Cosmions in the nonsymmetric gravitational theory

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(Received 9 May 1988)

A phenomenological model is presented for the conserved fermion-number current in the nonsymmetric gravitational theory (NGT) based on the numbers of stable protons, neutrons, neutrinos, and cosmions (weakly interacting massive particles). By a suitable choice of the coupling constants, it is shown that the NGT is consistent with terrestrial observations and with the data for the solar system. A fit to the data of seven nondegenerate binary systems, including the data for DI Herculis and AS Cam, is given. The overall results are in accord with a solution of the problem of the cosmological missing dark matter. Agreement is also achieved with the data for the pulsar system PSR 1913 + 16 and the close cataclysmic binary system 4U 1820-30, involving gravitational radiation.

I. INTRODUCTION AND BASIC RESULTS IN THE NONSYMMETRIC GRAVITATIONAL THEORY

The nonsymmetric gravitational theory (NGT) is a complete and dynamically self-consistent theory of space-time that contains general relativity in a well-defined limit.[1–9] It provides the most general description of space-time consistent with our notions of a continuum, since no unjustifiable assumption about the symmetry of the fundamental tensor $g_{\mu\nu}$ and the connection $\Gamma^{\lambda}_{\mu\nu}$ is made. The nonsymmetric tensor $g_{\mu\nu}$ leads to a nontrivial extension of the local gauge group SO(3,1) of general relativity to the local gauge group GL(4,R) (Refs. 8 and 9). The latter group describes the most general transformations of the linear frames associated with the tangent fiber bundle that contains the homogeneous Lorentz group as a subgroup. At the same time, the theory retains covariance under the diffeomorphism group of general coordinate transformations in the manifold; this group contains GL(4,R) as a subgroup comprising a global symmetry of the spacetime manifold. The physical motivation of the NGT is to construct the most general classical description of space-time that contains general relativity as a limiting theory. NGT makes precise predictions that can be tested experimentally. In this way, we can learn whether or not nature chooses the more general NGT to describe space-time. The group GL(4,R) contains only infinite-dimensional spinor representations. This leads to the idea that particles are described as extended objects,[9] which is a radical departure from Einstein's theory of gravitation, in which point-particle fermions are conventionally described by finite, nonunitary representations of the homogeneous Lorentz group SO(3,1). Thus, fermions have a special status in NGT and, therefore, will play a fundamental role in the theory in determining the structure of space-time and the nature of gravitation.

By using infinite boson and fermion representations of GL(4,R), which describe extended particles, a finite quantum field theory including gravitation can be constructed using perturbation theory. The S matrix is unitary and a microcausality condition for the nonlocal field operators exists.[9]

The main purpose of this paper is to include cosmions [weakly interacting massive particles (WIMP's)] in the fermion-number current that acts as a source in NGT in addition to the energy-momentum tensor. With the addition of the cosmions, it is possible to fit all the presently available experimental gravitational data, including the perihelion shift of Mercury that would arise if the quadrupole moment coefficient J_2 of the Sun were found to be larger than $J_2 \geq 5\times10^{-6}$, and the anomalously low periastron shifts observed for the binary systems DI Herculis and AS Cam. The latter data cannot be fitted by a fermion-number current that corresponds to normal matter consisting of protons, neutrons, light neutrinos, and electrons as was shown in a previously published paper.[6] NGT predicts the existence of a repulsive force between fermion and antifermion particles that could play a fundamental role in cosmology and astrophysics. In particular, it can produce a new suppression mechanism for cosmion annihilation.

In the following, we shall adopt the notation and conventions used in Refs. 5 and 6. For the sake of completeness, we shall review the basic formulas needed to analyze the currently available observational data. In addition to a conserved nonsymmetric energy-momentum tensor $T^{\mu\nu}$, the theory contains a conserved-vector-current density S^{μ}. The conservation of this current arises by Noether's theorem from the invariance of the Lagrangian density under the transformations of an Abelian U(1) group. This current has been identified physically with the conserved particle (fermion) number of a body,[2,4] given by

$$F \equiv l^2 = \int \mathbf{S}^0 d^3x \ , \tag{1}$$

where $\mathbf{S}^{\mu} = (-g)^{1/2} S^{\mu}$ and l has the dimensions of a length. We have

$$S^{\mu} = \sum_i f_i^2 N_i u^{\mu} \ , \tag{2}$$

where f_i^2 is a coupling constant for each species i of fermions, N_i denotes the stable fermion particle number, and $u^\mu = dx^\mu/ds$ denotes the proper-time velocity of the particle.

The equations of motion of test particles and extended bodies with mass have been derived from the conservation laws of NGT (Ref. 6). They can be obtained from the variational principle[7]

$$\delta \int_{t_1}^{t_2} \left[\left[g_{(\mu\nu)} \frac{u^\mu}{u^0} \frac{u^\nu}{u^0} \right]^{1/2} + \frac{1}{6} \frac{l^2}{m} W_\mu \frac{u^\mu}{u^0} \right] dt = 0 , \quad (3)$$

where

$$W_\mu = \tfrac{1}{2}(W^\lambda_{\mu\lambda} - W^\lambda_{\lambda\mu}) \quad (4)$$

is the skew contraction of the nonsymmetric connection $W^\lambda_{\mu\nu}$. Equation (3) is invariant under the transformation

$$W_\mu \to W_\mu + \lambda_{,\mu} \quad (5)$$

provided that the end points t_1 and t_2 are held fixed.

The difference in the acceleration of two test particles falling in a gravitational field is given by[5]

$$\Delta a = \frac{2 m_s l_s^2 c^2}{R_s^5} \Delta \left[\frac{l_t^2}{m_t} \right] , \quad (6)$$

where $\Delta a = a_1 - a_2$ and we choose a to be positive. The m_s, m_t denote the source and test particle masses and l_s^2, l_t^2 denote the source and test particle fermion-number charges, respectively. The composition-dependent NGT contribution to the force is *attractive*. The gradient above or at the surface of a body is given to leading order by

$$\frac{dg}{dh} = \frac{10 m_s l_s^2 c^2}{R_s^6} \left[\frac{l_s^2}{m_s} - \frac{l_t^2}{m_t} \right] . \quad (7)$$

From the equations of motion of two massive particles, we obtain the formula for the periastron shift of a binary star:[6]

$$\dot\omega_{\rm NGT} = \dot\omega_{\rm GR} \lambda , \quad (8)$$

where $\dot\omega_{\rm GR} = d\omega_{\rm GR}/dt$ is the general-relativistic periastron precession given by

$$\dot\omega_{\rm GR} = \frac{3 G^{2/3} M^{2/3}}{\mathcal{P}^{5/3} c^2 (1-e^2)} = 1.5682 \times 10^6 \frac{m^{2/3}}{\mathcal{P}^{5/3}(1-e^2)} \ (\deg\,{\rm yr}^{-1}) , \quad (9)$$

where G is Newton's constant of gravitation, $M = M_1 + M_2$ is the sum of the masses of the primary and secondary components, respectively, m is the mass in solar units, $\mathcal{P} = P/2\pi$ where P is the orbital period, and e is the eccentricity; we have used cgs units. The parameter λ is given by

$$\lambda = 1 - \frac{K^4 c^4 (1+e^2/4)}{(GM)^{8/3} \mathcal{P}^{4/3} (1-e^2)^2} = 1 - 1.7667 \times 10^{-28} \frac{K^4 (1+e^2/4)}{\mathcal{P}^{4/3}(1-e^2)^2 m^{8/3}} , \quad (10)$$

where

$$K = [m (l_1^2 - l_2^2) d]^{1/4} \quad (11)$$

and

$$d = l_1^2/m_1 - l_2^2/m_2 . \quad (12)$$

We observe that if the two stars in a binary system have the same composition and mass, then $l_1 = l_2$ and $m_1 = m_2$ and the parameter d vanishes identically. The *periastron shift is then the same as that predicted in* GR.

The classical Newtonian contribution to the periastron shift is given by[10,11]

$$\dot\omega_{\rm cl} = \frac{360^\circ}{P} \left\{ k_{2,1} r_1^5 \left[\frac{m_2}{m_1} 15 f_2(e) + \left[\frac{\omega_{r,1}}{\omega_k} \right]^2 \left[1 + \frac{m_2}{m_1} \right] (1-e^2)^{-2} \right] + k_{2,2} r_2^5 \left[\frac{m_1}{m_2} 15 f_2(e) + \left[\frac{\omega_{r,2}}{\omega_k} \right]^2 \left[1 + \frac{m_1}{m_2} \right] (1-e^2)^{-2} \right] \right\} , \quad (13)$$

where

$$f_2(e) = (1-e^2)^{-5} (1 + \tfrac{3}{2} e^2 + \tfrac{1}{8} e^4) , \quad (14)$$

and ω_r/ω_k denotes the ratio of the angular velocity of axial rotation of a star to that of the orbital motion. The $k_{2,i}$ $(i=1,2)$ denote the coefficients of composition, while r_i are the fractional radii of the stars. We shall use the notation

$$\dot\omega^{\rm tot}_{\rm GR} = \dot\omega_{\rm cl} + \dot\omega_{\rm GR} , \quad (15a)$$

$$\dot\omega^{\rm tot}_{\rm NGT} = \dot\omega_{\rm cl} + \dot\omega_{\rm NGT} . \quad (15b)$$

Krisher[12] has derived the formula for gravitational radiation in NGT. The total change in the orbital period of a binary is given by

$$\dot P / P = (\dot P/P)_{\rm GR} + (\dot P/P)_{\rm dipole} , \quad (16)$$

where $(\dot P/P)_{\rm GR}$ is the GR result

$$(\dot P/P)_{\rm GR} = -(96/5) \mu {\rm m}^{2/3} \mathcal{P}^{-8/3} f_{\rm GR}(e) , \quad (17a)$$

$$f_{\rm GR}(e) = (1-e^2)^{-7/2} (1 + \tfrac{73}{24} e^2 + \tfrac{37}{96} e^4) . \quad (17b)$$

Here μ is the reduced mass of the system. The dipole contribution in NGT is given by

$$(\dot{P}/P)_{\rm dipole} = -88\mu d^2 \mathcal{P}^{-4} f_{\rm dipole}(e) \;, \tag{18a}$$

$$f_{\rm dipole}(e) = (1-e^2)^{-11/2}(1+\tfrac{19}{2}e^2+\tfrac{69}{8}e^4+\tfrac{9}{16}e^6) \;, \tag{18b}$$

where the dipole parameter d is defined in Eq. (12). The dipole contribution given in Eq. (18a) results in a secular decrease in the orbital period of a binary in addition to that predicted by GR. If the two stars in the binary system have the same composition and the same mass, then this dipole contribution *vanishes identically* and the predicted gravitational radiation is the same as in GR.

For the solar system, it is convenient to use the test-particle formula for the perihelion shift of the planetary orbits:[5]

$$\Delta\omega = \frac{6\pi G M_\odot}{c^2 p}\lambda_{\odot\rm planet} \;, \tag{19}$$

where

$$\lambda_{\odot\rm planet} = 1 + \frac{J_2 R_\odot^2 c^2}{2GM_\odot p} - \frac{K^4_{\odot\rm planet} c^4 (1+e^2/4)}{G^2 M_\odot^2 p^2} \;. \tag{20}$$

Here $p = a(1-e^2)$ where a is the semimajor axis of the orbit and

$$K_{\odot\rm planet} = (l_\odot^2 M_\odot d_{\odot\rm planet})^{1/4} \tag{21}$$

and

$$d_{\odot\rm planet} = l_\odot^2/M_\odot - l^2_{\rm planet}/M_{\rm planet} \;. \tag{22}$$

Moreover, J_2 is the quadrupole moment coefficient of the Sun. When $K_\odot$ is zero, we regain the standard perihelion advance formula of GR, and when $K_\odot > 0$ there will be a retrograde contribution to the perihelion shift of the planet. Other important formulas in NGT that are used to carry out relativistic tests of the theory are given in Ref. 5.

II. MODEL FOR THE CONSERVED FERMION-NUMBER CURRENT

In order to make quantitative predictions in NGT, we have to construct a phenomenological model for the fermion-number current. This means that we have to rely on our understanding of how many different species of stable fermions exist in the Universe and what constitutes their fundamental properties. This task is made difficult by the fact that we do not have a complete theory of elementary particles and their interactions. We do not even known whether there exists new physics beyond the current standard model of particles. Hopefully new high-energy accelerators and recent nonaccelerator experiments will shed light on this issue.

Our model for the conserved fermion number takes the form

$$F \equiv l^2 = f_p^2 Z + f_n^2 N + f_\nu^2 N_\nu + f_c^2 N_c \;, \tag{23}$$

where $f_p^2, f_n^2, f_\nu^2, f_c^2$ are universal coupling constants, with the dimensions of (length)^2, which measure the strengths of the couplings of the protons, neutrons, light (or massless) neutrinos and cosmions (WIMP's), respectively. The coupling constants f_i^2 can be positive or negative. Moreover, Z, N, N_ν, and N_c denote the numbers of protons, neutrons, neutrinos, and cosmions, respectively. We have taken into account in (23) that for an electrically neutral body, the number of electrons equals the number of protons $Z=N_e$. The light neutrinos have a zero or small mass $m_\nu \le 10$ eV compared to their typical kinetic energy in stars $E_\nu \sim 13$ MeV, whereby the ratio of neutrinos to baryons in stars will be very small. Moreover, we expect that the coupling to light neutrinos will be of the same order of magnitude as the proton, neutron, and electron couplings, since particle number has to be conserved in beta decay: $N \to P + e^- + \bar{\nu}_e$ and $N \to P + \mu^- + \bar{\nu}_\mu$. Thus, for our present purposes, we can ignore the neutrino contribution in Eq. (23).

The heavy neutrinos, that have been called cosmions or WIMP's in the literature, have been introduced to solve the missing-dark-matter problem and the solar-neutrino problem.[13–27] Several detailed calculations which determine the balance between their capture rate in the galactic halo and their annihilation and evaporation rates have placed their mass in the range 4.4 GeV $\le m_c \le$ 10 GeV. The critical cross section for their interaction with nuclei is $\sigma_{\rm crit} \sim 4\times 10^{-36}$ cm^2, a cross section that produces the required energy transport of WIMP's in the Sun, to solve the solar-neutrino problem. The evidence is mounting that most of the matter in the Universe is invisible. Over scales ranging from $10^6 M_\odot$ to $10^{15} M_\odot$, the ratio of dynamical mass to luminous mass is approximately ~ 10. The most promising candidate for dark matter is referred to as cold dark matter. If cold dark matter is composed of elementary particles they are typically quite massive, interact weakly, thus avoiding the production of electromagnetic radiation, and they decouple from ordinary matter at times preceding the epoch of recombination. The latter condition is necessary to give the dark-matter adiabatic density perturbations sufficient time to grow and form galaxies. The ratio of cosmion to baryon density is $\rho_{\rm cos}/\rho_{\rm bary} = m_{\rm cos}/m_{\rm bary} \sim 5-10$, which is just what is needed to solve the missing-dark-matter problem.

Several particle physics models have been proposed to describe cosmions.[20,24,25] Raby and West[24] have suggested that the cosmion is a Dirac neutrino that is part of a fourth generation and that carries a conserved lepton number that guarantees its stability. It will possess radiative corrections that generate an anomalous magnetic moment, which allows it to interact electromagnetically, producing a sufficiently large cross section to solve the solar-neutrino problem.

It is clear that our model for the fermion-number charge is sensitive to the existence of massive stable fermions, and in our calculations we shall find that two distinct levels of fermion interactions in NGT are required to fit all the data.

The total mass of a body is given by

$$\begin{aligned} M &= m_p Z + m_n N + m_c N_c \\ &= N[m_p(Z/N) + m_n + m_c(N_c/N)] \;, \end{aligned} \tag{24}$$

where we have neglected the neutrinos and m_p and m_n denote the proton and neutron masses, respectively. We have that $N=(M/2m_p)Y=(M/2m_n)(1-X)$ and $Z=(M/m_p)X+(M/2m_p)(1-X)$, where X and Y denote the mass fractions of hydrogen and helium, respectively. Several detailed calculations[16–18,26] have shown that the expected ratio of cosmions to baryons required to solve the solar-neutrino problem is $N_c/N_B \sim 10^{-11}$, where $N_B=Z+N$ denotes the baryon number. For a cosmion of mass $m_c \sim 4-10$ GeV, we can neglect the last term in (24) and write (23) approximately as

$$l^2=(Y/2)(M/m_p)[f_p^2(Z/N)+f_n^2+f_c^2(N_c/N_B)N_r] , \tag{25}$$

where $Y=2/[(Z/N)+1]$ and $N_r=N_B/N$.

III. TERRESTRIAL EXPERIMENTS

Let us first consider several recent terrestrial experiments designed to detect a difference in the acceleration of bodies falling in the gravitational field of the Earth. We shall ignore the contribution of the cosmions in the Earth. The difference in the composition of two test particles is expressed by the formula

$$\Delta(l_t^2/m_t)=(f_p^2/m_H)\Delta(Z/\mu)+(f_n^2/m_H)\Delta(N/\mu) , \tag{26}$$

where $\mu=m_t/m_H$ and m_H is the mass of hydrogen. For Earth, we have $Z \sim N$, and the difference in the acceleration of two test particles falling in Earth's gravitational field is given by

$$\Delta a/g=\left[\frac{M_\oplus^2 c^2}{m_H^2 R_\oplus^5 g}\right][f_p^4\Delta(Z/\mu)+f_n^4\Delta(N/\mu)]\beta$$
$$=1.11\times 10^{77}[f_p^4\Delta(Z/\mu)+f_n^4\Delta(N/\mu)]\beta , \tag{27}$$

where β is a correction factor, which for free-fall experiments is unity. For torsion balance experiments, when Earth is the dominant source, the force is directed towards the center of Earth, so the angle between the new force and the torsion wire is given by $\sin\delta \sim 10^{-3}$. For an experiment carried out at a latitude of 45°, we have $\beta=1.4/980.66=1.428\times 10^{-3}$.

We must assume a specific coupling for protons and neutrons; e.g., we could couple to baryon number or some other combination of Z and N. Let us assume that the coupling to protons is dominant. This will not seriously affect our conclusions, since the present terrestrial data are still uncertain,[28] and it is not yet possible to put definite limits on the coupling constants. In the free-fall experiment of Niebauer, McHugh, and Faller,[29] using copper and uranium test particles, a null result is obtained and they quote the upper bound

$$(\Delta a/g)_{\text{Cu-U}}<5.1\times 10^{-10} \tag{28}$$

with $\Delta(Z/\mu)=5.6\times 10^{-2}$. This yields

$$f_p^2<2.9\times 10^{-43}\ \text{cm}^2 . \tag{29}$$

Using, for Earth,

$$l_\oplus^2 \simeq (f_p^2/2m_H)M_\oplus , \tag{30}$$

we get the upper bound $l_\oplus<0.23$ km. In the case where Earth dominates the Eötvös-type torsion balance experiment,[30] inserting the correction factor β and using (28) yields the upper bound

$$(\Delta a/g)_{\text{Cu-U}}^{\text{tors bal}}<7.3\times 10^{-13} . \tag{31}$$

From Eqs. (7), (29), and (30), we obtain for the maximum possible value of the gradient of the gravitational acceleration at the surface of Earth:

$$(dg/dh)_{\max}=3.5\times 10^{-14}/\text{sec}^2 , \tag{32}$$

a result that is well below the values quoted by Stacey *et al.*[31] in their bore-hole mine experiments, and also below the gravimeter measurement of Ekhardt *et al.*[28] For the cliff experiments of Stubbs *et al.*,[32] Adelberger *et al.*,[33,28] Boynton *et al.*,[34,28] Thieberger,[35,28] and Bizetti *et al.*,[28] we must use the formula

$$(\Delta a/g)_{\text{cliff}}=f_p^4\left[\frac{M_c^2c^2}{m_H^2R_c^5g}\right]\Delta(Z/\mu)$$
$$=3.28\times 10^{65}f_p^4(M_c^2/R_c^5)\Delta(Z/\mu) , \tag{33}$$

where M_c and R_c denote, respectively, the mass of the cliff and the distance from the center of the cliff, treated as a spherically symmetric source. Since our formulas are extracted from an exact spherically symmetric solution of the NGT vacuum field equations,[5] or, equivalently, from the post-Newtonian expansions of the field equations in the approximation of spherically symmetric bodies,[6] we can at present obtain only a rough estimate of the cliff mass and the distance R_c. We note that for a constant cliff mass density ρ_c, the cliff mass is of order $M_c \simeq (4\pi/3)\rho_c R_c^3$, and it follows from (33) that

$$(\Delta a/g)_{\text{cliff}} \sim 5.76\times 10^{66}f_p^4\rho_c^2R_c\Delta(Z/\mu) . \tag{34}$$

The NGT predictions based on (29) and (34) are not consistent with the Boynton and Thieberger cliff experimental data for reasonable values of R_c.

IV. BINARY PULSAR DATA AND NEUTRON STARS

Let us now turn to the data for the binary pulsar PSR 1913+16 (Refs. 36 and 37) and the close cataclysmic binary 4U 1820-30 (Ref. 38). An extensive study of the former system has been performed and, provided that it is *assumed* that the companion star is a neutron star or a slowly rotating white dwarf, then good agreement between GR and the data[37] is obtained for the inferred masses of the two components: $M_1=(1.42\pm0.03)M_\odot$ and $M_2=(1.40\pm0.03)M_\odot$. The latter values for the masses agree well with the Chandrasekhar mass limit for neutron stars. If the two stars do, indeed, have the same mass and the same composition, as implied by the fit of the data to GR, then from Eqs. (11), (12), and (18a), we see that the NGT corrections to the periastron shift and the gravitational radiation formula of GR vanish. In this case, the data do not place any bound on the neutron-star value of l and the binary pulsar data do not constitute a useful test of NGT. If the companion star is a slowly rotating white dwarf, then the upper bound obtained by

Krisher[12] for the value of l for the unseen companion is $l_{\rm comp} < 350$ km. Using this upper bound on $l_{\rm comp}$, we obtain from Eq. (25), ignoring the cosmion contribution for white dwarfs and neutron stars, the upper bound $f_p^2 < 1.45 \times 10^{-41}$ cm^2, a value that is 2 orders of magnitude above the bound (29) obtained for f_p^2 from the experiment of Niebauer, McHugh, and Faller.[29]

A useful bound on the l values for neutron stars comes from stellar model calculations performed in NGT (Refs. 39 and 40). By using various equations of state $p = p(\rho)$ and adopting central values ρ_c and S_c^0, the generalized Oppenheimer-Volkoff equations of NGT can be solved numerically. It is found that stable neutron stars exist only for $l_{\rm NS} \leq 12$ km. For neutron stars, we have, from (25),

$$l_{\rm NS}^2 = \{M_{\rm NS}/m_H[(Z/N)+1]\}[f_p^2(Z/N)+f_n^2] . \quad (35)$$

For typical neutron stars $M_{\rm NS} \sim 1.32 M_\odot$ and $Z/N \sim 0.05$ and ignoring the neutron coupling contribution, we get

$$f_p^2 \leq 2.3 \times 10^{-44} \text{ cm}^2 , \quad (36)$$

a value that is an order of magnitude below the bound obtained from the Niebauer *et al.* free-fall experiment. An evaluation of the differential acceleration toward the center of Earth for copper-uranium using (27) and (36), yields the upper bound

$$(\Delta a/g)_{\rm Cu\text{-}U} \leq 3.6 \times 10^{-12} . \quad (37)$$

If we use the maximum l value allowed for the neutron stars which, according to the NGT stellar model calculations, has the value $l_{\rm NS} \sim 12$ km, then the periastron shift formula (8) for the binary pulsar PSR 1913+16 gives a value for $\dot\omega_{\rm NGT}$ that equals $\dot\omega_{\rm GR}$ to within the limits of any foreseeable observations. Thus, although the PSR 1913+16 binary pulsar data agree with the predictions of NGT, they do not constitute a critical test of the theory.

Krisher[41] has used the data for the close cataclysmic binary system 4U 1820-30 (Refs. 38 and 42) to establish a bound on the l value for the white dwarf companion star with a mass $M_{\rm WD} \sim 0.055 M_\odot$:

$$l_{\rm WD} \leq 70 \text{ km} . \quad (38)$$

For white dwarfs $Z/N \sim 1$ and using (25) and (38), ignoring again the neutron contribution, we obtain

$$f_p^2 \leq 1.65 \times 10^{-42} \text{ cm}^2 , \quad (39)$$

a result that is an order of magnitude above (29).

V. NONDEGENERATE BINARIES AND THE SOLAR SYSTEM

We shall now concentrate on the nondegenerate binary systems and the relativistic data for the solar system. Eclipsing binary systems are an important source of astrophysical information. Periodically occurring eclipses result in short but measurable decreases in the total apparent brightness of the binary system, ranging from a few hundredths of a magnitude up to a few magnitudes. However, even in the absence of eclipses, important data can be acquired through spectroscopic analyses. Precise information about the apsidal motion of the orbit can be obtained from spectroscopic studies. About 3000 eclipsing binaries have been studied so far. For eccentric binary orbits the secondary minimum, which occurs when the hotter star eclipses the cooler star, is displaced from the half-period point in proportion to the product of the eccentricity and the cosine of the longitude of periastron. The eclipse timings are a sensitive measure of ω, and over many years they reveal, through changes in ω, the rate of periastron motion, $\dot\omega$. It is important to study systems that have a relativistic periastron shift that is comparable to or greater than the Newtonian apsidal motion, caused by the deviation of the stars from spherical symmetry. These deviations arise from tidal and rotational deformations of the binary components, which depend on the fractional radii, their axial rotation rates, and their internal mass distributions, as seen in formula (13). We shall find, however, that the NGT contribution to the apsidal motion for *massive* binaries can be a significant fraction of the total periastron shift. Therefore, it is also important to study such massive systems, even though the tidal effects dominate the GR periastron shift contribution. But for such massive binaries, the orbit is often circular, preventing any measurement of a periastron shift.

Attention was first called by Rudkjøbing[43] to the importance of the eighth magnitude, eccentric eclipsing binary DI Herculis as a test of GR, since the GR component is greater than the classical component. DI Her (=HD175227) consists of two main-sequence BV stars moving in an eccentric orbit with $e=0.49$ and with an orbital period of $P=10.55$ days. The orbital inclination is 89° and the fractional radii are small: $r_1 = 0.0622 \pm 0.0011$ and $r_2 = 0.0575 \pm 0.0011$. The eclipses are deep and narrow, permitting a very accurate determination of the periastron shift from the measurement of the displacement of the secondary minimum from the primary minimum. Because of the work of Martynov and Khalliulin[44] and the more recent extensive studies made by Guinan and Maloney,[45,46] and the excellent determinations of the orbit and stellar parameters of the system,[47] the theoretical classical and GR components of the apsidal motion have been determined with reasonable accuracy[45] to be

$$\dot\omega_{\rm cl} = (1.93° \pm 0.26°)/100 \text{ yr}$$

and (40)

$$\dot\omega_{\rm GR} = (2.34° \pm 0.15°)/100 \text{ yr} .$$

The total periastron shift is

$$\dot\omega_{\rm GR}^{\rm tot} = (4.27° \pm 0.3°)/100 \text{ yr} . \quad (41)$$

The GR periastron shift is nearly 200 times greater than the 43 arcsec/100 yr advance of the perihelion predicted for the orbit of Mercury. The observed periastron shift obtained from least-squares solutions of the timings of primary and secondary eclipse minima, extending over an 84-yr interval, yield a small periastron shift[45] of

$$\dot{\omega}_{\rm obs}=(0.65°\pm0.18°)/100\ {\rm yr}\ . \qquad (42)$$

A least-squares solution that weighs heavily against the less accurate nonphotometric data yields[48]

$$\dot{\omega}_{\rm obs}=(0.92°\pm0.2°)/100\ {\rm yr}\ . \qquad (43)$$

We shall use the result (42) as the accepted observed value of the periastron shift, since the use of the result (43) would not alter our conclusions significantly. We note that the data for DI Her is inconsistent with GR for any positive value of $\dot{\omega}_{\rm cl}$.

One of the clear, parameter-free predictions of NGT is that the periastron precession of two bodies does not contain any NGT corrections if the two bodies have the same composition and mass. We are, therefore, interested in studying binary systems in which the two stars have the same composition and mass. Three such systems have been the subject of careful observation: V889 Aql ($P=11.2d$, $e=0.38$), V1143 Cyg ($P=7.64d, e=0.54$), and V541 Cyg ($P=15.34d$, $e=0.49$) (Refs. 49–54). These three systems have the same spectral classification and the same mass. They show periastron shifts that agree well with GR. The system V541 Cyg is unique in that $\dot{\omega}_{\rm GR}/\dot{\omega}_{\rm cl}\sim5.5$, so that it provides a reliable test of the relativistic corrections to the periastron shift.[53,54] The system V889 Aql is also favorable from this point of view, for $\dot{\omega}_{\rm GR}/\dot{\omega}_{\rm cl}\sim3.6$.

Three other systems that possess good observational data are AS Cam ($P=3.43d$, $e=0.17$), AG Per ($P=2.03d, e=0.079$), and α-Vir (Spica) ($P=4.01d$, $e=0.146$). These three systems have different spectral classification and mass. AS Cam is another eighth magnitude eccentric, eclipsing binary with well-determined orbital and physical properties.[55,56] However, in contrast with DI Her, this system has larger fractional radii and the classical contribution to the periastron shift is greater than that predicted by GR. Nonetheless, the system is well understood and the observed periastron shift is one-third that expected from GR and Newtonian theory. The bright star Spica (α Virginis) was one of the first to be known as a spectroscopic binary. Because this star is bright and has been well observed, it has become an important source of information about massive stars.[57] Since this double-lined, spectroscopic binary can also be resolved as a visual binary by using interferometry techniques, accurate orbital data can be obtained.[58] In fitting NGT to the data, we shall use values of the composition coefficients $k_{2,i}$ ($i=1,2$) obtained from Jeffery.[59,60]

In fitting the solar-system data and the data for the nondegenerate binary systems, the cosmions will play a significant role. Indeed, as shown in Ref. 7 and as we have seen in the previous section, the f^2 corresponding to normal matter consisting of baryons, electrons, and light neutrinos cannot influence the motion of the planets in the solar system or the dynamics of nondegenerate binaries in a significant way. We shall choose the cosmion mass to be $m_c\sim4-10$ GeV and we shall choose for the Sun $(N_c/N_B)_\odot=10^{-11}$, a value for the cosmion-baryon ratio that can solve the solar-neutrino problem. If the density of cosmions in the core of the Sun is found to be smaller than the value required to solve the solar-neutrino problem, then we would have to lower the value of $(N_c/N_B)_\odot$ and increase the value of f_c^2. If we assume that the cosmions belong to a fourth generation of particles with an associated more massively charged companion and a new conserved lepton number, then the coupling strength to the fermion-number current can be significantly different compared to that experienced by the protons, neurons, and neutrinos. The new conserved lepton number signals the onset of a new scale of physics in which the cosmion is the lightest stable fermion. Because the cosmions are massive, they will lie well above the threshold energy for standard β decay. The quanta associated with the NGT interactions have spins 0, 1, and 2 and they are massless. However, since the coupling constants f_i^2 have the dimensions of a [length]^2, they can be associated with typical mass scales for the spin-0 and -1 interactions. For the protons and neutrons, the mass scale is $\geq10^{16}$ GeV^2, while for the cosmions it is ~400 GeV^2, corresponding to $f_p^2\sim f_n^2\leq10^{-44}$ cm^2 and $f_c^2\sim10^{-30}$ cm^2, respectively. In the linear approximation to NGT (Ref. 61), it can be shown that the coupling of the weak skew field $g_{[\mu\nu]}$ to the fermion current S^μ scales like $f_i^2/G^{1/2}$. But a calculation reveals that, due to the symmetries of the Feynman diagrams, the first-order coupling of $g_{[\mu\nu]}$ to the fermion current vanishes.

A fit to the solar system with $J_2=5.53\times10^{-6}$ is obtained using the values $f_c^2=8.75\times10^{-30}$ cm^2, $(N_c/N_B)_\odot=10^{-11}$, $f_p^2=6.47\times10^{-46}$ cm^2, $f_n^2=4.68\times10^{-46}$ cm^2, $Y_\odot=0.25$. With these values, a simultaneous fit is obtained to the observed periastron shifts for the four binary systems DI Her, AS Cam, AG Per (Ref. 62), and α Vir, while the data for the three systems V889 Aql, V541 Cyg, and V1143 Cyg agree with the predictions of both GR and NGT, because those systems have the same spectral classification and the same mass, leading to the vanishing of the NGT periastron shift contributions. The results of this fitting procedure for the binaries are displayed in Tables I, II, and III. The predicted value of l for a neutron star is $l_{\rm NS}=8.9$ km, using a typical value $Z/N=0.05$. If we choose f_c^2 to have a lower value, then the solar quadrupole moment coefficient J_2, required to fit Mercury's perihelion precession, will decrease below its expected value for a more slowly rotating solar interior: $J_2\simeq10^{-6}$. A fit to the solar system with $J_2=1.02\times10^{-6}$ and $K_\odot=2.11\times10^3$ km, and to the data for DI Her, is obtained for $f_c^2=3.75\times10^{-30}$ cm^2, $(N_c/N_B)_\odot=10^{-11}$ and $(N_c/N_B)_1=3.0\times10^{-10}$, $(N_c/N_B)_2=1.86\times10^{-10}$, $Y_\odot=0.25$, $Y_1=0.235$ and $Y_2=0.222$. The remaining binaries can also be fitted with this value of f_c^2 using given values of $(N_c/N_B)_i$. The precise choice of the coupling constants f_p^2 and f_n^2 is not important for our fitting procedure and they could be chosen to be equal and smaller in value if so desired.

We see from the results for the massive system α Virginis that NGT predicts significant contributions to the periastron shift that could play a major role in stellar-evolution theory. Model stellar calculations have been performed[63] using the usual assumptions of evolution that could not reproduce the apsidal motion for Spica. By increasing the nonelectron-scattering opacity or mix-

TABLE I. System parameters.

System	Spectral class	m_1	m_2	r_1	r_2
V1143 Cyg	F5V+F5V	1.3	1.3	0.062	0.054
		±0.03	±0.03	±0.003	±0.003
V889 Aql	B9V+B9V	2.5	2.5	0.0535	0.0535
		±0.5	±0.5	±0.002	±0.002
V541 Cyg	B9V+B9V	2.69	2.6	0.0441	0.0425
				±0.0003	±0.0004
AS Cam	B8V+B9V	3.3	2.5	0.149	0.111
		±0.1	±0.1	±0.0004	±0.0004
DI Her	B4V+B5V	5.15	4.52	0.0622	0.0575
		±0.10	±0.06	±0.0011	±0.0011
AG Per	B4V+B5V	4.53	4.12	0.207	0.187
		±0.4	±0.06	±0.004	±0.004
α Vir	B11V+B3V	10.9	6.8	0.292	0.184
		±1.0	±1.0	±0.018	±0.036
		$k_{2,1}$	$k_{2,2}$	$\omega_{r,1}/\omega_k$	$\omega_{r,2}/\omega_k$
V1143 Cyg		0.006	0.009	1.0	1.0
		±0.0005	±0.001		
V889 Aql		0.0085	0.0085	1.6	1.6
		±0.002	±0.002		
V541 Cyg		0.0055	0.0055	3.0	3.3
		±0.0010	±0.0010		
AS Cam		0.0056	0.0056	1.0	1.0
		±0.0010	±0.0010		
DI Her		0.0087	0.0078	3.5	3.8
		±0.0010	±0.0010	±1.1	±1.3
AG Per		0.0063	0.0063	2.5	2.5
		±0.0013	±0.0013		
α Vir		0.0031	0.0074	1.7	1.74
		±0.0015	±0.002	±0.34	±0.35

ing beyond the convective opacity core, it has been possible to produce models with the correct apsidal motion constant. Odell[63] found models using Watson's opacities[64] that include autoionization lines of metals, although the Cox and Stewart[65] opacities do not lead to agreement with the apsidal motion. This problem has also been studied by Stothers[66] and by Jeffery.[59] The compositions of massive binaries and their relation to the observed apsidal motion need further study in the light of our NGT results.

Solutions for lower values of f_c^2 and J_2 lead, of course, to different cosmion-baryon ratios for the binaries. Only careful calculations of the allowed ratios of cosmions to baryons for main-sequence stars could distinguish among these possible solutions for the solar system. Probably no significant distinctions among the solutions can be made at present. The value $J_2=5.53\times10^{-6}$ agrees with the result obtained from solar oscillation measurements by Hill, Bos, and Goode,[67] Hill and Rosenwald,[68] and the more recent measurements using visual techniques obtained by Dicke, Kuhn, and Libbrecht.[69] A lower value of J_2 would agree with the conflicting smaller values quoted by other authors.[70] It remains an important issue to obtain a reliable measurement of the Sun's J_2, e.g., by a close solar satellite fly-by. By using $f_c^2=8.75\times10^{-30}$ cm^2, we find that the parameters $K_{\odot\,\mathrm{Merc}}$ and $l_\odot$ in Eq. (21) have the values

$$K_{\odot\mathrm{Merc}}=3.23\times10^3\ \mathrm{km}\ , \tag{44}$$

$$l_\odot=3.23\times10^3\ \mathrm{km}\ . \tag{45}$$

TABLE II. Parameters used in the NGT calculation. We have used $f_p^2=6.47\times10^{-46}$ cm^2, $f_n^2=4.68\times10^{-46}$ cm^2, $f_c^2=8.75\times10^{-30}$ cm^2, $Y_\odot=0.25$, and $(N_c/N)_\odot=10^{-11}$.

System	K ($\times10^4$ km)	Y_1	Y_2	$(N_c/N_B)_1$ ($\times10^{-10}$)	$(N_c/N_B)_2$ ($\times10^{-10}$)
AS Cam	1.32	0.235	0.222	0.80	0.47
DI Her	1.98	0.235	0.222	1.0	0.49
AG Per	1.80	0.235	0.222	0.80	0.31
α Vir	4.56	0.250	0.222	8.58	8.0

TABLE III. Predicted and observed $\dot{\omega}$

System	$\dot{\omega}_{\rm cl}$ (deg/100 yr)	$\dot{\omega}_{\rm GR}$ (deg/100 yr)	$\dot{\omega}_{\rm NGT}$ (deg/100 yr)
V1143 Cyg	2.4	1.8	1.8
V889 Aql	0.33	1.20	1.20
V541 Cyg	0.15	0.82	0.82
AS Cam	34.1	8.47	−17.70
DI Her	1.94	2.33	−1.25
AG Per	692	25.9	−156
α Vir	521	13.6	−228
	$\dot{\omega}^{\rm tot}_{\rm GR}$ (deg/100 yr)	$\dot{\omega}^{\rm tot}_{\rm NGT}$ (deg/100 yr)	$\dot{\omega}_{\rm obs}$ (deg/100 yr)
V1143 Cyg	4.2	4.2	3.4 ±0.2
V889 Aql	1.53	1.53	1.5 ±0.5
V541 Cyg	0.97	0.97	0.90 ±0.13
AS Cam	42.57	16.40	16.0 ±1.3
DI Her	4.27	0.690	0.65 ±0.18
AG Per	718	536	554 ±39
α Vir	535	293	290 ±30

These values lead to agreement with the data for all the other NGT predictions for the solar system such as the bending of light, time-delay, and red-shift measurements.[5]

VI. CONSTRAINTS ON THE COSMION PARTICLE MODEL AND THE COSMION ANNIHILATION CROSSSECTION

If we choose the simplest solution to the phenomenological analysis of NGT, based on the two coupling constants $f_N^2 \equiv f_p^2 = f_n^2$ and f_c^2, then a strong case can be made, using NGT, for the existence of a new regime of physics starting at an energy around 4 GeV, which is the lowest value permitted by the solution of the solar-neutrino problem using WIMP's. One possibility is to choose $F=B-L$, where B and L are separately violated in grand unification theories (GUT's). In the effective low-energy theory, B is conserved while the baryon number B_c associated with the new regime is violated. The latter condition is needed to suppress the number of heavy baryons, since searches for heavy "protons" with mass less than 350 GeV have shown that their ratio to baryons is less than 10^{-21} (Ref. 71). We also require that the new lepton number L_c be conserved in the low-energy region, corresponding to a stable cosmion, thereby suppressing the loss rate of cosmions in the Sun.[24]

We must concern ourselves with the question whether enough cosmions can be present in a star to play a significant role as a source of fermion particle number. The total number of cosmions per baryon number in a star is given by[16]

$$N_c/N_B \sim \frac{\rho_c}{M} R^2 \frac{v_{\rm esc}^2}{\bar{v}} \frac{\sigma}{\sigma_{\rm crit}} \tau \frac{m_p}{m_c} , \qquad (46)$$

where ρ_c is the ambient cosmion mass density in the Galaxy, $v_{\rm esc}=(2\,GM/R)^{1/2}$ is the escape velocity from a star of radius R and mass M, $\bar{v}$ is the cosmion velocity dispersion, $\sigma_{\rm crit}$ is a critical cross section for cosmion capture which is 4×10^{-36} cm^2 for the Sun, and τ is the lifetime of the star. Let us assume that the star has reached a steady state, that $\sigma\sim\sigma_{\rm crit}$ and that $\rho_c\sim\rho_{c\odot}$. Then, the ratio of the number of cosmions to baryons of a star to that of the Sun is of the order

$$\delta = \frac{N_c}{N_B} \Bigg/ \left[\frac{N_c}{N_B}\right]_\odot \sim \frac{M_\odot R^2 v_{\rm esc}^2}{M R_\odot^2 v_{\rm esc\odot}^2} = R/R_\odot . \qquad (47)$$

We can choose for a neutron star $R_{\rm NS}\sim 8$ km, $M_{\rm NS}\sim M_\odot$ and using the cosmion coupling constant $f_c^2=8.75\times10^{-30}$ cm^2, we have $(N_c/N_B)_{\rm NS}\sim(N_c/N_B)_\odot\delta \sim 10^{-16}$, $N_r\sim 1$, and

$$l_{\rm NS}\sim f_c(M_{\rm NS}/m_p)^{1/2}(N_c/N)^{1/2}_{\rm NS}\sim 10 \text{ km} . \qquad (48)$$

This result is consistent with the stellar model calculations in NGT (Refs. 39 and 40). We have made assumptions about the size of the cosmion density ρ_c in the Galaxy, the amount of cosmion evaporation and annihilation that take place in a neutron star and that the cosmion-nuclear forces are spin independent. Moreover, not enough is known about the mechanisms of gravitational collapse or supernova explosions to make accurate estimates of the cosmion content of compact stars. We do see from this rough calculation that a considerable reduction of the l values for stars can take place after gravitational collapse.

For the primary component in the binary system DI Herculis, we have $R\sim 2.7R_\odot$, and (47) gives $(N_c/N_B)_{\rm DI\ Her}\sim(N_c/N_B)_\odot\delta\sim 3\times10^{-11}$, which is in rough agreement with the value used in our fits to the data for the DI Herculis binary system. However, this assumes that a steady state for the cosmion content of the stars has been reached, an assumption that requires further investigation. The estimated age of DI Herculis is $\sim10^7$ yr. At present, not enough is understood about the evolution of stars containing cosmions to make definite estimates of their cosmion contents. In our fits to the binary systems, we have used N_c/N_B ratios which are larger for the more massive components in a binary system, since this would be expected to be true for stars that have reached a cosmion steady state.

An important constraint on the theory is that the cosmion-anticosmion annihilation in the Sun must be highly suppressed.[17] Even if all the cosmions impinging on the Sun are captured, the cosmion annihilation will dilute the cosmion concentration below $N_c/N_B\sim10^{-11}$. The equilibrium concentration is determined by the balance of capture and annihilation rates and gives the constraint

$$\epsilon\sigma_A \le 10^{-4}\sigma_S \sim 10^{-40} \text{ cm}^2 , \qquad (49)$$

where ϵ is the mean thermal speed of the cosmion,

$\epsilon \sim 10^{-3}$, σ_A is the annihilation cross section, and σ_S is the mean scattering cross section per baryon in the Sun. A solution to this problem is found within the framework of NGT, since the NGT force is repulsive between cosmions and anticosmions.

The NGT acceleration between two particles i and j is given in post-Newtonian order by[5,6]

$$\mathbf{a}_{\rm NGT}=\frac{2K_{ij}\mathbf{r}c^2}{r^6}\ , \tag{50}$$

where

$$\mathbf{r}=\mathbf{r}_i-\mathbf{r}_j,\quad \mathbf{a}=\mathbf{a}_i-\mathbf{a}_j\ , \tag{51}$$

and

$$K_{ij}=m(l_i^2-l_j^2)d_{ij},\quad m=m_i+m_j\ ,\\ d_{ij}=l_i^2/m_i-l_j^2/m_j\ . \tag{52}$$

For two identical fermions or antifermions $l_i^2=l_j^2$ and $m_i=m_j$ and we see that $\mathbf{a}_{\rm NGT}$ vanishes identically in the post-Newtonian order of approximation. A weak attractive force between identical fermions will occur in higher orders of approximation. For antifermions we have that $l_{\bar{i}}^2=-l_i^2$ and choosing $l_j^2=l_{\bar{i}}^2$ in (50) and (52), we get

$$\mathbf{a}_{\rm NGT}=\frac{16f_i^4\mathbf{r}c^2}{r^6}\ , \tag{53}$$

where for a fermion particle we have set $l_i^2=f_i^2$. A consequence of this result in NGT is that a proton will fall faster in a gravitational field than an antiproton.

If we substitute into (53) the value for the cosmion coupling constant $f_c^2=8.75\times10^{-30}$ cm^2, used to obtain our fits to the solar system and the binary system data, then we find that the relative acceleration of a cosmion and an anticosmion at a distance, 10^{-13} cm, is of the order 10^{29} cm/s^2, which is about 5 orders of magnitude smaller than the coulomb acceleration of two protons at this distance. Thus, the NGT repulsive force is strong enough to reduce the cosmion annihilation in the Sun.

We have argued that cosmions are necessary to solve the missing-dark-matter problem, and that cosmions are cosmologically stable particles with a mass between 4 and 10 GeV with weak interactions and a coupling to the NGT skew field $g_{[\mu\nu]}$ that is stronger than the coupling of ordinary light baryons (quarks) to $g_{[\mu\nu]}$. It is noteworthy that such particles are the very ones that survive the big bang with cosmological relic densities $\Omega\sim\frac{1}{10}-1$ (Ref. 72). Other possible consequences of NGT in cosmology such as galaxy formation and cosmic strings will be considered elsewhere.

VII. CONCLUSIONS

We have derived a model for the conserved fermion-number current in NGT that serves as the basis for a complete analysis of data, ranging from neutron stars and binary pulsars to terrestrial experiments, the solar system, and nondegenerate binaries. As far as the terrestrial experiments are concerned, it would be important to attempt a free-fall experiment with an accuracy that could detect a differential acceleration toward the center of the Earth with $\Delta a/g\leq10^{-12}$. The proposed spacecraft experiment of Worden and Everitt[73] could achieve such an accuracy. In NGT, the ratio M_G/M_I of inertial to gravitational mass is unity up to the first post-Newtonian order of approximation.[6] Therefore, there is no Nordtvedt effect observable to this order as in GR, and we cannot use linear ranging data to detect a differential acceleration of the moon falling toward the Earth.[74]

The agreement of the predictions of NGT with the data for the binary pulsar PSR 1913+16 and the data for the nondegenerate binary systems such as DI Herculis can be taken as experimental confirmation of NGT. The nondegenerate binary data presently disagree seriously with GR. The observed anomalously low periastron shift for DI Herculis cannot be reconciled with the predictions of GR and Newtonian theory. No astrophysical explanation has been put forward for this anomaly that agrees in a reasonable way with the available data for this system. But further observational work on other binaries is required to increase the evidence for the correctness of NGT.

The use of precise astronomical data for the orbits of binary systems and the planets in the solar system, based on the role of cosmions in NGT, is a new and interesting way to explore the problem of the cosmological missing dark matter. A detection of cosmions by presently ongoing experiments[75] would be desirable, as well as a better understanding of the physics of the cosmions and its connection to the particle-physics structure of the NGT fermion-number current.

ACKNOWLEDGMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada and by Centre International des Étudiants et Stagiaires, France. I thank Professor K. Chadan for his kind hospitality during may stay at Complexe Scientifique D'Orsay, Orsay. I thank E. F. Guinan and A. Gould for helpful discussions. Laboratoire de Physique Théorique et Hautes Énergies is Laboratoire associé au CNRS.

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