Radiative muon capture in hydrogen and nucleon excitation

Douglas S. Beder
Department of Physics, University of British Columbia, Vancouver, British Columbia, Canada V6T 2A6

Harold W. Fearing
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3
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We extend our previous calculations of radiative muon capture on a nucleon and present detailed calculations of the role of the $\Delta(1232)$ using an improved Δ-nucleon-γ vertex and for a variety of values of the induced pseudoscalar coupling g_P. We also present calculations of the photon-muon spin asymmetry and examine effects of the $\Delta(1232)$ there.

I. INTRODUCTION

Motivated by imminent precision experiments on radiative muon capture in hydrogen[1] which are planned at TRIUMF and which are intended to improve our knowledge of the induced pseudoscalar coupling g_P appearing in the weak current, we have previously examined effects of the $\Delta(1232)$, $J^\pi=\frac{3}{2}^+$, nucleon resonance on the photon spectrum in the radiative muon-capture process.[2] We found there that predicted photon spectra are modified by amounts of up to 8% at the high-energy end of the spectrum by nucleon excitation amplitudes involving the $\Delta(1232)$. Reliable knowledge of these effects is essential to extraction in a precision way of weak-interaction details such as the induced pseudoscalar strength g_P.

In this Brief Report we extend these calculations by considering three features in further detail: (1) We consider an improved description of the $\Delta N\gamma$ vertex which properly generates both magnetic dipole and electric quadrupole couplings as determined from fits to pion photoproduction data;[3] (2) we examine the magnitude of the $\Delta(1232)$ effects for various values of g_P to see if there is a correlation between the size of these effects and the value of g_P; (3) we additionally consider the photon asymmetry to see if there is any increased sensitivity there to such effects.

II. FORMALISM

We use a completely relativistic formalism based in principle on an effective Lagrangian. In practice the amplitudes are obtained from a standard set of Feynman diagrams. For the main terms, consisting of radiation from the muon, proton, neutron, and intermediate pion generating the induced pseudoscalar term, and a gauge term, the amplitude is given explicitly in Ref. 4. We use the notation of that paper throughout. The Δ contributions were included[2] using a standard Rarita-Schwinger formalism. Thus the virtual Δ is described via the usual covariant spin-$\frac{3}{2}$ Feynman propagator with a complex mass. Such a propagator exhibits not only resonant $J=\frac{3}{2}$ propagation but also a $J=\frac{1}{2}$ nonresonant contribution. Such contributions are well known and required, for example, in descriptions of the low-energy $\pi^- p\to\gamma n$ process. They follow from the requirement of relativistic covariance, which dictates the inclusion of both time orderings in the propagators.[5] The rationale for such an approach and the methods we used for determining the couplings, and particularly the signs of the Δ couplings are described in Ref. 2. Thus the contribution of the Δ to the matrix element is given by Eq. (2) of Ref. 2 which for orientation purposes we reproduce here:

$$\begin{aligned}M_{fi}^{\Delta}=&-\epsilon_\mu L_\alpha \bar{u}_n \Gamma_{\rm EM}^{\delta\mu}(k)P_{\delta\beta}(n+k)\Gamma_{\rm wk}^{\beta\alpha}(n+k-p,n+k)u_p\\&-\epsilon_\mu L_\alpha \bar{u}_n \Gamma_{\rm wk}^{\beta\alpha}(p-k-n,p-k)\\&\times P_{\beta\delta}(p-k)\Gamma_{\rm EM}^{\delta\mu}(k)u_p\ .\end{aligned}\tag{1}$$

Here ϵ_μ and L_α are the photon polarization and the leptonic current, respectively, P is the Δ propagator, $\Gamma_{\rm wk}$ is the weak $N\Delta$ vertex, and $\Gamma_{\rm EM}$ is the $\gamma N\Delta$ vertex.

With regard to the $\gamma N\Delta$ vertex $\Gamma_{\rm EM}(k)$, we used previously a simple form containing a single coupling $g_{\gamma\Delta N}$ which we took to have the numerical value[3] $g_{\gamma\Delta N}=-2.4/m$, with m the nucleon mass. This is not the most general form for the vertex and is in fact inadequate to represent the strong suppression of the electric quadrupole excitation of the nucleon which appears in pion photoproduction and which has been recently discussed, for example, by Davidson, Mukhopadhyay, and Wittman.[3]

We thus modified our calculation to use the more general form for the $\gamma N\Delta$ vertex given in Ref. 3. Specifically we replace $\Gamma_{\rm EM}^{\delta\mu}(k)$ in Eq. (4) of Ref. 2 by the expression

$$\begin{aligned}\Gamma_{\rm EM}^{\delta\mu}(k,p)=&g_{\gamma\Delta N}^{(1)}(k^\beta\gamma_\beta g^{\delta\mu}-k^\delta\gamma^\mu)\gamma_5\\&+g_{\gamma\Delta N}^{(2)}(k^\delta p^\mu-k\cdot p\, g^{\delta\mu})\gamma_5\ .\end{aligned}\tag{2}$$

Then in Eq. (1) above the first $\Gamma_{\rm EM}^{\delta\mu}(k)$ is replaced by $\Gamma_{\rm EM}^{\delta\mu}(k,-n)$ and the second by $\Gamma_{\rm EM}^{\delta\mu}(k,p)$. The structure of the first term of $\Gamma_{\rm EM}^{\delta\mu}(k,p)$ is the same as that used before for $\Gamma_{\rm EM}(k)$ and the coupling $g_{\gamma\Delta N}^{(1)}$ is the same as $g_{\gamma\Delta N}$ used before and is taken as $-2.4/m$ from Ref. 3. The new coupling $g_{\gamma\Delta N}^{(2)}$ is taken as $-1.4/m^2$ also from

Ref. 3. Both values are obtained from fits to pion photoproduction and generate a small electric quadrupole contribution.[3] If we take $g^{(2)}_{\gamma\Delta N} = g^{(1)}_{\gamma\Delta N}/m_\Delta$ where m_Δ is the Δ mass, we suppress the quadrupole contribution exactly, as can be seen from the explicit expressions for the multipole amplitudes.[3] This choice leads to the characteristic angular distribution $(2+3\sin^2\theta)$ corresponding to pure $M1$ excitation, provided just the s-channel pole is evaluated at resonance. It thus provides a useful limiting case against which to check our numerical computations.

For consistency with the conserved-vector-current (CVC) theory we must at the same time modify the weak Δ-nucleon vertex given in Eq. (5) of Ref. 2. Thus we add the same structure as in Eq. (2) above to it, replacing $g_{V\Delta N}$ by $g^{(1)}_{V\Delta N}$ and adding a coupling $g^{(2)}_{V\Delta N}$. In both cases the conserved-vector-current hypothesis implies that the weak couplings are the negatives of the electromagnetic ones.

III. PHOTON SPECTRUM

Using our modified coupling for the $\gamma N\Delta$ vertex we have recalculated photon spectra for various muon-nucleon spin states including the statistical combination, the triplet state, and the ortho $p\mu p$ molecular state appropriate for capture in liquid hydrogen. In Fig. 1 we first show typical spectra as a function of the induced pseudoscalar weak coupling g_P without contributions from the Δ. We use the ratio of g_P to its standard value, denoted by R, to label the various cases. The well-known sensitivity of the photon spectrum to g_P is evident. Note however that for the ortho molecular $(p\mu p)_{1/2}$ case one can scarcely distinguish between the $R=-1$ and $R=+1.5$ spectra.

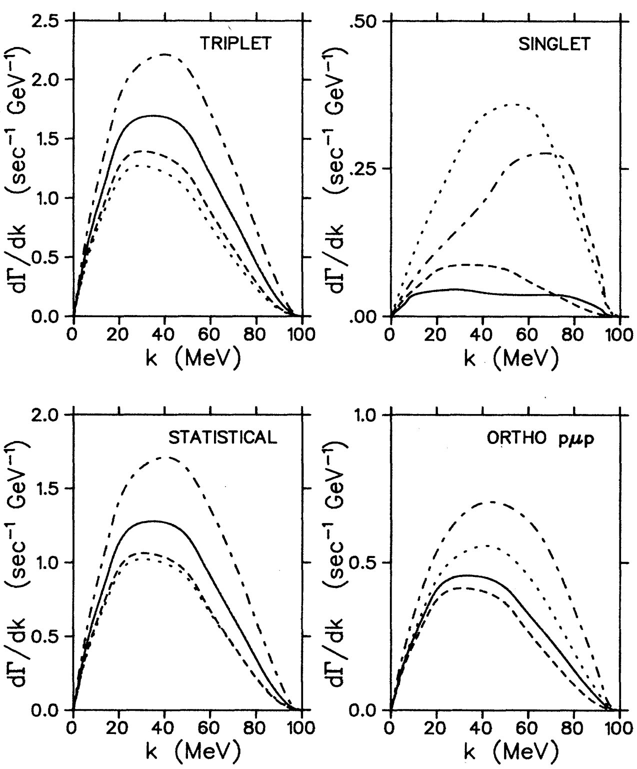

FIG. 1. Rates $d\Gamma/dk$ vs photon momentum k for radiative muon capture in hydrogen in units of $\sec^{-1}\mathrm{GeV}^{-1}$ for various ratios R of the induced pseudoscalar coupling constant g_P to its standard Goldberger-Treiman value $6.6g_A$. The four parts correspond to the triplet, singlet, statistical, and $p\mu p$ ortho molecular cases. The dotted, dashed, solid, and dot-dashed lines correspond, respectively, to $R=-1$, 0, 1, and 2. Δ effects are not included.

In Fig. 2 we show the percentage change in the spectrum produced by the contributions of the Δ for various values of g_P as labeled by the ratios R. For the triplet case the qualitative effect of the Δ is pretty much independent of R, but for the $p\mu p$ case it seems to be proportional to g_P and changes sign as g_P does. The singlet amplitudes are unusually small and hence unusually sensitive to Δ effects.

Comparison of the analogous curves in Ref. 2 with the $R=1$ case here shows that the use of the more complete $\gamma N\Delta$ vertex makes no qualitative difference in the results. The contributions arising from $g^{(2)}_{\gamma\Delta N}$ and $g^{(2)}_{V\Delta N}$ amount normally to less than 10% of the complete Δ contribution. If one holds the Δ decay width fixed, which requires a slight change in the value of $g^{(1)}_{\gamma\Delta N}$ when $g^{(2)}_{\gamma\Delta N}$ is dropped, then the results with and without $g^{(2)}_{\gamma\Delta N}$ are even closer.

IV. PHOTON ASYMMETRY

The angular distribution of the photon relative to the muon spin direction is experimentally measurable in spin-zero nuclei such as ^{12}C or ^{40}Ca but not normally in practice for hydrogen (or other spin-$\frac{1}{2}$ nuclei) since the muon is depolarized by ground-state hyperfine interactions $\mathbf{S}_\mu \cdot \mathbf{S}_p$ between the muon and proton spins or by molecular spin-dependent forces at higher densities.

This angular distribution is given, for fully polarized muons, in terms of an asymmetry parameter A as

$$\frac{d\Gamma}{d\Omega_\gamma dk} \propto (1+A\cos\theta) , \qquad (3)$$

where $\cos\theta = \hat{\mathbf{S}}_\mu \cdot \hat{\mathbf{k}}$ is the angle between the muon spin and the photon momentum. Thus A is given by the rate for $\mathbf{S}_\mu$ parallel to $\mathbf{k}$ minus the rate for $\mathbf{S}_\mu$ antiparallel to $\mathbf{k}$ divided by the sum of these two rates.

Because this asymmetry is rather sensitive to g_P, its measurement has been attempted in nuclei in some of the recent experiments measuring radiative muon capture[6-8] but with somewhat limited success so far. Various theoretical calculations[9,10] in nuclei do not appear fully in agreement with each other either, though this may mostly be attributable to nuclear structure effects. However no modern benchmark calculation which can be used for purposes of comparison seems to exist for hydrogen. Thus we have a dual motivation to calculate A: namely, (1) to provide a benchmark for comparison in a simple system, which is sometimes useful for checking certain aspects or limiting cases of the computer programs used

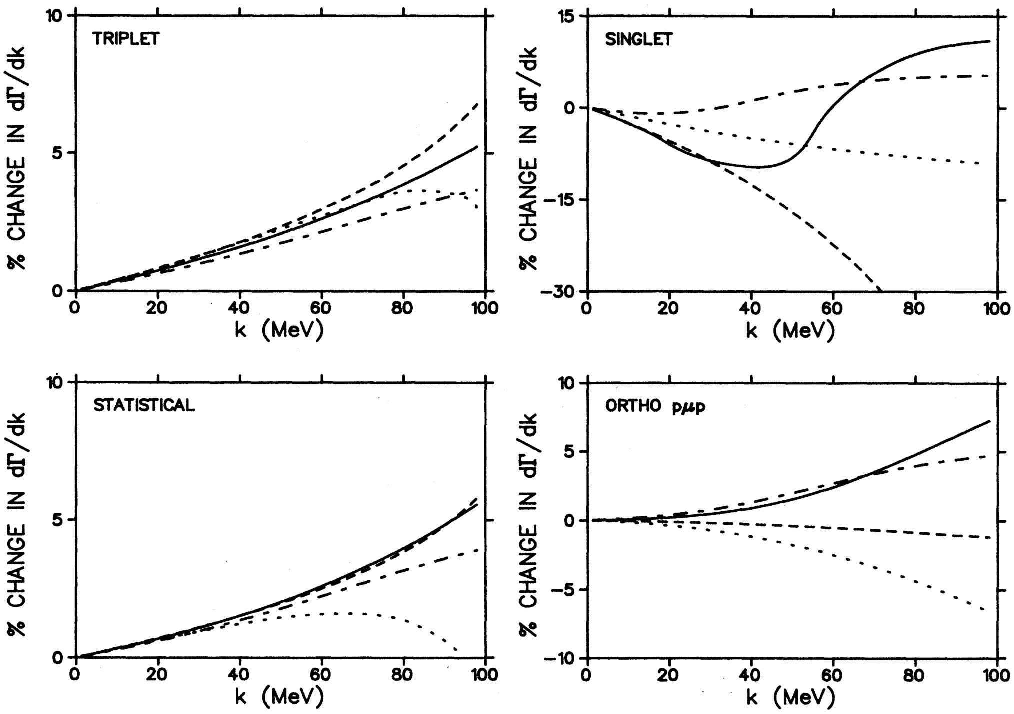

FIG. 2. The percentage change in $d\Gamma/dk$ vs the photon momentum k arising from inclusion of the Δ excitation amplitudes. This is calculated from the ratio of rates (with Δ − without Δ)/(with Δ). The graphs are labeled as in Fig. 1.

for nuclear calculations; and (2) to check the sensitivity of the asymmetry to Δ contributions, so as to get some indication of whether or not it would be of value to pursue such calculations in nuclei. Such calculations are easily executed since our computer procedures already calculate, separately, all possible spin amplitudes and so it is just a matter of combining the amplitudes in the proper way to get A.

Our results are shown in Fig. 3. The asymmetry is sensitive to g_P but seems to be very insensitive to contributions from the Δ. Such contributions change A by only a few percent in most cases. One can actually understand this insensitivity, which holds also for other types of corrections[10] in the following way. The asymmetry can be written as $A = 1-\epsilon$ and it can be shown in general[11] that the interesting physics is all contained in ϵ and that it is of order $1/m^2$ and dominated by terms involving g_P. Corrections which make, say, a 10% change in the rate tend to have a similar size effect on ϵ rather than on A directly and thus translate into a smaller effect on the measurable quantity A.

V. CONCLUSIONS

We have extended our previous calculation of Δ effects in radiative muon capture on hydrogen to include a more elaborate form for the $\gamma N \Delta$ and weak Δ nucleon vertices which properly reproduces both magnetic dipole and electric quadrupole amplitudes in pion photoproduction. We have also examined such effects for a variety of values of g_P. The results do not seem to be sensitive to the particular form of the Δ vertices chosen, but the magnitude of the Δ effect does depend significantly on g_P in some spin states. The overall contribution of the Δ is not large but should be included in the analysis of high-precision experiments.

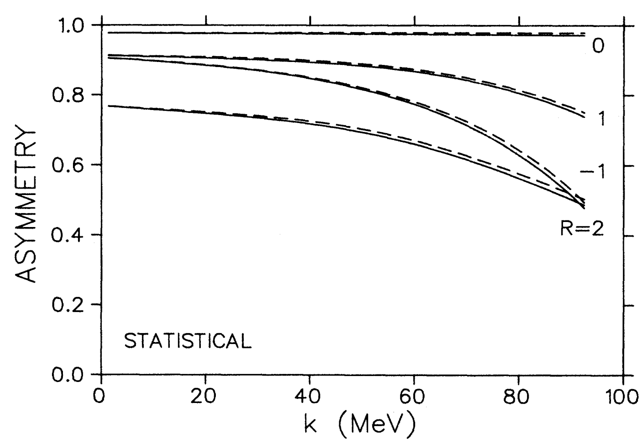

FIG. 3. The photon-muon-spin asymmetry parameter A vs the photon momentum k calculated for various values of the ratio R of g_P to its standard Goldberger-Treiman value. The solid curves correspond to no Δ contribution and the dashed curves, which are almost indistinguishable, include the Δ contributions. The curves correspond to $R = -1$, 0, 1, and 2 as labeled.

We have also examined the correlation between muon spin and photon direction. The corresponding asymmetry parameter A is quite sensitive to g_P as was well known but is not sensitive to types of Δ contributions we have evaluated here.

ACKNOWLEDGMENTS

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