PHYSICAL REVIEW D VOLUME 39, NUMBER 12 15 JUNE 1989

Experimental constraints on a minimal and nonminimal violation of the equivalence principle in the oscillations of massive neutrinos

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(Received 2 February 1989)

The negative results of the oscillations experiments are discussed with the hypothesis that the various neutrino types are not universally coupled to gravity. In this case the transition probabiltiy between two different flavor eigenstates may be affected by the local gravitational field present in a terrestrial laboratory, and the contribution of gravity can interfere, in general, with the mass contribution to the oscillation process. In particular it is shown that even a strong violation of the equivalence principle could be compatible with the experimental data, provided the gravity-induced energy splitting is balanced by a suitable neutrino mass difference.

I. INTRODUCTION

It has been recently pointed out that the laboratory experiments on neutrino oscillations probe the validity of the Einstein principle of equivalence for quantum and relativistic test particles.[1] If this principle were violated, in fact, the terrestrial gravitational potential could contribute to the oscillation process: the negative results of the experiments thus constrain possible violations of the equivalence principle in neutrino interactions, providing bounds which are rather stringent in the hypothesis that neutrinos are massless.[1] The aim of this paper is to extend the analysis of Ref. 1, by considering the cases in which the mass contribution to neutrino oscillations is not negligible with respect to the gravitational one.

We start by recalling that, according to the equivalence principle, the coupling of gravity to matter fields has to be minimal and universal: it can be represented formally by a covariant derivative, in which the coefficient of the connection (corresponding to the effective gravitational charge) is the same for all kinds of matter. This feature is essential in order that the effects of the field may be locally eliminated, for all the particles species, by the choice of a suitable accelerated frame.

A deviation from the equivalence principle corresponds to a situation in which different species of particles are differently affected by gravity, as if they had different gravitational charges. The deviation can be parametrized by assuming that the parametrized-post-Newtonian (PPN) parameters, in the expansion for a given metric, have a particle-dependent value,[2,3] representing thus the effective coupling constants of the various kinds of particles to the given geometry. If we work, in particular, in the weak-field limit of a static source, then, to first order in the Newtonian potential $\phi = GM/r \ll 1$, we can consider the effective metric

$$g_{44} = 1 - 2\alpha\phi, \quad g_{ij} = -\delta_{ij}(1+2\gamma\phi) . \tag{1.1}$$

The parameters α and γ are theory dependent (for example, $\alpha = 1 = \gamma$ in general relativity), but, for any given theory, their value is universal (the same for all kinds of matter) only if the principle of equivalence is satisfied. Otherwise their value could be different for test particles with different internal quantum numbers, for example, and (or) different energies. In the particular case of the neutrino field it has been shown recently,[2,3] by considering the time delays produced by the gravitational field of our Galaxy, that the value of γ is the same, to an accuracy of about 0.1%, for neutrinos and photons received from the supernova 1987A and that, in the hypothesis that neutrinos are massless, is the same for neutrinos of different energies[2] (ranging from 7 to 40 MeV), up to an accuracy of one part in 10^6 (to the same accuracy, γ is the same also for neutrinos and antineutrinos[4]). As regards photons moreover we know, from radar-echo delay experiments performed on a planetary scale,[5] that $|\gamma - 1| \lesssim 10^{-3}$.

It is important to stress that all these results have been obtained in the hypothesis $\alpha = 1$. This assumption is irrelevant when considering experimental data relative to one kind of particle only, as a possible deviation of α from 1 can be absorbed by redefining the mass of the source; it could be no longer justified, however, if we compare different particles or particles with different energies, since also the value of α could be particle dependent, or energy dependent, just like γ.

The laboratory experiments on neutrino oscillations, testing the universality of the gravitational red-shift, which depends on g_{44} only, provide information on the possibility that the value of α be different for different neutrino types.[1] If there are deviations from universality in the value of α, in fact, the energy splitting induced by gravity may contribute in general to the transition probability between different flavors, as we shall see in Sec. II, even if gravity is not the primary source of oscillations, that is, even if the energy eigenstates are unchanged by the influence of the weak gravitational field present in the laboratory.

A limit on a maximal violation of the equivalence principle, valid for the case in which gravity is the only source of oscillations, was obtained in a previous paper.[1] If neutrinos are massive, however, we may have simultaneously both the contribution of mass and gravity to the transition probability. In this paper, the bounds one can obtain from the experimental data, in the case of massive neutrinos, will be discussed by considering in particular two cases of physical interest. First by assuming, in Sec. III, that the mass and gravitational part of the total energy are diagonal in the same basis; then by considering, in Sec. IV, a minimal violation of the equivalence principle in which the eigenstates of the weak and gravitational interaction coincide. The main results of this paper will be summarized finally in Sec. V.

II. GRAVITATIONAL CONTRIBUTIONS TO THE OSCILLATION PROBABILITY

In the experimental tests on neutrino oscillations, performed in the laboratory, the neutrinos propagate not in vacuum, but through the gravitational potential locally present at the Earth's surface. In a constant gravitational field one can always choose, according to general relativity, a reference frame in which the components of the metric tensor do not contain explicitly the temporal coordinate t, which is called then universal time. In this frame, the conserved energy E_0 which determines, in the approximation of geometric optics, the temporal evolution of an energy eigenstate with respect to the universal time, is related to the energy E, measured by a local observer, by [6] $E_0=(g_{44})^{1/2}E$. For an ultrarelativistic particle of mass m, propagating through the weak field described by the metric (1.1), the conserved energy (to first order in ϕ and m^2/p^2) is then

$$E_0=p+\frac{m^2}{2p}-\alpha\phi p \ , \tag{2.1}$$

where $p=mv(1-v^2)^{-1/2}\gg m$, and v is the particle velocity measured with respect to the proper time of a local observer, at rest at a given point where ϕ is the value of the external potential.[6]

It is well known that if neutrinos are massive, and the mass contribution to the total energy (2.1) is not diagonal with respect to the weak flavor eigenstates $|\nu_w\rangle$, then oscillations can occur even in vacuum.[7] In that case the $|\nu_w\rangle$ states are superpositions of the mass eigenstates $|\nu_M\rangle$ and we can set (considering for simplicity a two-component mixing only) $|\nu_w\rangle=R(\theta_M)|\nu_M\rangle$, where

$$|\nu_w\rangle=\begin{bmatrix}\nu_e\\ \nu_\mu\end{bmatrix}, \quad |\nu_M\rangle=\begin{bmatrix}\nu_{1M}\\ \nu_{2M}\end{bmatrix}, \qquad R(\theta)=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} . \tag{2.2}$$

The energy splitting between the two mass eigenvalues

$$E_{1M}=m_1^2/2p, \quad E_{2M}=m_2^2/2p \tag{2.3}$$

defines then the oscillation length $L_M=2\pi/(E_{2M}-E_{1M})$.

In the presence of a gravitational field there is a further possibility of oscillations, as we may see from Eq. (2.1), if the coupling of the neutrino field to the external potential, characterized by the phenomenological parameter α, is not diagonal in the $|\nu_w\rangle$ basis. In order to have a gravitational contribution to the oscillation mechanism, however, a violation of the equivalence principle is needed. Suppose in fact that the flavor eigenstates are linear superpositions of the states which diagonalize the last term in Eq. (2.1). If we call $|\nu_G\rangle$ these states, then we have $|\nu_w\rangle=R(\theta_G)|\nu_G\rangle$, where

$$|\nu_G\rangle=\begin{bmatrix}\nu_{1G}\\ \nu_{2G}\end{bmatrix}, \quad E_{1G}=-\alpha_1 p\phi, \quad E_{2G}=-\alpha_2 p\phi \tag{2.4}$$

are the components of the gravitational basis and the corresponding eigenvalues. If the coupling of gravity to the neutrino field is universal, i.e., $\alpha_1=\alpha_2$, the gravitational part of the energy becomes a multiple of the identity: it is then already diagonal in the $|\nu_w\rangle$ basis, so that no significant mixing of states can be induced by gravity.

Only if there is a violation of the equivalence principle, and the energies of different neutrino types are differently red-shifted ($E_{1G}\neq E_{2G}$) when the neutrino types propagate, with the same momentum, through the same external potential, we may have then a nontrivial mixing, $\theta_G\neq 0$, and gravity-induced oscillations[1] with oscillation length $L_G=2\pi/\Delta E_G$, where $\Delta E_G=E_{2G}-E_{1G}$. A violation of the equivalence principle, in the context of neutrino oscillations, can thus be characterized, in general, by two phenomenological parameters: the mixing angle θ_G, which relates gravitational and weak-interaction eigenstates, and the difference $\Delta\alpha=\alpha_1-\alpha_2$, which determines the gravitational energy splitting. In particular the violation will be called "minimal" if the coupling is not universal, $\Delta\alpha\neq 0$, but the gravity contribution is diagonal in the $|\nu_w\rangle$ basis, i.e., $\theta_G=0$; it will be called "nonminimal" if the gravity eigenstates are different from the weak-interaction ones.

In the most general situation in which neutrinos are massive, and the principle of equivalence is violated, we may have $|\nu_w\rangle\neq|\nu_M\rangle\neq|\nu_G\rangle$. The eigenstates of the total energy are obtained then by diagonalizing the matrix which includes the contributions of the mass and gravitational terms. Starting from their diagonal expression (2.3) and (2.4), the total matrix can be written, in the $|\nu_w\rangle$ basis (modulo a multiple of the identity, which only contributes to an overall phase and does not affect oscillations),

$$\begin{bmatrix} -\frac{\Delta m^2}{2p}\cos^2\theta_M-\Delta E_G\cos^2\theta_G & \frac{\Delta m^2}{4P}\sin2\theta_M+\frac{\Delta E_G}{2}\sin2\theta_G \\ \frac{\Delta m^2}{4P}\sin2\theta_M+\frac{\Delta E_g}{2}\sin2\theta_G & -\frac{\Delta m^2}{2p}\sin^2\theta_M-\Delta E_G\sin^2\theta_G \end{bmatrix} , \tag{2.5}$$

where $\Delta m^2 = m_2^2 - m_1^2$, and $\Delta E_G = p\phi\Delta\alpha$. The components of $|\nu_w\rangle$ can be expressed, as usual, in terms of the eigenstates of this matrix, so that if we have, as initial state at $t=0$, a pure ν_e, at a time t later the state will be a mixture of ν_e and ν_μ. The transition probability is then

$$P_{\nu_e \to \nu_\mu}(t) = \sin^2 2\theta \sin^2(\pi t/L) , \qquad (2.6)$$

where L is the total oscillation length which, if the contributions of mass and gravity are both nonvanishing, is given by

$$L = L_M L_G [L_M^2 + L_G^2 + 2L_M L_G \cos 2(\theta_M - \theta_G)]^{-1/2} \qquad (2.7)$$

and θ is the rotation angle which diagonalizes the matrix (2.5):

$$\sin^2 2\theta = L^2 (L_M^{-1} \sin 2\theta_M + L_G^{-1} \sin 2\theta_G)^2 . \qquad (2.8)$$

The experimental tests on neutrino oscillations, measuring P, provide simultaneously information on θ and L. In the absence of positive results,[8] however, an upper bound for P does not fix any value for θ and L separately: it determines only an allowed region in the (θ, L) plane. In the following sections we shall discuss nevertheless the possibility of extracting, from the experimental data, significant information on a violation of the equivalence principle, by considering in particular the allowed values of $\Delta\alpha$, and relating them to the unknown neutrino mass splitting Δm^2, at some fixed value of the gravitational mixing angle θ_G.

III. MASS-DEPENDENT VIOLATION OF THE EQUIVALENCE PRINCIPLE

We shall consider, first of all, a nonminimal violation of the equivalence principle in which the gravitational part of the energy is diagonal in the $|\nu_M\rangle$ basis, that is, $|\nu_G\rangle = |\nu_M\rangle$. This is a natural choice to discuss the phenomenological consequences of a mass-dependent violation of the equivalence principle, $\alpha = \alpha(m)$, like that expected in the context of gravitational theories which include the effects of finite temperature,[9] or of a finite limit for the proper accelerations.[10,11]

In this case $\theta_G = \theta_M$, so that, from Eqs. (2.7) and (2.8),

$$\theta = \theta_M = \theta_G, \quad L = 2\pi(\Delta m^2/2p + p\phi\Delta\alpha)^{-1} . \qquad (3.1)$$

It is convenient to define the dimensionless variables

$$x = \Delta m^2/2p^2, \quad y = \phi\Delta\alpha . \qquad (3.2)$$

The experimental bounds on P define then, for any given value of $\sin^2 2\theta$, a number k such that

$$|x + y| \le k . \qquad (3.3)$$

As the value of k turns out to depend inversely on the neutrino momentum p, it follows that the best experimental constraints on y (i.e., $\Delta\alpha$), corresponding to the lowest possible values of k in Eq (3.3), are obtained from the experiments performed on accelerator neutrinos,[8] with $p \sim 1$ GeV (instead of reactor neutrinos, whose energies are in the MeV range). This is a consequence of the fact that the energy splitting due to a violation of the equivalence principle is proportional to the neutrino energy ($\Delta E_G = p\phi\Delta\alpha$), unlike the mass-induced splitting, which is proportional to p^{-1}.

We are interested, in this paper, in pointing out the most stringent conditions on $\Delta\alpha$ which can be obtained from the negative results of the experiments, so that we shall concentrate, in what follows, mainly on large values of the vacuum mixing angle θ. In this case, the best experimental data presently available on $\nu_e \leftrightarrow \nu_\mu$ oscillations are the results of the Big European Bubble Chamber (BEBC) experiment,[12] corresponding to neutrinos of average energy $p = 1.5$ GeV; in the case of maximal mixing ($\theta = \pi/4$) one can then obtain for k, from Ref. 12, the value $k_1 \equiv k(\pi/4) = 2 \times 10^{-20}$.

If the mass contribution to the oscillations is negligible ($x \simeq 0$), Eq. (3.3) then becomes $|y| \le k$ and we find, for maximal mixing ($k = k_1$), the result

$$|\Delta\alpha| \le k_1/\phi \simeq 2.9 \times 10^{-11} (0.69 \times 10^{-9}/\phi) , \qquad (3.4)$$

which coincides with the bound, presented in a previous paper,[1] on a maximal violation of the equivalence principle for massless neutrinos (provided we insert the value corresponding to the average terrestrial potential at Earth's surface, $\phi_E \simeq 0.69 \times 10^{-9}$).

The condition $|\Delta\alpha| \le k/\phi$ is still valid, as a limit case, for a violation of the equivalence principle such that x and y have the same sign (see the dashed area in Fig. 1). Otherwise it is possible to reconcile the negative experimental results even with values $|\Delta\alpha| \gg k/\phi$, provided the violation of the equivalence principle is balanced by a suitable splitting in the neutrino masses, according to Eq. (3.3) (this requires, however, fine-tuning; see Fig. 1 where the region within the two parallel lines shows part of the allowed values for x and y, measured in units of k).

Irrespective of the relative sign of x and y, however, it is possible to constrain $\Delta\alpha$ through the limit on the mass of the muon neutrino,[13] obtained from pion decays, which gives $\Delta m^2 \le 0.0625$ MeV^2 independently of the re-

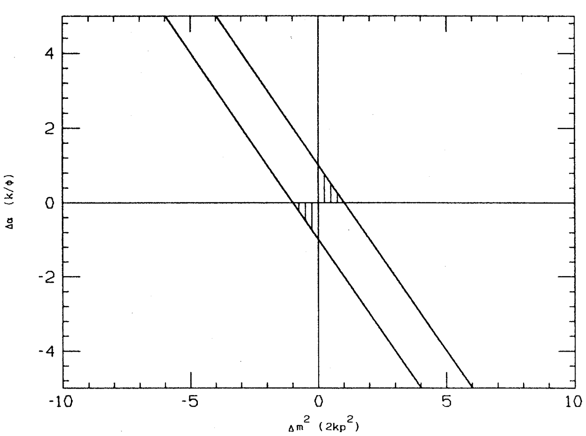

FIG. 1. The allowed values of $\Delta\alpha$, according to Eq. (3.3), in the case $\theta_G = \theta_M = \theta$ (the excluded region is outside the two parallel lines).

sults of the oscillations experiments. For neutrinos of energy $p = 1.5$ GeV we have then the condition

$$|x| \leq x_0 = 1.4\times 10^{-8} , \tag{3.5}$$

which implies, when combined with Eq. (3.3), the following restriction on y:

$$|y| \leq k + x_0 \tag{3.6}$$

valid even if $xy < 0$. In the case of a maximal violation of the equivalence principle ($k = k_1 \ll x_0$) this gives $|y| \leq x_0$, i.e.,

$$|\Delta\alpha| \leq x_0/\phi \simeq 0.2\times(0.69\times 10^{-9}/\phi) . \tag{3.7}$$

The bound $|\Delta\alpha| \lesssim 0.2$, for typical values of the terrestrial potential at Earth's surface, is not very stringent, especially when compared with the corresponding one achieved in the case that x and y have the same sign [Eq. (3.4)]. It should be noted, however, that the bound on $\Delta\alpha$ depends on the value of the total effective gravitational potential at the place where the experiment is performed. If the violations of the equivalence principle are induced by forces with very long or infinite range,[2-4,14] then the laboratory potential affecting neutrinos is the sum of the terrestrial, solar, and galactic potentials. In this case the galactic contribution is the dominant one, $\phi_G \simeq 6\times 10^{-7}$, and the bound on $\Delta\alpha$ turns out to be improved by a factor 10^3, thus becoming comparable with that on $\Delta\gamma$ obtained from the supernova neutrinos.[2-4]

Moreover, if one accepts the limit on the muon neutrino mass recently derived from the duration of the neutrino burst from the supernova,[15] i.e., $m_{\nu_\mu} \lesssim 40$ keV, instead of[13] $m_{\nu_\mu} \lesssim 250$ keV, then the value of x_0, and the corresponding bound on $\Delta\alpha$, are to be lowered by a factor 0.0256, irrespective of the value of ϕ.

Finally we must note also that if the mixing is not maximal, $\theta < \pi/4$, the value of k obtained from the experiments is larger than k_1 and approaches infinity[8] in the limit $\theta \to 0$. As θ is decreasing down to zero, therefore, the allowed region in the (x,y) plane grows up to cover all the band $|x| \leq x_0$. No limit can be obtained then on $\Delta\alpha$ for a minimal violation of the equivalence principle ($\theta_G = 0$), as in the case θ and θ_G coincide [see Eq. (3.2)].

IV. CONSTRAINTS ON A MINIMAL VIOLATION

In order to discuss a flavor-dependent violation of the equivalence principle in which the gravitational basis coincides with that of weak interactions, $|\nu_G\rangle = |\nu_w\rangle$ (the so-called minimal violation, see Sec. II), we shall consider now the general case in which the mass eigenstates are different from the gravitational ones, $|\nu_M\rangle \neq |\nu_G\rangle$. In this case $\theta_G = 0$ does not imply $\theta = 0$, and the general expression (2.6) for the transition probability, putting $\theta_G = 0$, reduces to

$$P = (x^2/A)\sin^2 2\theta_M \sin^2(t\sqrt{A}/2) , \tag{4.1}$$

where

$$A = x^2 + y^2 + 2xy\cos 2\theta_M . \tag{4.2}$$

To each value of $\sin^2 2\theta$ corresponds therefore a number k, determined by the negative results of the oscillation experiments, such that

$$x^2 + y^2 + 2xy\cos 2\theta_M \leq k^2 , \tag{4.3}$$

$$x^2(1 - \sin^2 2\theta_M/\sin^2 2\theta) + y^2 + 2xy\cos 2\theta_M = 0 , \tag{4.4}$$

where we have assumed $\sin^2 2\theta \neq 0$ (no constraint is obtained on a minimal violation in the limit $\theta \to 0$, as already seen in the case $\theta = \theta_G$ previously discussed). These two conditions determine, in general, the allowed values of $\Delta\alpha$, as a function of Δm^2 and θ_M, corresponding to a given value of θ. We shall consider, in particular, the case $\theta = \pi/4$, because it corresponds to the lowest value of k and provide then the most stringent limits on a violation of the equivalence principle.

If we set $\sin^2 2\theta = 1$, Eq. (4.4) becomes

$$x\cos 2\theta_M + y = 0 . \tag{4.5}$$

One can immediately see that in this case a minimal violation of the equivalence principle is forbidden ($y = 0$) if the mixing of the mass eigenstates is maximal ($\theta_M = \pi/4$) and, for all θ_M, also if the sign of x and y is the same (by convention the missing angles range from $-\pi/4$ to $\pi/4$).

A minimal violation of the equivalence principle, such that $\Delta\alpha$ and Δm^2 are of opposite sign, and related by Eq. (4.5), is allowed; it is constrained, however, by the condition [obtained by eliminating x in Eq. (4.3)]

$$y \leq k_1^2 \cos^2\theta_M/\sin^2 2\theta_M , \tag{4.6}$$

where k_1 is the previously introduced experimental value of k corresponding to $\theta = \pi/4$. The allowed values of $|\Delta\alpha|$, in units of k_1/ϕ, are shown in Fig. 2 as a function of θ_M.

The experimental limit (3.5) on x, independently obtained from the upper bound on the mass of the muon neutrino, moreover implies, through Eq. (4.5),

$$|y| \leq x_0 \cos 2\theta_M . \tag{4.7}$$

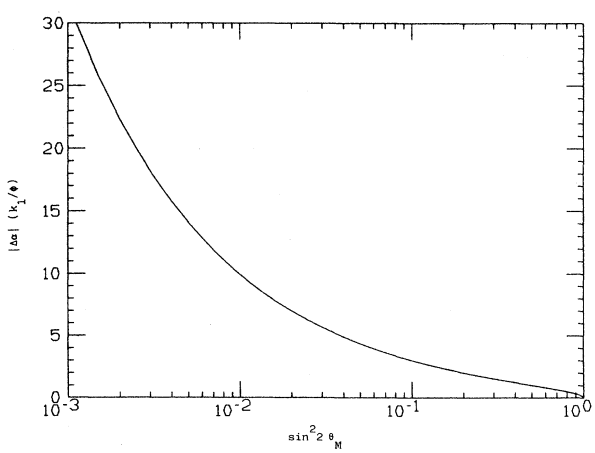

FIG. 2. The allowed values of $|\Delta\alpha|$, according to Eq. (4.6), in the case $\theta_G = 0$ and $\theta = \pi/4$ (valid for $\Delta\alpha\Delta m^2 < 0$). The area above the curve is excluded.

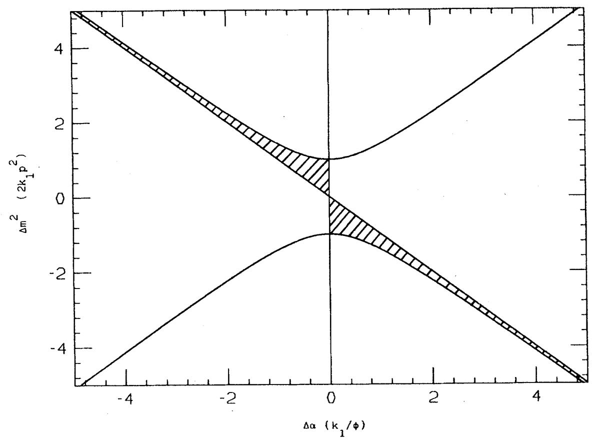

FIG. 3. The cross-hatched area shows the allowed values of $\Delta\alpha$ as a function of Δm^2, according to Eqs. (4.8) and (4.9), in the case $\theta_G=0$ and $\theta=\pi/4$.

In the limit $\theta_M \to 0$, therefore, we find again the condition $|y| \le x_0$, which provides the bound $|\Delta\alpha| \le x_0/\phi$ previously reported in Eq. (3.7).

To each allowed value of $\Delta\alpha$, the corresponding allowed values of Δm^2 are given by Eq. (4.5). These values can be simultaneously visualized by eliminating θ_M in Eq. (4.3), by means of the condition (4.5). We get then

$$x^2-y^2 \le k_1^2 \tag{4.8}$$

which, together with the constraints

$$-y/x \le 1, \quad xy<0 \tag{4.9}$$

[according to Eq. (4.5)], determines the allowed region in the (x,y) plane, part of which is shown in Fig. 3. It is important to stress again that values of Δm^2 even much larger than $2k_1p^2$ could become compatible with the negative results of the oscillations experiments, provided that $\Delta\alpha \neq 0$. This requires, however, an appropriate finetuning in the violation of the equivalence principle, according to Eq. (4.5).

V. CONCLUSIONS

If there are violations of the equivalence principle in neutrino interactions, in the sense that the energies of different neutrino types are differently red-shifted by gravity, then the transition probability between the various neutrino flavors can be affected by the presence of an external gravitational potential.

The violation may be minimal or nonminimal: in the first case it induces only an energy splitting between the usual weak flavor eigenstates, in the second case the eigenstates of the gravitational part of the total energy are different from the weak-interaction eigenstates. The results of the experimental tests on neutrino oscillations, performed at Earth's surface, are then to be analyzed, in general, in terms of four unknown parameters: Δm^2 and $\Delta\alpha$, which determine, respectively, the mass and gravitational energy splitting, and the two angles θ_M, θ_G, which relate, respectively, the mass and gravitational eigenstates to the eigenstates of weak interaction.

In this paper we have considered, in particular, the case $\theta_G=\theta_M$ (mass-dependent violation of the equivalence principle) and $\theta_G=0$ (minimal violation, flavor dependent). In both cases it has been shown that, if the mass and gravitational energy splitting are of opposite sign, no bound on the modulus of $\Delta\alpha$ can be obtained directly from an upper limit on the transition probability. The negative results of the oscillation tests, in fact, provide only a relation between $\Delta\alpha$ and Δm^2, from which we are allowed to constrain $\Delta\alpha$ only indirectly, by means of independent limits on the neutrino mass splitting. The experimental constraints on $\Delta\alpha$ are instead much more stringent, at a given fixed value of the vacuum mixing angle θ, if the violation of the equivalence principle is such that the sign of $\Delta\alpha$ and Δm^2 is the same.

Aside from the particular cases one may wish to discuss, however, there are two points, valid in general, which should be stressed. The first is that the limits on the neutrino mass difference, usually reported as results of the oscillation experiments,[8] are valid provided that $\Delta\alpha=0$, i.e., only if the equivalence principle is satisfied. Otherwise it is possible to reconcile the negative experimental results even with very large values of Δm^2 by assuming that the mass contribution to the oscillation probability is balanced by a suitable (fined-tuned) value of $\Delta\alpha$.

The second point is that the experiments on neutrino oscillations may be regarded, in general, also as tests of gravitational theories which predict deviations from universality in the coupling of leptons to gravity (a "fifth force" vector, for example, could be coupled, at least in principle, not only to baryon number[16]). A positive result in the oscillation experiments would provide a relation between $\Delta\alpha$ and Δm^2. The effective violation of the equivalence principle would be determined, therefore, only if the energy splitting between the mass eigenstates would be known from independent experiments. A direct measurement of the transition probability in vacuum, even with $\Delta\alpha$ and Δm^2 separately unknown, would give however decisive information on the possibility of solving the solar-neutrino puzzle by means of resonant oscillations, inside the solar matter, in the presence of both the contributions of gravity and weak interactions.[1]

ACKNOWLEDGMENTS

It is a pleasure to thank A. Bottino for his kind interest and a useful discussion. I am very grateful also to M. Caselle for his patient and generous help in preparing the computer programs for the figures displayed in this work.

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