Will the observation of $D_s^+ \to \omega\pi^+$ be a signal for the annihilation mechanism?

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In the factorization model $D_s^+ \to \omega\pi^+$ is forbidden due to the absence of a spectator term, conserved vector current, and the absence of second-class axial-vector currents. We show that $B(D_s^+ \to \omega\pi^+)$ up to 3% can nevertheless be generated by final-state interactions. Hence a large ($\approx 3\%$) branching ratio for $D_s^+ \to \omega\pi^+$ may not necessarily be a signal for the annihilation mechanism in this decay mode.

I. INTRODUCTION

Considerable progress has been made in our understanding of charmed meson (D^0, D^+, D_s^+) decays in recent years. There exist several theoretical models[1–8] to explain hadronic two-body decays of charmed mesons. These models broadly agree and are reasonably successful in explaining experimental data.[9] A point of contention is the contribution of the annihilation process. It used to be thought[10] that the observation of $D^0 \to \bar{K}^0\phi$ would establish the existence of an annihilation amplitude. It has now been shown[3,11,12] that final-state interactions can generate $B(D^0 \to \bar{K}^0\phi)$ at the level of $\approx 1\%$ in the absence of the annihilation term. It has recently been argued[13] that a diagrammatic analysis[1] of two-body decays of D^+, D^0, and D_s^+ requires a significant annihilation term. Based on this it is predicted[13] that $B(D_s^+ \to \omega\pi^+) \gtrsim B(D_s^+ \to \phi\pi^+)$. Since[9] $B(D_s^+ \to \phi\pi^+) \approx 3\%$, it implies a significantly large branching ratio for $D_s^+ \to \omega\pi^+$. The estimates of other models for $B(D_s^+ \to \omega\pi^+)$ differ greatly. Blok and Shifman[7] include nonfactorizable contributions, but ignore factorizable annihilation, and estimate $B(D_s^+ \to \omega\pi^+) = 0.3\%$.

The mode $D_s^+ \to \omega\pi^+$ is particularly interesting for the following reason. In a factorization model, there is no contribution from the spectator diagram for this mode. Although an annihilation contribution would appear to be possible at the quark level, it in fact vanishes; the vector part of the $(\bar{u}d)$ current makes no contribution due to the conserved-vector-current (CVC) hypothesis and a first-class axial-vector current cannot connect the vacuum to a $\omega\pi^+$ state which has even G parity. The question then arises: What should be the magnitude of $B(D_s^+ \to \omega\pi^+)$?

II. METHOD AND CALCULATION

We investigate this problem in a factorization model. In the absence of final-state interactions, as argued above, $B(D_s^+ \to \omega\pi^+)$ should be zero. However, final-state interactions can change the picture substantially; $\omega\pi^+$ could be generated by coupling to other final states. Since the strong interactions responsible for final-state interactions conserve G parity, $\omega\pi^+$ will couple only to G-even states, i.e., to $\phi\pi^+$ and $|K^*K\rangle_{G=+1}$; Even though $\phi\pi^+$ has even G parity, $\phi\pi^+ \leftrightarrow \omega\pi^+$ is disallowed by Okubo-Zweig-Iizuka (OZI) rule.[14] In the K^*K channel, the even and odd G-parity states are given by the symmetric and antisymmetric combinations, respectively:

$$|K^*K\rangle_{S,A} = \frac{1}{\sqrt{2}}(|K^{*+}\bar{K}^0\rangle \pm |K^+\bar{K}^{*0}\rangle) .$$

Hence, the symmetric $|K^*K\rangle_S$ can couple to the $\omega\pi^+$ state. This interchannel coupling is achieved through the unitarization scheme described below.

In two-body scattering of n (open) coupled channels a convenient parametrization of a unitary S matrix is, in terms of the K matrix,

$$\mathbf{S}(s) = [1 - i\mathbf{K}(s)]^{-1}[1 + i\mathbf{K}(s)] , \tag{1}$$

where $\mathbf{K}(s)$ is an $n \times n$ Hermitian matrix. The ununitarized amplitudes $\mathbf{A}^0(s)$ ($\sqrt{s}$ is equal to the charmed-meson mass) are unitarized through the prescription[11,15]

$$\mathbf{A}^u(s) = [1 - i\mathbf{K}(s)]^{-1}\mathbf{A}^0(s) . \tag{2}$$

The normalization is such that in the limit the strong interactions are turned off $[\mathbf{K}(s) \to 0]$, the unitarized amplitudes $\mathbf{A}^u(s)$ become equal to the un-unitarized amplitudes $\mathbf{A}^0(s)$.

Assuming factorization, the un-unitarized amplitudes are generated through the Cabibbo-angle-favored Hamiltonian[3]

$$H_w = \frac{G_F}{\sqrt{2}}\cos^2\theta_C[C_1(\bar{u}d)_H(\bar{s}c)_H + C_2(\bar{s}d)_H(\bar{u}c)_H] , \tag{3}$$

where θ_C is the Cabibbo angle and the subscript H denotes hadron field operators. C_1 and C_2 are related to the short-distance QCD factors C_+ and C_- by

$$(C_1, C_2) = \tfrac{1}{2}[(C_+ \pm C_-) + \xi(C_+ \mp C_-)] , \tag{4}$$

where ξ, the color factor [$\frac{1}{3}$ for SU(3)$_c$], is treated here as a free parameter. Our C_1 and C_2 are the coefficients a_1 and a_2 of Ref. 3. The un-unitarized amplitudes derived from Eq. (3) are listed in Table I. We have used the form factors evaluated in Refs. 3 and 16. We have also includ-

TABLE I. Multiply each amplitude by $G_F/\sqrt{2}\cos^2\theta_C C_2$. R_s is an annihilation parameter. We use the normalization from Ref. 3: $f_K=0.162$ GeV, $f_\pi=0.133$ GeV, $g_V=0.221$ GeV, $h_\phi=0.7$, $h'_{K^*}=0.634$, $h_{K'}=0.692$.

Mode	Un-unitarized amplitudes
$D_s^+\to K^{*+}\bar{K}^0$	$[4f_K m_{K^*} h'_{K^*}/(1-m_K^2/m_D^2)-R_s]/2$
$D_s^+\to K^+\bar{K}^{*0}$	$[4g_V m_{K^*} h'_K/(1-m_{K^*}^2/m_{D^*}^2)+R_s]/2$
$D_s^+\to\rho^+\pi^0$	$R_s/\sqrt{2}$
$D_s^+\to\rho^0\pi^+$	$-R_s/\sqrt{2}$
$D_s^+\to\phi\pi^+$	$2\frac{C_1}{C_2}f_\pi m_\phi h_\phi$
$D_s^+\to\omega\pi^+$	0

ed a weak annihilation amplitude, denoted by R_s, treated here as a free parameter. The un-unitarized amplitudes depend on C_1, C_1/C_2, and the annihilation parameter R_s. For a chosen value of the ratio C_1/C_2, we evaluate C_2 from Eq. (4), by using the perturbative constraint[17] $C_+^2 C_-\approx 1$. This leads to

$$C_2=\frac{(1+\xi)^{2/3}(1-\xi)^{1/3}}{(C_1/C_2+1)^{2/3}(C_1/C_2-1)^{1/3}}\ . \tag{5}$$

Once the ratios C_1/C_2 and ξ are chosen, C_2 is calculated through (5) and used in the amplitudes $A^0(s)$ shown in Table I. The unitarized amplitudes $A^u(s)$ are generated through (2), and finally the branching ratios are calculated from

$$B(D_s^+\to VP)=\tau_{D_s}\frac{|A^u(D_s^+\to VP)|^2k^3}{8\pi m_V^2}\ . \tag{6}$$

We now describe the parametrization of the K matrix. A coupled-channel analysis for $D_s^+\to VP$ has already been performed in Ref. 11. The mode $D_s^+\to\omega\pi^+$ was excluded in the discussion in Ref. 11. In the G-even state, we extend the K matrix of Ref. 11 from a 2×2 matrix to a 3×3 real-symmetric matrix, thereby including the $\omega\pi^+$ mode, as follows:

$$\mathbf{K}=\begin{bmatrix} k_1 b & (k_1k_2)^{1/2}c & (k_1k_3)^{1/2}f \\ (k_1k_2)^{1/2}c & k_2 a & (k_2k_3)^{1/2}d \\ (k_1k_3)^{1/2}f & (k_2k_3)^{1/2}d & k_3 e \end{bmatrix} \tag{7}$$

with channel labels $i=(1,2,3)$ belonging to $\phi\pi^+$, $|K^*K\rangle_S$, and $\omega\pi^+$, respectively. The parameters a, b, c, d, e, and f are chosen to be energy independent (zero range approximation[18]), since no known resonances with $G=+1$ and spin zero appear to exist; k_1, k_2, and k_3 are the c.m. momenta in the three channels, respectively. Since $\pi^+\phi\leftrightarrow\pi^+\phi$ and $\omega\pi^+\leftrightarrow\pi^+\phi$ transitions are disallowed by the OZI rule, we set b and f equal to zero as an approximation. Clearly, this disallows the OZI-violating transitions in the lowest order in the K matrix.

In the $G=-1$ channel, there exists an unconfirmed π-like resonance at 1770 MeV, i.e., close to the D_s^+ mass. We, therefore, parametrize the K matrix in G-odd state through a resonant form

$$\mathbf{K}(s)=\frac{1}{m_R^2-s}\begin{bmatrix} k_1\Gamma_{11} & (k_1k_2)^{1/2}\Gamma_{12} \\ (k_1k_2)^{1/2}\Gamma_{12} & k_2\Gamma_{22} \end{bmatrix} \tag{8}$$

with $m_R=1770$ MeV, the total width $\Gamma_R=300$ MeV. The channel labels $i=(1,2)$ belong to $\rho\pi$ and $|K^*K\rangle_A$, respectively.

Our model K matrix has four parameters a, c, d, and e in the G-even state and one parameter Γ_{11} in the G-odd state. The reduction of the number of parameters in Eq. (8) is accomplished by requiring factorization for the T matrix derived from Eq. (8) and fixing the total width $\Gamma_R=300$ MeV. The reader is referred to Ref. 11 for details.

We vary the parameters of our model and search for fits to ARGUS (Ref. 9) and E691 data.[9] For $(d,e)=0$, the $\omega\pi^+$ channel decouples from the other two channels and one gets $B(D_s^+\to\omega\pi^+)=0$. For fits to data, in the case when $D_s^+\to\omega\pi^+$ is decoupled from the other two channels, the reader is referred to Ref. 11. Because of the large number of parameters, an exact branching ratio for $D_s^+\to\omega\pi^+$ cannot be predicted. However, we find that, for reasonable values of d and e, it is possible to produce $B(D_s^+\to\omega\pi^+)$ up to 3%, keeping the other branching ratios (i.e., for $\bar{K}^{*0}K^+,\rho^0\pi^+,\phi\pi^+$ modes) within the experimental limits.

III. RESULTS AND DISCUSSION

Note that in our model, $D_s^+\to\omega\pi^+$ is being generated via final-state interactions only and not directly from annihilation in the $\omega\pi$ channel. We do, however, include an annihilation parameter in the K^*K amplitude, which feeds into the $\omega\pi^+$ final state. In the following discussion the reader is reminded that by annihilation term we mean

TABLE II. Fits to ARGUS data (Ref. 9) for Cabibbo-angle-favored $D_s^+\to VP$ decays. The various parameters in the model are selected to maximize $B(D_s^+\to\omega\pi^+)$ consistent with data. The parameters d,e take values greater than a,c and lie in the range 0.1–1 GeV^{-1}. All branching ratios are in percent.

Branching ratio	Theory $\xi=0$ $C_1/C_2=-2.0$ $R_s=0.15$ GeV^2	Theory $\xi=0$ $C_1/C_2=-2.2$ $R_s=0.17$ GeV^2	Theory $\xi=\frac{1}{3}$ $C_1/C_2=-2.0$ $R_s=0.15$ GeV^2	Theory $\xi=\frac{1}{3}$ $C_1/C_2=-2.2$ $R_s=0.17$ GeV^2	ARGUS data
$B(D_s^+\to\bar{K}^{*0}K^+)$	3.86	3.71	4.32	4.13	5.0±1.3
$B(D_s^+\to\rho^0\pi^+)$	0.59	0.56	0.66	0.63	<0.77
$B(D_s^+\to\phi\pi^+)$	3.57	3.45	4.00	3.83	3.2±0.7±0.5
$B(D_s^+\to\omega\pi^+)$	1.66	0.74	1.86	0.83	

TABLE III. Fits to E691 data (Ref. 9) for Cabibbo-angle-favored $D_s^+ \to VP$ decays. The various parameters in the model are selected to maximize $B(D_s^+ \to \omega\pi^+)$ consistent with data. The parameters d,e take values greater than a,c and lie in the range 0.1–1 GeV^{-1}. The values in parentheses correspond to $R_s = 0.0\ \text{GeV}^2$. All branching ratios are in percent.

Branching ratio	Theory $\xi=0$		Theory $\xi=\frac{1}{3}$		E691 data
	$C_1/C_2=-2.0$ $R_s=0.10\ \text{GeV}^2$	$C_1/C_2=-2.2$ $R_s=0.11\ \text{GeV}^2$	$C_1/C_2=-2.0$ $R_s=0.09\ \text{GeV}^2$	$C_1/C_2=-2.2$ $R_s=0.11\ \text{GeV}^2$	
$B(D_s^+ \to \bar{K}^{*0}K^+)$	2.24	2.11	2.13	2.10	2.6±0.5
	(2.30)	(2.11)	(2.57)	(2.14)	
$B(D_s^+ \to \rho^0\pi^+)$	0.27	0.24	0.25	0.27	<0.28
	(0.006)	(0.004)	(0.006)	(0.005)	
$B(D_s^+ \to \phi\pi^+)$	3.64	3.43	3.52	3.43	3.5±0.8
	(3.76)	(3.43)	(4.21)	(3.51)	
$B(D_s^+ \to \omega\pi^+)$	2.48	1.73	2.75	2.02	
	(1.84)	(0.89)	(2.05)	(1.15)	

the annihilation parameter R_s (see Table I) which appears in the decay amplitudes for $D_s^+ \to K^{*+}\bar{K}^0$ and $\bar{K}^{*0}K^+$. Note also that the un-unitarized amplitude for $D_s^+ \to \omega\pi^+$ is zero (Table I).

A fit to ARGUS data[9] requires that the annihilation parameter R_s be nonzero, even if we require $B(D_s^+ \to \omega\pi^+)=0$. Although fits to E691 data[9] may be obtained for vanishing R_s, nonvanishing R_s is also allowed by the data; larger $B(D_s^+ \to \omega\pi^+)$ being obtained for $R_s \neq 0$. The *maximum* value of $B(D_s^+ \to \omega\pi^+)$ consistent with a fit to ARGUS data requires an annihilation term $\simeq 40\%$ of the spectator term in $D_s^+ \to K^{*+}\bar{K}^0$ and that for E691 $\simeq 25\%$ of the spectator term. For E691 data, even *with the annihilation parameter set equal to zero,* final-state interactions *alone* can generate $B(D_s^+ \to \omega\pi^+)$ up to about 2%.

In Table II we show a fit to the ARGUS data (for $\xi=0$ and $\xi=\frac{1}{3}$), where the various parameters are chosen such that $B(D_s^+ \to \omega\pi^+)$ is at its maximum value. Table III lists the same for E691 data. Note that the E691 data, where $B(D_s^+ \to \bar{K}^{*0}K^+)$ is lower than in ARGUS data, allows a larger value of $B(D_s^+ \to \omega\pi^+)$ than the ARGUS data. This presumably results from more of the $\bar{K}^{*0}K^+$ rate being siphoned off into the $\omega\pi^+$ mode.

Lastly, a word about our choice of the ratio C_1/C_2. We note that $B(D_s^+ \to \omega\pi^+)$ falls as C_1/C_2 increases in magnitude. In Ref. 3, from a fit to $D \to K\pi$ data the ratio C_1/C_2 was estimated to lie in the range $-3.3 \le C_1/C_2 \le -2.0$. We chose C_1/C_2 close to the maximum of this range in order to generate as large a value for $B(D_s^+ \to \omega\pi^+)$ as possible. A next-to-leading-log (NLL) calculation with reasonable values of QCD parameters μ and Λ also gives[11] C_1/C_2 roughly in the range $-3.3 \le C_1/C_2 \le -2.0$ for $\xi=0$. For $\xi=\frac{1}{3}$ a value of C_1/C_2 in the same range can be secured[11] in an NLL calculation only by raising Λ to about 0.5 GeV or by lowering μ below 1.2 GeV.

In summary, we find that in a factorization model, where the quark level amplitude for $D_s^+ \to \omega\pi^+$ is zero, final-state interactions can generate $B(D_s^+ \to \omega\pi^+)$ as large as $\sim 3\%$. An observation of a signal at this level will not necessarily constitute an evidence for an annihilation term in the decay amplitude for $D_s^+ \to \omega\pi^+$ at the quark level. A measurement of this branching ratio will be very desirable.

Note added in proof. In a revised version of Ref. 13, the prediction of $B(D_s^+ \to \omega\pi^+)$ has been lowered to $\frac{1}{10}B(D_s^+ \to \phi\pi^+)$. See L. L. Chau and H. Y. Cheng, University of California, Davis, Report No. UCD-88-12 (unpublished). We thank Dr. Cheng for bringing this report to our attention.

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