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Bulk viscosity of hot neutron-star matter and the maximum rotation rates of neutron stars

Raymond F. Sawyer
Department of Physics, University of California, Santa Barbara, California 93106
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The bulk viscosity of neutron-star matter, arising from the time lag in achieving beta equilibrium as the density is changed, is calculated. In the model used in standard cooling calculations, it is found, for the case of normal neutron matter, that the bulk viscosity goes as the sixth power of the temperature (as compared with a T^{-2} dependence for the shear viscosity), and that at temperatures above 10^9 K the bulk viscosity may dominate the dissipation term which regulates the gravitational-wave instability of rapidly rotating neutron stars. This raises the possibility that in the first years of a neutron star's life the star could become unstable as the bulk viscosity decreases through cooling, with potentially observable consequences.

The viscosity of the matter within neutron stars may affect potentially observable properties in a number of ways, for example, by damping vibrations created in the formation of a new star, or in a seismic event in an established star. The viscosity also enters in criteria which have been developed for the growth of gravitational-wave instabilities in rapidly rotating neutron stars.[1-6] The maximum possible rate of rotation may be controlled by these instabilities, and in turn by the viscosity of the matter.

Cutler and Lindblom[7] have estimated the effects of viscous dissipation on the gravitational-wave instability, using shear viscosities calculated by Flowers and Itoh[8] for the case of ordinary neutron matter, with modifications for the possible case of nucleon superfluidity. Their considerations have further shown that the effects of bulk viscosity are likely to be negligible if the coefficient of bulk viscosity is no larger than the coefficient of shear viscosity.

The purpose of the present note is to point out that at temperatures higher than 10^9 K the bulk viscosity for neutron matter, in models which have been used for cooling calculations, is considerably larger than the shear viscosity. The bulk viscosity increases rapidly with increasing temperature, in contrast to the shear viscosity, which decreases rapidly with increasing temperature. There may be interesting consequences for a hot, young, rapidly rotating neutron star, such as expected at the core of SN 1987A. The source of the bulk viscosity is the deviation from beta equilibrium induced by compressions or rarefactions of the matter. The essential physics in what follows is similar to that in the 1968 paper by Finzi and Wolf[9] on the damping of pulsations of neutron stars; the differences are in the present work's detailed calculations of reaction rates, in the casting of the results in terms of the bulk viscosity, and in the application to the maximum rate of rotation.

We shall consider small deviations from chemical equilibrium in nearly degenerate matter composed of neutrons, protons, and electrons. The measure of the departure from equilibrium is the chemical potential difference $\delta\mu = \mu_n - \mu_p - \mu_e$, which will be taken to be small compared to $k_B T$, itself small compared to the chemical potentials of the individual species. The four pieces of microphysical data which are required are as follows:

(i) The reaction rates which push the system with $\delta\mu \neq 0$ toward beta equilibrium, that is, the difference, induced by $\delta\mu$, between the rates of production of electron neutrinos and electron antineutrinos as the star cools:

$$\Gamma_\nu = \Gamma_{\bar\nu} = (\delta\mu)\lambda + O((\delta\mu)^2) , \tag{1}$$

where Γ_ν and $\Gamma_{\bar\nu}$ are the rates per unit volume at which electron neutrinos and antineutrinos, respectively, are radiated by the hot medium. In the medium composed of n, p, e, the rate Γ_ν derives, in effect, from the reaction $e^- + p \to n + \nu$ and $\Gamma_{\bar\nu}$ from the reaction $n \to e + p + \bar\nu$. However, under the conditions which prevail in degenerate neutron matter, both reactions need a momentum-absorbing spectator nucleon to proceed, in order to balance energy and momentum while respecting the Fermi occupancy factors of the medium. When the system is in chemical equilibrium, we have $\Gamma_\nu = \Gamma_{\bar\nu}$.

(ii) The coefficient governing the response of $\delta\mu$ to a small change in the chemical composition, with the overall density of the medium fixed:

$$\delta\mu = B\delta\left(\frac{\rho_n-\rho_p}{\rho}\right) \quad (\delta\rho=0)\,, \tag{2}$$

where ρ_n, ρ_p, and ρ are the neutron, proton, and total nucleon number densities, respectively.

(iii) The coefficient governing the response of $\delta\mu$ to a change of density, where the weak interactions are kept frozen:

$$\delta\mu = C\frac{\delta\rho}{\rho} \quad \left[\delta\left(\frac{\rho_n-\rho_p}{\rho}\right)=0\right]. \tag{3}$$

(iv) The coefficient giving the response of the pressure to a chemical perturbation, at fixed density:

$$\delta P = D\delta\left(\frac{\rho_n-\rho_p}{\rho}\right) \quad (\delta\rho=0)\,. \tag{4}$$

Now consider a periodic perturbation of the nucleon density,

$$\rho(t)-\rho_0=(\delta\rho)e^{i\omega t}\,, \tag{5}$$

where ρ_0 is the equilibrium density. Defining the chemical response by δX, where

$$\delta\left(\frac{\rho_n-\rho_p}{\rho}\right)=\delta X e^{i\omega t}\,, \tag{6}$$

(i), (ii), and (iii) give an equation for δX,

$$i\omega\rho_0\delta X = 2\lambda(B\delta X + C\rho_0^{-1}\delta\rho)\,. \tag{7}$$

From (iv) we then can determine that part of the pressure variation resulting from the chemical response

$$\delta P_{\rm chem} = 2CD\lambda\rho_0^{-2}(\delta\rho)(i\omega-2\lambda B\rho_0^{-1})^{-1}e^{i\omega t}\,. \tag{8}$$

The rate of dissipation per unit volume is given by

$$\left.\frac{dE}{dt}\right|_{\rm diss} = (\mathrm{vol}\times T)^{-1}\int_0^T P d(\mathrm{vol})$$
$$= \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt[\mathrm{Re}P(t)](-\omega\sin\omega t)(\delta\rho)\rho_0^{-1}\,. \tag{9}$$

The only term in the pressure which contributes to the dissipation is $\delta P_{\rm chem}$. The bulk viscosity is given by

$$\zeta \equiv 2\frac{dE}{dt}\left(\frac{\delta\rho}{\rho_0}\right)^{-2}\omega^{-2}$$
$$= -2CD\lambda(\omega^2+4\lambda^2B^2\rho_0^{-2})^{-1}\rho_0^{-1}\,. \tag{10}$$

It remains to determine the coefficients λ, B, C, D. The beta interaction rate Γ_ν has been calculated, from the three-body process $e+p+n \to \nu+n+n$, in Refs. 10 and 11, but only for the case $\delta\mu=0$. It is, however, easy to retrace the steps through these calculations to obtain the result for the case $\delta\mu\neq 0$. Using the matrix element of Friman and Maxwell,[11] we obtain the rate

$$\Gamma_\nu(\delta\mu) = 3.65\times 10^{-20}(\rho_{15})^{2/3} \times \int \frac{d^3p}{(2\pi)^3\hbar^4}\left(\frac{k_BT}{1\ \mathrm{MeV}}\right)^4 J(p+c^{-1}\delta\mu)m_ec^2\,, \tag{11}$$

where

$$J(p)=(k_BT)^{-4}\int dE_1dE_2dE_edE_pdE_3[1-n_n(E_2)][1-n_n(E_3)]n_n(E_1)n_p(E_p)n_e(E_e)\delta(E_e+E_p+E_1-pc-E_2-E_3)$$
$$=(24)^{-1}[1+\exp(\beta pc)]^{-1}[(\beta pc)^4+10(\beta pc)^2\pi^2+9\pi^4]\left[1+O\left(\frac{k_BT}{\mu}\right)\right], \tag{12}$$

$\beta=(k_BT)^{-1}$, and ρ_{15} is the mass density in units of 10^{15} g cm^{-3}. Using the relation[12] $\Gamma_{\bar\nu}(\delta\mu)=\Gamma_\nu(-\delta\mu)$, taking the first term of the expansion of $J(p\pm c^{-1}\delta\mu)$ in powers of $\delta\mu$, and performing the integration over d^3p in (11), we obtain

$$\lambda = -1.62\times 10^{-18}\hbar^{-4}c^3m_e^3\left(\frac{k_BT}{1\ \mathrm{MeV}}\right)^6. \tag{13}$$

To estimate the coefficients B, C, and D, we use a free, nonrelativistic Fermi gas model at zero temperature. Keeping only the first term in an expansion in the reciprocal of the nucleon mass M, we obtain

$$B=\frac{\rho}{2}\left.\left(\frac{\partial\mu_n}{\partial\rho_n}+\frac{\partial\mu_p}{\partial\rho_p}+\frac{\partial\mu_e}{\partial\rho_e}\right)\right|_{\rm equil} = (\tfrac{4}{3})c^3\hbar^{-1}M^2(3\pi^2\rho_0)^{-1/3}[1+O(M^{-1})]\,, \tag{14}$$

$$C=\left.\left(\rho_n\frac{\partial\mu_n}{\partial\rho_n}-\rho_p\frac{\partial\mu_p}{\partial\rho_p}-\rho_e\frac{\partial\mu_e}{\partial\rho_e}\right)\right|_{\rm equil} = 6^{-1}\hbar^2M^{-1}(3\pi^2\rho_0)^{2/3}[1+O(M^{-1})]\,, \tag{15}$$

$$D=2^{-1}\rho_0C\,. \tag{16}$$

It follows from (13) and (14) that the term $(B\lambda\rho_0^{-1})^2$ in the denominator of (10) is small compared to ω^2, if ω is in the range of vibration frequencies of a neutron star. That is to say, the equilibration time scale for the medium is long compared to periods of vibration. The bulk viscosity is given in this limit by

$$\zeta = 1.46\times 10^{30}(\rho_{15})^2\omega^{-2}\left(\frac{k_BT}{1\ \mathrm{MeV}}\right)^6\ \mathrm{g\,cm^{-1}\,s^{-1}}. \tag{17}$$

We compare this result to the shear viscosity of the same state of matter, as given by an analytic fit[7] to the results of Flowers and Itoh,[8]

$$\eta = 1.3\times 10^{16}(\rho_{15})^{9/4}\left(\frac{k_BT}{1\ \mathrm{MeV}}\right)^{-2}. \tag{18}$$

The ratio of bulk to shear viscosity goes as the eighth power of temperature. For a density of $10^{15}\ \mathrm{g\,cm^{-3}}$ there is a crossover at about 10^9 K, for, e.g., a frequency of 1000 Hz. Since the models of Cutler and Lindblom indicate that shear viscosity is about sixty times as effective as bulk viscosity in damping the secular instability,[7] the effective crossover could be at twice this temperature.

These results indicate a possibility that, as a rapidly spinning neutron star cools, the decrease in viscosity could destabilize the star. Of course, the linear theory is inadequate to describe what happens then; but one could expect significant observable consequences; and there surely would be rapid angular momentum loss before the star reestablished itself in a stable, axially symmetric configuration.

Taking shear viscosity alone into consideration, Cutler and Lindblom[7] estimate that in no case does the gravitational-wave instability impose a limit on the rotation rate which is more than 20% less than that imposed by the elementary Kepler limit. Therefore, it could be unlikely that a neutron star would be found in the domain in which its rotation is limited by the gravitational-wave instability. On the other hand, it could well be that the initial rotation rate of a newly formed star has been set exactly by the necessity of avoiding the instability, and that the recurrence of the instability as the star cools is fairly inevitable.

The considerations of this note apply only to neutron matter in its normal state. At least in the denser regions the superfluid transition temperature is expected to be about (or even below) 10^9 K, and therefore, our considerations should be applicable at temperatures above 10^9 K. It is likely that the interior temperatures of the neutron star on the site of SN 1987A are presently in the neighborhood of 10^9 K.[13] Thus, there is a possibility that the neutron star at the core has already undergone a destabilization through viscosity decrease, or will undergo such an event in the near future.

I have not carried through the detailed calculation of bulk viscosity for other possible states of neutron-star matter. As one can see qualitatively from the analysis given in this note, the ingredients required to get a large bulk viscosity are (a) a large neutrino luminosity, and (b) a beta equilibrium which is density dependent. In the case of pion condensation, the neutrino luminosities are greatly enhanced over the neutron-matter values;[14,15] the condensate provides the momentum balance which the spectator nucleon provided in the normal state. Since the electron concentration at equilibrium depends on density in roughly the same way as in the normal case, we expect the bulk viscosity to be considerably larger in this case, and to go as the third rather than as the sixth power of the temperature.

For the case of quark matter composed of u, d, s quarks, with a significantly higher mass for the s quark than for u and d, there is a nonleptonic weak interaction which should dominate the viscosity terms, namely, the reaction $u+d \leftrightarrow s+u$, from the charged-current weak interaction. Momentum can be conserved on the Fermi surface in this reaction: The coefficient analogous to λ in Eq. (1) will go as the second power of the temperature. The analogues of the coefficients C and D will be comparable to those of the above calculation, if we choose the strange quark mass to be of the order of 100 MeV. We expect the bulk viscosity for this case to be much larger than in the neutron-matter case, for temperatures below 10 MeV.

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