# Dirac quantization of the vector superfield

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We use the Dirac quantization method in superspace to quantize the Abelian vector superfield in the Wess-Zumino gauge.

## I. INTRODUCTION

It has been shown that the Dirac quantization method[1,2] can be extended to superspace,[3-7] where superfields are taken as canonical variables.[8]

The consistency of this procedure was verified in supersymmetric quantum mechanics,[3] in the quantization of the chiral superfield,[4] and in supersymmetric theories in $1+1$ dimensions.[3,5-7] In the last case, the nonlinear $\sigma$ model is a nice example. We have quantized it in the usual version with $O(N)$ symmetry[3] and in the geometrical approach, without[6] and with torsion.[7]

The purpose of this paper is to use this method to quantize the vector superfield. We shall restrict our consideration to the Abelian case. There are two new facts which emerge here. First, the corresponding supersymmetric Lagrangian, in superfield formulation, contains higher-order derivatives. Second, we have now the inherent problems of a gauge theory. We will work in the Wess-Zumino gauge.[9] This choice does not completely fix the gauge. Additional assumptions have to be made. The condition we shall consider corresponds, in components, to the usual radiation gauge.[10]

The paper is organized as follows. In Sec. II we present the Lagrangian density of the theory and show how to incorporate the Wess-Zumino gauge in the Dirac formalism. In Sec. III we give a short discussion of the canonical formalism involving second-order derivatives. In Sec. IV the theory is quantized, starting with the vector superfield as a canonical variable. We show that the results are in agreement with the well-known ones in component fields. Some concluding remarks are left to Sec. V.

## II. DIRAC BRACKETS CONSISTENT WITH THE WESS-ZUMINO GAUGE

The action for the vector superfield $V$ has the form[11]

$$I=-\tfrac{1}{16}\int d^4x\, d^2\theta\, d^2\bar\theta(\bar D\bar D\, DV\, DV+\text{H.c.})\ , \tag{2.1}$$

where $V$ in terms of component fields is given by

$$\begin{aligned}V(x,\theta,\bar\theta)=&\,C(x)+i\theta\chi(x)-i\bar\theta\bar\chi(x)+\frac{i}{2}\theta\theta M(x)\\&-\frac{i}{2}\bar\theta\bar\theta M^*(x)-\theta\sigma^\mu\bar\theta A_\mu(x)+i\theta\theta\bar\theta\bar\lambda(x)\\&-i\bar\theta\bar\theta\theta\lambda(x)+\tfrac{1}{2}\theta\theta\bar\theta\bar\theta D(x)\ .\end{aligned}\tag{2.2}$$

$D_\alpha$ and $\bar D_{\dot\alpha}$ are the usual covariant derivatives

$$D_\alpha=\partial_\alpha+i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu,\quad \bar D_{\dot\alpha}=-\partial_{\dot\alpha}-i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\ . \tag{2.3}$$

The conventions used throughout in this paper are the same as those of the Wess and Bagger book, mentioned in Ref. 11.

We observe that the corresponding Lagrangian density

$$\mathcal{L}=-\tfrac{1}{16}(\bar D\bar D\, DV\, DV+\text{H.c.}) \tag{2.4}$$

contains derivatives up to third order acting on the vector superfield $V$.

In order to simplify the problem of higher derivatives we consider $V$ in the Wess-Zumino gauge,[9] where the component expansion has the form

$$\begin{aligned}V(x,\theta,\bar\theta)=&\,i\theta\theta\bar\theta\bar\lambda(x)-i\bar\theta\bar\theta\theta\lambda(x)-\theta\sigma^\mu\bar\theta A_\mu\\&+\tfrac{1}{2}\theta\theta\bar\theta\bar\theta D(x)\ .\end{aligned}\tag{2.5}$$

Now we observe that since $\bar\theta\bar\theta V$ and $\theta\theta V$ are zero, there are no more third-order derivatives. However, there are still second-order ones, because $\bar\theta_{\dot\alpha}\theta_\alpha V$ is not zero. So, canonical quantization of the vector superfield involves higher-order derivatives. The way of performing canonical quantization when higher-order derivative terms are present will be shortly discussed in Sec. III. Now, let us show how to incorporate the Wess-Zumino gauge in the canonical formalism.

The conventional way of dealing with constraints in the canonical quantization is first to calculate the momenta and get the primary constraints.[1,2] Then, by imposing that constraints do not evolve in time we may obtain what are called secondary constraints. The next step is to verify if there are constraints which have null Poisson brackets with all the others. If there are (these are called first-class constraints), this means that the theory presents some gauge symmetry. Then either we fix the gauge, or we may choose to work in a covariant formalism[12] related to the Becchi-Rouet-Stora-Tyutin (BRST) symmetry.[13] In this work we will adopt the gauge-fixing procedure.

For a question of simplicity, we avoid following all these steps. This is particularly possible here because any functional derivative with respect to a superfield, for example, $V$ can be written as

 

$$\frac{\delta}{\delta V(x,\theta,\bar\theta)}=\theta\theta\bar\theta\bar\theta\frac{\delta}{\delta C(x)}-2i\bar\theta\bar\theta\theta^{\alpha}\frac{\delta}{\delta\chi^{\alpha}(x)}+2i\theta\theta\bar\theta_{\dot\alpha}\frac{\delta}{\delta\bar\chi_{\dot\alpha}(x)}-2i\bar\theta\bar\theta\frac{\delta}{\delta M(x)}+2i\theta\theta\frac{\delta}{\delta M^{*}(x)}+2\theta\sigma^{\mu}\bar\theta\frac{\delta}{\delta A^{\mu}(x)}$$
$$+2i\theta^{\alpha}\frac{\delta}{\delta\lambda^{\alpha}(x)}-2i\bar\theta_{\dot\alpha}\frac{\delta}{\delta\bar\lambda_{\dot\alpha}(x)}+2\frac{\delta}{\delta D(x)}\ . \tag{2.6}$$

Thus, independently of the definition of the supermomentum [when there are higher-order derivatives the momentum expressions are different from the usual ones, see Eqs. (3.8) and (3.9)] and of the specific form of the Lagrangian, one may write that the canonical momentum related to the superfield $V$ has the general form

$$\Pi(x,\theta,\bar\theta)=\theta\theta\bar\theta\bar\theta P_c(x)-2i\bar\theta\bar\theta\theta\Pi_{\chi}(x)+2i\theta\theta\bar\theta\,\bar\Pi_{\bar\chi}(x)$$
$$-2i\bar\theta\bar\theta P_M(x)+2i\theta\theta P_{M^*}(x)+2\theta\sigma^{\mu}\bar\theta\Pi_{\mu}(x)$$
$$+2i\theta\Pi_{\lambda}(x)-2i\bar\theta\,\bar\Pi_{\bar\lambda}(x)+2P_D(x)\ , \tag{2.7}$$

where $P_c(x)$, $\Pi^{\alpha}_{\chi}(x)$, . . . are, respectively, the canonical momenta conjugated to $C(x)$, $\chi_{\alpha}(x)$, . . . .

The Wess-Zumino gauge eliminates the fields $C$, $M$, and $\chi$. So, it is a question of consistency that the corresponding canonical momenta have to be eliminated too. In superfield language, the elimination of these degrees of freedom in $V$ and in $\Pi$ is written as

$$\begin{aligned}&\delta^2(\theta)\delta^2(\bar\theta)V\approx 0\ ,\\&\delta^2(\theta)\delta^2(\bar\theta)\partial\partial V\approx 0\ ,\\&\delta^2(\theta)\delta^2(\bar\theta)\bar\partial\bar\partial V\approx 0\ ,\\&\delta^2(\theta)\bar\partial\bar\partial(\bar\theta_{\dot\alpha}V)\approx 0\ ,\\&\delta^2(\bar\theta)\partial\partial(\theta_{\alpha}V)\approx 0\ ;\end{aligned} \tag{2.8}$$

$$\begin{aligned}&\partial\partial\bar\partial\bar\partial\Pi\approx 0\ ,\\&\delta^2(\theta)\bar\partial\bar\partial_{\alpha}\Pi\approx 0\ ,\\&\delta^2(\bar\theta)\partial\partial\bar\partial_{\dot\alpha}\Pi\approx 0\ ,\\&\delta^2(\theta)\bar\partial\bar\partial\Pi\approx 0\ ,\\&\delta^2(\bar\theta)\partial\partial\Pi\approx 0\ ,\end{aligned} \tag{2.9}$$

where $\delta^2(\theta)=\theta\theta$ and $\delta^2(\bar\theta)=\bar\theta\bar\theta$. The symbol $\approx$ means weakly equal.[1,2]

These constraints are second class. The elimination of these degrees of freedom may be done by means of Dirac brackets,[1,2] whose general definition in superspace for any two dynamical quantities $A(x,\theta,\bar\theta)$ and $B(x,\theta,\bar\theta)$ is

$$\{A(z),B(z')\}^{*}_{x_0=x_0'}=\{A(z),B(z')\}_{x_0=x_0'}$$
$$-\int d^7z''d^7dz'''\{A(z),\Gamma_i(z'')\}_{x_0=x_0''}D_{ij}(z'',z''')\{\Gamma_j(z'''),B(z')\}_{x_0'''=x_0'}\ , \tag{2.10}$$

where $z=(x_{\mu},\theta_{\alpha},\bar\theta_{\dot\alpha})$ and $D$ is a matrix such that the constraints $\Gamma_i$ can be made strongly zero in the above brackets. See[3-6] how this matrix is built up when superfields are involved.

By using the set of constraints (2.8) and (2.9) one obtains the brackets

$$\{V(z),\Pi(z')\}^{*}_{x_0=x_0'}=[\delta^2(\theta)\delta^2(\bar\theta)-2\delta^2(\theta)\bar\theta\bar\theta'$$
$$-2\delta^2(\bar\theta)\theta\theta'+4\theta\theta'\bar\theta\bar\theta']\delta^3(\mathbf{x}-\mathbf{x}')\ ,$$
$$\{V(z),V(z')\}^{*}_{x_0=x_0'}=0=\{\Pi(z),\Pi(z')\}^{*}_{x_0=x_0'}\ . \tag{2.11}$$

These will be the fundamental brackets which we shall use in Sec. IV.

## III. CANONICAL FORMALISM WITH HIGHER-ORDER DERIVATIVES

In this section we give a short discussion of the canonical formalism involving second-order derivatives. We shall deal only with scalar fields. The extension to superfields will be illustrated in the next section.

Let us consider the following kind of Lagrangian density:[14]

$$\mathcal{L}=\mathcal{L}(\phi,\partial_{\mu}\phi,\partial_{\mu}\partial_{\nu}\phi)\ . \tag{3.1}$$

The Euler-Lagrange equation of motion is obtained by the usual variational principle

$$\delta I=\delta\int_{t1}^{t2}dt\int d^3\mathbf{x}\,\mathcal{L}=0\ , \tag{3.2}$$

where one considers that at the instants $t_1$ and $t_2$ the system is characterized by

$$\delta\phi(t_1)=\delta\phi(t_2)=0,\quad \delta\dot\phi(t_1)=\delta\dot\phi(t_2)=0\ . \tag{3.3}$$

The equation of motion is then found to be

$$\frac{\partial\mathcal{L}}{\partial\phi}-\partial_{\mu}\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\phi)}+\partial_{\mu}\partial_{\nu}\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\partial_{\nu}\phi)}=0\ . \tag{3.4}$$

We introduce canonical momenta by considering variations in the action with only one of the extremes held fixed, for instance,

$$\delta\phi(t_1)=0,\quad \delta\dot\phi(t_1)=0\ , \tag{3.5}$$

but in such a way that only classical trajectories are allowed.[15] The canonical momenta $\pi$ and $s$, conjugated to $\phi$ and $\dot\phi$, respectively, are given by

$$\delta I=\int d^3x(\pi\phi+s\dot\phi)\ , \tag{3.6}$$

where $\delta\phi$ and $\delta\dot\phi$ denote variations at $t_2=t$. In the case of Lagrangian densities such as (2.1) we have

$$\delta I=\int d^3\mathbf{x}\left[\left[\frac{\partial\mathcal{L}}{\partial\dot\phi}-\frac{\partial}{\partial t}\frac{\partial\mathcal{L}}{\partial\ddot\phi}-2\partial_i\frac{\partial\mathcal{L}}{\partial(\partial_i\dot\phi)}\right]\delta\phi+\frac{\partial\mathcal{L}}{\partial\ddot\phi}\delta\dot\phi\right]\ . \tag{3.7}$$

By comparing these two last expressions, one obtains

$$\pi=\frac{\partial\mathcal{L}}{\partial\dot\phi}-\frac{\partial}{\partial t}\frac{\partial\mathcal{L}}{\partial\ddot\phi}-2\partial_i\frac{\partial\mathcal{L}}{\partial(\partial_i\dot\phi)}\ , \tag{3.8}$$

$$s=\frac{\partial\mathcal{L}}{\partial\ddot\phi}\ . \tag{3.9}$$

The next step is to define the canonical Hamiltonian. This is done in the standard way, that is

$$H_c=\int d^3x(\pi\dot\phi+s\ddot\phi-\mathcal{L})\ . \tag{3.10}$$

One verifies that the canonical Hamiltonian is a functional of $\phi$, $\dot\phi$, $\pi$, and $s$. (These play the role of canonical variables at the present formalism.) We can thus obtain the Hamiltonian equations of motion

$$\begin{aligned}\dot\phi(x)&=\frac{\delta Hc}{\delta\pi(x)},\quad \ddot\phi(x)=\frac{\delta Hc}{\delta s(x)}\ ,\\ \dot\pi(x)&=-\frac{\delta Hc}{\delta\phi(x)},\quad \dot s(x)=-\frac{\delta Hc}{\delta\dot\phi(x)}\ .\end{aligned} \tag{3.11}$$

Let $A[\phi,\dot\phi,\pi,s]$ be some dynamical functional quantity. The total time derivative of $A$ is

$$\dot A=\int d^3\mathbf{x}\left[\frac{\delta A}{\delta\phi}\dot\phi+\frac{\delta A}{\delta\dot\phi}\ddot\phi+\frac{\delta A}{\delta\pi}\dot\pi+\frac{\delta A}{\delta s}\dot s\right]\ . \tag{3.12}$$

Using the Hamilton equations of motion (3.11), we have

$$\dot A=\{A,H_c\}\ , \tag{3.13}$$

where

$$\{A,H_c\}=\int d^3\mathbf{x}\left[\frac{\delta A}{\delta\phi}\frac{\delta Hc}{\delta\pi}+\frac{\delta A}{\delta\dot\phi}\frac{\delta Hc}{\delta s}-\frac{\delta A}{\delta\pi}\frac{\delta Hc}{\delta\phi}-\frac{\delta A}{\delta s}\frac{\delta Hc}{\delta\dot\phi}\right] \tag{3.14}$$

is the Poisson brackets of $A$ and $H_c$. For any two functionals $A$ and $B$, the Poisson brackets will be defined analogously to (3.14).

The fundamental nonvanishing Poisson brackets are

$$\begin{aligned}\{\phi(x),\pi(x')\}_{x_0=x_0'}&=\delta^3(\mathbf{x}-\mathbf{x}')\ ,\\ \{\dot\phi(x),s(x')\}_{x_0=x_0'}&=\delta^3(\mathbf{x}-\mathbf{x}')\ .\end{aligned} \tag{3.15}$$

Other Poisson brackets such as $\{\phi,\dot\phi\}$ and $\{\pi,s\}$ are zero. If there are constraints involved, the corresponding Dirac brackets might be different from zero. For examples see Ref. 16.

## IV. CANONICAL QUANTIZATION OF THE THEORY

As we have seen in Sec. II, even using the Wess-Zumino gauge, the Lagrangian density (2.4) exhibits second-order derivatives. Thus, according to Eqs. (3.8) and (3.9) we introduce the following supermomenta:

$$\Pi=\frac{\partial\mathcal{L}}{\partial\dot V}-\frac{\partial}{\partial t}\frac{\partial\mathcal{L}}{\partial\ddot V}-2\partial_i\frac{\partial\mathcal{L}}{\partial(\partial_i\dot V)}\ , \tag{4.1}$$

$$S=\frac{\partial\mathcal{L}}{\partial\ddot V}\ . \tag{4.2}$$

Since $V$ is been considered in the Wess-Zumino gauge, we immediately infer that $\dot V$ and $S$ also satisfy similar brackets such as (2.11):

$$\begin{aligned}\{\dot V(z),S(z')\}^*_{x_0=x_0'}&=[\delta^2(\theta)\delta^2(\bar\theta)-2\delta^2(\theta)\bar\theta\bar\theta'\\ &\quad-2\delta^2(\bar\theta)\theta\theta'+4\theta\theta'\bar\theta\bar\theta']\\ &\quad\times\delta^3(\mathbf{x}-\mathbf{x}')\ ,\\ \{\dot V(z),\dot V(z')\}^*_{x_0=x_0'}&=0=\{S(z),S(z')\}^*_{x_0=x_0'}\ .\end{aligned} \tag{4.3}$$

Using (2.4), we obtain, from the momentum definitions (4.1) and (4.2),

$$\begin{aligned}\Pi(x,\theta,\bar\theta)=&-\tfrac12\dot V+\tfrac78(\theta\partial+\bar\theta\bar\partial)\dot V+\tfrac12(\bar\theta\bar\sigma^{i0}\bar\partial+\theta\sigma^{0i}\partial)\partial_iV\\ &+\tfrac34(\bar\theta\bar\sigma^0\partial)(\theta\sigma^0\bar\partial\dot V)+\tfrac34(\theta\sigma^i\bar\partial)(\bar\theta\bar\sigma^0\partial)\partial_iV\\ &-\frac{i}{4}(\bar\theta\bar\sigma^0\partial)\bar\partial\bar\partial V-\frac{i}{4}(\theta\sigma^0\bar\partial)\partial\partial V\ ,\end{aligned} \tag{4.4}$$

$$\begin{aligned}S(x,\theta,\bar\theta)=&\tfrac14(\theta\partial+\bar\theta\bar\partial)V-\frac{i}{8}\theta\theta(\bar\theta\bar\sigma^0\partial\dot V)-\frac{i}{8}\bar\theta\bar\theta(\theta\sigma^0\bar\partial\dot V)\\ &-\tfrac18(\theta\sigma^0\bar\partial)(\bar\theta\bar\sigma^0\partial V)\ ,\end{aligned} \tag{4.5}$$

where $V$ is in the Wess-Zumino gauge.

Relations (4.4) and (4.5) lead to the primary constraints

$$\begin{aligned}&\Gamma_1=\delta^2(\theta)\delta^2(\bar\theta)\Pi\approx0\ ,\\ &\Gamma_{2\dot\alpha}=\delta^2(\theta)\left[\bar\theta_{\dot\alpha}\Pi-\frac{i}{8}\delta^2(\bar\theta)(\partial\sigma^0)_{\dot\alpha}\bar\partial\bar\partial V\right]\approx0\ ,\\ &\Gamma_3^\alpha=\delta^2(\bar\theta)\left[\theta^\alpha\Pi-\frac{i}{8}\delta^2(\theta)(\bar\partial\bar\sigma^0)^\alpha\partial\partial V\right]\approx0\ ,\\ &\Gamma_4=\delta^2(\theta)\delta^2(\bar\theta)(\bar\partial\sigma^0\partial\Pi-\bar\partial\bar\sigma^i\partial\partial_i\dot V)\approx0\ ,\\ &\Gamma_5^i=\delta^2(\theta)\delta^2(\bar\theta)(\bar\partial\bar\sigma^i\partial\Pi+2\bar\partial\bar\sigma^0\partial\partial^iV+\tfrac32\bar\partial\bar\sigma^i\partial\dot V)\approx0\ ,\\ &\Gamma_6=\delta^2(\theta)\delta^2(\bar\theta)S\approx0,\quad \Gamma_7^\alpha=\delta^2(\bar\theta)\theta^\alpha S\approx0\ ,\\ &\Gamma_{8\dot\alpha}=\delta^2(\theta)\bar\theta_{\dot\alpha}S\approx0,\quad \Gamma_9=\theta\sigma^0\bar\theta S\approx0\ ,\\ &\Gamma_{10}^i=\theta\sigma^i\bar\theta S-\tfrac18\delta^2(\theta)\delta^2(\bar\theta)\bar\partial\bar\sigma^i\partial V\approx0\ .\end{aligned} \tag{4.6}$$

Each one of these constraints has a specific meaning in the component language. For example, $\Gamma_1\approx0$ means that the canonical momentum related to the component field $D(x)$ is zero. $\Gamma_2 \approx 0$ is the usual constraint of the fermionic momentum $\Pi_\lambda$, and so on.

In order to look for secondary constraints we would have to construct the primary Hamiltonian[1,2] and impose the consistency condition that constraints do not evolve in time. Alternatively, they can also be obtained directly from the equations of motion. From the action (2.1) one obtains

$$DD\bar{D}\bar{D}\,V=0\ . \tag{4.7}$$

Thus, the secondary constraints are

$$\begin{aligned}
&\Gamma_{11}=\delta^2(\theta)\delta^2(\bar\theta)\partial\partial\bar\partial\bar\partial V\approx 0\ ,\\
&\Gamma_{12}=\delta^2(\theta)\delta^2(\bar\theta)\partial\partial\bar\partial\bar\partial \dot V\approx 0\ ,\\
&\Gamma_{13}^{\dot\alpha}=\delta^2(\theta)\delta^2(\bar\theta)(\bar\sigma^{\mu}\partial)^{\dot\alpha}\bar\partial\bar\partial\partial_\mu V\approx 0\ ,\\
&\Gamma_{14\alpha}=\delta^2(\theta)\delta^2(\bar\theta)(\sigma^{\mu}\bar\partial)_\alpha\partial\partial\partial_\mu V\approx 0\ ,\\
&\Gamma_{15}=\delta^2(\theta)\delta^2(\bar\theta)(\partial\sigma^0\bar\partial\nabla^2 V+\partial\sigma^i\bar\partial\partial_i\dot V)\approx 0\ .
\end{aligned}\tag{4.8}$$

$\Gamma_{11}$ and $\Gamma_{12}$ are the equations of motion for the auxiliary fields $D(x)$ and $\dot D(x)$. $\Gamma_{13}^{\dot\alpha}$ and $\Gamma_{14\alpha}$ are the Dirac equations of motion for the photino fields $\bar\lambda_{\dot\alpha}$ and $\lambda_\alpha$. (In the higher-order formalism, these are constraints.) Finally, $\Gamma_{15}$ is the Maxwell equation for $A^0$.

Constraints $\Gamma_4$, $\Gamma_9$, and $\Gamma_{15}$ are first class. They occur because there is a residual gauge freedom in the theory. The Wess-Zumino gauge does not completely fix the gauge. We make the gauge choice

$$\begin{aligned}
&\Gamma_{16}=\theta\sigma^0\bar\theta V\approx 0\ ,\\
&\Gamma_{17}=\theta\sigma^0\bar\theta \dot V\approx 0\ ,\\
&\Gamma_{18}=\theta\sigma^i\bar\theta\partial_i V\approx 0\ .
\end{aligned}\tag{4.9}$$

These correspond, in components, to the radiation gauge[10] ($A^0\approx 0$ and $\nabla\cdot\mathbf{A}\approx 0$, including also $\dot A^0\approx 0$).

Altogether, with these three last constraints, we have that constraints $\Gamma_1-\Gamma_{18}$ are second class. This means that if we construct the total Hamiltonian[1,2] and impose the consistency condition that constraints do not evolve in time, we will not obtain new constraints, just the Lagrange multiplier superfields in terms of the dynamic fields of the theory.

We now have to use all these constraints to construct the Dirac brackets. This can be done iteratively.[1-7] Particularly, we have chosen for these iterations the following subsets of constraints: $\Gamma_1$, $\Gamma_6$, $\Gamma_{11}$, $\Gamma_{12}$; $\Gamma_{2\dot\alpha}$, $\Gamma_3^\alpha$; $\Gamma_7^\alpha$, $\Gamma_{8\dot\alpha}$, $\Gamma_{13}^{\dot\alpha}$, $\Gamma_{14\alpha}$; $\Gamma_9$, $\Gamma_{17}$; $\Gamma_{15}$, $\Gamma_{16}$; $\Gamma_4$, $\Gamma_{18}$; $\Gamma_5^i$, $\Gamma_{10}^i$.

The calculation of Dirac brackets in superspace has been exhaustively discussed in previous works.[3-7] It is just a matter of algebraic calculation to find out that the nonvanishing Dirac brackets of the theory are

$$\begin{aligned}
&\{V(z),\Pi(z')\}^D_{x_0=x_0'}=\left[-\delta^2(\theta)\bar\theta\bar\theta'-\delta^2(\bar\theta)\theta\theta'+\tfrac{3}{2}(\theta\sigma^i\bar\theta)(\theta'\sigma^j\bar\theta')\left[-\eta_{ij}+\frac{1}{4\pi}\partial_i\partial_j\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right]\right]\delta^3(\mathbf{x}-\mathbf{x}')\ ,\\
&\{V(z),V(z')\}^D_{x_0=x_0'}=i[\delta^2(\bar\theta)\delta^2(\theta')\theta\sigma^0\bar\theta'-\delta^2(\theta)\delta^2(\bar\theta')\theta'\sigma^0\bar\theta]\delta^3(\mathbf{x}-\mathbf{x}')\ ,\\
&\{\Pi(z),\Pi(z')\}^D_{x_0=x_0'}=i(\theta\sigma^0\bar\theta'-\theta'\sigma^0\bar\theta)\delta^3(\mathbf{x}-\mathbf{x}')\ ,\\
&\{\dot V(z),S(z')\}^D_{x_0=x_0'}=\tfrac{1}{2}(\theta\sigma^i\bar\theta)(\theta'\sigma^j\bar\theta')\left[-\eta_{ij}+\frac{1}{4\pi}\partial_i\partial_j\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right]\delta^3(\mathbf{x}-\mathbf{x}')\ ,\\
&\{\dot V(z),\Pi(z')\}^D_{x_0=x_0'}=[\tfrac{1}{4}(\theta\sigma^i\bar\theta)(\theta'\sigma^j\bar\theta')\partial_i+2\delta^2(\bar\theta)\theta\sigma^i\bar\sigma^0\theta\partial_i-2\delta^2(\theta)\bar\theta\bar\sigma^0\sigma^i\bar\theta'\partial_i]\delta^3(\mathbf{x}-\mathbf{x}')\ ,\\
&\{V(z),\dot V(z')\}^D_{x_0=x_0'}=(\theta\sigma^i\bar\theta)(\theta'\sigma^j\bar\theta')\left[\eta_{ij}-\frac{1}{4\pi}\partial_i\partial_j\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right]\delta^3(\mathbf{x}-\mathbf{x}')\ ,\\
&\{\Pi(z),S(z')\}^D_{x_0=x_0'}=\tfrac{3}{4}(\theta\sigma^i\bar\theta)(\theta'\sigma^j\bar\theta)\left[\eta_{ij}-\frac{1}{4\pi}\partial_i\partial_j\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right]\delta^3(\mathbf{x}-\mathbf{x}')\ .
\end{aligned}\tag{4.10}$$

Since there are no ambiguity problems with ordering operators in the right-hand side of the above relations, one can finally obtain the canonical quantization of the theory with the usual prescription

$$i\ (\text{Dirac brackets})\rightarrow(\text{commutators})\ . \tag{4.11}$$

The consistency of these results may be verified if one writes $V$, $\dot V$, $\Pi$, and $S$ in components. From the relations (4.11) we obtain

$$\begin{aligned}
&[A^i(x),\dot A^j(x')]_{x_0=x_0'}=i\left[-\eta^{ij}+\frac{1}{4\pi}\partial^i\partial^j\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right]\times\delta^3(\mathbf{x}-\mathbf{x}')\ ,\\
&[\lambda_\alpha(x),\bar\lambda_{\dot\alpha}(x')]^+_{x_0=x_0'}=-\sigma^0_{\alpha\dot\alpha}\delta^3(\mathbf{x}-\mathbf{x}')\ .
\end{aligned}\tag{4.12}$$

Other commutators and anticommutators vanish. These are exactly the relations which we would have obtained if we had started with the action (2.1) written in component fields and worked in the radiation gauge.

## V. CONCLUSION

We have applied the Dirac method for constrained systems to quantize the Abelian vector superfield in the Wess-Zumino gauge. The quantization using the vector superfield as a canonical variable leads to a natural example of higher-order derivative formalism. The results we have found are consistent when we go to component fields.

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