Covariant BRS quantization of the ten-dimensional $N=1$ Brink-Schwarz superparticle

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We perform the covariant Becchi-Rouet-Stora quantization of the $N=1$ Brink-Schwarz superparticle in ten dimensions. The key ideas used are (1) the Batalin-Fradkin formalism for second-class constraints, (2) nonsquare vielbeins relating spinor indices to internal SO(8) indices, and (3) the special properties of Γ matrices in ten dimensions. Our work generalizes Eisenberg and Solomon's solution of the $N=2$ case to $N=1$.

I. INTRODUCTION

Recently, a number of papers have appeared[1–9] on the subject of the covariant quantization of the Brink-Schwarz (BS) superparticle.[10] While a noncovariant quantization presents no difficulties, the model has long resisted attempts at covariant quantization, due to the difficulty of covariantly separating the first- and second-class constraints, which together transform as an irreducible representation of the Lorentz group. The preferred solution should be firmly rooted in the covariant quantization scheme of Batalin, Fradkin, and Vilkovisky[11,12] (BFV), the most elegant tool yet developed for quantization of first-class constrained systems. A natural generalization of this scheme to encompass second-class constraints has recently been developed by Batalin and Fradkin[13] (BF), which makes use of auxiliary canonical variables further supplementing the extended phase space of BFV.

In this paper we perform the canonical quantization of the ten-dimensional $N=1$ BS superparticle, motivated by the recent trend to employ auxiliary variables in this problem, and especially by the work of Eisenberg and Solomon[1,2] (ES), in which elements of twistor geometry[8,9] and the BF scheme were used to canonically quantize the $N=2$ BS superparticle.

Our work relies heavily on the special features of ten dimensions, such as the existence of a real Weyl representation of the Γ matrices and a simple form of the charge-conjugation matrix C. These features enable us to present an economical solution using two Majorana-Weyl (MW) vielbeins mapping the Lorentz spinor indices to internal SO(8) indices. This mapping covariantly isolates the second-class part of the original constraints. The BF idea can then be applied by adding auxiliary self-conjugate SO(8) spinors to convert the second-class constraints to first class, thus closing the constraint algebra. The vielbeins must satisfy certain algebraic constraints, which however do not remove all of the vielbein degrees of freedom. The remaining degrees of freedom must then be gauged away by imposing additional constraints, which form a closed algebra with the original constraints. The new constraints have simple interpretations as generators of rotations on the internal SO(8) manifolds, and generators of "chiral" rotations relating the two vielbeins to each other.

We also extend our construction to the $N=2$ BS model in ten dimensions. Two possibilities exist for such a model, the symmetric model (type IIA), and the chiral model (type IIB), depending on whether the two supersymmetry generators have opposite or equal handedness. The chiral model IIB has been thoroughly discussed by ES (Ref. 1 and 2), and our results for this case are similar to those found in Ref. 2. The symmetric model IIA, like the $N=1$ case, has not yet been discussed using the approach of ES; we briefly discuss the solution here. For an explanation of our conventions, see the Appendix.

II. THE MODEL

The action for the Brink-Schwarz superparticle[10,14] is

$$S=\int \frac{1}{2e}\omega^2 , \tag{2.1}$$

where

$$\omega^\mu=\dot{x}^{\mu}-i\bar{\theta}^i\gamma^\mu\dot{\theta}^i . \tag{2.2}$$

Here, θ^i is a Majorana-Weyl 32-component spinor, where i runs from 1 to N, the number of supersymmetries. In this paper, we mainly consider the case $N=1$, and suppress the index i.

In 16-component notation (see the Appendix), $\bar{\theta}\gamma^\mu\dot{\theta}=\chi^T\hat{\sigma}^{\mu}\dot{\chi}$ if θ is right handed, and $\bar{\theta}\gamma^\mu\dot{\theta}=\chi^T\sigma^\mu\dot{\chi}$ if θ is left handed. Assuming, for definiteness, that θ is left handed, we have

$$\omega^\mu=\dot{x}^{\mu}-i\chi^T\sigma^\mu\dot{\chi} \tag{2.3}$$

and we compute the canonical momenta:

$$q=\frac{\partial L}{\partial\dot{e}}=0 , \tag{2.4}$$

$$p_\mu=\frac{\partial L}{\partial\dot{x}^{\mu}}=\frac{\omega_\mu}{e} , \tag{2.5}$$

$$\rho_\alpha=\frac{\partial_R L}{\partial\dot{\chi}^{\alpha}}=-i\chi^\beta p\cdot\sigma_{\beta\alpha} , \tag{2.6}$$

where the derivative in (2.6) acts from the right. We find the Hamiltonian

$$H_0=q\dot{e}+p_\mu\dot{x}^\mu+\rho_\alpha\dot{\chi}^\alpha-L=\tfrac{1}{2}ep^2 . \quad (2.7)$$

The canonical variables satisfy the graded Poisson brackets

$$\{q,e\}=1 , \quad (2.8)$$

$$\{p_\mu,x^\nu\}=\delta^\nu_\mu , \quad (2.9)$$

$$\{\rho_\alpha,\chi^\beta\}=\delta^\beta_\alpha . \quad (2.10)$$

Requiring the constraint (2.4) to commute with H_0, we find the secondary constraint[15]

$$\{q,H_0\}=\tfrac{1}{2}p^2\approx 0 . \quad (2.11)$$

From the form of the Hamiltonian in (2.7), we see that the einbein e plays the role of a Lagrange multiplier for the secondary constraint (2.11). With this interpretation, we can, for the moment, drop the constraint (2.4), since it will be reinstated implicitly by quantizing the system using the BFV formalism.

We then have the system of constraints and the Hamiltonian:

$$\phi\equiv p^2\approx 0 , \quad (2.12)$$

$$\phi_\alpha\equiv\rho_\alpha+ip\cdot\sigma_{\alpha\beta}\chi^\beta\approx 0 , \quad (2.13)$$

$$H_0\approx 0 . \quad (2.14)$$

The only nonvanishing Poisson bracket is then

$$\{\phi_\alpha,\phi_\beta\}=2ip\cdot\sigma_{\alpha\beta} . \quad (2.15)$$

We note that

$$(p\cdot\sigma)(p\cdot\hat{\sigma})=-p^2 . \quad (2.16)$$

In view of (2.12), this implies that the 16×16 matrix $p\cdot\sigma$ has eight null eigenvalues, so that the 16 constraints ϕ_α can be split into eight first-class and eight second-class constraints (although this cannot be done covariantly). Now first-class constraints require gauge fixing. In Dirac's method of quantization, this means the introduction of subsidiary constraints. On the other hand, second-class constraints can be understood as gauge fixing themselves, so no subsidiary constraints are needed. The eight first-class constraints, together with their subsidiary constraints, thus remove 16 degrees of freedom, while the eight second-class constraints remove another 8. Altogether, the constraints ϕ_α remove 24 degrees of freedom from the 32 components of the canonical pair $(\rho_\alpha,\chi^\alpha)$, leaving eight physical components. This explains the fundamental difficulty in the covariant quantization of the superparticle—there is no eight-dimensional spinor representation of the Lorentz group SO(1,9).

If there were no covariance difficulties, then we could deal with the second-class constraints straightforwardly, using a technique invented by Faddeev and Shatashvili[16] and discussed in the Becchi-Rouet-Stora (BRS) framework by Batalin and Fradkin[13] (BF).

The essence of the procedure can be illustrated by a simple example.[3,16] Consider a system with the constraints p and $x\approx 0$, where p and x are canonically conjugate. These constraints are second class:

$$\{p,x\}=1 . \quad (2.17)$$

Now add a canonical pair of auxiliary variables, satisfying $\{q,y\}=1$. The BF prescription for this simple case amounts to shifting the original constraints as follows:

$$p+y\approx 0,\quad x+q\approx 0 . \quad (2.18)$$

These constraints are now first class, and can be quantized according to the usual BFV formalism, *without the use of Dirac brackets.* The number of auxiliary variables corresponds to the number of original second-class constraints, a fact of general validity.

In applying this technique to our problem, we need eight auxiliary variables to convert our eight second-class constraints to first class. The question is how to introduce these covariantly. In recent papers,[1,2] Eisenberg and Solomon have presented a technique which works for the $N=2$ superparticle. We show here that the same general idea can be applied to our case, $N=1$, *despite the loss of the holomorphic representation exploited in Refs. 1 and 2.* To facilitate comparison of results, our notation parallels these two papers fairly closely.

Before discussing the procedure in detail, we make some general comments about how it works. The main ingredient[1,2] is the introduction of dynamical bosonic objects V^α_a and $\hat{V}_{a,\alpha}$, having canonical conjugates $W_{a,\alpha}$ and $\hat{W}^\alpha_a$:

$$\{W_{a,\alpha},V^\beta_b\}=\delta_{ab}\delta^\beta_\alpha,\quad \{\hat{W}^\alpha_a,\hat{V}_{b,\beta}\}=\delta_{ab}\delta^\alpha_\beta . \quad (2.19)$$

V^α_a and $\hat{V}_{a,\alpha}$ can be understood as "vielbeins," converting spinor indices α to internal SO(8) indices a. This enables us to redefine the set of constraints characterizing the superparticle model so that they have only SO(8) indices. We note that the vielbeins are not square matrices, and, hence, are not invertible.

It is useful to observe that $p\cdot\hat{\sigma}^{\alpha\beta}\phi_\beta$ is first class and reducible, due to (2.16). We use the vielbein $\hat{V}_{a,\alpha}$ to convert these to irreducible first-class constraints and we use V^α_a to convert ϕ_α to irreducible second-class constraints which can be treated by the BF procedure. Consistency imposes certain restrictions on the vielbeins, eliminating some degrees of freedom. We then impose additional constraints to gauge away the remaining vielbein degrees of freedom. One set of these constraints is a pair of SO(8) rotation generators. Another pair is analogous to "chiral" symmetry generators between the two SO(8) representations. An important technical problem is to show that the constraints we impose on the vielbeins form a set of irreducible, first-class constraints. By using a somewhat different set of constraints from that used by ES, we simplify the solution to this problem. The application of the standard BFV quantization is then straightforward.

III. THE SOLUTION

We now begin a more technical discussion of the solution. Using the vielbeins, the fermionic constraints can be written with SO(8) indices:

$$d_a \equiv V_a^\alpha \phi_\alpha + iS_a \approx 0 \ , \tag{3.1}$$

$$\hat{d}_a \equiv \hat{V}_{a,\alpha} p \cdot \hat{\sigma}^{\alpha\beta} \phi_\beta \approx 0 \ , \tag{3.2}$$

where (3.1) incorporates a shift by a self-conjugate spinor S_a, satisfying

$$\{S_a, S_b\} = i\delta_{ab} \ . \tag{3.3}$$

This shift makes it possible for (3.1) to be first class, as we now show. The constraints (3.1) and (3.2) have Poisson brackets:

$$\{d_a, d_b\} = iX_{ab} \ , \tag{3.4}$$

$$\{\hat{d}_a, \hat{d}_b\} = -i(\hat{X}_{ab} + \delta_{ab})p^2 \ , \tag{3.5}$$

$$\{d_a, \hat{d}_b\} = -2iV_a^\alpha \hat{V}_{b,\alpha} p^2 \ , \tag{3.6}$$

where we impose the constraints

$$X_{ab} \equiv 2V_a^\alpha p \cdot \sigma_{\alpha\beta} V_b^\beta - \delta_{ab} \approx 0 \ , \tag{3.7}$$

$$\hat{X}_{ab} \equiv 2\hat{V}_{a,\alpha} p \cdot \hat{\sigma}^{\alpha\beta} \hat{V}_{b,\beta} - \delta_{ab} \approx 0 \ . \tag{3.8}$$

Closure of the algebra (3.4)–(3.6) *required* that we impose (3.7) as a constraint. Symmetry suggests that (3.8) should also be imposed, and our analysis will show the utility of this constraint.

Equations (3.7) and (3.8) are *algebraic* constraints, and eliminate only a fraction of the vielbein degrees of freedom. We would like to treat the remaining vielbein components as pure-gauge variables. Operationally, this means that we need to find *differential* constraints to impose. A natural idea is to use the SO(8) rotation generators as constraints:

$$R_{ab} \equiv V_a^\alpha W_{b,\alpha} - V_b^\alpha W_{a,\alpha} - iS_a S_b \approx 0 \ , \tag{3.9}$$

$$\hat{R}_{ab} \equiv \hat{V}_{a,\alpha} \hat{W}_b^{\ \alpha} - \hat{V}_{b,\alpha} \hat{W}_a^{\ \alpha} \approx 0 \ . \tag{3.10}$$

These obey the SO(8) algebra

$$\{R_{ab}, R_{cd}\} = \delta_{bc} R_{ad} + \delta_{ad} R_{bc} - \delta_{ac} R_{bd} - \delta_{bd} R_{ac} \ . \tag{3.11}$$

It is easy to check closure of the algebra of the constraints (2.12), (3.1), (3.2), and (3.7)–(3.10), due to the relations

$$\{R_{ab}, d_c\} = \delta_{bc} d_a - \delta_{ac} d_b \ , \tag{3.12}$$

$$\{R_{ab}, X_{cd}\} = \delta_{bc} X_{ad} + \delta_{bd} X_{ac} - \delta_{ac} X_{bd} - \delta_{ad} X_{bc} \ , \tag{3.13}$$

and identical relations for the quantities with carets.

The SO(8) rotations, by themselves, are not enough to gauge away all of the remaining vielbein degrees of freedom. We introduce "chiral"-symmetry operators connecting the careted and uncareted SO(8) systems:

$$P_{ab} \equiv \hat{V}_{a,\alpha} p \cdot \hat{\sigma}^{\alpha\beta} W_{b,\beta} \approx 0 \ , \quad \hat{P}_{ab} \equiv V_a^\alpha p \cdot \sigma_{\alpha\beta} \hat{W}_b^{\ \beta} \approx 0 \ , \tag{3.14}$$

which generate, respectively, $V_b^\beta \to p \cdot \hat{\sigma}^{\beta\alpha} \hat{V}_{a,\alpha}$ and $\hat{V}_{b,\beta} \to p \cdot \sigma_{\beta\alpha} V_a^\alpha$. Imposing the constraints (3.14) turns out to gauge away the remaining degrees of freedom, as we now show.

Such a proof requires that we check three points: (1) the constraint algebra closes; (2) the constraints on the vielbeins are as numerous as the vielbein components; (3) the constraints are irreducible (i.e., independent).

One easily checks that the constraint algebra generated by the complete set of constraints (2.12), (3.1), (3.2), (3.7)–(3.10), and (3.14) closes. We have already presented part of the algebra. The remaining nonvanishing Poisson brackets are

$$\{P_{ab}, \hat{P}_{cd}\} = (\delta_{ad} V_c^\alpha W_{b,\alpha} - \delta_{bc} \hat{V}_{a,\alpha} \hat{W}_d^{\ \alpha}) p^2 \ , \tag{3.15}$$

$$\{P_{ab}, d_c\} = \delta_{bc} \hat{d}_a, \quad \{\hat{P}_{ab}, \hat{d}_c\} = -\delta_{bc}(d_a - iS_a) p^2 \ , \tag{3.16}$$

$$\{P_{ab}, X_{cd}\} = -2(\delta_{bc} \hat{V}_{a,\alpha} V_d^\alpha + \delta_{bd} \hat{V}_{a,\alpha} V_c^\alpha) p^2 \ , \tag{3.17}$$

$$\{P_{ab}, R_{cd}\} = \delta_{bc} P_{ad} - \delta_{bd} P_{ac} \ , \tag{3.18}$$

$$\{P_{ab}, \hat{R}_{cd}\} = \delta_{ac} P_{db} - \delta_{ad} P_{cb} \tag{3.19}$$

plus the careted versions of (3.17)–(3.19).

Let us now make a detailed comparison of the total number of degrees of freedom with the total number of constraints. For completeness, we consider not only the vielbeins, but also the other variables.

In the fermionic sector, phase space consists of $2\times16=32$ fermionic components (from the original MW spinor and its conjugate momentum), and eight components from the self-conjugate SO(8) object S_a. These are constrained by $8+8=16$ first-class constraints (3.1) and (3.2). Since first-class constraints each remove 2 degrees of freedom (after gauge fixing), we eliminate 32 fermionic degrees of freedom, leaving 8.

The bosonic variables x^μ and p_μ must satisfy the single first-class constraint (2.12) and so contribute $2\times10-2=18$ degrees of freedom.

Finally, each vielbein contributes $8\times16=128$ components; another 128 come from their conjugate momenta. This gives $4\times128=512$ auxiliary degrees of freedom. We have 36 independent constraints (due to symmetry) from each of the X_{ab} and $\hat{X}_{ab}$, and 28 from each SO(8) rotation generator R_{ab} and $\hat{R}_{ab}$. Each chiral-symmetry operator P_{ab} and $\hat{P}_{ab}$ defines 64 constraints. In sum, we have $2\times(36+28+64)=256$ first-class constraints, which therefore eliminate all 512 auxiliary degrees of freedom, *if these constraints are irreducible.*

To decide the irreducibility question, we consider these constraints in the particular Lorentz frame defined by $p_i=0$, where $i=1,2,\ldots,8$. Since $p^2=0$, we can also choose $p_- \equiv p_0 - p_9 \neq 0$, which fixes $p_+=0$. In this frame, using the conventions explained in the Appendix,

$$p \cdot \sigma = \begin{bmatrix} p_- & 0 \\ 0 & 0 \end{bmatrix}, \quad p \cdot \hat{\sigma} = \begin{bmatrix} 0 & 0 \\ 0 & p_- \end{bmatrix} . \tag{3.20}$$

Then the constraint $X_{ab} \approx 0$ leads to

$$V_a^A V_b^A \approx \frac{\delta_{ab}}{2p_-} \ , \tag{3.21}$$

where we follow the notation of Ref. 1 in parametrizing α by $\alpha=(A,\dot{A})$, with $A,\dot{A}=1,2,\ldots,8$. Equation (3.21) shows that V_a^A is essentially an 8×8 orthogonal matrix, with 28 degrees of freedom. These can be gauged away, using the 28 constraints defined by the antisymmetric SO(8) generator R_{ab}.

Similar results follow for the careted variables:

$$\hat{X}_{ab}\approx 0 \Longrightarrow \hat{V}_{a,\dot{A}}\hat{V}_{b,\dot{A}}\approx \frac{\delta_{ab}}{2p_-} \tag{3.22}$$

so $\hat{V}_{a,\dot{A}}$ is essentially an orthogonal matrix and $\hat{R}_{ab}$ can gauge away the remaining degrees of freedom, as above.

To complete the argument, we examine the constraints (3.14):

$$P_{ab}\approx 0 \Longrightarrow p_-\hat{V}_{a,\dot{A}}W_{b,\dot{A}}\approx 0 \Longrightarrow W_{b,\dot{A}}\approx 0 \;, \tag{3.23}$$

$$\hat{P}_{ab}\approx 0 \Longrightarrow p_-V_a^A\hat{W}_b^{\ A}\approx 0 \Longrightarrow \hat{W}_b^{\ A}\approx 0 \;. \tag{3.24}$$

Here, we have used the fact that V_a^A and $\hat{V}_{a,\dot{A}}$ are nonsingular matrices.

This proves that the constraints (3.7)–(3.10) and (3.14) are linearly independent. Thus, the constraints on the vielbiens eliminate *all* the vielbein degrees of freedom.

We can also examine the fermionic constraints (3.1) and (3.2) in this frame:

$$d_a\approx 0 \Longrightarrow V_a^A\phi_A+V_a^{\dot{A}}\phi_{\dot{A}}+iS_a\approx 0 \;, \tag{3.25}$$

$$\hat{d}_a\approx 0 \Longrightarrow p_-\hat{V}_{a,\dot{A}}\phi_{\dot{A}}\approx 0 \;. \tag{3.26}$$

Hence, $\phi_{\dot{A}}\approx 0$, while ϕ_A is completely determined in terms of S_a. This clarifies the counting; the independent fermionic variables are precisely the SO(8) variables S_a.

In the quantum-mechanical limit, the relation (3.3) satisfied by the remaining fermionic variables S_a defines an eight-dimensional Clifford algebra. Such an algebra can be represented concretely by matrices acting on a 16-dimensional carrier space. The 16-dimensional fundamental representation decomposes into two irreducible parts, $\mathbf{8}_c$ and $\mathbf{8}_s$, consisting of eight fermionic and eight bosonic states.[14,17] Hence, our $N=1$ model reproduces the supersymmetric Yang-Mills multiplet in ten dimensions. This fact also follows directly from a Fock-space analysis of the states defined by the corresponding U(4) fermionic creation operators.[14]

IV. THE BRS CHARGE

If we were interested in performing a Dirac quantization,[15] we would want to find subsidiary constraints to fix the gauge. It is straightforward to work out a suitable set of noncovariant subsidiary constraints, using the particular Lorentz frame used in Sec. III. However, we are interested in using the covariant quantization scheme of BFV, using the BRS charge. This scheme allows one to proceed directly to the second-quantized formalism.

In working out the BRS charge, two problems present themselves. First, some of our structure functions are not constant. This means that the BRS charge might not take the usual form for a "rank-1" theory:[11,12]

$$Q=\eta^a\phi_a+\tfrac{1}{2}(-)^a\eta^a\eta^b U_{ba}^c\mathcal{P}_c \;, \tag{4.1}$$

where

$$\{\phi_a,\phi_b\}=U_{ab}^c\phi_c \;. \tag{4.2}$$

Note that our structure functions U_{ab}^c are constant, *except* when ϕ_c corresponds to $p^2\approx 0$. It is easy to prove in such a case[1] that Q in (4.1) is nilpotent, using the Jacobi identity,[12] $\{q,\{q,q\}\}=0$, where $q\equiv\eta^a\phi_a$.

The second problem is that some of our constraints have symmetric or antisymmetric indices. This causes some confusion about double counting and about the correct Poisson brackets for the corresponding ghosts. The easiest way to resolve this confusion is to temporarily combine the symmetric and antisymmetric constraints:

$$\phi_{ab}\equiv X_{ab}+R_{ab},\quad \hat{\phi}_{ab}\equiv\hat{X}_{ab}+\hat{R}_{ab} \;. \tag{4.3}$$

The indices of ϕ_{ab} and $\hat{\phi}_{ab}$ then have no symmetry properties and neither do the corresponding ghosts. The ghost Poisson brackets is therefore obvious. One easily computes the BRS charge, with no double-counting difficulties. Using (4.3), one can then rewrite Q in terms of X_{ab} and R_{ab}, etc. This naturally selects the symmetric and antisymmetric combinations of the ghosts and determines the Poisson brackets for these combinations.

The results are as follows. To each constraint, we associate ghosts and antighosts having the Poisson brackets:

$$\phi\leftrightarrow\{\mathcal{P},\eta\}=1 \;, \tag{4.4}$$

$$d_a\leftrightarrow\{\mathcal{P}_a,\eta^b\}=\delta_a^b \;, \tag{4.5}$$

$$X_{ab}\leftrightarrow\{\mathcal{P}_{(ab)},\eta^{(cd)}\}=\tfrac{1}{2}(\delta_a^c\delta_b^d+\delta_a^d\delta_b^c) \;, \tag{4.6}$$

$$R_{ab}\leftrightarrow\{\mathcal{P}_{[ab]},\eta^{[cd]}\}=\tfrac{1}{2}(\delta_a^c\delta_b^d-\delta_a^d\delta_b^c) \;, \tag{4.7}$$

$$P_{ab}\leftrightarrow\{\mathcal{P}_{ab},\eta^{cd}\}=\delta_a^c\delta_b^d \;. \tag{4.8}$$

Equations (4.5)–(4.8) have analogs for the constraints $\hat{d}_a$, $\hat{X}_{ab}$, $\hat{R}_{ab}$, and $\hat{P}_{ab}$. Here, for convenience, we have used a slightly misleading notation. $\eta^{(ab)}$, $\eta^{[ab]}$, and η^{ab} are completely independent quantities, distinguished by the presence or absence of ( ) and [ ]. Also, note that η^a and $\mathcal{P}_a$ are bosonic ghosts.

For the BRS charge, we find

$$Q=q_\phi+(q_d+q_X+Q_R+q_P)+(\hat{q}_d+\hat{q}_X+\hat{Q}_R+\hat{q}_P) \;, \tag{4.9}$$

where the individual terms are

$$q_\phi\equiv\eta\phi-\frac{i}{2}[\eta^a\eta^b\mathcal{P}_{(ab)}-\hat{\eta}^a\hat{\eta}^b\mathcal{P}(\hat{X}_{ab}+\delta_{ab})]-\eta^{ab}\eta^b\hat{\mathcal{P}}_a+\hat{\eta}^{ab}\hat{\eta}^b\mathcal{P}(d_a-iS_a) \;, \tag{4.10}$$

$$q_d\equiv\eta^a d_a+2\eta^{[ab]}\eta^a\mathcal{P}_b+i\eta^a\hat{\eta}^b\mathcal{P}(V_a^\alpha\hat{V}_{b,\alpha}) \;, \tag{4.11}$$

$$q_X\equiv\eta^{(ab)}X_{ab}-4\eta^{[ab]}\eta^{(bc)}\mathcal{P}_{(ac)} \;, \tag{4.12}$$

$$Q_R\equiv\eta^{[ab]}R_{ab}-2\eta^{[ab]}\eta^{[bc]}\mathcal{P}_{[ac]} \;, \tag{4.13}$$

$$q_P\equiv\eta^{ab}P_{ab}+(V_a^\alpha W_c^\alpha)\hat{\eta}^{ab}\eta^{bc}\mathcal{P}+4(\hat{V}_a^{\ \alpha}V_c^\alpha)\eta^{ab}\eta^{(bc)}\mathcal{P}-2\eta^{ab}(\eta^{[bc]}\mathcal{P}_{ac}+\hat{\eta}^{[ac]}\mathcal{P}_{cb}) \;. \tag{4.14}$$

The careted terms $\hat{q}_d$, $\hat{q}_X$, $\hat{Q}_R$, and $\hat{q}_P$ follow from (4.11)–(4.14) by applying the caret operation, using the rule that the ghost $\mathcal{P}$ remains invariant. Note that Q is nilpotent, as are Q_R and $\hat{Q}_R$, while the other terms in (4.9) are not.

V. THE $N=2$ CASE

Finally, we briefly discuss the extension from $N=1$ to $N=2$. As we mentioned earlier, there are two cases to consider.

First, we discuss the chiral $N=2$ case (IIB), with both fermions having the same handedness. Thus, we double the number of original fermionic constraints (2.13), so that the algebra is now

$$\{\phi^i_\alpha,\phi^j_\beta\}=2ip\cdot\sigma_{\alpha\beta}\delta^{ij} \tag{5.1}$$

with $i,j=1,2$. Correspondingly, we define two MW auxiliary spinors S^i_a satisfying

$$\{S^i_a,S^j_b\}=i\delta_{ab}\delta^{ij}\ , \tag{5.2}$$

which are used to shift the SO(8) fermionic constraints

$$d^i_a\equiv V^\alpha_a\phi^i_\alpha+iS^i_a\approx 0\ . \tag{5.3}$$

It is *not* necessary to double the number of vielbeins. We also have

$$\hat{d}^{\,i}_a\equiv\hat{V}_{a,\alpha}p\cdot\hat{\sigma}^{\alpha\beta}\phi^i_\beta\ . \tag{5.4}$$

It is convenient to pass to the holomorphic notation:[1,2]

$$d_a\equiv\frac{1}{\sqrt{2}}(d^1_a+id^2_a),\quad \bar{d}_a\equiv\frac{1}{\sqrt{2}}(d^1_a-id^2_a)\ , \tag{5.5}$$

$$\hat{d}_a\equiv\frac{1}{\sqrt{2}}(\hat{d}^{\,1}_a+i\hat{d}^{\,2}_a),\quad \hat{\bar{d}}_a\equiv\frac{1}{\sqrt{2}}(\hat{d}^{\,1}_a-i\hat{d}^{\,2}_a)\ , \tag{5.6}$$

$$\phi_\alpha\equiv\frac{1}{\sqrt{2}}(\phi^1_\alpha+i\phi^2_\alpha),\quad \bar{\phi}_\alpha\equiv\frac{1}{\sqrt{2}}(\phi^1_\alpha-i\phi^2_\alpha)\ , \tag{5.7}$$

$$\xi_\alpha\equiv\frac{i}{\sqrt{2}}(S^1_a+iS^2_a),\quad \zeta_a\equiv-\frac{1}{\sqrt{2}}(S^1_a-iS^2_a)\ . \tag{5.8}$$

Note that

$$\{\xi_a,\zeta_b\}=\delta_{ab}\ . \tag{5.9}$$

Then we find that the holomorphic analog of (3.1) and (3.2) is

$$d_a=V^\alpha_a\phi_\alpha+\xi_a,\quad \bar{d}_a=V^\alpha_a\bar{\phi}_\alpha-i\zeta_a\ , \tag{5.10}$$

$$\hat{d}_a=\hat{V}_{a,\alpha}p\cdot\hat{\sigma}^{\alpha\beta}\phi_\beta,\quad \hat{\bar{d}}_a=\hat{V}_{a,\alpha}p\cdot\hat{\sigma}^{\alpha\beta}\bar{\phi}_\beta\ . \tag{5.11}$$

One easily sees that the bosonic sector of constraints is practically identical to the $N=1$ case. Only in the definition of the uncareted SO(8) gauge generator do we encounter a slight difference, due to the lack of an i in (5.9):

$$R_{ab}=V^\alpha_aW_{b,\alpha}-V^\alpha_bW_{a,\alpha}+(\xi_a\zeta_b-\xi_b\zeta_a)\ . \tag{5.12}$$

Hence, the proof given above for the pure-gauge character of the bosonic auxiliary variables extends automatically to the chiral $N=2$ case.

Next, we discuss the symmetric $N=2$ case (IIA), in which the fermions have opposite handedness. This case is not discussed in the literature, since it is technically similar to the $N=1$ problem. Here, we have the constraints

$$\phi^L_\alpha\equiv\rho^L_\alpha+ip\cdot\sigma_{\alpha\beta}\chi^\beta_L\approx 0\ , \tag{5.13}$$

$$\phi^\alpha_R\equiv\rho^\alpha_R+ip\cdot\hat{\sigma}^{\alpha\beta}\chi^R_\beta\approx 0\ , \tag{5.14}$$

which satisfy

$$\{\phi^L_\alpha,\phi^L_\beta\}=2ip\cdot\sigma_{\alpha\beta}\ , \tag{5.15}$$

$$\{\phi^\alpha_R,\phi^\beta_R\}=2ip\cdot\hat{\sigma}^{\alpha\beta}\ . \tag{5.16}$$

Again, we introduce the usual vielbeins V^α_a, $\hat{V}_{a,\alpha}$ and their canonical conjugates $W_{b,\beta}$, $\hat{W}^\beta_b$, which we use to convert the constraints to SO(8) indices:

$$d^L_a\equiv V^\alpha_a\phi^L_\alpha+iS^L_a\approx 0\ , \tag{5.17}$$

$$\hat{d}^{\,L}_a\equiv\hat{V}_{a,\alpha}p\cdot\hat{\sigma}^{\alpha\beta}\phi^L_\beta\approx 0\ , \tag{5.18}$$

$$\hat{d}^{\,R}_a\equiv\hat{V}_{a,\alpha}\phi^\alpha_R+iS^R_a\approx 0\ , \tag{5.19}$$

$$d^R_a\equiv V^\alpha_a p\cdot\sigma_{\alpha\beta}\phi^\beta_R\approx 0\ . \tag{5.20}$$

The nonvanishing Poisson brackets of this system of constraints are

$$\{d^L_a,d^L_b\}=iX_{ab}\ , \tag{5.21}$$

$$\{\hat{d}^{\,L}_a,\hat{d}^{\,L}_b\}=-i(\hat{X}_{ab}+\delta_{ab})p^2\ , \tag{5.22}$$

$$\{d^L_a,\hat{d}^{\,L}_b\}=-2i(V^\alpha_a\hat{V}_{b,\alpha})p^2\ , \tag{5.23}$$

$$\{\hat{d}^{\,R}_a,\hat{d}^{\,R}_b\}=i\hat{X}_{ab}\ , \tag{5.24}$$

$$\{d^R_a,d^R_b\}=-i(X_{ab}+\delta_{ab})p^2\ , \tag{5.25}$$

$$\{d^R_a,\hat{d}^{\,R}_b\}=-2i(V^\alpha_a\hat{V}_{b,\alpha})p^2\ , \tag{5.26}$$

where X_{ab} and $\hat{X}_{ab}$ are defined exactly as in (3.7) and (3.8).

Clearly, we are now forced to impose *both* X_{ab} and $\hat{X}_{ab}$ as constraints. The removal of the remaining vielbein degrees of freedom proceeds as in the $N=1$ case, with the proviso that the SO(8) rotation generators now take the form

$$R_{ab}=V^\alpha_aW_{b,\alpha}-V^\alpha_bW_{a,\alpha}-iS^L_aS^L_b\ , \tag{5.27}$$

$$\hat{R}_{ab}=\hat{V}_{a,\alpha}\hat{W}^\alpha_b-\hat{V}_{b,\alpha}\hat{W}^\alpha_a-iS^R_aS^R_b\ . \tag{5.28}$$

The constraints P_{ab} and $\hat{P}_{ab}$, and the counting of degrees of freedom, are unchanged from the $N=1$ case.

Note added in proof. After this paper had been sent for publication we came across the following relevant papers: E. Nissimov, S. Pacheva, and S. Solomon, Weizmann Institute Reports Nos. WIS-88/23 and 24/MAY-PH (unpublished); Y. Eisenberg, Weizmann Institute Report No. WIS-88/52/MAY-PH (unpublished); and A. Dresse, J. Fisch, M. Henneaux, and C. Schomblond, Phys. Lett. B **210**, 141 (1988).

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APPENDIX: CONVENTIONS

Throughout this paper, a dagger means Hermitian conjugation, an asterisk means complex conjugation, and T means transpose, when these symbols appear as superscripts. We use the Minkowski metric $\eta^{\mu\nu} = \text{diag}(-++\cdots+)$.

We look for a set of Γ matrices Γ^μ and a charge-conjugation matrix C in ten dimensions, satisfying

$$\{\Gamma^\mu,\Gamma^\nu\} = -2\eta^{\mu\nu} , \quad \text{(A1)}$$

$$(\Gamma^0)^\dagger = \Gamma^0, \quad (\Gamma^i)^\dagger = -\Gamma^i , \quad \text{(A2)}$$

$$C(\Gamma^\mu)^T C^{-1} = -\Gamma^\mu . \quad \text{(A3)}$$

Many such representations exist, but ours will be chosen to simplify the Majorana and Weyl conditions on spinors.

From Appendix 5.B of Ref. 14, we know that one can construct a set of real-symmetric 16×16 matrices γ^i, where $i = 1, 2, \ldots, 9$, satisfying

$$\{\gamma^i,\gamma^j\} = 2\delta^{ij} . \quad \text{(A4)}$$

We take the sign of γ^9 opposite that in Ref. 14, so that

$$\gamma^9 = -\gamma^1\gamma^2 \cdots \gamma^8 = \begin{bmatrix} -1 & \\ & 1 \end{bmatrix} . \quad \text{(A5)}$$

We then define the real-symmetric matrices σ^μ and $\hat\sigma^\mu$, where $\mu = 0, 1, \ldots, 9$, by

$$\sigma^\mu = (1,\gamma^i), \quad \hat\sigma^\mu = (1,-\gamma^i) . \quad \text{(A6)}$$

Using these, we define a set of pure-real Γ matrices:

$$\Gamma^\mu = \begin{bmatrix} & \sigma^\mu \\ \hat\sigma^\mu & \end{bmatrix} . \quad \text{(A7)}$$

One checks easily that these satisfy (A1) and (A2). Moreover, $\Gamma^{11} \equiv \Gamma^0\Gamma^1 \cdots \Gamma^9$ is diagonal, and C also takes a simple form:

$$\Gamma^{11} = \begin{bmatrix} 1 & \\ & -1 \end{bmatrix}, \quad C = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix} . \quad \text{(A8)}$$

It is easy to check that $C^{-1} = C^T = C^\dagger = -C$ and that (A3) is satisfied.

We now consider the Majorana and Weyl conditions on 32-component complex spinors. The Weyl condition states that $\Gamma^{11}\psi_w = \pm\psi_w$ where the eigenvalue is $+$ $(-)$ for right- (left-) handed components. The Majorana condition states that $\psi_m = C(\Gamma^0)^T\psi_m^*$. In our representation, $C(\Gamma^0)^T = \Gamma^{11}$, so that a Majorana-Weyl spinor satisfies the simple conditions

$$\Gamma^{11}\psi_{mw} = \pm\psi_{mw}, \quad \psi_{mw} = \Gamma^{11}\psi_{mw}^* . \quad \text{(A9)}$$

The solutions for right- and left-handed spinors are

$$\psi_R = \begin{bmatrix} \chi \\ 0 \end{bmatrix}, \quad \psi_L = \begin{bmatrix} 0 \\ i\chi \end{bmatrix} , \quad \text{(A10)}$$

where χ is a real, 16-component spinor. Using the representation (A7), it is easy to work out the generators of Lorentz transformations in 16-component notation.

[1] Y. Eisenberg and S. Solomon, Nucl. Phys. **B309**, 709 (1988).

[2] S. Solomon, Phys. Lett. B **203**, 86 (1988).

[3] M. Moshe, in *Frontiers in Particle Theory,* proceedings of the 11th John Hopkins Workshop, Lanzhou, People's Republic of China, 1987, edited by Y. Duan, G. Domokos, and S. Kovesi-Domokos (World Scientific, Singapore, 1988).

[4] E. Nissimov, S. Pacheva, and S. Solomon, Nucl. Phys. **B296**, 462 (1988); **B299**, 183 (1988).

[5] E. Nissimov and S. Pacheva, International Center for Theoretical Physics Report No. IC/87/389 (unpublished).

[6] L. Brink, M. Henneaux, and C. Teitelboim, Nucl. Phys. **B293**, 505 (1987).

[7] T. Hori, K. Kamimura, and M. Tatewaki, Phys. Lett. B **185**, 367 (1987); T. Hori and C. B. Kim, *ibid.* **207**, 44 (1988).

[8] A. K. H. Bengtsson, I. Bengtsson, M. Cederwall, and N. Linden, Phys. Rev. D **36**, 1766 (1987).

[9] T. Shirafuji, Prog. Theor. Phys. **70**, 18 (1983).

[10] L. Brink and J. H. Schwarz, Phys. Lett. **B100**, 310 (1981).

[11] E. S. Fradkin and G. A. Vilkovisky, Phys. Lett. **55B**, 224 (1975); Report No. CERN-TH2332, 1977 (unpublished); I. A. Batalin and G. A. Vilkovisky, Phys. Lett. **69B**, 309 (1977).

[12] M. Henneaux, Phys. Rep. **126**, 1 (1985).

[13] I. A. Batalin and E. S. Fradkin, Nucl. Phys. **B279**, 514 (1987).

[14] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, England, 1987).

[15] P. A. M. Dirac, *Lectures in Quantum Mechanics* (Yeshiva University Press, New York, 1964).

[16] L. D. Faddeev and S. L. Shatashvili, Phys. Lett. **167B**, 225 (1986).

[17] B. de Wit, in *Supersymmetry, Supergravity and Superstrings '86,* proceedings of the Trieste Spring School, Trieste, Italy, 1986, edited by B. deWit, P. Fayet, and M. Grisaru (World Scientific, Singapore, 1986).