34**, 3214 (1986).

[8]A. Casher, *On the Breaking of $N=1$ Global Supersymmetry in Gauge Theories,* University Libre de Bruxelles report, 1982 (unpublished); A. Casher, in *Supersymmetry,* proceedings of the NATO Advanced Study Institute, Bonn, West Germany, 1984, edited by K. Dietz *et al.* (NATO ASI Series B, Vol. 125) (Plenum, New York, 1985), p. 577.

[9]A proof that the SUSY algebra is conserved was claimed to have been found in Ref. 5.

[10]J. A. Bagger, in *Supersymmetry* (Ref. 8), p. 45.

[11]Y. Aharonov, A. Casher, and S. Yankielowicz, Phys. Rev. D **30**, 386 (1984).

[12]Quantization of the one-instanton sector and factorization of the contributions of multi-instanton configurations are extensively studied in the literature within the semiclassical approximation. See, for example, R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1982), and references therein. Some special features of gauge theories are discussed in Ref. 13.

[13]A. Casher, Y. Shamir, and Y. Aharonov, Tel-Aviv University Report No. TAUP 1563-87, 1987 (unpublished).

[14]In fact, $\det^{1/2}[E_{mn}(\beta)]$ is the analog of the 't Hooft effective Lagrangian, see G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).

[15]H. B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).

[16]In Eq. (2.35) the background field is that of a single instanton. Hence it is legitimate to use a regular gauge.

[17]Y. Frishman and S. Yankielowicz, Phys. Rev. D **19**, 540 (1979); I. Affleck, Nucl. Phys. **B191**, 429 (1981).

[18]A. J. Niemi and G. W. Semenoff, Nucl. Phys. **B269**, 131 (1986) (and references therein).

[19]Notice, however, that due to our Majorana notation Ψ_1 and Ψ_2 are *four-component* spinors.