Coupled-channel effects in the decay $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$

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The spectrum of the decay $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ is fitted assuming interference between a multipole amplitude containing an Adler zero and a constant nonmultipole amplitude. It is argued that the coupling to b-flavored channels may be large enough to account for the nonmultipole effect.

The dipion spectra of the transitions $\psi'\to J/\psi\pi^+\pi^-$ (Ref. 1) and $\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-$ (Ref. 2) are described well by a model[3] in which the heavy quarks emit two gluons by means of a QCD multipole coupling and in which the conversion of gluons into pions proceeds in accordance with current algebra. The most significant feature of this model is the fact that the current-algebra amplitude is proportional to $M_{\pi\pi}^2$; i.e., it contains an "Adler zero." The resulting distribution $d\Gamma/dM_{\pi\pi}\sim M_{\pi\pi}^5\times$(phase space), giving the pronounced peak at large $M_{\pi\pi}$ that is observed in these decays.

The spectrum of the decay $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ (Refs. 4 and 5) however, have a peculiar doubled-peaked structure. The most recent CLEO data[5] have greatly improved the statistics for this decay, so that the shape of the spectrum can be regarded as well established. Lipkin and Tuan[6] have suggested that this decay proceeds through coupling $B\bar{B}$, $B^*\bar{B}^*,\ldots,$ intermediate states, e.g., through the channel $\Upsilon(3S)\to B\bar{B}\to B^*\bar{B}\pi\to B\bar{B}\pi\pi\to\Upsilon(1S)$. They proposed that the double peak arises as follows. Working in the heavy-quarkonium limit, in which recoil of the final-quarkonium state is neglected, they argue that the amplitude will contain a term proportional to $\mathbf{p}_1\cdot\mathbf{p}_2\propto\cos\theta_{12}$, where θ_{12} is the angle between the pions ($\mathbf{p}_1$ and $\mathbf{p}_2$ are the pion three-momenta), times some coefficient that is a function of the kinematic invariants. If the coefficient function were a constant, then the double peak in the distribution $d\Gamma/d\cos\theta_{12}\propto\cos\theta_{12}^2$ would result in a double peak in the closely related distribution $d\Gamma/dM_{\pi\pi}$.

The suggestion of the importance of $B\bar{B}$ intermediate states has a change of being right, since the $\Upsilon(3S)$ lies closer to the b-flavor threshold than the 1S or 2S and so is more likely to have a significant coupling to the flavored sector. However, a quantitative fit to the data was not given in Ref. 6, and the assumptions there that the $\cos\theta_{12}$ term dominates, and that all of the $M_{\pi\pi}$ dependence comes from the $\cos\theta_{12}$, rather than from the unknown function multiplying it, seem somewhat arbitrary. Also, the authors of Ref. 6 did not give even an order-of-magnitude estimate of the corresponding rate; one might worry that the effect is completely negligible due to the small coupling of, say, the $\Upsilon(1S)$ to the flavored sector.

Moreover, the multipole mechanism must continue to operate, and for the decay $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ it is known that most models for the quarkonium part of the amplitude predict a total multipole rate that is an order of magnitude too large.[7] (An exception is one of the models of Kuang and Yan.[3]) Thus, if the coupled-channel mechanism is thought of as dominating the multipole one, then the rate will be even larger, exacerbating the problem of the total rate for $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$.

In view of this, it seems likely that what is needed is for the multipole and coupled-channel amplitudes to interfere. That is, the amplitude could have the form $F_{\pi\pi}^2 F(M_{\pi\pi}^2)-B(M_{\pi\pi}^2,M_{\Upsilon\pi}^2)$, where the first term ($A$ is a known constant) is the multipole amplitude exhibiting the Adler zero and the second term, associated with the coupled-channel process, is some unknown function of the kinematic invariants $M_{\pi\pi}^2$ and $M_{\Upsilon\pi}^2$. The first term in the amplitude is modified by the form factor for $\pi^+\pi^-$ final-state interactions:[8]

$$F(q^2)=\frac{g_{\sigma\pi\pi}^2+\lambda(m_\sigma^2-q^2)}{\{(m_\sigma^2-q^2)[1-\lambda\zeta(q^2)]-g_{\sigma\pi\pi}^2\zeta(q^2)\}(8\pi f_\pi^2)^{-1}q^2}\,, \tag{1}$$

where $f_\pi=0.094$ GeV is the pion decay constant, $\lambda=-0.73$, $g_{\sigma\pi\pi}=0.64$ GeV, $m_\sigma=0.71$ GeV, and the function $\zeta(q^2)$ is

$$\zeta(q^2)=\frac{2}{\pi}\left[1-\left[1-\frac{4m_\pi^2}{q^2}\right]^{1/2}\times\left[\ln\frac{\sqrt{q^2}+(q^2-4m_\pi^2)^{1/2}}{2m_\pi}-\frac{i\pi}{2}\right]\right]. \tag{2}$$

In this case the spectrum has the form

$$\frac{d\Gamma}{dM_{\pi\pi}}=|AM_{\pi\pi}^2F(M_{\pi\pi}^2)-B|^2\times M_{\pi\pi}(M_{\pi\pi}^2-4m_\pi^2)^{1/2}(E_f^2-M_f^2)^{1/2}\,, \tag{3}$$

where M_i and M_f are, respectively, the initial- and final-quarkonium masses, and $E_f=(M_i^2-M_f^2-M_{\pi\pi}^2)/2M_{\pi\pi}$ is the energy of final-quarkonium state in the $\pi^+\pi^-$ rest frame. The constant A is completely fixed,[17] given a model for the quarkonium part of the amplitude.

In the absence of a dynamical model for the coupled-channel effects, let us assume B to be a (complex) constant. (We are simply assuming that the dependence of B on the variable $M^2_{\pi\pi}$ is not very strong over the range considered. In terms of the variables $\mathbf{p}_1, \mathbf{p}_2$ appropriate to the heavy-quarkonium limit, we assume that the amplitude is not dominated by the term containing $\mathbf{p}_1 \cdot \mathbf{p}_2 \propto \cos\theta_{12}$, but that the other tensor structures conspire to give a distribution in $M_{\pi\pi}$ that is more or less flat.) We obtain a reasonable fit (Fig. 1) to the $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$ data[5] using $\mathrm{Re}B/A = 0.2196$, $\mathrm{Im}B/A = -0.2983$, with a χ^2 per degree of freedom of 1.27. (The χ^2 was computed by averaging over the bins, to take into account the fact that the highest-energy bin extends beyond the phase-space boundary.) While the introduction of the constant B may seem like a naive way to obtain a zero in the amplitude other than at $M_{\pi\pi}=0$, the fact that we completely know the multipole amplitude gives us an idea of how big the coupled-channel amplitude should be. The conclusion is that in order to fit the $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$ spectrum the coupled-channel amplitude must be of the same order of magnitude as the multipole amplitude, or a bit smaller.

Thus it is of interest to ask if it is plausible for the coupled-channel amplitude to be as large as the multipole amplitude. Since only a crude estimate of the coupled-channel rate will be possible, let us restrict ourselves, in doing the phase space, to the heavy-quarkonium limit where the rate is given by[7] $\Gamma \approx |A|^2 \Delta^3/48\pi^3$, where the amplitude A depends on the process considered, and $\Delta = M_i - M_f$. We also omit the effect of $\pi^+\pi^-$ final-state interactions. For the multipole process we have[3,7] $A \approx I_{\mathrm{mult}} M^2_{\pi\pi} \approx I_{\mathrm{mult}} \Delta^2$, with $|I_{\mathrm{mult}}| \sim 1\ \mathrm{GeV}^{-3}$ for a typical $b\bar{b}$ transition. Hence

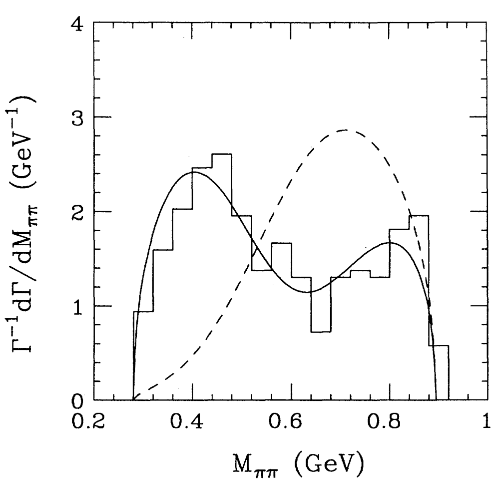

FIG. 1. Spectrum of the decay $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$. Dashed line: current-algebra result modified by $\pi^+\pi^-$ final-state interactions (Ref. 8). Solid line: result of interference of current-algebra amplitude with a constant nonmultipole amplitude, Eq. (3) with $\mathrm{Re}B/A = 0.2196$, $\mathrm{Im}B/A = -0.2983$.

$$\Gamma_{\mathrm{mult}} \approx \frac{|I_{\mathrm{mult}}|^2 \Delta^7}{48\pi^3} \tag{4}$$

[~ 10 keV for $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$].

For the coupled-channel process we have to take account of the probabilities that the initial and final Υ states lie in the $B\bar{B}$ sector; let us denote these by Z_i and Z_f. The corresponding amplitude $A \sim \langle B|B^*\pi\rangle\langle B^*|B\pi\rangle$. Since for $B^* \to B\pi$ we are dealing with a transition from a (virtual) vector state to a pair of pseudoscalars, we have $\langle B^*|B\pi\rangle \sim \mathrm{const} \times \mathbf{p} \sim \mathrm{const} \times \Delta$, where $\mathbf{p}$ is a typical pion momentum. Following the method of Ref. 9, we estimate the magnitude of the constant by assuming that the amplitude is independent of the heavy-quark mass, so that $\langle B^*|B\pi\rangle \approx \langle D^*|D\pi\rangle \approx \langle K^*|K\pi\rangle \equiv C\mathbf{p}$. (It is assumed that the strange quark can be treated as heavy.)

Our normalization is such that the rate for $K^* \to K\pi$ is (up to an isospin factor which we omit for the present purpose)

$$\Gamma(K^* \to K\pi) = \frac{|\mathbf{p}|^3\ |(2M_{K^*} 2E_K 2E_\pi)^{1/2} C|^2}{24\pi M^2_{K^*}} . \tag{5}$$

From the observed value of the rate for $K^* \to K\pi$, one extracts[9] $|C| \sim 10\ \mathrm{GeV}^{-3/2}$. Then,

$$\Gamma_{\mathrm{CC}} \sim \frac{Z_i Z_f |I_{\mathrm{CC}}|^2 \Delta^7}{48\pi^3} , \tag{6}$$

with $|I_{\mathrm{CC}}| \sim 100\ \mathrm{GeV}^{-3}$, so $\Gamma_{\mathrm{CC}}/\Gamma_{\mathrm{mult}} \sim 10^4 Z_i Z_f$ for two-pion transitions between S-wave Υ states.

Of course, this very crude estimate of Γ_{CC} may well be off by an order of magnitude in either direction, but the important point is that I_{CC} may be considerably larger than I_{mult}, and this may counteract the effect of having small probabilities Z_i, Z_f. Unfortunately, these probabilities have not been tabulated in the literature and are likely to be highly model dependent. In the case of charmonium, Ref. 9 gives the total probabilities that the J/ψ and ψ' lie in the flavored sector as 0.034 and 0.209, respectively, while in other studies this mixing with the flavored continuum is found to be much smaller. (Some coupled-channel models are reviewed in Ref. 10.)

At any rate, we see that in the Υ's the coupled-channel effect might well be large enough to interfere with the multipole amplitude, as long as $Z_{1S}Z_{3S}$ is not much smaller than 10^{-4}, which does not seem unreasonable. It would be useful to see explicit results for the probabilities (or the corresponding complex mixing coefficients) in various coupled-channel models.

In conclusion, we can consider the following picture of why the coupled-channel effect has been seen only in the decay $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$. It is possible that the effect is present to some extent in the decays $\psi' \to J/\psi\pi^+\pi^-$ and $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$ (and may account for the discrepancy[8] between the theoretical and observed spectra near threshold), but is largely overwhelmed by the multipole amplitude. For $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$, however, the multipole amplitude is somewhat suppressed, perhaps due to the zeros in the $3S$ wave function,[7] and so the multipole and coupled-channel amplitudes can interfere, giving the observed double-peaked structure.

This picture opens the possibility that the coupled-channel mechanism might also be seen in other decays, such as $\Upsilon(3S)\to\Upsilon(1^1P_1)\pi^+\pi^-$ (Ref. 11), $\Upsilon(1^3D_1)\to\Upsilon\pi^+\pi^-$ (Ref. 7), and $\psi(3770)\to J/\psi\pi^+\pi^-$ (Ref. 12), for which the multipole amplitude is suppressed for other reasons. Further theoretical study of the small mixings between $b\bar{b}$ states and states with open flavor, as well as of the dynamics of the mechanism proposed by Lipkin and Tuan, may shed light on this possibility.

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