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Conformal properties of the superstring-ghost Thirring model: Short-distance structure and bosonization

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It is known that the free theory of superstring-ghost fields can be extended by adding the superconformal-invariant quartic interaction $QJ\bar{J}$ where $J=bc+\beta\gamma$ is the total ghost number current. We use functional techniques to compute correlation functions of the elementary fields and currents of the theory to all orders in the coupling constant Q, and we use these to obtain the important operator products. The model is then shown to bosonize in terms of the same fields used for the free system, and we obtain bosonization formulas such as $c(z,\bar{z})=:\exp[a_1\varphi_1(z)+a_2\varphi_2(z)+b_1\bar{\varphi}_1(\bar{z})+b_2\bar{\varphi}_2(\bar{z})]:$, where a_i and b_i are algebraic functions of Q, such that a_2, b_1, and b_2 vanish in the free limit. The model thus exhibits the nonholomorphic behavior characteristic of the Thirring model as well as novel mixing of the scalar fields φ_1 and φ_2. It does not seem possible to construct an operator in the interacting theory with the properties of the Becchi-Rouet-Stora-Tyutin current.

I. INTRODUCTION

Two years ago a generalization of the standard free field theory of superstring ghosts was obtained[1] containing superconformal-invariant quartic interactions. The motivation was to develop possible new superstringlike theories with interacting ghosts. Shortly thereafter a superspace version of the theory appeared,[2,3] and the conformal anomaly was calculated.[3]

In this paper we explore the structure of the theory as a two-dimensional superconformal field theory. Specifically we show how to calculate exactly correlation functions on the Minkowski plane among the elementary fields and important composite operators such as currents and stress tensor. This determines the operator structure of the theory. We then bosonize the theory on the Euclidean sphere by generalizing the known bosonization construction[4] for free superstring ghosts. We will report elsewhere on the bosonization of the model using the superfields[5] of Martinec and Sotkov,[6] and on the extension of the bosonization to the cylinder and the torus and the calculation of the partition function in both the fermionic and the bosonic versions of the theory.[7]

Before presenting the supersymmetric model, it is instructive to discuss first the bosonic string truncation of the model. This theory was studied earlier[8] using an auxiliary-field formulation in which the action is

$$S[b^{\mu\nu},c_\mu,A_\mu]=\int d^2x\sqrt{-g}\left[\frac{1}{2\pi Q}g^{\mu\nu}A_\mu A_\nu - ib^{\mu\nu}(\nabla_\mu+A_\mu)c_\nu\right]. \tag{1.1}$$

Here $b^{\mu\nu}$ and c_ν are trace-free symmetric tensor antighost and vector ghost fields, respectively, and both are real and anticommuting. The world-sheet metric $g_{\mu\nu}$ has the Lorentz signature $(+-)$, and ∇_μ is the covariant derivative with metric connection. The vector auxiliary field A_μ can be integrated out to produce an equivalent action with current-current coupling $\frac{1}{2}\pi QJ_\mu J^\mu$ with $J^\mu=b^{\mu\nu}c_\nu$. This makes it clear that the ghost theory is very closely related to the spin-$\frac{1}{2}$ Thirring model, and, indeed, the correlation functions and bosonization of the two theories are very similar.

The complete action[1] of the superstring-ghost Thirring model is very complicated in its general form which involves the coupling to $N=1$ world-sheet supergravity and requires two vector auxiliary fields. In this paper we consider only situations where the world-sheet gravitino is absent. The action of Ref. 1 then simplifies considerably and can be written as

$$S[b^{\mu\nu},c_\mu,\bar{\beta}^\mu,\gamma]=\int d^2x\sqrt{-g}\left[\frac{1}{2\pi Q}g^{\mu\nu}A_\mu A_\nu - ib^{\mu\nu}(\nabla_\mu+A_\mu)c_\nu - i\bar{\beta}^\mu(\nabla_\mu+A_\mu)\gamma\right], \tag{1.2}$$

where $\bar{\beta}^\mu$ is a Γ traceless vector spinor antighost and γ a spinor ghost field. Both are real and commuting. Integration of A_μ produces the current-current coupling $\frac{1}{2}\pi QJ_\mu J^\mu$ with $J^\mu=b^{\mu\nu}c_\nu+\bar{\beta}^\mu\gamma$.

The $N=2$ supersymmetry algebra of the free theory[4] generalizes to the interacting system. The action obtained from (1.2) after elimination of A_μ is invariant under the two distinct superconformal transformations:

$$\delta_{\pm}c_{\mu}=\bar{\epsilon}\Gamma_{\mu}\gamma\pm\tfrac{1}{2}\pi Q\bar{\epsilon}\Gamma_{\nu}\Gamma_{\mu}\beta\cdot cc^{\nu}\ ,$$

$$\delta_{\pm}\gamma=\pm\left[\frac{i}{2}\nabla_{\mu}c_{\nu}\Gamma^{\nu}\Gamma^{\mu}\epsilon-ic_{\nu}\Gamma^{\nu}\Gamma^{\mu}\nabla_{\mu}\epsilon+\tfrac{1}{4}\pi Q\Gamma_{\mu}\Gamma_{\nu}\gamma\bar{\epsilon}\Gamma^{\nu}\Gamma^{\mu}\beta\cdot c\right],$$

$$\delta_{\pm}b^{\mu\nu}=\pm\left[\frac{i}{2}\bar{\epsilon}\Gamma^{\rho}\Gamma^{\nu}\nabla_{\rho}\beta^{\mu}+\frac{3i}{2}\nabla_{\rho}\bar{\epsilon}\Gamma^{\rho}\Gamma^{\nu}\beta^{\mu}-\tfrac{1}{2}\pi Q\bar{\epsilon}\Gamma^{\nu}\Gamma_{\rho}\beta\cdot cb^{\rho\mu}\right],\tag{1.3}$$

$$\delta_{\pm}\beta^{\mu}=b^{\mu\nu}\Gamma_{\nu}\epsilon\mp\tfrac{1}{2}\pi Q\beta_{\rho}\bar{\epsilon}\Gamma^{\mu}\Gamma^{\rho}\beta\cdot c\ .$$

Invariance holds on a curved world sheet for spinor parameters $\epsilon(x)$ satisfying the conformal Killing condition $\nabla_{\mu}\epsilon=\frac{1}{2}\Gamma_{\mu}\Gamma\cdot\nabla\epsilon$. The Q-dependent terms in (1.3) can be interpreted as a field-dependent gauge transformation[1] and this simplifies the proof of invariance. One should note that the space-time translation term appears with opposite signs in the commutators $[\delta_{\pm},\delta_{\pm}]$, so that the $N=2$ algebra does not have unitary representations.

The auxiliary-field formulation of Thirring-type models provides a convenient method to calculate correlation functions and anomalies, and as recently shown, the partition functions.[9] The method is to integrate out the dynamical fields, using Pauli-Villars regularization or the families index theorem to obtain the effective action $\Gamma[g_{\mu\nu},A_{\nu}]$ (Ref. 8). Fortunately, the results presented in Ref. 8 for a general spin b-c system allow us to treat the system (1.2) with little new work required.

We now sketch the properties of the correlation functions and the operator bosonization of the two models (1.1) and (1.2). Results for (1.1) are taken from Ref. 8 while the results for (1.2) are derived in Secs. II and III below. The chiral-anomaly term in the effective action is very important for the dynamical properties of the models. We choose the finite local counterterm arising from the integration of the dynamical fields, so that the ghost vector current is anomalous. This is appropriate for string ghost fields and is automatic in Pauli-Villars regularization.[8] The anomaly term is the nonlocal expression

$$\Gamma_{\text{anomaly}}=\mp\frac{1}{2\pi}\int d^2x\sqrt{-g}\,\partial\cdot A\frac{1}{\Box}\partial\cdot A\ .\tag{1.4}$$

The magnitude of the coefficient is independent of the spin of the b-c system which is integrated, and the sign depends only on the statistics, $-$ ($+$) corresponding to the anticommuting (commuting) fields. Thus the anomaly term occurs with coefficient ($-$) in the "bosonic" ghost Thirring model (1.1) and coefficient zero in the supersymmetric model (1.2), because this model contains two b-c systems of opposite statistics.

It is this simple fact which leads to very different behavior of operator products, conformal and current anomalies, and the bosonization formulas of the two models. For the "bosonic" model (1.1), the anomalies are

$$T^{\mu}{}_{\mu}=\frac{-1}{24\pi}\left[26-\frac{27Q}{1+Q}\right]R\ ,\qquad D_{\mu}(b^{\mu\nu}c_{\nu})=\frac{i}{4\pi}\frac{3}{1+Q}R\ ,\tag{1.5}$$

while the operator product of conjugate elementary fields is

$$b(z,\bar{z})c(w,\bar{w})\sim\frac{1}{|z-w|^{Q^2/2(1+Q)}(z-w)}\ .\tag{1.6}$$

The model is bosonized by a single anti-Hermitian scalar $\phi(z,\bar{z})$ with the action

$$S=\frac{1}{2\pi}\int d^2z\sqrt{g}\left[(\partial\phi)^2+\frac{3i}{2\sqrt{1+Q}}R\phi\right]\tag{1.7}$$

and the operator formulas

$$c(z,\bar{z})\sim\,:e^{\alpha\varphi(z)+\beta\bar{\varphi}(\bar{z})}:\ ,\qquad J(z)=\frac{1}{\sqrt{1+Q}}\partial_z\varphi(z)\ ,\tag{1.8}$$

where $\varphi(z)$ and $\bar{\varphi}(\bar{z})$ are the chiral and antichiral parts of ϕ, and

$$\alpha=\frac{1}{2}\left[\frac{1}{\sqrt{1+Q}}+\sqrt{1+Q}\right],\qquad \beta=\frac{1}{2}\left[\frac{1}{\sqrt{1+Q}}-\sqrt{1+Q}\right].\tag{1.9}$$

The results (1.6), (1.8), and (1.9) are similar to the spin-$\frac{1}{2}$ Thirring model.

In the supersymmetric model, the anomalies are

$$T^{\mu}{}_{\mu}=-\frac{1}{24\pi}(15-3Q)R\ ,\qquad D_{\mu}(b^{\mu\nu}c_{\nu})=\frac{i}{4\pi}(3-Q)R\ ,\qquad D_{\mu}(\bar{\beta}^{\mu}\gamma)=\frac{i}{4\pi}(-2+Q)R\tag{1.10}$$

and the operator products of b,c and β,γ are

$$b(z,\bar{z})c(w,\bar{w})\sim\frac{1}{z-w}\ ,\qquad \beta(z,\bar{z})\gamma(w,\bar{w})\sim\frac{-1}{z-w}\ .\tag{1.11}$$

The fact that these operator products have the same value as in the free theory is a striking feature of this model, although such behavior can only occur because the theory is not unitary.

The supersymmetric model is bosonized using a pair of anti-Hermitian scalars $\phi_1(z,\bar{z})$ and $\phi_2(z,\bar{z})$, and the $c=-2$ fermion pair $\xi(z),\eta(z)$ and their conjugates. The ξ-η system appears in exactly the same way as in the noninteracting case $Q=0$ of the standard superstring ghost bosonization,[4] but ϕ_1 and ϕ_2 (which correspond to σ and ϕ of Ref. 4) mix for $Q\neq 0$, and $b(z,\bar{z})$, $c(z,\bar{z})$, etc., are nonholomorphic as is characteristic of Thirring models. The ϕ_1,ϕ_2 action is

$$S=\frac{1}{2\pi}\int d^2z\sqrt{g}\,[(\partial\phi_1)^2-(\partial\phi_2)^2+2R(k_1\phi_1-k_2\phi_2)]\ .\tag{1.12}$$

The currents bosonize as

$$:bc:=r_1\partial_z\varphi_1+r_2\partial_z\varphi_2,\qquad :\beta\gamma:=s_1\partial_z\varphi_1+s_2\partial_z\varphi_2\ . \tag{1.13}$$

The bosonization of the fields is generically of the form

$$c(z,\bar z)\sim :\exp[a_1\varphi_1(z)+a_2\varphi_2(z)+b_1\bar\varphi_1(\bar z)+b_2\bar\varphi_2(\bar z)]:\ . \tag{1.14}$$

The exponential part of the bosonization of $\gamma(z,\bar z)$ is similar to (1.14) with a_1 and a_2 interchanged, while $b(z,\bar z)$ and $\beta(z,\bar z)$ bosonize with exponentials whose arguments are opposite in sign to that of $c(z,\bar z)$ and $\gamma(z,\bar z)$, respectively. The constants k_i, a_i, b_i, r_i, and s_i in (1.13) and (1.14) are algebraic functions of Q given in Table II below and in (3.6) and (3.15). The operator product (1.11) is reproduced in the bosonized theory because of algebraic relations between the coefficients, e.g., $a_1^2-a_2^2=1$, $b_1^2=b_2^2$, etc. There is a striking contrast between the complicated Q dependence of many coefficients in the bosonization formulas and simple linear dependence of the anomalies (1.10).

The conformal components of stress tensor $T(z)$ and supercurrents $S_\pm(z)$ [corresponding to the symmetries $\delta_\pm$ of (1.3)] are independent of Q, after equations of motion are used:

$$T(z)=-\tfrac{1}{2}(b\overleftrightarrow{\partial}_z c+\beta\overleftrightarrow{\partial}_z\gamma)-\tfrac{3}{2}\partial_z(bc)-\partial_z(\beta\gamma)\ , \tag{1.15}$$

$$S^\pm(z)=\mp\tfrac{3}{2}\beta\partial_z c\mp\partial_z\beta c+\tfrac{1}{2}b\gamma\ . \tag{1.16}$$

Despite this, the standard calculation of operator-product coefficients among these operators and with elementary fields is invalid because we have an interacting theory. Instead we determine these coefficients from regularized functional calculations using the action (1.2). In this way we find that anomalous dimensions of all elementary fields are also linear in Q; see Table I. One way to describe these results is to say that (Pauli-Villars-regulated calculations of) correlation functions such as $\langle T(w)b(y,\bar y)c(z,\bar z)\rangle$ are one-loop exact. The reason for this is that at one-loop order the contributing bc and $\beta\gamma$ bubble graphs have different numerical coefficients, while higher-order corrections involve a sum of bc and $\beta\gamma$ bubbles which is regular at short distances due to opposite statistics. This cancellation is a consequence of the supersymmetric interactions.

This brief summary of the properties of the supersymmetric Thirring ghost model shows that it is a conformal field theory, albeit nonunitary, with some novel properties. The correlation functions and operator products of the theory are discussed in Sec. II below, and the bosonization is discussed in Sec. III. In Sec. IV we return briefly to the original motivation of the theory, namely, the possible use of interacting ghosts in string theory, and we discuss several unsuccessful attempts to construct an operator with the properties of the Becchi-Rouet-Stora-Tyutin (BRST) current.

II. CORRELATION FUNCTIONS OF THE MODEL

In this section we outline the calculation of the correlation functions and anomalies of the superstring-ghost Thirring model. We use the results of Pauli-Villars regularized functional calculations of Ref. 8, which are compatible with earlier work.[4] In a conformally flat background metric, $g_{\mu\nu}(x)=e^{2\sigma(x)}\eta_{\mu\nu}$, $\sqrt{-g}R=-2\Box\sigma$, and current source $A_\mu(x)$, the effective actions of the free b-c and β-γ systems, defined by

$$e^{i\Gamma_1[g,A]}\equiv\int\mathcal{D}b^{\mu\nu}\mathcal{D}c_\nu\exp\left[\int d^2x\sqrt{-g}\,b^{\mu\nu}(\nabla_\mu+A_\mu)c_\nu\right], \tag{2.1}$$

$$e^{i\Gamma_2[g,A]}\equiv\int\mathcal{D}\beta^\mu\mathcal{D}\gamma\exp\left[\int d^2x\sqrt{-g}\,\bar\beta^\mu(\nabla_\mu+A_\mu)\gamma\right] \tag{2.2}$$

are given by

$$\Gamma_1[g,A]=\frac{1}{2\pi}\int d^2x\sqrt{-g}\left[\tfrac{26}{12}(\partial_\mu\sigma)^2-3A_\mu\partial^\mu\sigma-\nabla\cdot A\frac{1}{\Box}\nabla\cdot A\right], \tag{2.3}$$

$$\Gamma_2[g,A]=\frac{1}{2\pi}\int d^2x\sqrt{-g}\left[-\tfrac{11}{12}(\partial_\mu\sigma)^2+2A_\mu\partial^\mu\sigma+\nabla\cdot A\frac{1}{\Box}\nabla\cdot A\right]. \tag{2.4}$$

TABLE I. Charges and conformal dimensions of the elementary fields in the model. See (2.18) and (2.19). The coefficients $e_{i-}(\chi)$ and $h_-(\chi)$ are obtained by replacing $+\leftrightarrow-$ above, e.g., $e_{1-}(c^+)=e_{1+}(c^-)=\frac{1}{2}Q$.

χ	b_{++}	b_{--}	c^+	c^-	β_+	β_-	γ_+	γ_-
$e_{1+}(\chi)$	$1-\frac{Q}{2}$	$-\frac{Q}{2}$	$-1+\frac{Q}{2}$	$\frac{Q}{2}$	$-\frac{Q}{2}$	$-\frac{Q}{2}$	$\frac{Q}{2}$	$\frac{Q}{2}$
$e_{2+}(\chi)$	$\frac{Q}{2}$	$\frac{Q}{2}$	$-\frac{Q}{2}$	$-\frac{Q}{2}$	$1+\frac{Q}{2}$	$\frac{Q}{2}$	$-1-\frac{Q}{2}$	$-\frac{Q}{2}$
$e_+(\chi)$	1	0	-1	0	1	0	-1	0
$h_+(\chi)$	$2-\frac{Q}{4}$	$-\frac{Q}{4}$	$-1+\frac{Q}{4}$	$\frac{Q}{4}$	$\frac{3}{2}-\frac{Q}{4}$	$-\frac{Q}{4}$	$-\frac{1}{2}+\frac{Q}{4}$	$\frac{Q}{4}$

Since ghost mass terms, e.g., $m\epsilon^{\mu\nu}c_\mu c_\nu$ or $m\bar{\gamma}\Gamma_5\gamma$, are invariant under dual ghost number transformations, but not ordinary ghost number transformations (e.g., invariant under $\delta c_\mu \sim \epsilon_\mu{}^\nu c_\nu$ and $\delta\gamma \sim \Gamma_5\gamma$, not $\delta c_\mu \sim c_\mu$ and $\delta\gamma \sim \gamma$), it is an automatic consequence of Pauli-Villars regularization that these effective actions are invariant under $\delta A_\mu = \epsilon_\mu{}^\nu \partial_\nu \theta(x)$. In other regularization methods one may choose the coefficient of the finite local counterterm $\int d^2x\sqrt{-g}\,A^2_\mu$ to achieve this.

The next step is to implement the Thirring interaction via the vector auxiliary field in (1.2) and to define

$$e^{i\Gamma[g,C_1,C_2]} = \int \mathcal{D}A_\mu \exp\left[i\Gamma_1[g, A_\mu + C_{1\mu}] + i\Gamma_2[g, A_\mu + C_{2\mu}] + \frac{i}{2\pi Q}\int d^2x\sqrt{-g}\,A_\mu^2 \right] . \quad (2.5)$$

Looking back to (2.1) and (2.2) we see that $C_{1\mu}$ and $C_{2\mu}$ are sources for the conserved currents $J_1^\mu = b^{\mu\nu}c_\nu$ and $J_2^\mu = \bar{\beta}^\mu\gamma$. The Gaussian integration over A_μ can be performed by a shift of variables discussed in Sec. IV of Ref. 8 with the result

$$\Gamma[g,C_1,C_2] = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[-(1-Q)\nabla\cdot C_1 \frac{1}{\Box}\nabla\cdot C_1 - 2Q\nabla\cdot C_1\frac{1}{\Box}\nabla\cdot C_2 + (1+Q)\nabla\cdot C_2\frac{1}{\Box}\nabla\cdot C_2 - (3-Q)C_1^\mu\partial_\mu\sigma + (2-Q)C_2^\mu\partial_\mu\sigma + \tfrac{1}{4}(5-Q)(\partial_\mu\sigma)^2 \right] . \quad (2.6)$$

This immediately implies that anomalies (1.10) and the current two-point functions

$$\begin{aligned} \langle TJ_{1\mu}(x)J_{1\nu}(y)\rangle &= \frac{i}{\pi}(1-Q)\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\Delta(x,y) , \\ \langle TJ_{1\mu}(x)J_{2\nu}(y)\rangle &= \frac{i}{\pi}Q\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\Delta(x,y) , \\ \langle TJ_{2\mu}(x)J_{2\nu}(y)\rangle &= -\frac{i}{\pi}(1+Q)\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\Delta(x,y) , \end{aligned} \quad (2.7)$$

where $\Delta(x,y)$ is the scalar Green's function which satisfies $\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\Delta(x,y)) = -\delta(x,y)$ and is given for the flat metric, $g_{\mu\nu}=\eta_{\mu\nu}$, by $\Delta(x,y) = (i/4\pi)\ln[-m^2(x-y)^2 + i\epsilon]$.

We now discuss correlation functions of conjugate dynamical fields using light-cone coordinates $x^\pm = (1/\sqrt{2})(x^0 \pm x^1)$ in the flat plane. To calculate these we again follow Ref. 8 and consider the generating functional

$$W[B_{\mu\nu}, C^\rho] = \int \mathcal{D}A_\mu \mathcal{D}b^{\mu\nu}\mathcal{D}c_\rho \mathcal{D}\beta^\mu \mathcal{D}\gamma \exp\left[\int d^2x \left[\frac{i}{2\pi Q}A_\mu^2 + b^{\mu\nu}(\partial_\mu + A_\mu)c_\nu + \bar{\beta}^\mu(\partial_\mu + A_\mu)\gamma + b_{\mu\nu}B^{\mu\nu} + C_\rho c^\rho \right]\right] . \quad (2.8)$$

The ghost integrals are regulated and after a shift, they are evaluated as in (2.1)–(2.4) with $\sigma(x)=0$. The chiral anomaly cancels due to opposite statistics of b,c and β,γ, and we obtain

$$W[B_{\mu\nu}, C^\rho] = \int \mathcal{D}A_\mu \exp\left[\int d^2x \left[\frac{i}{2\pi Q}A_\mu^2 - C^+\frac{1}{\partial_+ + A_+}B_{++} - C^-\frac{1}{\partial_- + A_-}B_{--} \right]\right] . \quad (2.9)$$

The A_μ integral may be evaluated using the representation $A_\mu(x) = \partial_\mu s(x) + \epsilon_\mu{}^\nu\partial_\mu p(x)$, or equivalently $A_\pm = \partial_\pm(s\pm p)$ and using the results

$$A_\mu^2 = (\partial_\mu s)^2 - (\partial_\mu p)^2 + 2\partial_\mu(\epsilon^{\mu\nu}s\partial_\nu p) , \quad (2.10)$$

$$(\partial_\pm + A_\pm)^{-1} = e^{-(s\pm p)}\partial_\pm^{-1}e^{(s\pm p)} . \quad (2.11)$$

Converting the functional integral over s and p to Dyson-Wick form and differentiating with respect to sources, we find the correlation functions

$$\langle Tb^{\pm\pm}(x)c_\pm(y)\rangle = \exp[-\langle s(x)s(y)\rangle - \langle p(x)p(y)\rangle]\left[-\frac{1}{\partial_\pm}\right]_{x,y} . \quad (2.12)$$

However, the Wick contractions satisfy $\langle s(x)s(y)\rangle = -\langle p(x)p(y)\rangle$ because of the relative $-$ sign in (2.10) and the two-point functions are therefore independent of the coupling constant Q. Note that

$$\left[\frac{1}{\partial_\pm}\right]_{x,y} = 2\partial_\pm^x\frac{1}{\Box}(x,y) = \frac{i}{\pi}\frac{(x-y)^\pm}{-(x-y)^2 + i\epsilon} = \frac{-i}{2\pi}\frac{1}{(x-y)^\mp} . \quad (2.13)$$

If the chiral anomaly had not canceled, the coefficient of $(\partial s)^2$ and $(\partial p)^2$ would not be of equal magnitude, but opposite sign in the integral which replaces (2.9). The result (2.12) would hold, but the Wick contractions would lead to a Q-dependent anomalous dimension term in the correlation function.

An identical calculation with sources for β^μ and γ gives the correlation function

$$\langle T\beta_\pm(x)\gamma_\pm(y)\rangle=\frac{1}{\partial_\pm}=\frac{-i}{2\pi}\frac{1}{(x-y)^\mp}\ , \tag{2.14}$$

which is again equal to its value in the standard theory of free superstring ghosts. (We use $\beta_\pm$ to denote the spin $\pm\frac{3}{2}$ component of the field $\bar{\beta}_{\mu\alpha}$.)

We also need three-point correlation functions of the currents and ghost fields, such as $\langle TJ_1^\mu(x)b^{\pm\pm}(y)c_\pm(x)\rangle$. This may be obtained from the generating functional

$$\mathcal{J}^\mu[x,B_{\nu\rho},C^\lambda]=\int \mathcal{D}A_\mu\mathcal{D}b^{\mu\nu}\mathcal{D}c_\mu\mathcal{D}\beta^\mu\mathcal{D}\gamma\exp\left[\frac{i}{2\pi Q}\int d^2y\ A_\rho^2\right]$$
$$\times\frac{\delta}{\delta Z_\mu(x)}\exp\left[\int d^2z[b^{\nu\rho}(\partial_\nu+A_\nu+Z_\nu)c_\rho+\bar{\beta}^\nu(\partial_\nu+A_\nu)\gamma+b^{\nu\rho}B_{\nu\rho}+C^\lambda c_\lambda\right]\Bigg|_{Z=0}\ . \tag{2.15}$$

The integral can be evaluated by straightforward extension of the techniques used previously. One finds results such as

$$\langle TJ_1^\mu(x)b^{\pm\pm}(y)c_\pm(z)\rangle=-\frac{i}{2\pi}[(1-Q)g^{\mu\nu}\mp\epsilon^{\mu\nu}]\left[\frac{x_\nu-y_\nu}{-(x-y)^2+i\epsilon}-\frac{x_\nu-z_\nu}{-(x-z)^2+i\epsilon}\right]\langle Tb^{\pm\pm}(y)c_\pm(z)\rangle\ ,$$
$$\langle TJ_2^\mu(x)b^{\pm\pm}(y)c_\pm(z)\rangle=\frac{-i}{2\pi}Q\left[\frac{x^\mu-y^\mu}{-(x-y)^2+i\epsilon}-\frac{x^\mu-z^\mu}{-(x-z)^2+i\epsilon}\right]\langle Tb^{\pm\pm}(y)c_\pm(z)\rangle\ . \tag{2.16}$$

As discussed in Ref. 8, the Q-dependent terms in these expressions describe rescalings of the vector ghost charges due to the Thirring interaction. From (2.16) one can extract operator-product relations such as

$$J_{1\pm}(x)c^\pm(y)\sim-(1-\tfrac{1}{2}Q)\frac{i}{2\pi}\frac{c^\pm(y)}{(x-y)^\pm}\ ,$$
$$J_{2\pm}(x)c^\pm(y)\sim-\frac{1}{2}Q\frac{i}{2\pi}\frac{c^\pm(y)}{(x-y)^\pm}\ . \tag{2.17}$$

Let $\chi(y)$ denote a chiral component of one of the antighost or ghost fields. Then the operator-product relations which can be extracted from the full set of three-point functions of the type given in (2.16) can be written generically as

$$J_{i\pm}(x)\chi(y)\sim e_{i\pm}(\chi)\frac{i}{(2\pi)}\frac{\chi(y)}{(x-y)^\pm}\ . \tag{2.18}$$

The charges $e_{i\pm}(\chi)$ are given in Table I. One can observe using the table that the charges of the total ghost number current $J^\mu=J_1^\mu+J_2^\mu$ are independent of the coupling Q.

Let us now turn our attention to the conformal properties of the ghost fields. The conformal dimensions $h_\pm(\chi)$ of the field χ are specified by the coefficient of the second-order pole in the operator-product expansion with the stress tensor: namely,

$$T_{\pm\pm}(x)\chi(y)\sim\frac{h_\pm(\chi)\chi(y)}{[(x-y)^\pm]^2}+\frac{1}{(x-y)^\pm}\frac{\partial}{\partial y^\pm}\chi(y)\ . \tag{2.19}$$

These operator products could be directly obtained from correlation functions of the form $\langle TT_{\pm\pm}(x)\chi_1(y)\chi_2(z)\rangle$. However, the functional methods developed in Ref. 8 cannot be directly applied to calculate the correlation functions of $T_{\pm\pm}$ and we therefore use an indirect method to determine the $h_\pm(\chi)$ using previous results together with the "charge conjugation" symmetry of the action (1.2).

We first note that from the action of the Virasoro generator L_0 and $\bar{L}_0$ on the two-point correlation functions (2.12) and (2.14) we learn the following information about the conformal dimensions:

$$h_+(b_{++})+h_+(c^+)=1\ ,$$
$$h_-(b_{++})+h_-(c^+)=0\ ,$$
$$h_+(\beta_+)+h_+(\gamma_+)=1\ ,$$
$$h_-(\beta_+)+h_-(\gamma_+)=0 \tag{2.20}$$

together with four identical relations with $+\leftrightarrow-$.

The stress tensor and supercurrent of the Thirring superghost model can be obtained by functional derivatives of the supergravity-coupled action of Ref. 1 with respect to zweibein and gravitino. This gives

$$T_{\mu\nu}=-2ib_{\mu\rho}\partial_\nu c^\rho-i\partial_\nu b_{\mu\rho}c^\rho-\frac{3i}{2}\bar{\beta}_\mu\partial_\nu\gamma$$
$$-\frac{i}{2}\partial_\nu\bar{\beta}_\mu\gamma+\tfrac{1}{2}\pi Qg_{\mu\nu}J_\lambda J^\lambda\ ,$$
$$S_\mu^\pm=\mp\tfrac{3}{2}\beta_\nu\partial_\mu c^\nu\mp\partial_\mu\beta_\nu c^\nu-\frac{i}{2}b_{\mu\nu}\Gamma^\nu\gamma$$
$$\pm\frac{i}{4}\pi Q[J^\nu(\beta_\mu c_\nu-\beta_\nu c_\mu)-J_\mu\beta\cdot c]\ . \tag{2.21}$$

The stress tensor is conserved symmetric and traceless on shell, while the supercurrent is conserved and Γ traceless. The light-cone components of $T_{\mu\nu}$ can be written as

$$T_{\pm\pm}=T^{(0)}_{\pm\pm}-\tfrac{3}{2}i\partial_{\pm}J_{1\pm}-i\partial_{\pm}J_{2\pm}\ ,$$
$$T^{(0)}_{\pm\pm}=-\frac{i}{2}(b_{\pm\pm}\overleftrightarrow{\partial}_{\pm}c^{\pm}+\beta_{\pm}\overleftrightarrow{\partial}_{\pm}\gamma_{\pm}). \tag{2.22}$$

Next we observe that the action S in (1.2) is invariant under the conjugation transformation $b_{\pm\pm}\leftrightarrow c^{\pm}$ and $\beta_{\pm}\leftrightarrow-\gamma_{\pm}$ and $A_{\pm}\to-A_{\pm}$. The currents $J_{i\pm}(x)$ are odd under this symmetry, while $T_{\pm\pm}(z)$ in (2.22) is expressed as a sum of $T^{(0)}_{\pm\pm}(z)$, which is even, plus odd terms involving the currents. Invariance of the whole theory under this symmetry requires that the set of operator-product expansions (OPE's) transform into itself. Then, for example, applying the symmetry transformation to

$$T_{++}(x)b_{++}(y)\sim\frac{h_{+}(b_{++})b_{++}(y)}{[(x-y)^{+}]^{2}}+\frac{1}{(x-y)^{+}}\frac{\partial}{\partial y^{+}}b_{++}(y) \tag{2.23}$$

gives

$$[T^{(0)}_{++}(x)+\tfrac{3}{2}i\partial_{+}J_{1+}(x)+i\partial_{+}J_{2+}(x)]c^{+}(y)$$
$$\sim\frac{h_{+}(b_{++})c^{+}(y)}{[(x-y)^{+}]^{2}}+\frac{1}{(x-y)^{2}}\frac{\partial}{\partial y^{+}}c^{+}(y)\ . \tag{2.24}$$

But from the definition of $h_{+}(c^{+})$ contained in (2.19) we also have

$$[T^{(0)}_{++}(x)-\tfrac{3}{2}i\partial_{+}J_{1+}(x)-i\partial_{+}J_{2+}(x)]c^{+}(y)$$
$$\sim\frac{h_{+}(c^{+})c^{+}(y)}{[(x-y)^{+}]^{2}}+\frac{1}{(x-y)^{+}}\frac{\partial}{\partial y^{+}}c^{+}(y)\ . \tag{2.25}$$

In the difference between these two equations, only the operator products of $\partial_{+}J_{i+}(x)$ with $c^{+}(y)$ contribute, and these may be evaluated by taking the derivative of the current-field OPE (2.17). In this way we obtain

$$h_{+}(b_{++})-h_{+}(c^{+})=3(1-\tfrac{1}{2}Q)+2\tfrac{1}{2}Q=3-\tfrac{1}{2}Q\ . \tag{2.26}$$

Proceeding in a similar fashion using (2.18) and the charges $e_{i\pm}(\chi)$ in Table I, we obtain the additional results

$$h_{-}(b_{++})-h_{-}(c^{+})=-\tfrac{1}{2}Q\ ,$$
$$h_{+}(\beta_{+})-h_{+}(\gamma_{+})=2-\tfrac{1}{2}Q\ , \tag{2.27}$$
$$h_{-}(\beta_{+})-h_{-}(\gamma_{+})=-\tfrac{1}{2}Q\ .$$

There are four additional relations obtained from (2.26) and (2.27) by $+\leftrightarrow-$. It is now easy to solve (2.20), (2.26), and (2.27) to obtain all the conformal dimensions $h_{\pm}(\chi)$, which we have listed in the last row of Table I. Note that the difference between $h_{+}(\chi)$ and $h_{-}(\chi)$ for each field χ is independent of Q and is equal to the integer or $\frac{1}{2}$-integer spin of the field. One further check of the consistency of these conformal weights can be made using the fact that the conformal supersymmetry algebra requires that the difference between h_{+} for any positive-chirality field and for its supersymmetry partner should be $\frac{1}{2}$ and that the h_{-} value of such a pair of fields should be equal. This checks with $h_{+}(c^{+})-h_{+}(\gamma_{+})=-\frac{1}{2}$ and $h_{-}(c^{+})=h_{-}(\gamma_{+})$ and with $h_{+}(\beta_{+})-h_{+}(b_{++})=-\frac{1}{2}$ and $h_{-}(\beta_{+})=h_{-}(b_{++})$.

III. BOSONIZATION

In this section we study the bosonized representation of the superstring Thirring model. Using the operator-product expansions of fields and currents we express the field operators in terms of two scalar fields ϕ_1 and ϕ_2, and the fermion fields ξ,η. Our derivation combines the bosonization of the standard Thirring model[10] (and/or the bosonic string Thirring ghosts[8]) with the bosonization of the free superghosts of Ref. 4. In the first part of this section we will work in the Lorentzian signature plane and derive the bosonization rules for currents using the results from the previous section. In the second part we perform the Euclidean continuation and use conventions similar to Ref. 4. This should allow the reader a more straightforward comparison of our results in the interacting model with the corresponding ones in the free model.

Let us first consider the bosonization of the ghost number currents J_1 and J_2 whose two-point correlation functions are given in (2.7). It is convenient to take linear combinations of these currents,

$$K_1=\cos\theta J_1+\sin\theta J_2\ ,$$
$$K_2=-\sin\theta J_1+\cos\theta J_2\ , \tag{3.1}$$

where the mixing angle θ is chosen so that $\langle TK_{1\mu}(x)K_{2\nu}(y)\rangle=0$. This gives $\tan2\theta=Q$ and the two-point functions

$$\langle TK_{1\mu}(x)K_{1\nu}(y)\rangle=\frac{f_{+}}{\pi}i\frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial y^{\nu}}\Delta(x,y)\ ,$$
$$\langle TK_{2\mu}(x)K_{2\nu}(y)\rangle=-\frac{f_{-}}{\pi}i\frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial y^{\nu}}\Delta(x,y)\ , \tag{3.2}$$

where $f_{\pm}=f_{\pm}(Q)=(1+Q^2)^{1/2}\mp Q$. Note that $f_{+}f_{-}=1$. Thus K_1 and K_2 can be bosonized using two anti-Hermitian scalar fields ϕ_1 and ϕ_2 with two-point functions

$$\langle T\phi_1(x)\phi_1(y)\rangle=-\langle T\phi_2(x)\phi_2(y)\rangle=i\Delta(x,y). \tag{3.3}$$

Comparing (3.3) with (3.2) we identify

$$K_{1\mu}=\left[\frac{f_{+}}{\pi}\right]^{1/2}\partial_{\mu}\phi_1,\quad K_{2\mu}=\left[\frac{f_{-}}{\pi}\right]^{1/2}\partial_{\mu}\phi_2\ . \tag{3.4}$$

In a conformally flat background the usual equations of motion of ϕ_1 and ϕ_2 are determined by the anomaly (1.10) in the ghost number currents which yields

$$\Box\phi_i=k_iR,\quad i=1,2\ , \tag{3.5}$$

$$k_1=\frac{i}{8\sqrt{\pi}}\left[(1+Q^2)^{1/4}+\frac{5-Q}{(1+Q^2)^{1/4}}\right]\ ,$$
$$k_2=\frac{i}{8\sqrt{\pi}}\left[(1+Q^2)^{1/4}-\frac{5-Q}{(1+Q^2)^{1/4}}\right]\ . \tag{3.6}$$

Equations (3.3) and (3.5) correspond to the actions

$$S[\phi_i]=\epsilon_i\int d^2x\sqrt{-g}\,(\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i+k_iR\phi_i)\ ,\qquad \epsilon_1=+1,\quad \epsilon_2=-1 \tag{3.7}$$

from which we find the energy-momentum tensors

$$T^{(i)}_{\mu\nu}=\epsilon_i(\partial_\mu\phi_i\partial_\nu\phi_i-\tfrac{1}{2}g_{\mu\nu}g^{\rho\sigma}\partial_\rho\phi_i\partial_\sigma\phi_i -2k_i\partial_\mu\partial_\nu\phi_i+2k_ig_{\mu\nu}\Box\phi_i)\ . \tag{3.8}$$

Adding the traces of $T^{(i)}_{\mu\nu}$'s we find that the trace anomaly, including the quantum contribution from each scalar field, is

$$\langle T^\mu{}_\mu\rangle=\left[2k_1^2-2k_2^2+\frac{2}{24\pi}\right]R =\frac{1}{24\pi}(-13+3Q)R\ . \tag{3.9}$$

The value of the conformal anomaly in the superstring Thirring ghost model was previously calculated in Ref. 3 and its coefficient turned out to be $-15+3Q$. This shows that the difference with (3.9) is precisely the same as in the free model and thus will be compensated by adding the standard ξ-η fermion system.[4] If this is indeed correct we should now be able to express the fields of the original theory using the scalar fields ϕ_1, and ϕ_2 and the fermion fields ξ,η.

At this point it is convenient to make the continuation to the Euclidean plane with complex coordinates $z,\bar z$ ($z=x^1+x^2$), and make the following identification of the fields:

$$c^+(z,\bar z)\to\frac{1}{\sqrt{\pi}}c(z,\bar z),\quad c^-(z,\bar z)\to\frac{1}{\sqrt{\pi}}\bar c(z,\bar z)\ ,$$
$$b_{++}(z,\bar z)\to\frac{-i}{\sqrt{2\pi}}b(z,\bar z),\quad b_{--}(z,\bar z)\to\frac{i}{\sqrt{2\pi}}\bar b(z,\bar z)\ , \tag{3.10}$$

$$\gamma_+(z,\bar z)\to\frac{1}{\sqrt{\pi}}\gamma(z,\bar z)\ ,\quad \gamma_-(z,\bar z)\to\frac{1}{\sqrt{\pi}}\bar\gamma(z,\bar z)\ ,$$
$$\beta_+(z,\bar z)\to\frac{-i}{\sqrt{2\pi}}\beta(z,\bar z),\quad \beta_-(z,\bar z)\to\frac{i}{\sqrt{2\pi}}\bar\beta(z,\bar z)\ . \tag{3.11}$$

The new fields have the "canonical" OPE's

$$c(z,\bar z)b(w,\bar w)\sim\frac{1}{z-w},\quad \bar c(z,\bar z)\bar b(w,\bar w)\sim\frac{1}{\bar z-\bar w}\ , \tag{3.12}$$

$$\gamma(z,\bar z)\beta(w,\bar w)\sim\frac{1}{z-w},\quad \bar\gamma(z,\bar z)\bar\beta(w,\bar w)\sim\frac{1}{\bar z-\bar w}\ , \tag{3.13}$$

which trivially follow from (2.12) and (2.14). Similarly we define the chiral components of the scalar fields

$$\phi_1(z,\bar z)=-\frac{i}{2\sqrt{\pi}}[\varphi_1(z)+\bar\varphi_1(\bar z)]\ ,\qquad \phi_2(z,\bar z)=+\frac{i}{2\sqrt{\pi}}[\varphi_2(z)+\bar\varphi_2(\bar z)] \tag{3.14}$$

so that $\varphi_1(z)\varphi_1(w)\sim\ln(z-w)$ and $\varphi_2(z)\varphi_2(w)\sim-\ln(z-w)$. In terms of these fields the holomorphic components of the ghost number currents and the stress tensors are [note that $j_i(z)=-J_1(z)$ and $j_2(z)=-J_2(z)$, where $J_1(z)$ and $J_2(z)$ are the currents of Sec. II]

$$j_1(z)=\sqrt{f_+}\cos\theta\partial_z\varphi_1(z)+\sqrt{f_-}\sin\theta\partial_z\varphi_2(z)\ ,\qquad j_2(z)=\sqrt{f_+}\sin\theta\partial_z\varphi_1(z)-\sqrt{f_-}\cos\theta\partial_z\varphi_2(z) \tag{3.15}$$

and

$$T^{(1)}(z)=\tfrac{1}{2}\partial_z\varphi_1\partial_z\varphi_1+\omega_1\partial_z\partial_z\varphi_1\ ,\qquad T^{(2)}(z)=-\tfrac{1}{2}\partial_z\varphi_2\partial_z\varphi_2+\omega_2\partial_z\partial_z\varphi_2\ , \tag{3.16}$$

where $\omega_1=-2i\sqrt{\pi}k_1$ and $\omega_2=-2i\sqrt{\pi}k_2$.

It is straightforward to write the OPE's (2.17)–(2.19) for the Euclidean fields and currents. One obtains, e.g.,

$$j_1(z)c(w,\bar w)\sim(1-\tfrac{1}{2}Q)\frac{c(w,\bar w)}{z-w},\qquad j_1(z)\bar c(w,\bar w)\sim-\frac{1}{2}Q\frac{\bar c(w,\bar w)}{z-w} \tag{3.17}$$

and generically

$$j_i(z)\chi(w,\bar w)\sim-e_i(\chi)\frac{\chi(w)}{z-w}\ . \tag{3.18}$$

To establish the bosonization of the field operators let us consider the *Ansatz*

$$c(z,\bar z)=e^{\Phi_1(z,\bar z)}\ , \tag{3.19}$$

$$b(z,\bar z)=e^{-\Phi_1(z,\bar z)}\ , \tag{3.20}$$

$$\gamma(z,\bar z)=\eta(z)e^{\Phi_2(z,\bar z)}\ , \tag{3.21}$$

$$\beta(z,\bar z)=\partial_z\xi(z)e^{-\Phi_2(z,\bar z)}\ , \tag{3.22}$$

where

$$\Phi_1(z,\bar z)=a_1\varphi_1(z)+a_2\varphi_2(z)+b_1\bar\varphi_1(\bar z)+b_2\bar\varphi_2(\bar z)\ , \tag{3.23}$$

$$\Phi_2(z,\bar z)=c_1\varphi_1(z)+c_2\varphi_2(z)+d_1\bar\varphi_1(\bar z)+d_2\bar\varphi_2(\bar z)\ . \tag{3.24}$$

This generalizes the usual bosonization rules from the noninteracting model[4] in two respects. First, we accommodate the nonanalytic behavior characteristic of the Thirring interaction, and we incorporate the mixing of φ_1 and φ_2 necessary to produce currents of the form (3.15). The ξ-η system enters in the same way as in the conventional model in order to compensate for the conformal anomaly. The constant coefficients $a_1,\dots,d_2$ depend upon the coupling constant and can be uniquely determined by requiring that the fields in (3.18)–(3.21) reproduce the two-point functions (3.12) and (3.13) and the OPE (2.17) and (2.18) with the ghost number currents. The result is given in Table II. Finally, the conjugation of (3.19)–(3.22) gives the bosonization of the fields $\bar c,\dots,\bar\beta$.

We have written on purpose the bosonization rules in (3.19)–(3.22) in the same form as in the standard treatment of the free b-c and β-γ systems,[4] except that the scalar fields $\Phi_1(z,\bar z)$ and $\Phi_2(z,\bar z)$ are not purely holomorphic in the presence of the interaction. However, the antiholomorphic terms in Φ_1 and Φ_2 cancel in the ghost number currents and the energy-momentum tensor. Therefore,

TABLE II. Coefficients which appear in the bosonization formulas (3.16) and (3.19)–(3.24).

$$a_1=c_2=\frac{\cos^2\theta}{\sqrt{\cos 2\theta}}=\frac{1}{2}\left[(1+Q^2)^{1/4}+\frac{1}{(1+Q^2)^{1/4}}\right]$$

$$a_2=c_1=\frac{\sin^2\theta}{\sqrt{\cos 2\theta}}=\frac{1}{2}\left[(1+Q^2)^{1/4}-\frac{1}{(1+Q^2)^{1/4}}\right]$$

$$b_1=-b_2=d_1=-d_2=-\frac{1}{2}\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}=-\frac{1}{2}\frac{Q}{(1+Q^2)^{1/4}}$$

$$\omega_1=\tfrac{1}{2}(3a_1-2a_2+b_1)=\frac{1}{4}\left[(1+Q^2)^{1/4}+\frac{5-Q}{(1+Q^2)^{1/4}}\right]$$

$$\omega_2=\tfrac{1}{2}(3a_2-2a_1+b_2)=\frac{1}{4}\left[(1+Q^2)^{1/4}-\frac{5-Q}{(1+Q^2)^{1/4}}\right]$$

$$a_1^2-a_2^2=1,\quad a_1b_1-a_2b_2=-\tfrac{1}{2}Q$$

the algebraic structure of the operator-product expansions of these currents is the same as in the free theory, but with modified conformal dimensions of fields, as we have already seen in Sec. II.

To see this more explicitly we solve (3.23) and (3.24) for φ_1 and φ_2:

$$\varphi_1(z)=a_1\Phi_1(z,\bar z)-a_2\Phi_2(z,\bar z)-b_1[\bar\Phi_1(z,\bar z)-\bar\Phi_2(z,\bar z)]\ , \tag{3.25}$$

$$\varphi_2(z)=-a_2\Phi_1(z,\bar z)+a_1\Phi_2(z,\bar z)-b_1[\bar\Phi_1(z,\bar z)-\bar\Phi_2(z,\bar z)]\ .$$

Substituting (3.25) into (3.15) and (3.16) we obtain the energy-momentum tensor of the scalar fields:

$$T^{\Phi_1\Phi_2}(z)=\tfrac{1}{2}\partial_z\Phi_1\partial_z\Phi_1++(\tfrac{3}{2}-\tfrac{1}{4}Q)\partial_z\partial_z\Phi_1 -\tfrac{1}{2}\partial_z\Phi_2\partial_z\Phi_2+(-1+\tfrac{1}{4}Q)\partial_z\partial_z\Phi_2\ , \tag{3.26}$$

which depends only on the holomorphic part of Φ_1 and Φ_2. We also have

$$\langle\Phi_i(z,\bar z)\Phi_j(w,\bar w)\rangle=\epsilon_i\delta_{ij}\ln(z-w),\quad i,j,=1,2\ , \tag{3.27}$$

$$\langle\Phi_i(z,\bar z)\bar\Phi_j(w,\bar w)\rangle=-\frac{Q}{2}\ln|z-w|^2,\quad i,j,=1,2\ . \tag{3.28}$$

Comparing with the standard bosonization formulas[4] we see that the holomorphic parts of Φ_1 and Φ_2 can be viewed as the chiral scalar fields which bosonize free b-c and β-γ systems with the conformal weight $\lambda^{bc}=2-\frac{1}{4}Q$ and $\lambda^{\beta\gamma}=\frac{3}{2}-\frac{1}{4}Q$, respectively.

The ghost number currents are given by

$$\begin{aligned} j_1&=(1-\tfrac{1}{2}Q)\partial_z\Phi_1+\tfrac{1}{2}Q\partial_z\Phi_2\ ,\\ j_2&=\tfrac{1}{2}Q\partial_z\Phi_1-(1+\tfrac{1}{2}Q)\partial_z\Phi_2\ . \end{aligned} \tag{3.29}$$

Using (3.26) we calculate the OPE of these currents with the full energy-momentum tensor, $T(z)=T^{\Phi_1\Phi_2}(z)+T^{\xi\eta}(z)$:

$$\begin{aligned} T(z)j_1(w)&=\frac{Q-3}{(z-w)^3}+\frac{j_1(w)}{(z-w)^2}+\frac{\partial_w j_1(w)}{z-w}+\cdots\ ,\\ T(z)j_2(w)&=\frac{2-Q}{(z-w)^3}+\frac{j_2(w)}{(z-w)^2}+\frac{\partial_w j_2(w)}{z-w}+\cdots\ . \end{aligned} \tag{3.30}$$

This confirms the value for the ghost number anomalies in (1.10) and shows explicitly that $j_1(z)$ and $j_2(z)$ have conformal weight one. The total ghost number current is $j(z)=j_1(z)+j_2(z)$ and it does not depend explicitly on the coupling constant.

The Sugawara-Sommerfield form of the stress tensor can be obtained by solving (3.29) for $\partial_z\Phi_1$ and $\partial_z\Phi_2$ and substituting in (3.26). The result is

$$T^{\Phi_1\Phi_2}=\tfrac{1}{2}(j_1+j_2)[(1+Q)j_i-(1-Q)j_2] +\tfrac{3}{2}\partial j_1+\partial j_2\ . \tag{3.31}$$

It is also straightforward to construct the supercurrents $S^{\pm}(z)$ in terms of Φ_1, Φ_2, and ξ-η. One simply takes the bosonized supercurrent from the free theory[11,6] and substitutes the modified conformal weight. This gives

$$S^{\pm}(z)=\mp\tfrac{1}{2}\partial_z\xi\partial_z\Phi_1e^{\Phi_1-\Phi_2}\pm(-1+\tfrac{1}{4}Q)\partial_z(\partial_z\xi e^{\Phi_1-\Phi_2}) +\tfrac{1}{2}\eta e^{-\Phi_1+\Phi_2}\ . \tag{3.32}$$

Note that $\Phi_1(z,\bar z)-\Phi_2(z,\bar z)$ is purely holomorphic. It should be obvious that the OPE's of $T(z)$ and $S^{\pm}(z)$ are the standard ones of the $N=2$ superconformal algebra with the central charge $\hat c=-10+2Q$.

The antiholomorphic components in Φ_1 and Φ_2 become important when we consider the original elementary fields. For example, using (3.19) and (3.26)–(3.28) we find

$$T(z)c(w,\bar w)\sim\frac{h_c c(w,\bar w)}{(z-w)^2}+\frac{\partial_w c(w,\bar w)}{z-w} \tag{3.33}$$

as well as

$$\bar T(\bar z)c(w,\bar w)\sim\frac{\bar h_c c(w,\bar w)}{(\bar z-\bar w)^2}+\frac{\partial_{\bar w}c(w,\bar w)}{\bar z-\bar w}\ , \tag{3.34}$$

which show that the holomorphic and antiholomorphic conformal dimensions of $c(z,\bar z)$ are $h_c=-1+Q/4$ and $\bar h_c=Q/4$, respectively. Similarly, we can determine the conformal dimensions of other fields, and the result agrees with the previous calculation summarized in Table I. In our opinion this provides a nontrivial check of the bosonization.

The bosonized representation also allows us to define the composite operators in terms of the elementary fields. We will restrict our analysis to operators which are bilinear in the fields, and regularize the products of fields using the point-splitting method. It is known that the lack of analyticity of the elementary fields in the standard Thirring model makes the construction of the composite operators rather cumbersome. It seems that the supersymmetric theory considered here is less complicated and this can be traced to the simpler two-point functions (3.12) and (3.13) of elementary fields.

Let us first consider the ghost number currents j_1 and j_2. For $z \to w$ we find

$$b(z,\bar{z})c(w,\bar{w}) \sim : \left[-\partial_z \Phi_1(z,\bar{z}) - \frac{\bar{z}-\bar{w}}{z-w}\partial_{\bar{z}}\Phi_1(z,\bar{z}) + \frac{1}{z-w} \right] : + O(|z-w|) \,, \tag{3.35}$$

which shows that j_1 cannot be constructed from $b(z,\bar{z})$ and $c(z,\bar{z})$ alone. In addition one must use not only $\beta(z,\bar{z})$ and $\gamma(z,\bar{z})$ but also the complex-conjugated fields. The latter are necessary to cancel the terms with factor $\bar{z}-\bar{w}/z-w$ so that the final result is independent of how we evaluate the limit $w \to z$. We then find that the correct point-splitting prescription for j_1 and j_2 is

$$j_1(z) = \lim_{w\to z} \left[-(1-\tfrac{1}{2}Q)b(z,\bar{z})c(w,\bar{w}) + \tfrac{1}{2}Q\beta(z,\bar{z})\gamma(w,\bar{w}) + \frac{1}{z-w} - \frac{1}{2}Q\frac{\bar{z}-\bar{w}}{z-w}[\bar{b}(z,\bar{z})\bar{c}(w,\bar{w}) + \bar{\beta}(z,\bar{z})\bar{\gamma}(w,\bar{w})] \right] , \tag{3.36}$$

$$j_2(z) = \lim_{w\to z} \left[-\tfrac{1}{2}Qb(z,\bar{z})c(w,\bar{w}) - (1+\tfrac{1}{2}Q)\beta(z,\bar{z})\gamma(w,\bar{w}) - \frac{1}{z-w} + \frac{1}{2}Q\frac{\bar{z}-\bar{w}}{z-w}[\bar{b}(z,\bar{z})\bar{c}(w,\bar{w}) + \bar{\beta}(z,\bar{z})\bar{\gamma}(w,\bar{w})] \right] . \tag{3.37}$$

On the other hand, when we consider the total ghost number current, we find, in agreement with the previous observations,

$$j(z) = j_1(z) + j_2(z) = \lim_{w\to z}[b(z,\bar{z})c(w,\bar{w}) + \beta(z,\bar{z})\gamma(w,\bar{w})] \,, \tag{3.38}$$

which coincides with the prescription one has in the free theory.

The idea of *b*-*c* system with modified conformal weights can also be used to provide point-splitting prescriptions for the stress tensor and supercurrents. We simply consider the point-splitting formulas of the free theory and insert the conformal weights of Table I to obtain

$$T(z) = \lim_{w\to z}[(-2+\tfrac{1}{4}Q)b(z,\bar{z})\partial_w c(w,\bar{w}) + (-1+\tfrac{1}{4}Q)\partial_z b(z,\bar{z})c(w,\bar{w}) + (-\tfrac{3}{2}+\tfrac{1}{4}Q)\beta(z,\bar{z})\partial_w\gamma(w,\bar{w}) + (-\tfrac{1}{2}+\tfrac{1}{4}Q)\partial_z\beta(z,\bar{z})\gamma(w,\bar{w})] \,, \tag{3.39}$$

$$S^{\pm}(z) = \lim_{w\to z}[\pm(-\tfrac{3}{2}+\tfrac{1}{4}Q)\beta(z,\bar{z})\partial_w c(w,\bar{w}) \pm(-1+\tfrac{1}{4}Q)\partial_z\beta(z,\bar{z})c(w,\bar{w}) + \tfrac{1}{2}b(z,\bar{z})\gamma(w,\bar{w})] \,. \tag{3.40}$$

It is easy to check that the limits in (3.39) and (3.40) are well defined and that we reproduce the bosonized currents in (3.26) and (3.32)

The relation between the Q-dependent formulas (3.39) and (3.40) and the Q-independent formulas (1.15) and (1.16) for the same quantities is the following. Equations (1.15) and (1.16) are classical Noether currents which can be used in quantum theory defined by Pauli-Villars-regularized functional calculations as in Sec. II, while (3.39) and (3.40) are operator expressions for the quantum theory regularized by point splitting. Both sets of expressions lead to the same correlation functions.

The operators $T(z)$ and $S^{\pm}(z)$ transform in a multiplet of the $N=2$ supersymmetry together with a dimension-one current with classical Noether form $H^{\mu} = b^{\mu\nu}c_{\nu} + \frac{3}{2}\bar{\beta}^{\mu}\gamma$. The point-splitting prescription for this current is

$$H(z) = -(1-\tfrac{1}{2}Q)j_1(z) - (\tfrac{3}{2}-\tfrac{1}{2}Q)j_2(z) \,. \tag{3.41}$$

Using (3.36) and (3.37) this leads to the bosonized form

$$H(z) = -(1-\tfrac{1}{4}Q)\partial\Phi_1 + (\tfrac{3}{2}-\tfrac{1}{4}Q)\partial\Phi_2 \,. \tag{3.42}$$

The bosonized version of the theory is convenient for the direct calculation of operator products because φ_1 and φ_2 are free scalar fields. For example, one can easily compute

$$\bar{b}(z,\bar{z})c(w,\bar{w}) \sim |z-w|^{Q}\exp[\Phi_1(z,\bar{z}) - \bar{\Phi}_1(z,\bar{z})] \,. \tag{3.43}$$

This illustrates that it is only the OPE's of conjugate elementary fields, as in (2.12) and (2.14), which are unmodified by the Thirring interaction. Because of the interaction or, equivalently, the mixing of the fields φ_1 and φ_2 in the bosonic version, there are other OPE's with coupling-dependent singularity.

IV. ATTEMPTS TO CONSTRUCT A BRST CURRENT

The BRST symmetry of the conventional superstring action for space-time fields plus free ghosts plays an important role in superstring theory. At the classical level this symmetry is lost when the Thirring interaction is added. This is most easily seen if one notes that the current

$$\mathcal{J}^{\mu} = c_{\nu}T_X^{\mu\nu} - \bar{\gamma}S_X^{\mu} \,, \tag{4.1}$$

where T_X and S_X are the stress tensor and supercurrent of space-time fields, is not conserved but satisfies

$$\nabla_{\mu}\mathcal{J}^{\mu} = -i\pi Q J_{\mu}\mathcal{J}^{\mu} \tag{4.2}$$

if one uses the Euler-Lagrange equations of the action (1.2), where J_{μ} is the total ghost number current.

It does not seem possible to restore conservation by adding local terms to (4.1). Note, however, that the equations of motion have the formal solution

$$c^{+}(x^{+},x^{-})=\exp\left[-i\pi Q\int^{x^{-}}dy^{-}J_{-}(y^{-})\right]c'^{+}(x^{+}),$$
$$\gamma_{+}(x^{+},x^{-})=\exp\left[-i\pi Q\int^{x^{-}}dy^{-}J_{-}(y^{-})\right]\gamma'_{+}(x^{+}),\qquad(4.3)$$

where $c'^{+}(x^{+})$ and $\gamma'_{+}(x^{+})$ are arbitrary functions of the light-cone coordinate x^{+}. There are analogous expressions for all other field components. Thus a current $\mathcal{J}'^{\mu}$, of the same form as (4.1) but using c' and γ', is conserved classically, although it is nonlocal with respect to the original fields and carries anomalous dimension $1+\frac{1}{4}Q$. These properties are unusual and perhaps already a negative indication, but nevertheless we choose to investigate in a more precise way the existence of a BRST current at the quantum level.

As a generalization of the technique used in conventional string theory[12] we observe that the limit $w\to z$ of the operator $c(z)T(w)-\gamma(z)S(w)$, with the leading singular term subtracted, gives an expression in the bosonized theory of the form

$$\mathcal{J}=:[r_1(\partial\Phi_1)^2-r_2(\partial\Phi_2)^2+r_3\partial\Phi_1\partial\Phi_2+r_4\partial\partial\Phi_1 - r_5\partial\partial\Phi_2-r_6\eta\partial\xi]e^{\Phi_1}:\ +r_7:\eta\partial\eta e^{-\Phi_1+2\Phi_2}:\qquad(4.4)$$

We consider this as an *Ansatz* for the ghost contribution to a BRST-type current, and try to determine the coefficients $r_1,\ldots,r_7$ so that $\mathcal{J}$ satisfies the standard properties.[12] For example, we impose the requirement $\{Q_{\rm BRST},b(w)\}\sim T(w)$ [or equivalently that the simple pole term in the OPE $\mathcal{J}(z)b(w)$ is $T(w)/(z-w)$]. This leads to the equations

$$\begin{aligned}&r_1-r_4=-1\,,\\&r_1-r_4=-1-\tfrac{1}{6}Q\,,\\&r_2=\tfrac{1}{2}\,,\\&r_3+r_5=1-\tfrac{1}{4}Q\,,\\&r_6=1\,,\end{aligned}\qquad(4.5)$$

which are inconsistent unless $Q=0$.

To further underline this inconsistency we consider the sum of the *Ansatz* (4.4) plus the coordinate field BRST current (4.1) and study the operator-product expansions of $\mathcal{J}(z)\mathcal{J}(w)$. We demand that the simple pole term has a coefficient which is a total derivative in w, so that $(Q_{\rm BRST})^2=0$. The equations for the coefficients $r_1,\ldots,r_7$ obtained in this way are again consistent only for $Q=0$.

The *Ansatz* considered above is not purely holomorphic, but contains the overall antiholomorphic factor $\exp[b_1(\bar{\varphi}_1-\varphi_2)]=\exp[-\frac{1}{2}Q(\bar{\Phi}_1-\bar{\Phi}_2)]$. This operator has a nonsingular OPE with itself and with other operators involved in the calculation above. Thus the same inconsistent equations are obtained whether or not the nonholomorphic factor is included. Omission of this factor corresponds to the use of c' and γ' fields (4.3) to form a candidate BRST current, and it is unfortunate that this candidate fails the test.

An even more general *Ansatz* for $\mathcal{J}$ involving nonholomorphic operators, such as $\partial\Phi\,\bar{\partial}\Phi$, in addition to those in (4.4), and an exponential factor $\exp(\kappa_1\Phi_1+\kappa_2\Phi_2)$ with free parameters was also considered. One still finds that the equations to fix the parameters are inconsistent for $Q\neq0$. Since the various *Ansätze* we have considered do not seem to lead to the current with usual properties of the BRST current if $Q\neq0$, we conclude that it is unlikely that such a current exists for the Thirring superghost model.

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