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Finite quantum field theory based on superspin fields

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A field-theory formalism based on a free superspin field, which has an internal degree of infinite spin, is developed using perturbation theory. Interaction Lagrangians are constructed and ultraviolet-finite theories are obtained in four-dimensional spacetime. The finite property of the graphs, unitarity of the S matrix, and microscopic causality are demonstrated for a $g\Phi^4$ model for the interaction Lagrangian. A definition of locality of the fields is given that extends the concept of the local commutativity of strictly localizable fields and a formalism for massless superspin particles is presented.

I. INTRODUCTION

The problem of finding a consistent quantum theory of gravitation has been a long-standing problem. The attempt to obtain a renormalizable theory of gravitation based on supergravity failed, because the infinities in the loop graphs in the third order of approximation did not cancel. The supergravity formalism was based on point particles and strictly localizable fields.[1] There appears to be no general gauge symmetry beyond that of supersymmetry that can achieve a renormalizable theory of gravitation based on point particles. There has been a recent surge of interest in string theories[2] and, since the strings are extended objects that interact locally, there is good reason to believe that they lead to a finite theory of gravitation. However, the superstring theories are only well defined in a ten-dimensional spacetime and it is necessary to compactify the ten-dimensional theory to four dimensions. There seems at present to be no unique way to achieve this and, indeed, it has been demonstrated that the compactification of the heterotic string models to four dimensions leads to a very large number of possible string theories.[3] Since string theory does not initially take the form of a conventional field theory, attempts have been made to construct a field theory of strings with limited success.[4] Such field theories are nonlocal and they do not fit into the standard framework of axiomatic field theory, based on point particles and strictly localizable fields.[5–11] It is difficult to perform calculations in these theories due to the complicated structure of the string graphs.

It has recently been shown that the bosonic string model has a divergent perturbation series and is not Borel summable.[12] There are strong indications that the same holds true for the superstring model, although there is, as yet, no rigorous proof that this is true. These infinities correspond to perturbative instabilities of the flat-space vacuum. Thus, string theory does not appear to possess a perturbation-theory solution, putting it at present beyond the reach of experimental verification. These problems may well be caused by the fact that the basic mass scale is the Planck mass $\sim 10^{19}$ GeV.

Although it is not clear how serious these difficulties are for the future of string theory, it does show that problems exist when specific assumptions are made about the topological structure of the extended object that describes the basic building block of matter. Recent attempts have been made to construct a theory using membranes.[13] Such theories suffer from the lack of a conformal invariance that is possessed by string theories. The basic equations of motion of the membranes are nonlinear and are difficult to solve. There appears at present to be no guiding principle that tells us which kind of extended object to use to describe nature.

The renormalization program has enjoyed success in its application to gauge theories. Renormalizable gauge theories form the basis of our present understanding of strong, weak, and electromagnetic interactions. Computations in renormalizable field theories are normally accompanied by the presence of infinite quantities. The coefficients of a power series in a small coupling constant are typically infinite and are, therefore, meaningless. The renormalization theory allows one to extract physically correct results from these seemingly meaningless computations. The theory is regulated in some manner so that when a regularization parameter approaches infinity, the basic quantities in the theory diverge. These divergent quantities are removed by absorbing them into the parameters that define the initial unquantized theory. The renormalization program is only successful if a finite number of parameters is needed to absorb the infinite quantities to all orders in perturbation theory. Gravitation cannot be renormalized in this program, because an infinite number of parameters that cannot be determined experimentally is needed to define the theory after quantization. The renormalization program would appear to be only a makeshift method that will be replaced by a finite, consistent field theory.

Two essential physical features of the string theories are (1) particles are described as extended objects and not as structureless points and (2) the string vibrations give rise to infinite levels of masses for all spins. Is it possible to construct a consistent quantum field theory based on infinite-component (spin) fields that leads to a unitary S

matrix and that describes a finite quantum theory of gravity? Recently a program for constructing such a theory has been published.[14] In the following, we shall give the detailed formalism for this theory in which field theories of spins 0, 1, and 2 are constructed in four-dimensional spacetime which are finite, unitary, and satisfy a principle of microscopic causality. We shall introduce the idea of the "superspin field," which for spin-0, -1, and -2 particles possesses an "internal" infinite-spin degree of freedom. The particles are described by nonlocal extended objects that we call "superspin" particles.

Krasnikov[15] has recently suggested using infinite-component fields to remove the problem of ultraviolet infinities in field theory, but he did not offer any physical explanation for the underlying nature of the infinite-component structure of the fields.

The new degree of freedom associated with the infinite spin leads to propagators for the free fields that fall off sufficiently rapidly with increasing momentum to guarantee the ultraviolet finiteness of the perturbation theory to all orders for a given Lagrangian. The superspin fields are constructed to be free of ghost poles; they do not possess any "wrong" spin components. To overcome the problem of unphysical modes occurring for interacting spins-0, -1, and -2 superspin fields, we introduce the idea of infinite-spin gauge invariance. The superspin field propagators are described by entire functions of order $\geq \frac{1}{2}$ that do not possess any singularities in the finite complex momentum plane and do not violate the unitarity of the S matrix.

Since the superspin fields are associated with nonlocal objects, we must implement some features of nonlocal field theory. Consistent nonlocal extensions of the standard strictly local field-theory formalism have been developed[16–21] that can lead to a generalized microscopic causality for a suitable choice of the entire analytic functions generated by the superspin fields. Although the superspin fields are not *strictly* localizable, we can associate them with a definition of locality that leads to microscopic causality.

It is important to recognize that the physical mechanism that leads to a finite quantum field theory is the internal degree of *infinite* spin associated with particles. There is no need to implement supersymmetry as with finite-spin point-particle theory, although the formalism could be extended to include supersymmetry. We know from experiment that the leading Regge-pole trajectories appear to be linearly rising up to the highest detected spins,[22] which suggests that particles do form infinite-spin towers. In the nonsymmetric gravitation theory (NGT), the local gauge structure in the tangent bundle is GL(4,R) (Ref. 23). The latter group and its double cover only possess infinite-dimensional spinor representations.[24] Thus, NGT departs radically from general relativity (GR) in that fermions must have an infinite-spin degree of freedom associated with them. In GR, the local gauge group SO(3,1) contains finite-component spinor representations of the homogeneous Lorentz group corresponding to point particles, but this theory is unrenormalizable and the perturbation theory diverges at every order.

One of the problems of string theories is their lack of physical predictions that can be used to test the theories. This is, of course, in part due to the complexities associated with the required compactification of these theories to four dimensions and the necessity to break supersymmetry. Beyond the obvious aesthetic appeal of finite theories, how can we be sure that our scheme yields a physically superior theory when compared to the standard renormalization theory? There may, indeed, exist several alternative ways to construct a finite theory of gravity. How do we know that we have obtained the "correct" quantum gravity theory? The construction of a successful theory would necessarily have a serious influence on the formulation of the field theories of the other forces of nature. In the generalized electroweak theory, finite electroweak interactions would predict first-order perturbation theory results in agreement with experiment given the known values of the Glashow-Weinberg angle $\sin^2\theta_W$ and the intermediate vector boson W and Z^0 masses. The electroweak cutoff M_w is a physical parameter in the superspin field theory, since the Feynman graph loop integrals damp off exponentially fast above the value of M_w. The Higgs gauge hierarchy problem no longer exists and the Higgs sector becomes a nontrivial theory, because there does not exist any Landau singularity. Thus, the generalized electroweak theory becomes a fundamental field theory with a physical Higgs particle. The Higgs radiative corrections obtained from our finite superspin theory of electroweak interactions differ from their counterparts in the standard Weinberg-Salam-Glashow theory, and thus lead to different physical predictions that could be tested using high-energy accelerators.

No attempt is made here to construct a unified field theory including gravitation. The field theory is formulated in four-dimensional spacetime. It could be generalized to higher dimensions, opening the door to a Kaluza-Klein type of unification, but this would introduce an unacceptable degree of arbitrariness into the scheme. Some new guiding principle must be discovered to unify the fields that does not bring with it the arbitrariness associated with Kaluza-Klein compactification schemes.

II. PARTICLE AND FIELD INTERPRETATION FOR ARBITRARY SPIN

The physical states of particles are described by the Wigner basis states $|\mathbf{p},m,j\sigma\rangle$ for a unitary irreducible representation of the inhomogeneous Lorentz group (Poincaré group)[25] with $p^2=p_0^2-\mathbf{p}^2=m^2$. The spin j corresponds to the eigenvalues $J^2=j(j+1)$ and $J_3=\sigma$ $(\sigma=j,j-1,\ldots,-j)$. These states form the Hilbert space of the theory and they can be obtained from the rest states $|m,j\sigma\rangle$ by a unitary transformation

$$|\mathbf{p},m,j\sigma\rangle=[m/\omega(\mathbf{p})]^{1/2}U[L(\mathbf{p})]|m,j\sigma\rangle\ , \tag{2.1}$$

where $U[(\mathbf{p})]$ is a unitary operator associated with the pure Lorentz "boost" that takes $[0,m]$ into $[\mathbf{p},p^0]$ and is given by

$$L^i_j(\mathbf{p})=\delta_{ij}+\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j(\cosh\theta-1)\ , \tag{2.2a}$$

$$L^i_0(\mathbf{p})=L^0_i(\mathbf{p})=\hat{\mathbf{p}}_i\sinh\theta\ , \tag{2.2b}$$

$$L_0^0(\mathbf{p})=\cosh\theta\ . \tag{2.2c}$$

Here $\hat{\mathbf{p}}$ is the unit vector $\mathbf{p}/|\mathbf{p}|$, $\sinh\theta=|\mathbf{p}|/m$, and $\cosh\theta=\omega/m=(\mathbf{p}^2+m^2)^{1/2}/m$. Our normalization convention is

$$\langle \mathbf{p},\sigma|\mathbf{p}',\sigma'\rangle=\delta^3(\mathbf{p}-\mathbf{p}')\delta_{\sigma\sigma'}\ . \tag{2.3}$$

An arbitrary Lorentz transformation $\Lambda^\mu{}_\nu$, performed on these one-particle states, gives

$$\begin{aligned}U(\Lambda)|\mathbf{p},\sigma\rangle&=(m/\omega)^{1/2}\sum_{\sigma'}U[L(\Lambda\mathbf{p})]|\sigma'\rangle\\&\quad\times\langle\sigma'|U[L^{-1}(\Lambda\mathbf{p})\Lambda L(\mathbf{p})]|\sigma\rangle\\&=[\omega(\Lambda\mathbf{p})/\omega(\mathbf{p})]^{1/2}\\&\quad\times\sum_{\sigma'}|\Lambda\mathbf{p},\sigma'\rangle\mathcal{D}^j_{\sigma'\sigma}[L^{-1}(\Lambda\mathbf{p})\Lambda L(\mathbf{p})]\ ,\end{aligned} \tag{2.4}$$

where the coefficients $\mathcal{D}^j_{\sigma'\sigma}$ are

$$\mathcal{D}^j_{\sigma'\sigma}(R)=\langle\sigma'|U(R)|\sigma\rangle\ . \tag{2.5}$$

Here, R is the pure Wigner rotation $L^{-1}(\Lambda\mathbf{p})\Lambda L(\mathbf{p})$, so that $\mathcal{D}^j(R)$ is the familiar $(2j+1)$-dimensional unitary matrix representation of the rotational group. We have not assumed that there exists an explicit dependence of the mass m on the spin j. However, an extension of the formalism could be made to incorporate this feature of the mass spectrum.

We write for free fields

$$\phi_{j\sigma}(x)=\phi_{j\sigma}^{(+)}(x)+\phi_{j\sigma}^{(-)}(x)\ , \tag{2.6}$$

where

$$\begin{aligned}\phi_{j\sigma}^{(+)}(x)&=(2\pi)^{-3/2}\int\frac{d^3\mathbf{p}}{(2\omega)^{1/2}}\sum_{j'\sigma'}\mathcal{D}_{j\sigma,j'\sigma'}[L(\mathbf{p})]\\&\quad\times\tilde{b}^*(\mathbf{p},j'\sigma')e^{ip\cdot x}\ ,\end{aligned} \tag{2.7a}$$

$$\begin{aligned}\phi_{j\sigma}^{(-)}(x)&=(2\pi)^{-3/2}\int\frac{d^3\mathbf{p}}{(2\omega)^{1/2}}\sum_{j'\sigma'}\mathcal{D}_{-j\sigma,-j'\sigma'}[L(\mathbf{p})]\\&\quad\times a(\mathbf{p},-j'\sigma')e^{-ip\cdot x}\ ,\end{aligned} \tag{2.7b}$$

and $\phi_{j\sigma}^{(-)}(x)$ is the annihilation part for the particles.[26,27] The $\mathcal{D}[L(\mathbf{p})]$ is the representation matrix of SO(3,1) [or its double covering SL(2,C)] for a boost along $\hat{\mathbf{p}}$. The a^* and a are creation and annihilation operators operating on the vacuum state $|0\rangle$ with $|\mathbf{p},m,j\sigma\rangle=a^*(\mathbf{p},j\sigma)|0\rangle$. Moreover, the antiparticle operator is defined by $\tilde{b}^*_\sigma=\sum_{\sigma'}[(C^j)^{-1}]_{\sigma\sigma'}b^*(\sigma')$, where C is a $(2j+1)\times(2j+1)$ matrix with $C^*C=(-)^{2j}$ and $C^\dagger C=1$. C is used to define the ordinary complex conjugate of the finite Lorentz representation: $\mathcal{D}^j(R)^*=C\mathcal{D}^j(R)C^{-1}$. The a's and b's satisfy the standard free-particle Bose-Fermi commutation (anticommutation) relations

$$[a(\mathbf{p},j\sigma),a^*(\mathbf{p}',j'\sigma')]_\pm=\delta^3(\mathbf{p}-\mathbf{p}')\delta_{jj'}\delta_{\sigma\sigma'}\ . \tag{2.8}$$

The field operators $\phi_{j\sigma}(x)$ satisfy the correct statistics and crossing symmetry.[26] We have

$$[\phi_{j\sigma}(x),\phi^\dagger_{j\sigma'}(y)]_\pm=t_{\sigma\sigma'}^{\mu_1\mu_2\cdots\mu_{2j}}\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_{2j}}\Delta(x-y)\ , \tag{2.9}$$

where Δ is the standard free-particle causal function

$$\begin{aligned}\Delta(x-y)&=\frac{-i}{(2\pi)^3}\int\frac{d^3\mathbf{p}}{2\omega}(e^{-ip\cdot(x-y)}-e^{ip\cdot(x-y)})\\&=-\frac{1}{(2\pi)^3}\int\frac{d^3\mathbf{p}}{\omega}e^{ip\cdot(x-y)}\sin p_0(x_0-y_0)\\&=\frac{-i}{(2\pi)^3}\int d^4p\ \epsilon(p_0)\delta(p^2-m^2)e^{-ip\cdot(x-y)}\ ,\end{aligned} \tag{2.10}$$

where $\epsilon(p_0)=\theta(p_0)-\theta(-p_0)=p_0/|p_0|$. Moreover, $t_{\sigma\sigma'}^{\mu_1\cdots\mu_{2j}}$ is a symmetric, traceless tensor.

The $(2j+1)$-component fields will have left-handed and right-handed contributions with the helicity $\lambda=-j$ and $+j$ corresponding to the representations $(j,0)$ and $(0,j)$, respectively. The corresponding $(2j+1)$-component annihilation fields are $\phi_{j\sigma}^{(-)}(x)$ and $\bar{\phi}_{j\sigma}^{(-)}(x)$ and

$$\begin{aligned}\bar{\phi}_{j\sigma}^{(-)}(x)&=(2\pi)^{-3/2}\int\frac{d^3\mathbf{p}}{(2\omega)^{1/2}}\sum_{j'\sigma'}\bar{\mathcal{D}}_{j\sigma,j'\sigma'}[L(\mathbf{p})]\\&\quad\times a(\mathbf{p},j'\sigma')e^{-ip\cdot x}\ .\end{aligned} \tag{2.11}$$

The $\mathcal{D}^j(\Lambda)$ and $\bar{\mathcal{D}}^j(\Lambda)$ are the finite, nonunitary $(2j+1)\times(2j+1)$-dimensional matrices corresponding to Λ in the $(j,0)$ and $(0,j)$ representations, respectively.

In the standard way, we can construct finite-dimensional fields transforming under the Lorentz group as irreducible representations. The three rotation generators $\mathbf{J}$ and the three Lorentz-boost generators $\mathbf{K}$ satisfy the usual commutation relations

$$\begin{aligned}&[J_a,J_b]=i\epsilon_{abc}J_c,\quad [J_a,K_b]=i\epsilon_{abc}K_c\ ,\\&[K_a,K_b]=-i\epsilon_{abc}K_c\ .\end{aligned} \tag{2.12}$$

We can decouple these commutation relations by defining the two anti-Hermitian operators $\mathbf{A}=\frac{1}{2}(\mathbf{J}+i\mathbf{K})$ and $\mathbf{B}=\frac{1}{2}(\mathbf{J}-i\mathbf{K})$ that satisfy the commutation relations

$$\mathbf{A}\times\mathbf{A}=i\,\mathbf{A},\quad \mathbf{B}\times\mathbf{B}=i\mathbf{B},\quad [A_i,B_j]=0\ . \tag{2.13}$$

Then, a finite nonunitary, irreducible representation (a,b) is labeled by a and b defined by the eigenvalue equations $\mathbf{A}^2=a(a+1)$ and $\mathbf{B}^2=b(b+1)$; $a,b=0,\frac{1}{2},1,\frac{3}{2},\ldots$. The components of an irreducible tensor are given by a_3,b_3 of A_3,B_3 or by j and σ.

A calculation of the covariant propagator gives

$$D^{jc}_{\sigma\sigma'}(x-y)=\langle 0|T[\phi_{j\sigma}(x)\phi^{\dagger}_{j\sigma'}(y)]|0\rangle$$
$$=\theta(x-y)\langle 0|\phi_{j\sigma}(x)\phi^{\dagger}_{j\sigma'}(y)|0\rangle+(-1)^{2j}\theta(y-x)\langle 0|\phi^{\dagger}_{j\sigma'}(y)\phi_{j\sigma}(x)|0\rangle$$
$$=t^{\mu_1\mu_2\cdots\mu_{2j}}_{\sigma\sigma'}\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_{2j}}\Delta^c(x-y)\ , \tag{2.14}$$

where Δ^c is Feynman's propagator and we have neglected contact terms. By performing a Fourier transformation to momentum space, we get

$$D^{jc}_{\sigma\sigma'}(p)=\int d^4x\, e^{-ip\cdot x}D^{jc}_{\sigma\sigma'}(x)=\frac{1}{i}\frac{K^{j}_{\sigma\sigma'}(p)}{-p^2+m^2-i\epsilon}\ . \tag{2.15}$$

The functions K and $\bar{K}$ have the following properties.[26]

(a) They are scalars, since they satisfy

$$\mathcal{D}^j(\Lambda)K(p)\mathcal{D}^j(\Lambda)^{\dagger}=K(\Lambda p)\ , \tag{2.16a}$$
$$\bar{\mathcal{D}}^j(\Lambda)\bar{K}(p)\bar{\mathcal{D}}^j(\Lambda)^{\dagger}=\bar{K}(\Lambda p)\ . \tag{2.16b}$$

(b) K and $\bar{K}$ are related by inversion

$$\bar{K}(-\mathbf{p},p^0)=K(p)\ . \tag{2.17}$$

(c) The K and $\bar{K}$ are related by the transformation

$$\bar{K}^*(p)=CK(p)C^{-1}\ , \tag{2.18}$$

where

$$-\mathbf{J}^{j*}=C\mathbf{J}^jC^{-1}\ . \tag{2.19}$$

(d) The K and $\bar{K}$ are further related by

$$K(p)\bar{K}(p)=(p^2)^{2j}\ . \tag{2.20}$$

For p in the forward light cone,

$$K(p)=(p^2)^{2j}\exp[-2\theta(p)\hat{\mathbf{p}}\cdot\mathbf{J}^j]\ , \tag{2.21a}$$
$$\bar{K}(p)=(p^2)^{2j}\exp[2\theta(p)\hat{\mathbf{p}}\cdot\mathbf{J}^j]\ , \tag{2.21b}$$

with $\sinh\theta(p)=(|\mathbf{p}|^2/p^2)^{1/2}$.

An explicit calculation has been given by Weinberg[26] for the functions $K(p)$ using the fact that

$$\mathcal{D}^j_{\sigma\sigma'}[L(\mathbf{p})]=\exp(-\hat{\mathbf{p}}\cdot\mathbf{J}^j\theta)_{\sigma\sigma'}\ . \tag{2.22}$$

For integer j the result is, for arbitrary p,

$$K^j(p)=(p^2)^j+\sum_{n=0}^{j-1}\frac{(p^2)^{j-1-n}}{(2n+2)!}(2\mathbf{p}\cdot\mathbf{J})[(2\mathbf{p}\cdot\mathbf{J})^2-(2\mathbf{p})^2][(2\mathbf{p}\cdot\mathbf{J})^2-(4\mathbf{p})^2]\cdots[(2\mathbf{p}\cdot\mathbf{J})^2-(2n\mathbf{p})^2][2\mathbf{p}\cdot\mathbf{J}-(2n+2)p^0]\ . \tag{2.23}$$

For half-integer j, we have, for all p,

$$K^j(p)=(p^2)^{j-1/2}(p^0-2\mathbf{p}\cdot\mathbf{J})+\sum_{n=1}^{j-1/2}\frac{(p^2)^{j-n-1/2}}{(2n+1)!}[(2\mathbf{p}\cdot\mathbf{J})^2-\mathbf{p}^2][(2\mathbf{p}\cdot\mathbf{J})^2-(3\mathbf{p})^2]\cdots$$
$$\times\{(2\mathbf{p}\cdot J)^2-[(2n-1)\mathbf{p}]^2\}[(2n+1)p^0-2\mathbf{p}\cdot\mathbf{J}]\ . \tag{2.24}$$

The fields $\phi_{j\sigma}(x)$ satisfy the Klein-Gordon equation

$$(\Box+m^2)\phi_{j\sigma}(x)=0\ . \tag{2.25}$$

All the "wrong-spin" subsidiary conditions are *automatically satisfied* by the free fields $\phi_{j\sigma}(x)$ for the $(j,0)$ and $(0,j)$ representations. Thus, the free fields $\phi_{j\sigma}(x)$ do not have any unphysical ghost states. If we take the limit $m\to 0$ for the massless particles, then this limit only exists for the helicity choice $\lambda=B-A$ for the $(j,0)$ and $(0,j)$ representations.[26] In the massless case, the fields satisfy

$$\Box\phi_{j\sigma}(x)=0\ . \tag{2.26}$$

We shall also introduce the field $\psi_{j\sigma}(x)$ given by

$$\psi_{j\sigma}(x)=(2\pi)^{-3/2}\int d^3\mathbf{p}\left[\frac{m}{E(\mathbf{p})}\right]^{1/2}\sum_{j'\sigma'}\{\mathcal{D}_{j\sigma,j'\sigma'}[L(\mathbf{p})]b(\mathbf{p},j'\sigma')w(\mathbf{p})e^{-ip\cdot x}$$
$$+(\mathcal{D}C^{-1})_{j\sigma,j'\sigma'}[L(\mathbf{p})]d^*(\mathbf{p},j'\sigma')v(\mathbf{p})e^{ip\cdot x}\}\ , \tag{2.27}$$

where w and v are Dirac spinors. The free field $\psi_{j\sigma}(x)$ satisfies the Dirac equation

$$(-i\gamma^{\mu}\partial_{\mu}+m)\psi_{j\sigma}(x)=0 \tag{2.28}$$

and the b's and d's satisfy the standard commutation and anticommutation rules. A calculation of the covariant propagator yields the result

$$S^{jc}_{\sigma\sigma'}(x-y)=\langle 0|T[\psi_{j\sigma}(x)\psi^{\dagger}_{j\sigma'}(y)]|0\rangle=(i\gamma^{\mu}\partial_{\mu}+m)t^{\mu_1\mu_2\cdots\mu_{2j}}_{\sigma\sigma'}\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_{2j}}\Delta^c(x-y)\ . \tag{2.29}$$

Fourier transforming this result to momentum space gives

$$S^{jc}_{\sigma\sigma'}(p)=\int d^4x\, e^{-ip\cdot x}S^{jc}_{\sigma\sigma'}(x)=\frac{1}{i}\frac{K^j(p)}{m-p\cdot\gamma-i\epsilon}\ , \tag{2.30}$$

where we have used the notation $p\cdot\gamma=p^{\mu}\gamma_{\mu}$.

It is often preferable to use tensor suffix notation to do calculations with massless fields. For massless particles, the helicity must satisfy $B-A=\pm j$. For the representations $(0,j)$ and $(j,0)$, the fields $\phi_{j\sigma}(x)$ are written as[26]

$$F^{[\mu_1\nu_1]\cdots[\mu_j\nu_j]}_{\pm}(x)=i^j\int d^3\mathbf{p}(2|\mathbf{p}|)^{-1/2}[p^{\mu_1}e^{\nu_1}_{\pm}(\mathbf{p})-p^{\nu_1}e^{\mu_1}_{\pm}(\mathbf{p})]$$
$$\times\cdots\times[p^{\mu_j}e^{\nu_j}_{\pm}(\mathbf{p})-p^{\nu_j}e^{\mu_j}_{\pm}(\mathbf{p})][a(\mathbf{p},\pm j)e^{-ip\cdot x}+b^*(\mathbf{p},\mp j)e^{ip\cdot x}]\ . \tag{2.31}$$

The polarization tensors e satisfy

$$p_{\mu}e^{\mu}_{\pm}(\mathbf{p})=0,\quad e_{\pm\mu}(\mathbf{p})e^{\mu}_{\pm}(\mathbf{p})=0\ , \tag{2.32a}$$

$$\epsilon^{\mu\nu\rho\eta}p_{\rho}e_{\pm\eta}(\mathbf{p})=\mp i[p^{\mu}e^{\nu}_{\pm}(\mathbf{p})-p^{\nu}e^{\mu}_{\pm}(\mathbf{p})]\ , \tag{2.32b}$$

where $\epsilon^{\mu\nu\rho\eta}$ is the totally antisymmetric tensor with $\epsilon^{0123}\equiv 1$ and we have $e^0_{\pm}\equiv 0$. The F tensors satisfy the following conditions.[26]

(1) $F_{\pm}(x)$ are antisymmetric under the interchanges $\mu_r\leftrightarrow\nu_r$ within any one index pair:

$$F^{[\mu_1\nu_1]\cdots}_{\pm}=-F^{[\nu_1\mu_1]\cdots}_{\pm}\ . \tag{2.33}$$

(2) $F_{\pm}(x)$ are symmetric under the interchange of any two index pairs $[\mu_r\nu_r]\leftrightarrow[\mu_s\nu_s]$.

(3) $F_+(x)$ and $F_-(x)$ are, respectively, self-dual or anti-self-dual with respect to each index pair $[\mu_r\nu_r]$:

$$\epsilon^{\mu\nu\mu_1\nu_1}F^{[\mu_2\nu_2]\cdots}_{\pm[\mu_1\nu_1]}=\mp 2iF^{[\mu\nu][\mu_2\nu_2]\cdots}_{\pm}\ . \tag{2.34}$$

(4) The $F_{\pm}$ tensors are traceless. The complete contraction of any pair of suffixes $[\mu_r\nu_r],[\mu_s\nu_s]$ gives zero:

$$\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}F^{[\mu_1\nu_1][\mu_2\nu_2]\cdots}_{\pm}=0\ . \tag{2.35}$$

Moreover, any single trace vanishes:

$$\eta_{\mu_1\mu_2}F^{[\mu_1\nu_1][\mu_2\mu_2]\cdots}_{\pm}=0\ . \tag{2.36}$$

Here $\eta_{\mu\nu}$ is the Minkowski-space metric $\eta_{\mu\nu}=\text{Diag}(1,-1,-1,-1)$. Because of the four conditions listed above, the $F_{\pm}$ tensors each have at most $2j+1$ independent components.

The $F_{\pm}(x)$ are tensors under Lorentz transformations

$$U(\Lambda,a)F^{[\mu_1\nu_1]\cdots[\mu_j\nu_j]}_{\pm}(x)U(\Lambda,a)^{-1}$$
$$=\Lambda^{\mu_1}_{\rho_1}\Lambda^{\nu_1}_{\eta_1}\cdots\Lambda^{\mu_j}_{\rho_j}\Lambda^{\nu_j}_{\eta_j}F^{[\rho_1\eta_1]\cdots[\rho_j\eta_j]}_{\pm}(\Lambda x+a)\ . \tag{2.37}$$

The irreducible fields are determined uniquely by the representation (A,B) under which they transform

$$\phi^{AB}_{ab}(x)=(2\pi)^{-3/2}\int d^3\mathbf{p}(2|\mathbf{p}|)^{A+B-1/2}$$
$$\times\mathcal{D}^A_{a,-A}[R(\hat{\mathbf{p}})]\mathcal{D}^B_{b,B}[R(\hat{\mathbf{p}})]$$
$$\times[a(\mathbf{p},\pm j)e^{-ip\cdot x}$$
$$+b^*(\mathbf{p},\mp j)e^{ip\cdot x}]\ , \tag{2.38}$$

where the indices a and b run by unit steps from $-A$ to $+A$ and $-B$ to $+B$, respectively. When the particles are their own antiparticles, we set the operators a equal to b. The $F_{\pm}$ transform according to some irreducible or reducible representation of the homogeneous Lorentz group SL(2,C) with dimensionality $2j+1$. Of the irreducible representations (A,B) that satisfy the massless condition $B-A=\pm j$, the ones with the smallest dimensionality are the $(2j+1)$-dimensional representations $(j,0)$ for helicity $-j$, and $(0,j)$ for helicity $+j$. Thus, F_- and F_+ transform according to the $(j,0)$ and $(0,j)$ representations, which are just the $2j+1$ component fields obtained from (2.38) by setting $B=0$ or $A=0$, respectively.

The $F_{\pm}$ tensors are *gauge-invariant* quantities and they correspond to the Maxwell field strengths for $j=1$ and satisfy Maxwell's equations. In general, the $F_{\pm}(x)$ satisfy the field equations

$$\partial_{\mu_1}F^{[\mu_1\nu_1]\cdots}_{\pm}=0\ . \tag{2.39}$$

The five independent components of the tensor $F^{[\mu\nu][\rho\sigma]}_{\pm}$ can be identified with the left- or right-handed parts of the Riemann-Christoffel curvature tensor. The $F_{\pm}(x)$ tensors can be written as curls of potentials $A^{\mu_1\cdots\mu_j}_{\pm}(x)$. The latter satisfy the free-field equations

$$\partial_{\mu_1}A^{\mu_1\cdots\mu_j}_{\pm}=0 \tag{2.40}$$

and

$$\Box A^{\mu_1\cdots\mu_j}_{\pm}=0\ . \tag{2.41}$$

The $A_{\pm}$'s can be written as

$$A^{\mu_1\cdots\mu_j}_{\pm}(x)=(2\pi)^{-3/2}$$
$$\times\int d^3\mathbf{p}(2|\mathbf{p}|)^{-1/2}e^{\mu_1}_{\pm}(\mathbf{p})\cdots e^{\mu_j}_{\pm}(\mathbf{p})$$
$$\times[a(\mathbf{p},\pm j)e^{-ip\cdot x}$$
$$+(-)^j b^*(\mathbf{p},\mp j)e^{ip\cdot x}]\ . \tag{2.42}$$

In contrast with the $F_{\pm}$'s, the $A_{\pm}$'s are not Lorentz tensors, since their timelike components vanish. The noncovariance of the $A_{\pm}$'s manifests itself in the appearance of gradient terms in the Lorentz transformation law of the $A_{\pm}$'s, which disappear when we take the curls to form the $F_{\pm}$ tensors.[26]

A crucial difference between the $F_\pm$ tensors and the $A_\pm$ potentials is that the propagator calculated from the $F_\pm$'s, which corresponds to the one obtained previously from the $\phi_{j\sigma}(x)$ fields, has a quite different behavior as a function of the momentum p as $p \to \infty$. Indeed, whereas the propagator $D^{jc}(p)$, in Eq. (2.14), behaves like $(p^2)^{j-1}$ as $p \to \infty$, the covariant part of the propagator obtained from the potentials $A_\pm(x)$ will have a constant behavior as $p \to \infty$. For point particles, this difference is closely related to the long-range behavior of electromagnetic and gravitational forces and the existence of infrared divergences in the limit $p \to 0$. It will play an important role in the development of the superspin field theory.

III. CONSTRUCTION OF THE SUPERSPIN FIELDS

We shall construct fields depending on the spacetime coordinates x as well as four complex variables z_1, w_1 and z_2, w_2 arranged into two two-component spinors $\theta_i = (z_i, w_i)$ $(i = 1,2)$. The θ will form our spin-coordinate basis space that replaces the conventional tensor suffix basis space. This kind of basis space was introduced by Bargmann[28] and used by Abarbanel[29] and Rivers[30] to analyze Regge-pole scattering amplitudes. The superspin field is given by

$$\Phi(\theta,x) = \sum_{j=0}^{\infty} c_j \sum_{\sigma=-j}^{+j} u_{j\sigma}(\theta)\phi_{j\sigma}(\theta,x) \ , \tag{3.1}$$

where $\phi_{j\sigma}(\theta,x)$ contains a θ spinor space dependence through the creation and annihilation operators $a^*(\mathbf{p}, j\sigma, \theta)$ and $a(\mathbf{p}, j\sigma, \theta)$. Now the commutation relation for the creation and annihilation operators (2.8) contains a $\delta(\theta-\eta)$ on the right-hand side. The coefficients c_j are growth dampeners and the $u_{lm}(\theta)$ are orthonormal spin basis functions for angular momentum l and projection m:

$$u_{lm}(\theta) = \frac{z^{l+m} w^{l-m}}{[(l+m)!(l-m)!]^{1/2}} \ . \tag{3.2}$$

We can also exhibit the superspin field in terms of the diagonalized A_3 and B_3, rather than the j and σ eigenvalues:

$$\Phi(\theta_1,\theta_2;x) = \sum_{a,b=0}^{\infty} c_{ab} \sum_{a_3=-a}^{+a} u_{aa_3}(\theta_1) \sum_{b_3=-b}^{+b} u_{bb_3}(\bar\theta_2) \times \phi^{(a,b)}_{a_3 b_3}(x) \ , \tag{3.3}$$

where it is understood that $\phi^{(a,b)}_{a_3 b_3}(x)$ contains a θ_1 and θ_2 dependence. We shall define a representation of the Lorentz covering group SL(2,C) on the spinors $u_{aa_3}(\theta_1)$ and $u_{bb_3}(\bar\theta_2)$ and an operation Z_A:

$$\begin{aligned} Z_A u_{aa_3}(\theta_1) u_{bb_3}(\bar\theta_2) &= u_{aa_3}(A^{-1}\theta_1) u_{bb_3}(\bar A^{-1}\bar\theta_2) \\ &= \sum_{a_3' b_3'} u_{aa_3'}(\theta_1) u_{bb_3'}(\bar\theta_2) \times \mathcal{D}^{(a,b)}_{a_3' b_3', a_3 b_3}[\Lambda(A)] \ , \end{aligned} \tag{3.4}$$

where $\Lambda(A)$ is the Lorentz transformation corresponding to A. The superspin field then has the Lorentz transformation law

$$U(\Lambda)\Phi(\theta_1,\bar\theta_2;x)U(\Lambda)^{-1} = \Phi(A\theta_1, \bar A \bar\theta_2; \Lambda x) \ . \tag{3.5}$$

We can project out the $(a,0)$ component of the superspin field (3.3) by noting that $u_{lm}(\theta)$ is orthonormal with the measure[28]

$$\overline{d\mu}(\theta) = \frac{d^2z\, d^2w}{\pi^2} \exp(-|z|^2 - |w|^2) \tag{3.6}$$

so that

$$c_a \phi^{(a,0)}_{a_3}(x) = \int d\mu(\theta) u_{aa_3}(\bar\theta)\Phi(\theta,x) \ . \tag{3.7}$$

We can now define the Lorentz-scalar superspin field by

$$\Phi(x) = \int d\mu(\theta)\Phi(\theta,x) \ , \tag{3.8}$$

where $d\mu(\theta) = (d^2z\, d^2w/\pi)\exp(-\frac{1}{2}\bar\theta\cdot\theta)$. We must treat $\Phi(x)$ as an operator, since the definition (3.8) is only meaningful within a matrix element in which the measure integration acts on a spin-space-dependent property of a state vector $|A\rangle$. We can also construct a superspin field from the field operator $\xi^{(j/2,j/2)}(x)$ for the representations with $(a,b) = (j/2, j/2)$:

$$\Xi(\theta,\eta;x) = \sum_{j=0}^{\infty} \sum_{a_3,b_3} b_j u_{ja_3}(\theta) u_{jb_3}(\bar\eta) \xi^{(j/2,j/2)}_{a_3 b_3}(x) \tag{3.9}$$

and

$$\Xi(x) = \int d\mu(\theta) d\mu(\eta) \Xi(\theta,\eta;x) \ . \tag{3.10}$$

Let us now calculate the commutation relations for the superspin field $\Phi(x)$. We have

$$[\Phi(x),\Phi^\dagger(y)]_- = \sum_{j=0}^{\infty} \frac{|c_j|^2}{(2j)!} P_s^j(\partial)\Delta(x-y) \ , \tag{3.11}$$

where $P_s^j(\partial)$ is the spin-projection operator. The T product of two superspin fields leads to the following expression for the causal superspin propagator D_s^c:

$$\begin{aligned} D_s^c(x-y) &= \langle 0|T[\Phi(x)\Phi^\dagger(y)]|0\rangle \\ &= \sum_{j=0}^{\infty} \frac{|c_j|^2}{(2j)!} P_s^j(\partial)\Delta^c(x-y) \ , \end{aligned} \tag{3.12}$$

where we have neglected contact terms. In momentum space, this becomes

$$D_s^c(p) = \int d^4x\, e^{-ip\cdot x} D_s^c(x) = \frac{1}{i} \frac{\Pi(p)}{-p^2+m^2-i\epsilon} \ , \tag{3.13}$$

where

$$\Pi(p) = \sum_{j=0}^{\infty} |d_j|^2 K_s^j(p) \tag{3.14}$$

and the constant j dependence has been put into a constant denoted by d_j. The function $K_s^j(p)$ contains within it an integration over the spin basis space spinors.

We shall also have need for a spin-$\frac{1}{2}$ superspin field constructed from the $\psi_{j\sigma}(x)$ in (2.27):

$$\Psi(\theta,x)=\sum_{j=0}^{\infty} c_j \sum_{\sigma=-j}^{+j} u_{j\sigma}(\theta)\psi_{j\sigma}(x) \ . \tag{3.15}$$

The SL(2,C)-invariant spin-$\frac{1}{2}$ superspin field is then given by

$$\Psi(x)=\int d\mu(\theta)\Psi(\theta,x) \ . \tag{3.16}$$

The spinor superspin field propagator in momentum space is

$$S_s^c(p)=\frac{1}{i}\frac{\Pi(p)}{m-p\cdot\gamma-i\epsilon} \ , \tag{3.17}$$

where $\Pi(p)$ is defined by (3.14).

By using the explicit Weinberg formulas for $K^j(p)$, in Eqs. (2.23) and (2.24), we find that a sufficient condition for $\Pi(p)$ given by (3.14) to be an entire function of p is that, for large j, $|d_j|$ falls off as $j^{-j/a}$ with $a>0$. We shall take it as given that this somewhat weak condition is satisfied.

Let us now construct massless spin-1 and spin-2 superspin fields. Since the photon and graviton are both massless and neutral, we shall restrict ourselves to the case of massless particles that are identical to their antiparticles. It is convenient to define phases so that

$$b(\mathbf{p},\lambda)=(-)^j a(\mathbf{p},\lambda) \ . \tag{3.18}$$

We observe that $(e^{\mu}_{\pm})^*=e^{\mu}_{\mp}$, so we have

$$F_{\pm}^{[\mu_1\nu_1]\cdots[\mu_j\nu_j]\dagger}=F_{\mp}^{[\mu_1\nu_1]\cdots[\mu_j\nu_j]} \ . \tag{3.19}$$

Therefore, we can define fields $H(x)$ as

$$H(x)=H_+(x)+H_-(x) \ . \tag{3.20}$$

Let us begin by defining the field

$$h_{\mu}^{[\mu_1\nu_1]\cdots[\mu_j\nu_j]}(x)=i^j\int d^3\mathbf{p}(2\omega)^{-1/2}\sum_{\pm}[p^{\mu_1}e_{\pm}^{\nu_1}(\mathbf{p})-p^{\nu_1}e_{\pm}^{\mu_1}(\mathbf{p})] \times\cdots\times[p^{\mu_j}e_{\pm}^{\nu_j}(\mathbf{p})-p^{\nu_j}e_{\pm}^{\mu_j}(\mathbf{p})][a_{\mu}(\mathbf{p},\pm j)e^{-ip\cdot x}+a_{\mu}^*(\mathbf{p},\mp j)e^{ip\cdot x}] \ , \tag{3.21}$$

where we shall only consider massless integer-spin particles. Then, the spin-1 superspin field is defined by

$$\mathcal{A}_{\mu}(\theta,x)=\sum_{j=0}^{\infty} c_j \left[\prod_{i,k=1}^{j}(\bar{\theta},\sigma_{\mu_i\nu_k}\theta)\right] \times h_{\mu}^{[\mu_1\nu_1]\cdots[\mu_j\nu_j]}(x) \ , \tag{3.22}$$

where $\sigma_{\mu_i\nu_k}=(i/2)(\sigma_{\mu_i}\sigma_{\nu_k}-\sigma_{\nu_k}\sigma_{\mu_i})$. The Lorentz-covariant spin-1 field is obtained by using the measure integration

$$\mathcal{A}_{\mu}(x)=\int d\mu(\theta)\mathcal{A}_{\mu}(\theta,x) \ . \tag{3.23}$$

The superspin field strength for the massless "photon" field is

$$\mathcal{F}_{\mu\nu}=\partial_{\mu}\mathcal{A}_{\nu}-\partial_{\nu}\mathcal{A}_{\mu} \ . \tag{3.24}$$

Continuing this method of construction, consider now the tensor

$$R_{\mu\nu}^{[\mu_1\nu_1]\cdots[\mu_j\nu_j]}(x)=i^j\int d^3\mathbf{p}(2\omega)^{-1/2}\sum_{\pm}[p^{\mu_1}e_{\pm}^{\nu_1}(\mathbf{p})-p^{\nu_1}e_{\pm}^{\mu_1}(\mathbf{p})]\times\cdots\times[p^{\mu_j}e_{\pm}^{\nu_j}(\mathbf{p})-p^{\nu_j}e_{\pm}^{\mu_j}(\mathbf{p})] \times[a_{\mu\nu}(\mathbf{p},\pm j)e^{-ip\cdot x}+a_{\mu\nu}^*(\mathbf{p},\mp j)e^{ip\cdot x}] \ . \tag{3.25}$$

We now obtain the spin-2 superspin field

$$s_{\mu\nu}(\theta,x)=\sum_{j=0}^{\infty} c_j \left[\prod_{i,k=1}^{j}(\bar{\theta},\sigma_{\mu_i\nu_k}\theta)\right] R_{\mu\nu}^{[\mu_1\nu_1]\cdots[\mu_j\nu_j]}(x) \tag{3.26}$$

and the Lorentz-covariant spin-2 field

$$s_{\mu\nu}(x)=\int d\mu(\theta)s_{\mu\nu}(\theta,x) \ . \tag{3.27}$$

The spin-2 field $s_{\mu\nu}(x)$ satisfies the wave equation

$$\Box s_{\mu\nu}(x)=0 \ . \tag{3.28}$$

The free fields $\mathcal{A}_{\mu}(x)$ and $s_{\mu\nu}(x)$ contain no unphysical ghost states. The creation and annihilation operators for the superspin spin-1 fields satisfy the commutation relations

$$[a_{\mu}(\mathbf{p},j,\theta),a_{\nu}^*(\mathbf{p}',j',\eta)]_-=\delta^3(\mathbf{p}-\mathbf{p}')\delta(\theta-\eta)\delta_{jj'}\eta_{\mu\nu} \ , \tag{3.29}$$

while those for the superspin spin-2 fields obey

$$[a_{\mu\nu}(\mathbf{p},j,\theta),a_{\lambda\rho}^*(\mathbf{p}',j',\eta)]_- =\delta^3(\mathbf{p}-\mathbf{p}')\delta(\theta-\eta)\delta_{jj'}(\eta_{\mu\lambda}\eta_{\nu\rho}+\eta_{\mu\rho}\eta_{\nu\lambda}-\eta_{\mu\nu}\eta_{\lambda\rho}) \ . \tag{3.30}$$

The propagator for the superspin photon is given by

$$D^c_{s\mu\nu}(x-y)=\langle 0|T[\mathcal{A}_\mu(x)\mathcal{A}_\nu(y)]|0\rangle$$
$$=\eta_{\mu\nu}\sum_{j=0}^{\infty}|d_j|^2\int\frac{d^3\mathbf{p}}{2\omega}K^j_s(p)\times[\theta(x-y)e^{-ip\cdot x}+\theta(y-x)e^{ip\cdot x}] . \quad (3.31)$$

In momentum space, the propagator becomes

$$D^c_{s\mu\nu}(p)=-\frac{1}{i}\eta_{\mu\nu}\frac{\Pi(p)}{-p^2-i\epsilon} . \quad (3.32)$$

For the superspin "graviton" the propagator is

$$D^c_{s\mu\nu\lambda\rho}(x-y)=\langle 0|T[s_{\mu\nu}(x)s_{\lambda\rho}(y)]|0\rangle=(\eta_{\mu\lambda}\eta_{\nu\rho}+\eta_{\mu\rho}\eta_{\nu\lambda}-\eta_{\mu\nu}\eta_{\lambda\rho})\times\sum_{j=0}^{\infty}|d_j|^2\int\frac{d^3\mathbf{p}}{2\omega}K^j_s(p)[\theta(x-y)e^{-ip\cdot x}+\theta(y-x)e^{ip\cdot x}] \quad (3.33)$$

and in momentum space this becomes

$$D^c_{s\mu\nu\lambda\rho}(p)=\frac{1}{i}(\eta_{\mu\lambda}\eta_{\nu\rho}+\eta_{\mu\rho}\eta_{\nu\lambda}-\eta_{\mu\nu}\eta_{\lambda\rho})\frac{\Pi(p)}{-p^2-i\epsilon} . \quad (3.34)$$

IV. INTERACTION DENSITIES AND PERTURBATION THEORY

We shall use perturbation theory and assume that the S matrix can be calculated from Dyson's formula:[31,32]

$$S=\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{\infty}d^4x_1\cdots d^4x_n T[\mathcal{L}_I(x_1)\cdots\mathcal{L}_I(x_n)], \quad (4.1)$$

where in the interaction representation, the Lagrangian is described by

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_I \quad (4.2)$$

and

$$\mathcal{L}_I(x)=\exp(i\mathcal{L}_0 t)\mathcal{L}_I\exp(-i\mathcal{L}_0 t) . \quad (4.3)$$

We shall construct $\mathcal{L}$ by using our superspin fields. Let us consider first the massive superspin scalar field $phI(x)$ given by (3.1) and (3.8). Later, we shall use massless superspin gauge fields to construct the interaction Lagrangians. The free-field-Lagrangian is given by

$$\mathcal{L}_0=-\tfrac{1}{2}:\phi(x)\Box\phi(x): , \quad (4.4)$$

where $\phi(x)$ is the point-particle free field. An invariant coupling is

$$\mathcal{L}_I(x)=g:\Phi(x)^n:$$
$$=\int d\mu(\theta_1)d\mu(\theta_2)\cdots d\mu(\theta_n)\times:\Phi(\theta_1,x)\Phi(\theta_2,x)\cdots\Phi(\theta_n,x):\times I(\theta_1\times\theta_2,\theta_1\times\theta_3,\ldots) , \quad (4.5)$$

where I is the coupling function of all $(n/2)$ determinants $(\theta_1\times\theta_2),(\theta_1\times\theta_3),\ldots$. We can construct Feynman rules and define a Wick normal ordering in terms of N products. We can replace $I(\ )$ in (4.5) by the identity times g, which we are free to do because any Lorentz-invariant $I(\ )$ is acceptable. Then we only perform the θ integrations in Wick contractions. The only modification of perturbation theory is the two-point propagator.

Let us assume that the superspin field $\Phi(x)$ is given by[16]

$$\Phi(x)=\int dy\, B(x-y)\phi(y)=B(\partial_x)\phi(x) , \quad (4.6)$$

where $B(\partial)$ is some operator that depends on ∂, and $\phi(x)$ is the standard free field solution of the Klein-Gordon equation

$$(\Box+m^2)\phi(x)=0 , \quad (4.7)$$

which is given by

$$\phi(x)=(2\pi)^{-3/2}\int\frac{d^3\mathbf{p}}{(2\omega)^{1/2}}(a_{\mathbf{p}}e^{-ip\cdot x}+a^*_{\mathbf{p}}e^{ip\cdot x}) . \quad (4.8)$$

This solution obeys the commutation relations

$$[\phi(x),a^\dagger_{\mathbf{p}}]_-=(2\pi)^{-3/2}\frac{e^{-ip\cdot x}}{(2\omega)^{1/2}} , \quad (4.9a)$$

$$[a_{\mathbf{p}},\phi(x)]_-=(2\pi)^{-3/2}\frac{e^{ip\cdot x}}{(2\omega)^{1/2}} . \quad (4.9b)$$

For the superspin Dirac field $\Psi(x)$, we assume that

$$\Psi(x)=\int dy\, B(x-y)\psi(y)=B(\partial_x)\psi(x) , \quad (4.10)$$

where $\psi(x)$ is the free-particle solution of the Dirac equation

$$(-i\gamma^\mu\partial_\mu+m)\psi(x)=0 , \quad (4.11)$$

which is given by

$$\psi(x)=(2\pi)^{-3/2}\int d^3\mathbf{p}\left[\frac{m}{E(\mathbf{p})}\right]^{1/2}\times[b_{\mathbf{p}}w(\mathbf{p})e^{-ip\cdot x}+d^*_{\mathbf{p}}v(\mathbf{p})e^{ip\cdot x}] . \quad (4.12)$$

We shall define the N products according to the Wick theorem by using the "chronological" contraction:

$$D_s^c(x-y)=\Phi^{\cdot}(x)\Phi^{\cdot}(y)=B(\partial_x)B(\partial_y)\phi^{\cdot}(x)\phi^{\cdot}(y)\ , \quad (4.13)$$

or

$$D_s^c(x-y)=B(\partial_x)B(\partial_y)\Delta^c(x-y)=\frac{1}{(2\pi)^4 i}\int\frac{d^4p\,[B(p^2)]^2e^{ip\cdot(x-y)}}{-p^2+m^2-i\epsilon}\ . \quad (4.14)$$

Similarly, for the superspin Dirac field $\Psi(x)$, we have the chronological contraction

$$S_s^c(x-y)=\Psi^{\cdot}(x)\Psi^{\cdot}(y)=B(\partial_x)B(\partial_y)\psi^{\cdot}(x)\psi^{\cdot}(y)\ , \quad (4.15)$$

or

$$S_s^c(x-y)=B(\partial_x)B(\partial_y)S^c(x-y)=\frac{1}{(2\pi)^4 i}\int\frac{d^4p\,[B(p^2)]^2e^{ip\cdot(x-y)}}{m-p\cdot\gamma-i\epsilon}\ . \quad (4.16)$$

This is the form of our *postulated* "Wick T product" or T^* operation. In nonlocal field theory, the T^* operation cannot be defined in the same meaningful sense as is done in strictly local field theories. But this does not prevent us from postulating the rules (4.13) and (4.15) as a method of constructing our perturbation theory. As we shall see, by employing well-defined regularization techniques, all the physical requirements of a consistent field theory can be satisfied. We now use the usual methods of quantum field theory, leading to the standard perturbation series, except that our causal function will be of the form (4.14) and (4.16), and the operator $B(\partial)$ is such that $[B(p^2)]^2=\Pi(p)$, where $\Pi(p)$ is given in Eq. (3.14).

Let us cite the Feynman rules for our superspin scalar field $\Phi(x)$ given by (3.1) and (3.8) and for a $g:\Phi(x)^4:$ coupling.

(a) For each vertex include a factor $(-i)$ times

$$I(\theta_1\times\theta_2,\theta_1\times\theta_3,\ldots)\ . \quad (4.17)$$

(b) For each internal line running from a vertex x to a vertex y include a superspin propagator

$$D_s^c(x-y)=\frac{1}{(2\pi)^4 i}\int\frac{d^4p\,\Pi(p)e^{ip\cdot(x-y)}}{-p^2+m^2-i\epsilon}\ . \quad (4.18)$$

(c) For every external line corresponding to a superspin particle, include a wave function

$$\sum_{j=0}^{\infty}c_j\sum_{\sigma=-j}^{+j}u_{j\sigma}(\theta)\phi_{j\sigma}^{(-)}(x) \quad \text{(superspin particle destroyed)}\ , \quad (4.19)$$

$$\sum_{j=0}^{\infty}c_j\sum_{\sigma=-j}^{+j}u_{j\sigma}^{*}(\theta)\phi_{j\sigma}^{(+)}(x) \quad \text{(superspin particle created)}\ .$$

(d) Integrate over all the measure integration variables $\theta_1,\theta_2,\ldots$, and over all vertex positions x,y, etc.

These Feynman rules could easily be stated in momentum space. We have assumed that the scalar superspin particle $\Phi(x)$ is equal to its own antiparticle. If this is not true, then the rules are readily extended to include the antiparticle field operator.

The infinite-spin degree of freedom in the function $\Pi(p)$ will act as a regulator of the form factors in the perturbation series. As we shall show in the following sections, the integrals will be convergent and all the physical conditions required for the S matrix will be satisfied.

The infinite towers of particles that make up the superspin fields cannot be excited into observable particle states in interactions. The superspin field is an infinite, linear combination of spin-space scalars, so that superspin particles remain spin-space scalars in collision processes. Therefore, in superspin field theory the interactions do not produce unphysical ghost poles as in the standard couplings of higher-spin fields. The superspin field contains a *confined,* hidden degree of infinite spin that defines a nonlocal field operator.

V. GENERALIZED FUNCTIONS AND CLASSES OF ENTIRE FUNCTIONS

In the framework of axiomatic field theory,[5-11] the following basic requirements are made of relativistic field theory: (a) a Hilbert space of states, (b) the fields are covariant under the Poincaré group of transformations, (c) the fields satisfy local commutativity, (d) positive energy, and (e) particle interpretation.

A field $\varphi(x)$ is an operator-valued generalized function, averaged over a smooth test function $f(x)$:

$$\varphi(f)=\int dx\,\varphi(x)f(x)\ . \quad (5.1)$$

The standard physical requirements of relativistic field theory are obtained if we choose tempered test functions. The temperedness of functions reflects the symmetry between coordinate and momentum spaces. Moreover, temperedness leads to the scattering amplitude being analytic in s (for fixed $t<0$) in a cut plane and it possesses a polynomial behavior. These requirements comprise what we understand to be a *strictly* local field theory.

We can use nontempered test functions that are still consistent with the requirements (a)–(e). Such functions have been studied by Jaffe.[11] For these functions, the off-mass-shell scattering amplitudes can be allowed to grow, for large energies, faster than any polynomial. An example of such fields is those controlled by entire functions. The concept of a strictly localizable field is based on the existence of enough test functions with compact support in configuration space. The existence of test functions with compact support in configuration space requires test functions in momentum space, which decrease at infinity such as $\exp(-\|p\|^a)$ with $a<1$, where $\|p\|$ is the Euclidean norm.

In spite of the appeal of strictly localizable fields, the choice of test functions cannot be dictated by physical experiment. It is motivated by a consistent mathematical framework that leads, in the simplest way, to the results of local microscopic commutativity.

The idea of local commutativity can be widened to include values $a\leq 1$. It can be proved that the Wightman functions can grow arbitrarily fast near the light cone, even for fields that are not strictly localizable, and still satisfy a condition of microcausality.[16-21] The other physical requirements imposed by conditions (a)–(e) can still be satisfied for an extended definition of locality.

Let us study the properties of the operators $B(\partial)$. Let us represent the operators $B(\partial)$ as infinite series in powers of $\Box=\partial^\mu\partial_\mu$ (Ref. 16):

$$B(\Box)=\sum_{j=0}^{\infty}\left[\frac{|c_j|^2}{(2j)!}\right]^{1/2}\Box^j . \tag{5.2}$$

The Fourier transform of this operator is

$$B(t)=\sum_{j=0}^{\infty}\left[\frac{|c_j|^2}{(2j)!}\right]^{1/2}t^j , \tag{5.3}$$

were $t=-p^2$. The first requirement of $B(t)$ is that it is an *entire* function of the complex variable t. An entire function of t is such that no singularities of t occur in the finite complex t plane. This avoids the possibility that nonphysical singularities of $B(t)$ will appear in physical scattering amplitudes, thereby violating the unitarity of the S matrix.

Given that $B(t)$ is an entire function, we can distinguish three cases.

Case (I):

$$\limsup_{j\to\infty}|c_j|^{1/j}=0 . \tag{5.4a}$$

Case (II):

$$\limsup_{j\to\infty}|c_j|^{1/j}=\text{const}<\infty . \tag{5.4b}$$

Case (III):

$$\limsup_{j\to\infty}|c_j|^{1/j}=\infty . \tag{5.4c}$$

In case (I), the functions $B(t)$ are entire functions of order $\gamma<\frac{1}{2}$. This means that

$$|B(t)|<\exp(\alpha|t|^\gamma),\quad \gamma<\tfrac{1}{2} , \tag{5.5}$$

where α is a positive number. It can be proved[33] that for these functions there exist no directions in the complex t plane along which they decrease. Thus, these functions cannot lead to a finite perturbation theory. Moreover, these functions can be shown to lead to a strictly local field theory, emphasizing again that such a field theory cannot form the basis of an ultraviolet finite perturbation theory.

The functions $B(t)$ for case (II) are entire analytic functions of order $\gamma=\frac{1}{2}$ for which

$$|B(t)|\le\exp(\alpha|t|^{1/2}) . \tag{5.6}$$

These functions *can decrease along one direction* in the complex plane and can lead to a finite perturbation theory.

In case (III), the functions $B(t)$ are entire of order $\gamma>\frac{1}{2}$ and satisfy

$$|B(t)|\le\exp[g(|t|)] , \tag{5.7}$$

where $g(|t|)$ is a positive function satisfying the condition $g(|t|)>\alpha|t|^{1/2}$ as $|t|\to\infty$ for any $\alpha>0$. These functions can possess several directions along which they decrease as $|t|\to\infty$. They can lead to ultraviolet-finite field theories. But the fields corresponding to these entire functions do not necessarily possess any form of local commutativity. The behavior of the field operator at the point $x=0$ is determined by the behavior of the field operator over the whole of x space.

Efimov[16] has studied distributions of the type

$$B(x-y)=B(\Box_x)\delta^4(x-y) , \tag{5.8}$$

where the operator $B(\Box)$ has the integral representation

$$B(\Box)=\int_{\rho^2<\lambda^2}d^4\rho\,\kappa(\rho^2)\exp\left[i\rho_0\frac{\partial}{\partial x_0}+\boldsymbol{\rho}\cdot\frac{\partial}{\partial\mathbf{x}}\right]$$
$$=(2\pi)^2\int_0^\lambda d\beta\,\beta^2\kappa(\beta^2)\frac{J_1(\beta\Box^{1/2})}{\Box^{1/2}} \tag{5.9a}$$

or

$$B(\Box)=\int_{\rho^2<\lambda^2}d^4\rho\,\kappa(\rho^2)\exp\left[\rho_0\frac{\partial}{\partial x_0}+i\boldsymbol{\rho}\cdot\frac{\partial}{\partial\mathbf{x}}\right]$$
$$=(2\pi)^2\int_0^\lambda d\beta\,\beta^2\kappa(\beta^2)\frac{J_1(\beta(-\Box)^{1/2})}{(-\Box)^{1/2}} . \tag{5.9b}$$

Here $\kappa(\rho^2)$ is an integrable function of the Euclidean four-vector ρ with $\rho^2=\rho_0^2+\rho_1^2+\rho_2^2+\rho_3^2$ and $J_1(z)$ is a Bessel function. The parameter λ has the meaning of a fundamental length. The operators $B(\Box)$ are of type A or type B, depending upon whether they are of the form (5.9a) or (5.9b), respectively. In the momentum representation,

$$B(-p^2)=(2\pi)^2\int_0^\lambda d\beta\,\beta^2\kappa(\beta^2)\frac{J_1[\beta(-p^2)^{1/2}]}{(-p^2)^{1/2}}\quad\text{(type A)} , \tag{5.10a}$$

$$B(-p^2)=(2\pi)^2\int_0^\lambda d\beta\,\beta^2\kappa(\beta^2)\frac{J_1[\beta(p^2)^{1/2}]}{(p^2)^{1/2}}\quad\text{(type B)} . \tag{5.10b}$$

For operators $B(\Box)$ of type A, the functions $B(-p^2)$ decrease as $p^2\to-\infty$ and increase as $p^2\to+\infty$. For type-B operators, the functions $B(-p^2)$ decrease as $p^2\to+\infty$ and increase as $p^2\to-\infty$.

The class of test functions is denoted by $\mathcal{F}$ and the distributions by $\mathcal{F}^*$. The test functions $f(x)$ are real and decrease at infinity. For the class of test functions, we have

$$(B,f)=\int dy\,B(x-y)f(y)=B(\Box_x)f(x) . \tag{5.11}$$

This can be expressed in the form

$$(B,f)=\int_{\rho^2<\lambda^2}d^4\rho\,\kappa(\rho^2)\exp\left[i\rho_0\frac{\partial}{\partial x_0}+\boldsymbol{\rho}\cdot\frac{\partial}{\partial\mathbf{x}}\right]f(x)$$
$$=\int_{\rho^2<\lambda^2}d^4\rho\,\kappa(\rho^2)f(x_0+i\rho_0,\mathbf{x}+\boldsymbol{\rho})\quad\text{(type A)} \tag{5.12a}$$

and

$$(B,f)=\int_{\rho^2<\lambda^2} d^4\rho\,\kappa(\rho^2)f(x_0+\rho_0,\mathbf{x}+i\boldsymbol{\rho}) \quad \text{(type B)} . \tag{5.12b}$$

The class of test functions consists of entire analytic functions $f(z)$ which decrease along any direction in the z plane outside the region $|\mathrm{Re} z|<r$, where r is a given number. We choose the test functions to be described by a sequene $f_n(x,y)$ so that each $f_n(x,y)$ belongs to $\mathcal{F}$ and

$$f(x,y)=\lim_{n\to\infty} f_n(x,y) \tag{5.13}$$

does not belong to $\mathcal{F}$ and vanishes at all points $x\neq y$. Moreover, the sequence is normalized according to

$$\int d^4x\, f_n(x,y)=1 . \tag{5.14}$$

Efimov[16] has shown that there exist sequences $f_n(x-y)$ which satisfy (5.13) such that under the action of the operator $B(\Box)$ each sequence will transform into a new sequence. The new sequence reduces to a function which is zero outside a bounded region connected by a Lorentz transformation with the original sequence. All the bounded regions derived from the sequences $f_n(x-y)$ lie inside a hyperboloid defined by

$$-\lambda^2\leq(x-y)^2\leq\lambda^2 . \tag{5.15}$$

A field $\Phi(x)$ that appears and then disappears at a time y_0 at the spatial point $\mathbf{y}$ will only affect, through the action of the distributions $B(x-y)$, the regions with vanishing four-volume contained within the hyperboloid (5.15). These bounded regions define the extended superspin particles that could be strings, membranes or some topologically complicated objects.

Distributions are constructed by using an improper transition to the limit. Efimov[16] introduces a regularizing function $R^{\delta}(t)$ and approximates the distributions by the following regular functions:

$$B^{\delta}(x-y)=\frac{1}{(2\pi)^4}\int d^4p\, e^{ip\cdot x}B(-p^2)R^{\delta}(p^2) , \tag{5.16}$$

where

$$R^{\delta}(t)=\exp[-\delta(t+iM^2)^{1/2+\nu}e^{-i\pi\sigma}] , \tag{5.17}$$

where $0<\nu<\sigma<\frac{1}{2}$ and M^2 is a positive parameter. At large t, the regularizing function behaves as

$$|R^{\delta}(t)|\sim\exp\{-\delta|t|^{1/2+\nu}\cos[\pi\sigma-(\nu+\tfrac{1}{2})\arg t]\} . \tag{5.18}$$

Thus, R^{δ} is an analytic function that falls off faster than the linear exponential in the upper-half plane of the complex variable t. The integral (5.16) is convergent for $\delta>0$ and defines a well-behaved function $B^{\delta}(x-y)$. Products of distributions

$$G^{\delta}(x-y)=-iB^{(1)\delta}(x-y)B^{(2)\delta}(x-y) \tag{5.19}$$

can also be defined. In the limit $\delta\to 0$, there exists a function G such that the functional (G^{δ},f) is well defined for test functions f belonging to the class $\mathcal{F}$. The use of regularizing functions R^{δ} guarantees that we can perform a rotation over q_0 by an angle $\pi/2$ in the integral

$$(G^{\delta},f)=-i\int d^4p\, e^{ip\cdot x}\tilde{f}(p)\int d^4q\, B^{(1)\delta}(-q^2)\times B^{(2)\delta}(-(p-q)^2) , \tag{5.20}$$

where

$$B^{\delta}(-q^2)=B(-q^2)R^{\delta}(q^2) . \tag{5.21}$$

VI. FINITE PERTURBATION SERIES FOR THE S MATRIX

We shall follow the methods of Efimov[16] to analyze the properties of the S-matrix perturbation series, since the superspin propagators have the same characteristics as the propagators used by him. However, in our case, the *physical* mechanism that regularizes the propagators is the *infinite-spin degree of freedom* carried by the superspin scalar fields $\Phi(x)$. We shall establish the finiteness of the perturbation series using the scalar polynomial interaction Lagrangian (4.5), since the results obtained will apply in a similar way to the more complicated Lagrangians to be considered later.

The matrix element in x space of a process in the nth approximation of perturbation theory can be written as a sum of Feynman graphs:

$$M(x_1,\ldots,x_n)=\prod_{i,j} D_s^c(x_i-y_j) , \tag{6.1}$$

where i and j are integers from 1 to n. The amplitude $M(x_1,\ldots,x_n)$ is a distribution that belongs to class $\mathcal{F}^*$ and is integrable in the class $\mathcal{F}$ of test functions. We can write

$$D_s^c(x-y)=D^c(x-y)+N(x) , \tag{6.2}$$

where $D^c(x-y)$ is the standard Feynman propagator for free fields $\phi(x)$, and

$$N(x)=\frac{1}{(2\pi)^4 i}\int \frac{d^4p\, e^{ip\cdot x}}{-p^2+m^2-i\epsilon}[\Pi(p^2)-1] = \frac{1}{(2\pi)^4 i}\int d^4p\, N(-p^2)e^{ip\cdot x}=N(\Box_x)\delta^4(x) . \tag{6.3}$$

The operator $N(\Box)$ belongs to the same class of entire functions as $B(\Box)$ and $\Pi(\Box)$.

Consider now the regularized causal superspin propagator

$$\mathrm{Reg}D_s^c(x-y)=\frac{1}{(2\pi)^4 i}\int \frac{d^4p\, e^{ip\cdot(x-y)}}{-p^2+m^2-i\epsilon}\Pi(-p^2)R^{\delta}(p^2) . \tag{6.4}$$

Then, we have

$$\lim_{\delta\to 0}\int d^4x\, \mathrm{Reg}D_s^c(x)f(x)=\int d^4x\, D_s^c(x)f(x) , \tag{6.5}$$

where $f(x)$ belongs to $\mathcal{F}$. We write the Fourier transform of the regularized amplitude $M^{\delta}(x_1,\ldots,x_n)$ as

$$\tilde{M}^{\delta}(p_1,\ldots,p_n)=\int d^4x_1\cdots\int d^4x_n e^{i(p_1\cdot x_1+\cdots+p_n\cdot x_n)}\times M^{\delta}(x_1,\ldots,x_n)\ . \tag{6.6}$$

We obtain the convergent integral

$$\tilde{M}^{\delta}(p_1,\ldots,p_n)=(2\pi)^4\delta(p_1+\cdots+p_n)T^{\delta}(p_1,\ldots,p_n)\ , \tag{6.7}$$

where

$$T^{\delta}(p_1,\ldots,p_n)=\int\cdots\int\prod_i d^4l_i\prod_r\frac{\Pi(-k_r^2)}{k_r^2-m_r^2+i\epsilon}\times R^{\delta}(k_r^2)\ . \tag{6.8}$$

The l_i denote the four-momentum over which the integration is performed and the k_r is the four-momentum corresponding to a given line.

Since we have used the regularizing function R^{δ}, we can rotate the integral over $(l_i)_0$ by an angle $\pi/2$ for type-A functions $B(-k^2)$ and rotate the integral over the space components $(l_i)_1,(l_i)_2,(l_i)_3$ by an angle $-\pi/2$ for type-B functions. For type A, we have $B(-k^2)\to 0$ as $k^2\to-\infty$, while for type B, we have $B(-k^2)\to 0$ as $k^2\to+\infty$. For the regularized amplitudes, the essential singularity in the entire function in the two-point propagator, which occurs in one direction at infinity, does not cause problems for the Wick rotation analytic continuation to the Euclidean momentum plane.

After going to the Euclidean momenta for type-A functions and taking the limit $\delta\to 0$, we obtain an integral over the Euclidean four-momenta (l_i). We retain the Minkowski character of the external momenta. Since $\Pi(-k^2)=[B(-k^2)]^2=O(1/k^2)$ as $k^2\to-\infty$, the integrals corresponding to any Feynman diagram will converge. The same will hold true for type-B functions. The amplitude $T(p_1,\ldots,p_n)$ depends only on convergent integrals in the limit $\delta\to 0$ and on the scalar products of the Minkowski external momenta $p_1,\ldots,p_n$. Since the amplitude $T(p_1,\ldots,p_n)$ is an analytic function of the invariant variables, it can be considered as a function of the n Euclidean momenta $q_1,\ldots,q_n$. Using the theorem of the uniqueness of analytic continuation, we can obtain the physical amplitude $T(p_1,\ldots,p_n)$ by analytic continuation over the whole region for both spacelike and timelike four-momenta.

In the limit when $B(-p^2)\sim 1$, we retrieve standard point-particle field theory and the perturbation series for the S matrix will be ultraviolet divergent.

VII. UNITARITY OF THE S MATRIX

The unitarity of the S matrix is guaranteed if in each order of perturbation theory on the mass shell we have

$$\langle\alpha|SS^{\dagger}|\beta\rangle=\langle\alpha|\beta\rangle\ , \tag{7.1}$$

where $|\alpha\rangle$ and $|\beta\rangle$ are arbitrary physical states. For two operators O_1 and O_2, we assume that there exists a set of amplitudes $|n,\mathbf{k}\rangle$, which is complete so that

$$\langle\alpha|O_1O_2|\beta\rangle=\langle\alpha|O_1|0\rangle\langle 0|O_2|\beta\rangle+\sum_n\int d\mathbf{k}\langle\alpha|O_1|n,\mathbf{k}\rangle\langle n,\mathbf{k}|O_2|\beta\rangle\ . \tag{7.2}$$

Let us write the S matrix in the form

$$S=1+iA\ , \tag{7.3}$$

where we expand the amplitude A in a power series in the coupling constant g:

$$A=\sum_{n=1}^{\infty}g^nA_n\ . \tag{7.4}$$

We have that

$$(-i)\langle\alpha|(A-A^*)|\beta\rangle=\langle\alpha|AA^*|\beta\rangle\ . \tag{7.5}$$

In every order of the coupling constant

$$2\,\mathrm{Im}\langle\alpha|A_n|\beta\rangle=\sum_{m_1+m_2=n}\sum_N\int d^3\mathbf{k}_1\cdots\int d^3\mathbf{k}_N\times\langle\alpha|A_{m_1}|\mathbf{k}_1,\ldots,\mathbf{k}_N\rangle\times\langle\mathbf{k}_1,\ldots,\mathbf{k}_N|A^*_{m_2}|\beta\rangle, \tag{7.6}$$

where we have used $\langle\alpha|A_n|\beta\rangle=\langle\beta|A_n|\alpha\rangle$, which holds for the single-component scalar superspin field.

The amplitude $\langle\alpha|A_n|\beta\rangle$ is a sum of all possible Feynman graphs in nth order of perturbation theory, in which n_β lines finish and n_α lines begin. Equation (7.6) is precisely the structure of the T product of the S matrix:

$$S=T\exp\left[-i\int\mathcal{L}_I(x)d^4x\right]\ . \tag{7.7}$$

The Cutkosky theorem[34] guarantees this property of the perturbation-theory amplitudes. Since the only difference between the standard local quantum field theory and the superspin field formalism arises from the *entire* functions $\Pi(p^2)$ in the superspin propagators, we know that the amplitudes have the same singularities as in the standard local field theory. If Cutkosky's theorem holds true, then the S matrix will be unitary on the mass shell in every order of perturbation theory. This result has been proved by Efimov.[16]

Let us consider in more detail the structure of Cutkosky's theorem for the amplitudes, which decrease in the region of spacelike external momenta. We construct in the four-dimensional Euclidean momentum space the amplitude corresponding to some arbitrary Feynman diagram with n external lines. We choose the n external momenta q to be Euclidean, satisfying the conservation law

$$q_1+\cdots+q_n=0\ . \tag{7.8}$$

To every internal line is associated the superspin propagator

$$D_s^c(l^2)=\frac{\Pi(l^2)}{l^2+m^2}\ , \tag{7.9}$$

where l is the Euclidean four-momentum for the internal line. The function $\Pi(z)$ is an entire function in the complex z plane and decreases rapidly with $\text{Re}z\rightarrow+\infty$. The given diagram is described by the integral

$$\tilde{T}=\int\cdots\int\prod_i d^4l_i\prod_r\frac{\Pi_r(k_r^2)}{k_r^2+m_r^2}\ . \tag{7.10}$$

The k_r is the Euclidean four-momentum corresponding to a given line in the diagram, while m_r is the mass of the corresponding particle. The integration is performed over the four-dimensional Euclidean momentum space.

The integral will be convergent, for the function $\Pi_r(k_r^2)$ decreases rapidly for $k_r^2\rightarrow\infty$. The Euclidean amplitude $\tilde{M}$ coincides with the real physical amplitude in the Euclidean region of the spacelike external momenta p_r on which the physical amplitude depends. The transition to the physical region of the external momenta is performed by analytically continuing the amplitude with respect to the invariant momentum variables. All the masses will have negative imaginary corrections $m_r^*=m_r-i\epsilon$. If $\Pi(k^2)$ were simply polynomial in k^2, the amplitude $\tilde{M}$ would coincide with the Minkowski physical amplitude. The amplitude obtained by Landau[35] is such that the Euclidean and Minkowski expressions for the amplitude are immediately equivalent.

After performing a Feynman parametrization, we have

$$\tilde{M}=(N-1)!\int d\alpha_1\cdots\int d\alpha_N\delta\left[1+\sum_{i=1}^N\alpha_i\right]\times\int\cdots\int\frac{\prod_i d^4l_i\prod_r\Pi_r(k_r^2)}{\left[\sum_r\alpha_r(k_r^2+m_r^2)\right]^N}\ , \tag{7.11}$$

where N is the number of internal lines. Following the arguments of Landau[35] and Efimov,[16] we find that the change of variables eliminates from the denominator all the terms linear in l_r, yielding

$$\sum_{r=1}^N\alpha_r(k_r^2+m_r^2)=W(\alpha,q_i,q_r;m^2)+Q(\alpha,l')\ . \tag{7.12}$$

Here, W is the nonhomogeneous quadratic form of the vectors q_i that describes the free ends of the diagram, and Q is a homogeneous quadratic form of the new variables of integration l' with coefficients that depend only on the parameters α_r. The numerator dependence of the external momenta cannot produce any new singularities in the finite region of the invariant momentum variables, since the numerator is an entire function of the scalar products q_iq_r and the parameters α_r.

For the sake of completeness, we shall state the Cutkosky rule for normal thresholds.[34] Let the diagram corresponding to the amplitude $\tilde{M}$ be separated into two vertices $\tilde{M}_{\rm I}$ and $\tilde{M}_{\rm II}$ that are connected by r internal lines:

$$\tilde{M}=\int\cdots\int d^4k_1\cdots d^4k_s\tilde{M}_{\rm I}(q_r,k_i)\times\prod_{\mu=1}^s\frac{\Pi_\mu(k_\mu^2)}{k_\mu^2+m_\mu^2}\tilde{M}_{\rm II}(q_r',k_i)\times\delta^4(q-k_1-\cdots-k_s)\ , \tag{7.13}$$

where q_r $(r=1,\ldots,n_1)$ and q_r' $(r=1,\ldots,n_2)$ denote the external momenta corresponding to the vertices I and II, respectively. Also, $q=q_1'+\cdots+q_{n_2}'=-(q_1+\cdots+q_{n_1})$, where $n=n_1+n_2$ is the number of external lines. The functions $\tilde{M}_{\rm I}(q_rk_r)$ and $\tilde{M}_{\rm II}(q_r'k_r)$, which describe the vertices I and II, depend on the scalar products of the vectors q_r, q_r', and k_i.

The amplitude $\tilde{M}$, considered as a function of the complex variable $z=-q^2$, has a branch cut beginning at the point $z=(m_1+\cdots+m_s)^2$ and the discontinuity of the function $\tilde{M}$ along this branch cut is given by

$$\Delta\tilde{M}(z)=i(2\pi)^s\prod_{\mu=1}^s\Pi_\mu(-m_\mu^2)\int d^4\tilde{k}_1\cdots\int d^4\tilde{k}_s\prod_{\mu=1}^s\theta(\tilde{k}_{\mu_0})\delta(m_\mu^2+\tilde{k}_\mu^2)\delta^4(\tilde{q}-\tilde{k}_1-\cdots-\tilde{k}_s)\tilde{M}_{\rm I}(q_r,\tilde{k}_i)\tilde{M}_{\rm II}(q_r',\tilde{k}_i)\ . \tag{7.14}$$

The $\tilde{k}_i$ are the four-dimensional vectors with the components $(\mathbf{k}_i,ik_{i0})$ with $\tilde{k}_i^2=\mathbf{k}_i^2-k_{i0}^2$, $(\tilde{k}_iq_r)=\mathbf{k}_i\mathbf{q}_r+ik_{i0}q_{r0}$, and $d^4\tilde{k}_i=d\mathbf{k}_idk_{i0}$. Moreover, the vector $\tilde{q}$ satisfies the condition $\tilde{q}^2=\mathbf{q}^2-q_0^2=-z$. The functions $\tilde{M}_{\rm I}(q_r\tilde{k}_i)$ and $\tilde{M}_{\rm II}(q_r'\tilde{k}_i)$ are the analytic continuations with respect to the corresponding values of the scalar arguments $(q_r\tilde{k}_i),(q_r'\tilde{k}_i)$ of the initial functions $\tilde{M}_{\rm I}(q_rk_i)$ and $\tilde{M}_{\rm II}(q_r'k_i)$.

Equation (7.14) is a statement of the Cutkosky rule for normal thresholds for an arbitrary Feynman diagram in Euclidean momentum space, except that we have used the superspin entire functions $\Pi(q)$. In fact, when we set $\Pi(q)=1$, the equation that results from (7.14) is just the Cutkosky rule for the standard, local quantum field theory in Euclidean momentum space, and the transition to the physical region is implemented by an analytic continuation with respect to the invariant momentum variables. Anomalous singularities of the diagrams will arise in the usual way when the analytic properties of the vertices I and II are taken into account.

The appearance of the entire superspin functions $\Pi(k^2)$ will violate the immediate equivalence of the Euclidean and Minkowski formulations of the theory, since we cannot perform a rotation of the momentum variables into the Minkowski space due to the occurrence of an essential singularity at infinity. Efimov, however, has proved that this does not change the analytic properties of the theory in an arbitrary region of finite-momentum variables. Because of the validity of the Cutkosky rule, he was able to prove the unitarity of the S matrix by using the theorem of the uniqueness of analytic continuation. Moreover, the regularization of the amplitudes prior to the Wick rotation, guarantees that the rotation can be performed at infinity for a suitable choice of regularization function $R^{\delta}(p)$.

It is useful to illustrate how the transition to the Euclidean momenta in the amplitudes of the physical processes is performed, and how the unitarity condition for arbitrary external momenta is satisfied. We do this by studying the amplitude in the second order of perturbation theory:

$$\Lambda(p^2)=\lim_{\delta\to 0} i\int d^4k \frac{\Pi(-k^2)R^{\delta}(k^2)}{-k^2+m^2-i\epsilon}\frac{\Pi(-(k-p)^2)R^{\delta}((k-p)^2)}{-(k-p)^2+m^2-i\epsilon}$$
$$=\lim_{\delta\to 0} i\int d^3\mathbf{k}\int_{-\infty}^{\infty} dk_0 \frac{\Pi(-k_0^2+\mathbf{k}^2)R^{\delta}(k_0^2-\mathbf{k}^2)\Pi(-(k_0^2-p_0)^2+(\mathbf{k}-\mathbf{p})^2)R^{\delta}((k_0-p_0)^2-(\mathbf{k}-\mathbf{p})^2)}{(k_0-a_+)(k_0-a_-)[k_0-b_+(p_0)][k_0-b_-(p_0)]} \quad (7.15a)$$

and

$$\Lambda^*(p^2)=\lim_{\delta\to 0}(-i)\int d^3\mathbf{k}\int_{-\infty}^{\infty} dk_0 \frac{\Pi(-k_0^2+\mathbf{k}^2)R^{\delta^*}(k_0^2-\mathbf{k}^2)\Pi(-(k_0-p_0)^2+(\mathbf{k}-\mathbf{p})^2)R^{\delta^*}((k_0-p_0)^2-(\mathbf{k}-\mathbf{p})^2)}{(k_0-a_+^*)(k_0-a_-^*)[k_0-b_+^*(p_0)][k_0-b_-^*(p_0)]}, \quad (7.15b)$$

where $a_\pm=\pm\omega_{\mathbf{k}}\mp i\epsilon$, $\omega_{\mathbf{k}}=(\mathbf{k}^2+m^2)^{1/2}$, $b_\pm(p_0)=p_0\pm\omega_{\mathbf{p}-\mathbf{k}}\mp i\epsilon$, and $\omega_{\mathbf{p}-\mathbf{k}}=[(\mathbf{p}-\mathbf{k})^2+m^2]^{1/2}$. The unitarity condition dictates that

$$\Delta\Lambda(p^2)\equiv\Lambda(p^2)-\Lambda^*(p^2)=i(2\pi)^2[\Pi(-m^2)]^2\pi\left[\frac{p^2-4m^2}{p^2}\right]^{1/2}. \quad (7.16)$$

Consider now the singularities of the integrand of $\Lambda(p^2)$ in the complex plane k_0+ik_4. In the denominator, the singularities occur at the points $a_\pm$ and $b_\pm(p_0)$. The singularities in $R^{\delta}(k^2)$ and $R^{\delta}((k-p)^2)$ are found from the equations $k^2=-iM^2$ and $(k-p)^2=-iM^2$, where M is a parameter with the dimensions of mass and we choose $M^2\gg m^2$. The singularities occur at the points

$$v_\pm=\pm\frac{M^2}{(2\{[(\mathbf{k}^2)^2+M^4]^{1/2}-\mathbf{k}^2\})^{1/2}}\mp i\left[\frac{[(\mathbf{k}^2)^2+M^4]^{1/2}-\mathbf{k}^2}{2}\right]^{1/2} \quad (7.17a)$$

and

$$w_\pm(p_0)=p_0\pm\frac{M^2}{(2\{[(\mathbf{p}-\mathbf{k})^2]^2+M^4-(\mathbf{p}-\mathbf{k})^2\})^{1/2}}\mp i\left[\frac{\{[(\mathbf{p}-\mathbf{k})^2]^2+M^4\}^{1/2}-(\mathbf{p}-\mathbf{k})^2}{2}\right]^{1/2}. \quad (7.17b)$$

The branch cuts of the functions $R^{\delta}(k^2)$ and $R^{\delta}((k-p)^2)$ begin at the above-determined points. The singularities of the integrand of $\Lambda(p^2)$ occur at the points $a_\pm$, $b_\pm$, $v_\pm$, and $w_\pm$ and at $a_\pm^*$, $b_\pm^*$, $v_\pm^*$, and $w_\pm^*$ in the integrand of $\Lambda^*(p^2)$. The initial contour of integration runs along the real axis. Because the integrands in (7.15a) and (7.15b) decrease for $\delta>0$ in the region $k_0k_4>0$ for $\Lambda(p^2)$ and in the region $k_0k_4<0$ for $\Lambda^*(p^2)$, the contours of integration over k_0 can be rotated by $\pi/2$ into k_4 for $\Lambda(p^2)$ and by $-\pi/2$ into k_4 for $\Lambda^*(p^2)$.

The sheets of the functions $\Lambda(p^2)$ and $\Lambda^*(p^2)$ are defined so that these functions are real for $p^2<0$. It is now straightforward to show that for $\delta\to 0$, the singularities associated with the functions R^{δ} disappear and the integral converges, since the functions $\Pi(k_4^2+\mathbf{k}^2)$ decrease very rapidly as $k_4\to\pm\infty$. It then follows that

$$\Delta\Lambda(p^2)=0, \quad (7.18)$$

a result that should hold for $p^2<0$.

By similar manipulations, we can calculate $\Delta\Lambda(p^2)$ for $p^2>0$. The integrals around the poles b_- and b_-^* give the results

$$\Lambda=\lim_{\delta\to 0} i\int d^3\mathbf{k}\, 2\pi i\frac{\Pi(-b_-^2+\mathbf{k}^2)R^{\delta}(b_-^2-\mathbf{k}^2)\Pi(-m^2)R^{\delta}(m^2)}{(b_--a_-)(b_--a_+)(b_--b_+)} \quad (7.19a)$$

and

$$\Lambda^* = \lim_{\delta\to 0}(-i)\int d^3\mathbf{k}(-2\pi i)\frac{\Pi(-b_-^{*2}+\mathbf{k}^2)R^{\delta *}(b_-^{*2}+\mathbf{k}^2)\Pi(-m^2)R^{\delta *}(m^2)}{(b_-^*-a_-^*)(b_-^*-a_+^*)(b_-^*-b_+^*)} . \tag{7.19b}$$

A calculation gives the result

$$\begin{aligned}\Delta\Lambda(p^2) &= -2\pi\lim_{\delta\to 0}\int d^3\mathbf{k}\left[\frac{\Pi(-(p_1-\omega_{\mathbf{p-k}})^2+\mathbf{k}^2)\Pi(-m^2)R^{\delta}(\cdots)R^{\delta}(\cdots)}{2\omega_{\mathbf{p-k}}(p_0-\omega_{\mathbf{p-k}}+\omega_{\mathbf{k}})(p_0-\omega_{\mathbf{p-k}}-\omega_{\mathbf{k}}+2i\epsilon)}\right.\\ &\qquad\left.-\frac{\Pi(-(p_0-\omega_{\mathbf{p-k}})^2+\mathbf{k}^2)\Pi(-m^2)R^{\delta}(\cdots)R^{\delta}(\cdots)}{2\omega_{\mathbf{p-k}}(p_0-\omega_{\mathbf{p-k}}+\omega_{\mathbf{k}})(p_0-\omega_{\mathbf{p-k}}-\omega_{\mathbf{k}}-2i\epsilon)}\right]\\ &= -2\pi\int d^3\mathbf{k}\frac{\Pi(-(p_0-\omega_{\mathbf{p-k}})^2+\mathbf{k}^2)\Pi(-m^2)}{2\omega_{\mathbf{p-k}}(p_0-\omega_{\mathbf{p-k}}+\omega_{\mathbf{k}})}\left[\frac{1}{p_0-\omega_{\mathbf{p-k}}-\omega_{\mathbf{k}}+2i\epsilon}-\frac{1}{p_0-\omega_{\mathbf{p-k}}-\omega_{\mathbf{k}}-2i\epsilon}\right]\\ &= (2\pi)^2 i\int d^3\mathbf{k}\frac{\Pi(-(p_0-\omega_{\mathbf{p-k}})^2+\mathbf{k}^2)\Pi(-m^2)}{2\omega_{\mathbf{p-k}}(p_0-\omega_{\mathbf{p-k}}-\omega_{\mathbf{k}})}\delta(p_0-\omega_{\mathbf{p-k}}-\omega_{\mathbf{k}})\\ &= i(2\pi)^2[\Pi(-m^2)]^2\int\frac{d^3\mathbf{k}\,\delta(p_0-\omega_{\mathbf{p-k}}-\omega_{\mathbf{k}})}{2\omega_{\mathbf{p-k}}2\omega_{\mathbf{k}}}\\ &= i(2\pi)^2[\Pi(-m^2)]^2\pi\left[\frac{p^2-4m^2}{p^2}\right]^{1/2}\theta(p_0)\theta(p^2-4m^2) .\end{aligned} \tag{7.20}$$

This proves the unitarity condition in the second order of perturbation theory. We see that the transition to the limit $\delta\to 0$ can be performed for arbitrary values of the Euclidean momentum variables. This result can be extended to any order in perturbation theory.

VIII. THE MICROCAUSALITY CONDITION

Let us consider the local commutation relation for the superspin fields $\Phi(x)$. We use the improper limit for the regularized fields $\Phi^{\delta}(x)$, defined by the relation

$$\Phi^{\delta}(x)=\int dy\, B^{\delta}(x-y)\phi(x) , \tag{8.1}$$

where $B^{\delta}(x-y)$ is given by (5.16). The commutator becomes

$$\begin{aligned}[\Phi(x),\Phi(y)]_- &= \lim_{\delta\to 0}\int dy_1\int dy_2 B^{\delta}(x-y_1)B^{\delta}(y-y_2)\\ &\qquad\times[\phi(y_1),\phi(y_2)]_-\\ &= \lim_{\delta\to 0}[B(-m^2)R^{\delta}(m^2)]^2\Delta(x-y)\\ &= [\Pi(-m^2)]^2\Delta(x-y) .\end{aligned} \tag{8.2}$$

This is the standard local commutation relation for the free-scalar fields $\phi(x)$, which shows that the superspin fields obey the condition of microcausality.

A necessary and sufficient condition for a power series

$$g(z)=\sum_{n=0}^{\infty}a_n z^n \tag{8.3}$$

to be an entire function of order γ is

$$\gamma=\lim_{n\to\infty} n\frac{\ln n}{\ln(1/|a_n|)} . \tag{8.4}$$

A weak growth restriction for large j on the coefficients $|d_j|$ in Eq. (3.14), which guarantees the vanishing of the commutator (8.2) of the superspin fields outside the light cone, is given by

$$\ln(1/|d_j|^2)>2j\ln j . \tag{8.5}$$

Thus, even though we have an infinite sum of fields and, therefore, an infinite sum of derivatives of delta functions in the commutator of the fields, we can still retain microcausality by imposing suitable conditions on the derivatives.

The superspin formalism cannot incorporate the Bogoliubov-Shirkov[32] causality condition

$$\frac{\delta}{\delta\Phi(x)}\left[\frac{\delta S}{\delta\Phi(y)}S^{-1}\right]=0 , \tag{8.6}$$

which holds in strictly localizable field theories for $x\le y$. The reason is that the superspin fields do not have compact support in configuration space and, therefore, the condition (8.6) cannot be stated in a meaningful way. The same is, of course, true in string theory or in the nonlocal field theory versions of string theory. However, the microcausality condition (8.2) is the primary causality property of the theory, since the S matrix is a quantity derived from the basic fields $\Phi(x)$ and the interaction Lagrangian.

IX. EXAMPLE OF A SUPERSPIN THEORY

As an example of a specific superspin field theory, consider the choice $|c_j|=C^j$, where C is a constant. This corresponds to case (II) in (5.4b). Then, as follows from (3.12), (3.14), and (8.4), the $\Pi(p^2)$ are entire functions of order $\gamma=\frac{1}{2}$, and decrease rapidly in the Euclidean momentum plane. Let us assume a model theory in which the infinite tower of particles that makes up the representation from which we construct $\Pi(p^2)$ consists of integer-spin particles. Then, according to the results of Weinberg in (2.23), apart from spin basis space factors, the superspin propagator will have the large-p^2 behavior

$$D_s^c(p)\simeq\frac{1}{i}\sum_{j=0}^{\infty}\frac{(Cp)^{2j}}{(2j)!}\frac{1}{-p^2} . \tag{9.1}$$

By making a transition to the Euclidean momentum space, we get the asymptotic behavior for the superspin propagator

$$D_s^c(-k^2)\simeq\frac{1}{i}\sum_{j=0}^{\infty}\frac{(-C|k|)^{2j}}{(2j)!}\frac{1}{k^2}\simeq\frac{1}{i}\frac{\exp(-C|k|)}{k^2} . \tag{9.2}$$

This large-momentum behavior for the propagator will result in a finite perturbation theory and a unitary S matrix in the way that has been demonstrated in the previous sections. A similar result is found for the fermion propagator $S_s^c(p)$ associated with the superspin $\Psi(x)$ field.

The form factor corresponding to the large-momentum behavior of the superspin propagator (9.2) is consistent with the form-factor behavior obtained by Wu and Yang[36] for extended particles, and by Martin,[37] who used complex-variable techniques to obtain a bound on the decay of the form factor $F(t)$ that says that $F(t)$ cannot decrease faster than

$$F(t)\sim\exp(-a|t|^{1/2}) , \tag{9.3}$$

as $t\to-\infty$. Martin proposed that interactions are "minimal" in the sense that they correspond to the bound (9.3), which is allowed by general physical principles. This bound was also derived by Jaffe[11] using the principle of locality in field theory. It is interesting to note that the large-momentum behavior of the form factor in string theory is determined by the string vertex function and is not consistent with the bound (9.3) (Ref. 2). This is due to the fact that an assumption of locality in field theory, used by Martin and Jaffe, is relaxed in string theory.

Let us write the superspin causal propagator for the scalar field in the Euclidean region of the momentum space as

$$\begin{aligned} D_s^c(k)&=\int d^4x\, e^{-ikx}D_s^c(x)\\ &=\frac{\Pi(k^2)}{k^2+m^2}=\frac{1}{k^2+m^2}-N(-k^2) , \end{aligned} \tag{9.4}$$

where k^2 is the square of the Euclidean momentum four-vector. In the Euclidean x space, the superspin propagator (9.4) can be represented as

$$D_s^c(x)=\Delta^c(x)\zeta(x^2) , \tag{9.5}$$

where $\zeta(x^2)$ is a positive continuous function. In the Euclidean formulation, the generating functional of $g:\Phi(x)^n$: theory can be written as

$$\begin{aligned} Z[\mathcal{J}]=&\exp\left[-g\int d^4x\,[\delta/\delta\mathcal{J}(x)]^n\right]\\ &\times\exp\left[\tfrac{1}{2}\int\int d^4x_1d^4x_2\mathcal{J}(x_1)D_s^c(x_1-x_2)\mathcal{J}(x_2)\right], \end{aligned} \tag{9.6}$$

where $\mathcal{J}(x)$ is an external source of the field $\Phi(x)$, and the point-particle Feynman propagator $\Delta^c(x)$ is given by

$$\begin{aligned} \Delta^c(x_1-x_2)&=\frac{1}{(2\pi)^4}\int\frac{d^4k\,\exp[ik(x_1-x_2)]}{k^2+m^2}\\ &=\frac{m}{(2\pi)^2}\frac{K_1(m\sqrt{(x_1-x_2)})}{\sqrt{(x_1-x_2)}} , \end{aligned} \tag{9.7}$$

where $K_1(z)$ is the Hankel function of imaginary argument of order 1 and $K_1(z)=-(\partial/\partial z)K_0(z)$ with $K_0=(\pi/2)H_0^{(1)}(iz)$. The exact Green's functions of nth order are connected with the S matrix (9.6) through the relation

$$G_n(x_1,\ldots,x_n)=Z^{-1}[\mathcal{J}]\frac{\delta^n Z[\mathcal{J}]}{\delta\mathcal{J}(x_1)\cdots\delta\mathcal{J}(x_n)}\bigg|_{\mathcal{J}(x)=0.} \tag{9.8}$$

The Fourier transforms of the Green's functions give representations of the functions in Euclidean momentum space for spacelike external momenta. An analytic continuation in the invariant momentum variables to the physical region is performed to obtain the physical values of the Green's functions, using $p^2=-k^2$ with $p^2=p_0^2-\mathbf{p}^2$. Now (9.4) becomes

$$D_s^c(p^2)=\frac{1}{-p^2+m^2-i\epsilon}-N(p^2) . \tag{9.9}$$

The entire function $N(p^2)$ is expanded in terms of p^2:

$$N(p^2)=\frac{1}{m^2}\sum_{j=0}^{\infty}\frac{a_j}{j!}\left[\frac{p^2}{4m^2}\right]^j , \tag{9.10}$$

where the coefficients a_j are defined in terms of the c_j's by

$$a_j=\frac{|c_j|^2 j!}{(2j)!} . \tag{9.11}$$

We obtain from (9.4) the Fourier-transformed function $N(-k^2)$:

$$N(-k^2)=\int_0^{\infty}dv\, v^2\frac{J_1(v\sqrt{k^2})}{\sqrt{k^2}}[1-\zeta(x^2)]\frac{mK_1(mv)}{v} . \tag{9.12}$$

The coefficients a_j are determined by the inversion formula[16]

$$a_j=\frac{1}{2(j+1)!}\int_0^\infty dv\, v^{2+2j}\left[1-\zeta\left[\frac{v^2}{m^2}\right]\right]K_1(v)\ . \quad (9.13)$$

We must now restrict the form of the function $\zeta(x^2)$ by imposing the boundary conditions

$$\zeta(x^2)\underset{x^2\to 0}{\longrightarrow}(x^2)^\rho \quad (\rho\geq 0)\ , \quad (9.14a)$$

$$\zeta(x^2)\underset{x^2\to\infty}{\longrightarrow}1\ . \quad (9.14b)$$

The convergence properties of the coefficients c_j in (5.4a)–(5.4c) further restrict the form of the function $\zeta(x^2)$ by requiring that as $v\to\infty$, we have

$$|1-\zeta(v^2)|<K\exp[-b(v^2)^\beta]\ , \quad (9.15)$$

where K, b, and β are positive constants.

We must supply a scale of the dimensions of a length $l=1/M$ associated with the size of a particle. When $l\to 0$, we regain the standard ultraviolet-divergent perturbation theory. For $l|p|\ll 1$, the calculations in the superspin field theory will only be sensitive to the asymptotic values of the c_j coefficients for large values of j. We get the asymptotic values

$$c_j\equiv(\text{const})^j/\Gamma((1-1/\gamma)j) \quad (j\gg 1) \quad (9.16)$$

and if we require that

$$\lim_{j\to\infty}(a_j)^{1/j}=0\ , \quad (9.17)$$

then an example of a suitable $\zeta(x^2)$ function is

$$\zeta(x^2)=1-\exp[-(x^2/l^2)^\gamma]\ , \quad (9.18)$$

where the constant $\gamma>1$. Thus, by imposing the physical boundary conditions (9.14a) and (9.14b), and convergence conditions on the entire function $N(-k^2)$, we severely restrict the possible values of the coefficients c_j.

A superspin field-theory model will only contain one new parameter l associated with the scale of the Lagrangian density being considered. The scale l can be determined experimentally, when the signature of nonlocality sets in at high energies for $l|p|\simeq 1$. The low-energy predictions for $l|p|\ll 1$ will be insensitive to the c_j coefficients for nonasymptotic values of j and the value of l. Finite mass and charge renormalizations will be carried out at the lowest order of perturbation theory. Such a program, in which definite predictions can be made for cross sections, etc., cannot be carried out in the standard point-particle model of a nonrenormalizable theory such as Einstein's gravitational theory, since the coefficients associated with the counterterms that occur in each new order of perturbation theory must be determined ahead of time before meaningful calculations can be performed in any given order. Clearly this is not possible and no definite predictions can be made to any order in standard nonrenormalizable gravitational theory.

In pure gravitational theory, it is natural to adopt the Planck length as the fundamental length scale $l_G=(\hbar G/c^3)^{1/2}=1.2\times 10^{19}$ GeV. Hopefully, in a future unified field theory that includes gravitation, the length scales associated with electroweak and QCD interactions at energies well below the Planck mass will be determined in a fundamental way by the theory.

It should be stressed that standard regularization techniques, such as Feynman cutoff procedures or the Pauli-Villars regularization technique,[32] all suffer from violations of unitarity and causality, except in the limit that the cutoff parameters become infinite. Dimensional regularization only works in a fictitious, fractional dimensional space. Thus, none of these techniques help to produce meaningful perturbation-theory calculations in nonrenormalizable gravitation theory.

X. CONCLUSIONS

By assigning an internal degree of infinite spin to every particle, we have succeeded in developing a field-theory formalism that leads to a finite perturbation theory and a unitary S matrix for the basic spin-0, spin-1, and spin-2 fields of nature. The causality properties of a strictly localizable field were extended so that a condition of microcausality for the fields was satisfied. The other requirements of axiomatic field theory, such as the existence of a scattering theory, can also be included in the extended nonlocal field theory.

With the failure of point-particle field theory to resolve the infinities in standard quantum gravity, we seem to be forced into a theoretical picture in which particles are extended objects and field theory is intrinsically slightly nonlocal. The superspin field theory developed here is an example of a self-consistent field theory, based on nonlocal fields, that can remove the unsatisfactory features of standard strictly local field theory. More work remains to be done to investigate many of the fundamental ramifications of such a theory and its implications for future particle physics.

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