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General-relativistic domain walls

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Domain walls naturally arise in any model that has a spontaneously broken discrete symmetry. In the simplest example, symmetry breaking is accomplished by a real scalar field. We investigate infinite domain walls in general relativity using the full Einstein scalar-field equations. We study the gravitational effects on test particles both inside and outside the walls and find that matter is repelled by the walls. Also, we find that there is pressure in the direction perpendicular to the plane of the wall.

I. INTRODUCTION

In recent years there has been considerable interest in the topological defects that can presumably form during spontaneous symmetry breaking associated with a phase transition in the early Universe.[1] The objects possible include magnetic monopoles, cosmic strings, and domain walls,[2] as well as hybrids of these[3,4] and variants that can support superconducting currents.[5,6] To date, much of this interest has focused on cosmic strings which are viewed not only as cosmologically "safe" but as viable candidates for the seed fluctuations necessary to initiate the formation of a large-scale structure.[7] On the other hand, it is widely believed that domain walls and magnetic monopoles, if present, would have lead to a Universe radically different from the one we live in,[8,9] unless of course they are inflated away or become unstable during some subsequent phase transition.

Let us discuss the cosmological consequences of domain walls in a bit more detail. Domain walls occur when the vacuum manifold for the order parameter or scalar field driving the symmetry breaking has a discrete (Z_N) symmetry. At the time of the phase transition, there are both infinite and closed surface walls. As the system of walls evolves, the closed walls oscillate, lose energy via gravitational and particle radiation, and eventually disappear. Likewise, small-scale inhomogeneities in the infinite walls are smoothed out. Eventually, the characteristic scale for the domain-wall network becomes comparable to the horizon, so that there will typically be one domain in a given Hubble volume. A single domain wall stretching across the horizon leads to fluctuations in the microwave background with $\delta T/T \sim \sigma/Hm_{\rm Pl}^2$, where σ is the mass per unit area of the wall, H is the Hubble parameter, and $m_{\rm Pl}=1.2\times10^{19}$ GeV. For $\sigma>(10\ \text{MeV})^3$ these fluctuations would come in conflict with present-day observations that constrain $\delta T/T$ to be less than 10^{-4}. Therefore, domain walls that are present today must have an energy scale <10 MeV.

Recently, Hill, Schramm, and Fry[10] (HSF) have suggested that a late-time phase transition (one occurring after the decoupling of matter and radiation) can give rise to very light and therefore cosmologically safe domain walls. The phase transition might occur at an energy as low as $1-10^{-3}$ eV if, for example, it is the phase transition responsible for giving neutrinos small masses. The thickness of such a wall can be enormous. HSF discuss models that have low-mass pseudo-Goldstone bosons called schizons[11] that are very much akin to the axion. As with the axion, the mass of the schizon is $\sim m^2/8\pi^2 f$, where m is the mass of some associated fermion and f is a generic high-energy mass scale [e.g., grand-unified-theory (GUT) scale]. For $f=10^{15}$ GeV and $m=1$ eV ($m=10^{-3}$ eV) the schizon mass is 10^{-25} eV (10^{-31} eV). If there are domain walls produced due to some discrete symmetry in the schizon potential, then the thickness of the walls will be of order the Compton wavelength of the schizon. For $m=1$ eV (10^{-3} eV) the thickness is roughly 10 pc (10 Mpc). HSF suggest that these walls would provide large but acceptable density inhomogeneities that could generate large-scale structure formation. Furthermore, they contend that the variations in the microwave background would be small and well within the limits set by present-day observations.

Given a renewed interest in domain walls, we felt it necessary to reexamine the work on domain walls in general relativity. The metric for an infinite, static, plane-symmetric domain wall is rather peculiar. As was first noted by Vilenkin,[12] a domain wall of this type does *not* admit a static metric. Vilenkin[13] and Ipser and Sikivie[14] have solved Einstein's equations in the presence of a planar domain wall by approximating the stress energy of the wall as that of an infinitely thin plane with positive energy density and negative, homogeneous, and isotropic presure in the plane of the wall. The stress-energy tensor is taken to be

$$T_{\mu\nu}=\sigma\delta(z)(1,-1,-1,0)\ , \qquad (1.1)$$

where σ is the mass per unit area of the wall and the z axis is perpendicular to the wall. In suitable coordinates, the metric takes the form[13]

$$ds^2=e^{-\kappa|z|}[-dt^2+dz^2+e^{\kappa t}(dx^2+dy^2)]\ . \qquad (1.2)$$

The $z=$const hypersurfaces have the properties of $(2+1)$-dimensional de Sitter space. Furthermore, the

(t,z) part of the metric describes a $(1+1)$-dimensional Rindler space (i.e., flat space in the reference frame of a uniformly accelerating observer with proper acceleration $\kappa/2$).

There are two basic problems with the approach outlined above. First, the scalar field couples to gravity and one should therefore solve the Einstein scalar-field equations simultaneously. An obvious question along these lines is whether one can have a static wall constructed out of a scalar field in a nonstatic spacetime. Furthermore, one might wonder whether Eq. (1.1) properly describes a wall with finite thickness. As we shall see below, the *zz* component of the stress-energy tensor for a thick wall does *not* vanish.[15] (It does, however, vanish in the thin-wall limit.) The second problem is more relevant for understanding the astrophysical consequences of very light and very thick domain walls. Clearly, if one is to understand the motion of test particles in the gravitational field of a *megaparsec* thick wall, one requires the interior as well as exterior metric of the wall. Equation (1.2) however, gives only the exterior metric.

II. DOMAIN WALLS IN MINKOWSKI SPACE

In this section we discuss the structure of an infinite, plane-symmetric wall in flat space. We assume that the wall is homogeneous and isotropic in the x-y plane, symmetric about $z=0$, and static. Such a wall can arise in a theory with a real scalar field. Let $\Phi=f\phi$ be the scalar field responsible for the wall and $V(\Phi/f)=m^4U(\phi)$ be the scalar potential. Here m and f are energy scales and ϕ and $U(\phi)$ are dimensionless. $U(\phi)$ has degenerate minima at ϕ_+ and ϕ_- and a maximum at ϕ_M. We assume that $U(\phi_\pm)=0$ so that $U(\phi)\geq 0$ for all ϕ. Furthermore, we assume that the potential is symmetric about ϕ_M [i.e., $U(\phi_M-\phi)=U(\phi_M+\phi)$]. A particular example for $U(\phi)$, and one that we will use below, is

$$U(\phi)=1-\cos 2\phi, \quad 0\leq\phi<2\pi \ . \tag{2.1}$$

The Lagrange density for Φ is

$$\mathcal{L}=-\frac{f^2}{2}\partial_\mu\phi\,\partial^\mu\phi-m^4U(\phi) \tag{2.2}$$

and the stress-energy tensor is

$$T_{\mu\nu}=f^2\partial_\mu\phi\,\partial_\nu\phi+\eta_{\mu\nu}\mathcal{L} \ , \tag{2.3}$$

where $\eta_{\mu\nu}=\text{diag}(-1,1,1,1)$ is the flat-space metric. (We use high-energy physics units so that $\hbar=c=k_B=1$ and $G_N=m_{\text{Pl}}^{-2}$, where $m_{\text{Pl}}=1.2\times10^{19}$ GeV is the Planck mass.) The nonzero components of $T_{\mu\nu}$ for the static, plane-symmetric wall are

$$T^0_0=T^1_1=T^2_2=-\left[\frac{f^2}{2}(\phi')^2+m^4U(\phi)\right] , \tag{2.4a}$$

$$T^3_3=\frac{f^2}{2}(\phi')^2-m^4U(\phi) \ . \tag{2.4b}$$

(Here, and throughout, a prime will denote differentiation with respect to z and an overdot will denote differentiation with respect to t.) For the case at hand, the equation of motion for ϕ is

$$\phi''=\frac{m^4}{f^2}\frac{\partial U}{\partial\phi} , \tag{2.5}$$

with $\phi\to\phi_\pm$ for $z\to\pm\infty$. Integrating this equation, we find that

$$\tfrac{1}{2}(\phi')^2=\frac{m^4}{f^2}U(\phi) \ . \tag{2.6}$$

Using this result the stress-energy tensor takes the simple form

$$T^\mu_\nu=-2m^4U(\phi)\text{diag}(1,1,1,0) \ . \tag{2.7}$$

Consider now the potential given by Eq. (2.1) and let us assume that $\phi=0$ for $z\to-\infty$ and $\phi=\pi$ for $z\to\infty$. With this form for the potential, we can analytically solve for ϕ:

$$\tan\phi/2=e^{2m^2z/f} \ . \tag{2.8}$$

The stress-energy tensor, as a function of z, is then

$$T^\mu_\nu=-\frac{16m^4e^{4m^2z/f}}{(1+e^{4m^2z/f})^2}\text{diag}(1,1,1,0) \ . \tag{2.9}$$

The energy and pressures are localized in a plane of thickness $O(f/m^2)$ and mass per unit area is σ where

$$\sigma\equiv\int_{-\infty}^{\infty}T_{00}dz=4m^2f \ . \tag{2.10}$$

III. DOMAIN WALLS IN CURVED SPACE

To search for domain walls in curved space we assume a form for the metric that is homogeneous and isotropic in two spatial dimensions and symmetric about the $z=0$ plane where z is the coordinate orthogonal to the wall. Under these assumptions, the most general form for the metric is[16]

$$ds^2=A(t,|z|)(-dt^2+dz^2)+B(t,|z|)(dx^2+dy^2) \ . \tag{3.1}$$

The Lagrange density and stress-energy tensor are given by Eqs. (2.2) and (2.3) provided we make the usual substitution of $g_{\mu\nu}$ from Eq. (3.1) for $\eta_{\mu\nu}$. However, the stress-energy tensor does *not* take the simple form in Eq. (2.7). The nonzero components of $T_{\mu\nu}$ are

$$T^0_0=T^1_1=T^2_2=-\left[\frac{f^2}{2A}(\phi')^2+m^4U(\phi)\right] , \tag{3.2}$$

$$T^3_3=\frac{f^2}{2A}(\phi')^2-m^4U(\phi) \ . \tag{3.3}$$

As we will show below, T^3_3 is nonzero.

A. The Einstein scalar-field equations

Let us assume that $\phi=\phi(|z|)$ so that the domain wall is static in some chosen reference frame. The equation of motion is then

$$\phi''+\frac{B'}{B}\phi'-\frac{m^4}{f^2}A\frac{\partial U}{\partial\phi}=0 \ . \tag{3.4}$$

Differentiating with respect to t we find that

$$\phi' \frac{\partial}{\partial t}\left[\frac{B'}{B}\right] - \frac{m^4}{f^2}\frac{\partial U}{\partial \phi}\frac{\partial A}{\partial t} = 0 \ . \tag{3.5}$$

The simplest solution to this equation is

$$\frac{\partial}{\partial t}\left[\frac{B'}{B}\right] = 0 \ \text{ and } \ \frac{\partial A}{\partial t} = 0 \ . \tag{3.6}$$

so that

$$B = C(t)D(|z|) \ \text{ and } \ A = A(|z|) \ . \tag{3.7}$$

It is now straightforward to derive the Einstein field equations for the metric functions C, D, and A:

$$\frac{\dot{C}}{C}\left[\frac{D'}{D} - \frac{A'}{A}\right] = 0 \ , \tag{3.8a}$$

$$\frac{\ddot{C}}{C} - \frac{D''}{D} = \frac{16\pi m^4}{m_{\rm Pl}^2} A U(\phi) \ , \tag{3.8b}$$

$$\frac{\ddot{C}}{C} - \frac{1}{2}\left[\frac{\dot{C}}{C}\right]^2 + \frac{D''}{D} - \frac{1}{2}\left[\frac{D'}{D}\right]^2 - \frac{D'A'}{DA} = -\frac{8\pi f^2}{m_{\rm Pl}^2}(\phi')^2 \ , \tag{3.8c}$$

$$\frac{D''}{D} - \frac{1}{2}\left[\frac{D'}{D}\right]^2 + \frac{A''}{A} - \left[\frac{A'}{A}\right]^2 - \frac{\ddot{C}}{C} + \frac{1}{2}\left[\frac{\dot{C}}{C}\right]^2 = -\frac{8\pi}{m_{\rm Pl}^2}[f^2(\phi')^2 + 2m^4 A U] \ , \tag{3.8d}$$

From Eq. (3.8a) we see that either $\dot{C}=0$ or $D \propto A$. In the Appendix, we show that $\dot{C} \neq 0$ for a domain-wall solution if we assume that $U(\phi) > 0$ for all ϕ. We therefore have $D = A$ where the constant is absorbed by rescaling x and y.

By separation of variables in Eq. (3.8b), we see that

$$\frac{\ddot{C}}{C} = \kappa^2 \ . \tag{3.9}$$

Here, κ is a constant with dimensions of mass and, as will soon be apparent, $\kappa^2 > 0$. Equations (3.8b)–(3.8d) can now be combined to give

$$\left[\frac{D'}{D}\right]^2 = \kappa^2 + \frac{16\pi}{3m_{\rm Pl}^2}[f^2(\phi')^2 - 2m^4 D U(\phi)] \ , \tag{3.10a}$$

$$\frac{d}{dz}\left[\frac{D'}{D}\right] = -\frac{16\pi}{3m_{\rm Pl}^2}[f^2(\phi')^2 + m^4 D U(\phi)] \ . \tag{3.10b}$$

These two equations, together with the equation of motion for the scalar field,

$$\phi'' + \frac{D'}{D}\phi' - \frac{m^4}{f^2} D \frac{\partial U}{\partial \phi} = 0 \ , \tag{3.11}$$

completely describe the domain-wall spacetime. [In actuality, only two of these equations are necessary. It is easy to show that by differentiating Eq. (3.10a) and using Eq. (3.11) one can derive Eq. (3.10b).]

From the above equations, it is straightforward to derive an expression for κ. Integrating Eq. (3.10b) and using the symmetry properties of D, ϕ, and $U(\phi)$, we find that

$$\left.\frac{D'}{D}\right|_{z=L} = -\frac{16\pi}{3m_{\rm Pl}^2}\int_0^L [f^2(\phi')^2 + m^4 D U(\phi)]dz \ . \tag{3.12}$$

On the other hand, Eq. (3.10a) implies that

$$\left[\frac{D'}{D}\right]^2 = \kappa^2 \ \text{ for } z \to \infty \ . \tag{3.13}$$

It follows that

$$\kappa = \frac{16\pi}{3m_{\rm Pl}^2}\int_0^\infty [f^2(\phi')^2 + m^4 D U(\phi)]dz \ . \tag{3.14}$$

(The overall sign of κ is irrelevant.) We see that κ is real and therefore $\kappa^2 > 0$. Returning to Eq. (3.9) we find that $C = e^{\kappa t}$ (Ref. 17).

Though analytic solutions for D and ϕ are difficult if not impossible to obtain, we can easily estimate κ. The energy density in the wall is $O(m^4)$ and the thickness of the wall is $O(f/m^2)$. We therefore expect $\kappa = O(m^2 f/m_{\rm Pl}^2)$. A more accurate approximation is found by assuming that ϕ obeys its flat-space equation of motion and using the results in Sec. II. (As we shall see in the next section, this is valid so long as $f/m_{\rm Pl} \ll 1$.) We then find that $\kappa = 4\pi\sigma/m_{\rm Pl}^2$ where σ is given in Eq. (2.10). Finally, we note that $D \to e^{\mp \kappa z}$ for $z \to \pm\infty$ and therefore the metric far from the wall is given by Eq. (1.3) in agreement with the results of Ref. 13.

B. Near zone

We now discuss the solution to the Einstein scalar-field equations in the region near $z=0$. We begin by changing to the dimensionless variable $\zeta = zf$. We then find that

$$\frac{d}{d\zeta}\left[\frac{1}{D}\frac{dD}{d\zeta}\right] = -\frac{\epsilon^2}{3}\left[\left[\frac{d\phi}{d\zeta}\right]^2 + \frac{m^4}{f^4} D U(\phi)\right] \ , \tag{3.15}$$

$$\frac{d^2\phi}{d\zeta^2} + \frac{1}{D}\frac{dD}{d\zeta}\frac{d\phi}{d\zeta} - \frac{m^4}{f^4} D \frac{\partial U}{\partial \phi} = 0 \ , \tag{3.16}$$

where $\epsilon^2 = 16\pi f^2/m_{\rm Pl}^2$. ϵ^2 is roughly equal to $\sigma T/m_{\rm Pl}^2$, where T is the thickness of the wall. In what follows, we will assume that $\epsilon^2 \ll 1$. The $\epsilon \to 0$ limit corresponds to one of the following: (1) turning off gravity ($m_{\rm Pl}^2 \to \infty$); (2) holding σ fixed and letting $T \to 0$; (3) holding T fixed and letting $\sigma \to 0$.

To first order in ϵ^2,

$$D = 1 + \epsilon^2 \Delta, \quad \phi = \phi_0 + \epsilon^2 \sigma \ , \tag{3.17}$$

where ϕ_0 satisfies Eqs. (2.5) and (2.6). Equation (3.15) to first order in ϵ^2 gives

$$\frac{d^2\Delta}{d\zeta^2} = -\frac{m^4}{f^4} U(\phi_0) \ . \tag{3.18}$$

Assuming the form for the potential given by Eq. (2.1) we see that

$$\Delta = -\tfrac{1}{2}\ln(\cosh 2m^2 z/f) . \tag{3.19}$$

Consider first the case where $z \ll f/2m^2$ so that $\Delta \simeq -m^4 z^2/f^2$. The equation of motion for a slowly moving test particle is then $\ddot{z}=\beta^2 z$, where $\beta = 4\pi^{1/2}m^2/m_{\mathrm{Pl}}$. We then find that

$$z = Ae^{\beta t} + Be^{-\beta t} , \tag{3.20}$$

where A and B are constants that are determined by the particle's initial position and velocity. Suppose, for example, that a particle starts at rest at $z=z_0$, where $0<z_0 \ll f/m^2$. The particle will be accelerated in the plus z direction (away from the center of the wall) and will reach $z=f/2m^2$ at a time $t=\beta^{-1}\ln(f/m^2 z_0)$. Note that the time scale β^{-1} is $\sim 10^2$ (10^8) sec for $m=1$ eV (10^{-3} eV) and $f=10^{15}$ GeV.

Next we consider the region where $f/2m^2 \ll z \ll m_{\mathrm{Pl}}^2/16\pi f m^2$. From the first inequality we find $\Delta \simeq -m^2 z/f$. [The second inequality ensures that the small-ϵ expansions in Eqs. (3.17) are valid.] We now find that

$$\ddot{z} = \frac{8\pi m^2 f}{m_{\mathrm{Pl}}^2} . \tag{3.21}$$

The particle is uniformly accelerated away from the wall at a rate $8\pi m^2 f/m_{\mathrm{Pl}}^2 = 2\pi\sigma/m_{\mathrm{Pl}}^2$. This is in agreement with the results of Refs. 13 and 14 and also with the results obtained from the "exterior metric" Eq. (1.2).

C. Stress-energy tensor

We conclude by discussing the stress-energy tensor for a general-relativistic wall under the assumption $\epsilon^2 \ll 1$. To order ϵ^0, $T_{\mu\nu}$ is given by Eqs. (2.4) and (2.7) and, in particular, we find that $T_{33}=0$. Let us compute T_{33} to order ϵ^2. Integrating Eq. (3.11) by parts and using the definition of T_{33} [Eq. (3.3)], we have

$$\frac{dT_{33}}{dz} = -\frac{D'}{D}\left[f^2(\phi')^2 + \frac{m^4}{f^2} DU(\phi) \right] \tag{3.22}$$

so that

$$T_{33}(z=L) = \int_L^{\infty} \frac{D'}{D}\left[f^2(\phi')^2 + \frac{m^4}{f^2} DU(\phi) \right] dz . \tag{3.23}$$

To leading order in ϵ^2 this becomes

$$T_{33}(z=L) = \frac{48\pi m^4 f^2}{m_{\mathrm{Pl}}^2} \int_L^{\infty} \Delta' U(\phi_0) dz$$
$$= -\frac{24\pi m^4 f^2}{m_{\mathrm{Pl}}^2} \frac{1}{\cosh^2 2m^2 L/f} . \tag{3.24}$$

At the center of the wall, for example, we have

$$T_{33}(z=0) = -\frac{24\pi m^4 f^2}{m_{\mathrm{Pl}}^2} = O(\epsilon^2) T_{11}(z=0) . \tag{3.25}$$

We see that the pressure orthogonal to the wall is negative and is small compared to the other terms in the stress-energy tensor so long as $\epsilon^2 \ll 1$. This pressure has a simple physical interpretation. As discussed above, freely falling test particles are accelerated away from the wall. Likewise, the scalar-field ϕ feels a repulsive force due to its own gravitational field. The pressure T_{33} precisely balances this force.

IV. CONCLUSION

If domain walls are to have anything to do with the large-scale structure of the Universe then they must be either unstable (as is the case when the walls are bounded by strings) or very light. If the walls are very light, then they will also be very thick. Previous calculations of the gravitational effects of domain walls assumed that the walls were infinitely thin. Clearly, the results from these investigations are inapplicable for problems such as the motion of test particles in the interior of the wall.

In the present calculation, we obtain the metric for the entire spacetime of a realistic domain wall (i.e., one constructed from a theory of a real scalar field that is responsible for breaking a discrete symmetry) by considering the full coupled Einstein scalar-field equations. The motion of test particles can then be studied both in the interior and exterior of the wall. An interesting result that emerged in this work is that the pressure perpendicular to the wall is nonzero.

The main drawback with the present investigation is that it assumes a very special configuration for the wall, namely that the wall is plane symmetric and static. If walls exist in the Universe then there will be closed surface walls as well as infinite walls. Furthermore, the infinite walls are likely to be curved and folded. The spacetime of a Universe with domain walls will undoubtedly be very complicated and will, we hope, be the subject of future investigations.

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APPENDIX

In this appendix, we show that $\dot{C}=0$ leads to an unphysical metric. Let us assume that $\dot{C}=0$. Equation (3.8b) then gives

$$D'' = -\frac{16\pi m^4}{m_{\mathrm{Pl}}^2} A\, DU(\phi) , \tag{A1}$$

so that $D''<0$ for all z assuming that $U(\phi)>0$ and $D>0$. By the symmetry properties of the metric, we know that $D'(z=0)=0$. Therefore, $D' \to \mp K$ for $z \to \pm\infty$ where

$$K = \frac{16\pi m^4}{m_{\mathrm{Pl}}^2} \int_0^{\infty} A\, DU(\phi) dz \tag{A2}$$

and $K>0$. From this we would conclude that D changes sign and goes negative for large $|z|$ leading to an unphysical metric.

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