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Dynamical excited weak bosons and their observable signatures

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Within the framework based on the local "flavor"-$SU(2)_L^{loc}\times U(1)_Y^{loc}$ symmetry, excited states of the weak bosons W^* and Z^* are regarded as bound states of spinor constituents with the compositeness scale $\Lambda_{comp}\sim 1$ TeV, which are formed by the four-Fermi interactions of the Nambu–Jona-Lasinio–Bjorken type. From the complementarity viewpoint, which further requires a local "color"-$SU(2)_{\mathscr{C}}^{loc}$ symmetry, it is shown that the $SU(2)_L^{loc}\times U(1)_Y^{loc}$ model with W^* and Z^* is equivalent to an $SU(2)_L^{loc}\times U(1)_Y^{loc}\times SU(2)_{\mathscr{C}}^{loc}$ model realized in the Higgs phase of $SU(2)_{\mathscr{C}}^{loc}$. The masses of the weak bosons, $m_{W,Z}$, and of the excited weak bosons, m_{W^*,Z^*}, satisfy $m_W m_{W^*}=\cos\theta m_Z m_{Z^*}$ for θ being the mixing angle of the gauge particles. Phenomenological implications of the presence of W^* and Z^* are discussed.

I. INTRODUCTION

Composite models of quarks, leptons, and gauge bosons, in which "elementary" particles are further made of more fundamental particles called subquarks (or preons),[1] predict various exotic particles including excited states of quarks, leptons, and gauge bosons. Although the compositeness scale Λ_{comp} can be any value ranging from ~ 1 TeV to $\sim 10^{19}$ GeV, there is an expectation that it provides the Fermi scale $G_F^{-1/2}$ ($\simeq 300$ GeV), which is the energy scale accessible to the existing or planned high-energy colliders. If this is the case, exotic composites can be as light as ~ 1 TeV and participate in today's physics. In particular, the presence of the excited states of weak bosons[2] W^* and Z^* significantly affects charged- and neutral-current interactions. However, their interactions with quarks and leptons are not arbitrarily chosen but constrained not to disturb the well-established low-energy interaction phenomenology that has been described by the exchanges of the (standard) weak bosons W and Z.

The observed properties of W and Z are consistent with those of gauge particles of the standard $SU(2)_L^{loc}\times U(1)_Y^{loc}$ model. The measured masses turn out to be[3] $m_W=80.76\pm1.72$ GeV and $m_Z=91.59\pm2.14$ GeV, which are just the right order of $eG_F^{-1/2}$ of the spontaneous $SU(2)_L^{loc}\times U(1)_Y^{loc}$ breaking. Thus, it is quite conceivable that $SU(2)_L^{loc}\times U(1)_Y^{loc}$ for W and Z is still effective even in the presence of exotic composites. Once $SU(2)_L^{loc}\times U(1)_Y^{loc}$ is present, W and Z arise from the gauge fields $V_\mu^{(a)}$ of $SU(2)_L^{loc}$ and B_μ of $U(1)_Y^{loc}$, while W^* and Z^* can be introduced through $SU(2)_L^{loc}$-triplet matter fields[4] $V_\mu^{*(a)}$ ($a=1,2,3$). In the case of composite W and Z based on the γ-Z mixing scheme,[5] there have been lots of discussions on W^* and Z^* (Ref. 6). But, now, these composite W and Z must simulate the gauge bosons to meet $m_{W,Z}\simeq eG_F^{-1/2}$ (but not $\simeq G_F^{-1/2}$).

The $SU(2)_L^{loc}$-triplet matter fields $V^{*(a)}$ are assumed to be bound states of L-handed spinor subquarks w_{Li} ($i=1,2$), carrying the two weak charges[7] $V_\mu^{*(a)}\sim\overline{w_L}\gamma_\mu\tau^{(a)}w_L$. To generate the composite W^* and Z^*, we adopt the four-Fermi interactions of the Nambu–Jona-Lasinio–Bjorken type,[8] which will determine an effective theory for W^* and Z^*. The previous attempts can be found in the discussion on composite weak bosons of the $SU(2)_L^{loc}\times U(1)_Y^{loc}$ symmetry[9] or of the γ-Z mixing type[10] and on a composite leptonic gluon.[11] For $V^{*(a)}$, the kinetic mixing with gauge fields will be generated as in the γ-Z mixing,[5] where γ is replaced by $V^{(a)}$ and Z is replaced by $V^{*(a)}$. These bosons $V^{(a)}$, B, and $V^{*(a)}$ will be mixed into γ, $W^\pm$, Z, $W^{*\pm}$, and Z^*.

The excited weak bosons $V^{*(a)}$ ($\sim W^*$ and Z^*) especially generated by the four-Fermi interactions can be shown to be equivalent to massive gauge particles of a new "color"-$SU(2)_{\mathscr{C}}^{loc}$ symmetry.[12] The "flavor"-$SU(2)_L^{loc}\times U(1)_Y^{loc}$ model with W^* and Z^* is enlarged to an $SU(2)_L^{loc}\times U(1)_Y^{loc}\times SU(2)_{\mathscr{C}}^{loc}$ model with the composite "color" gauge bosons under the complementarity conditions that relate the "color" symmetry to the "flavor" symmetry.[13] By the transmutation[14] of $SU(2)_{\mathscr{C}}^{loc}$, the "color" gauge particles are converted into $V^{*(a)}$. Since this "color" symmetry can act as a mass-protection symmetry for W^* and Z^*, it is possible to generate light W^* and Z^* with masses of the order of Λ_{comp} times the $SU(2)_{\mathscr{C}}^{loc}$ coupling constant as far as the coupling is sufficiently small. It will be found that in most of our analyses the $SU(2)_L^{loc}\times U(1)_Y^{loc}\times SU(2)_{\mathscr{C}}^{loc}$ model can be regarded as an ordinary gauge model with the fundamental gauge particles (such as the L-R gauge model also with extra W and Z).[15]

Our phenomenology is affected by light particles with the masses $\ll 1$ TeV, which should imply the presence of mass-protection symmetries. Light particles of our model will consist of (1) the photon γ and the weak bosons W and Z, protected by the spontaneously broken "flavor"-$SU(2)_L^{loc}\times U(1)_Y^{loc}$ symmetry, (2) the excited weak bosons W^* and Z^* by the "color"-$SU(2)_{\mathscr{C}}^{loc}$ symmetry, and (3) quarks and leptons to be protected by the chiral version of the Pati-Salam symmetry.[16] The masses m_{W,Z,W^*,Z^*} are found to satisfy $m_W m_{W^*}=\cos\theta\, m_Z m_{Z^*}$ for θ being the mixing angle of the gauge bosons.[17] Since $\cos\theta \simeq m_W/m_Z$ is required from low-energy neutrino-induced reactions,[18] it yields $m_{W^*} \simeq m_{Z^*} \gtrsim 200$ GeV set by the UA1 and UA2 data.[19]

Relying upon the four-fermion interactions, we examine various properties of W^* and Z^*. In the next section, the transmutation of the local "color"-$SU(2)_{\mathscr{C}}^{loc}$ symmetry is explained. The underlying dynamics is given by nonlinear interactions of subquarks and the effective Lagrangian for W^* and Z^* is derived. In Sec. III, properties of W^* and Z^* are examined on the basis of the effective Lagrangian. In Sec. IV their phenomenological implications are discussed. The final section is devoted to summary and discussions.

II. MODEL

A. W^* and Z^* as massive gauge particles

Let us first review a specific aspect of composite vector bosons generated by four-Fermi interactions.[20] In a recent paper,[12] we have advocated that W^* and Z^* are massive gauge particles of a new local "color"-$SU(2)_{\mathscr{C}}^{loc}$ symmetry.[21] The "flavor"-$SU(2)_L^{loc}\times U(1)_Y^{loc}$ model with W^* and Z^* can be enlarged into an $SU(2)_L^{loc}\times U(1)_Y^{loc}\times SU(2)_{\mathscr{C}}^{loc}$ model with the $SU(2)_{\mathscr{C}}^{loc}$-triplet gauge fields,[22] in which $SU(2)_L^{loc}\times SU(2)_{\mathscr{C}}^{loc}$ is broken to its diagonal subgroup $SU(2)_D^{loc}$ in the Higgs phase or reduced to $SU(2)_L^{loc}$ in the confining phase. Following complementarity[13] that utilizes the physical equivalence between the confining phase and the Higgs phase at low energies,[23] one observes that W^* and Z^*, which lie in the confining phase, are the massive gauge bosons of $SU(2)_{\mathscr{C}}^{loc}$, which appear in the Higgs phase. In the following, we briefly discuss how the gauge bosons are converted into W^* and Z^*.

The ingredients[24] are the $SU(2)_{\mathscr{C}}^{loc}$-doublet spinor χ_{Lm} $(m=1,2)$ and the $SU(2)_L^{loc}$- and $SU(2)_{\mathscr{C}}^{loc}$-doublet scalars ξ_m^i $(i=1,2)$, which provide our subquarks $w_{Li}\sim(\xi^\dagger)_i^m\chi_{Lm}$ and our excited weak bosons (up to the "flavor" gauge fields) $V_{\mu i}^{*j}\sim(\xi^\dagger)_i^m i(\partial_\mu - ig_s\mathscr{G}_\mu)_m^n\xi_n^j$. The "flavor"-$SU(2)_L^{loc}$-invariant Lagrangian for generating V^* is given by

$$\mathscr{L}_{\rm conf}=i\overline{w_L}\gamma^\mu(\partial_\mu - igV_\mu)w_L - \frac{1}{8\Lambda_{V^*}^2}(\overline{w_L}\gamma^\mu\tau^{(a)}w_L)^2 , \tag{2.1}$$

where $\Lambda_{V^*}\sim 1$ TeV $(\sim\Lambda_{\rm comp})$, which is subsequently transformed into

$$\mathscr{L}_{\rm conf}=i\overline{w_L}\gamma^\mu(\partial_\mu - igV_\mu - ig^*V_\mu^*)w_L + \frac{\mu_{V^*}^2}{2}(V_\mu^{*(a)})^2 \tag{2.2}$$

with the auxiliary field $V_\mu^*\equiv(\tau^{(a)}/2)V_\mu^{*(a)}$ and the mass parameter μ_{V^*} defined by

$$V_\mu^{*(a)} = -\frac{1}{2g^*\Lambda_{V^*}^2}\overline{w_L}\gamma_\mu\tau^{(a)}w_L , \tag{2.3a}$$

$$\mu_{V^*}=g^*\Lambda_{V^*} . \tag{2.3b}$$

The transmutation of the local "color"-$SU(2)_{\mathscr{C}}^{loc}$ symmetry occurs if $(\xi^\dagger)_i^m\xi_m^j=\Lambda_{V^*}^2\delta_i^j$, also leading to $\xi_m^i(\xi^\dagger)_i^n=\Lambda_{V^*}^2\delta_m^n$. These conditions preserve the "color"-$SU(2)_{\mathscr{C}}^{loc}$ symmetry. Thus, we are in the confining phase. Converting V_μ^* into $\mathscr{G}_\mu$:$g^*V_\mu^*=\xi^\dagger(i\partial_\mu+g_s\mathscr{G}_\mu)\xi/\Lambda_{V^*}^2-gV_\mu$ with w_L into χ_L:$w_{Li}=(\xi^\dagger)_i^m\chi_{Lm}/\Lambda_{V^*}$ yields the manifestly $SU(2)_{\mathscr{C}}^{loc}$-invariant Lagrangian $\mathscr{L}_{\rm inv}$:

$$\mathscr{L}_{\rm inv}=i\overline{\chi_L}\gamma_\mu(\partial_\mu - ig_s\mathscr{G}_\mu)\chi_L + |(\partial_\mu - ig_s\mathscr{G}_\mu)\xi + ig\xi V_\mu|^2 . \tag{2.4}$$

Since the "gluons" $\mathscr{G}_\mu^{(a)}$ have no kinetic terms, it is fair to stress that "confining" only means "unbroken" and "hidden."[25]

From complementarity, $\mathscr{L}_{\rm conf}$ realized in the confining phase can be replaced by $\mathscr{L}_{\rm Higgs}$ in the Higgs phase, where $\langle\xi_m^i(x)\rangle=\Lambda_{V^*}\delta_m^i$ is assumed. Since all the Nambu-Goldstone scalars U in $\xi=\Lambda_{V^*}\exp(iU/\Lambda_{V^*})$ are absorbed by the gauge fields $\mathscr{G}_\mu^{(a)}$, these scalars do not appear. The Lagrangian $\mathscr{L}_{\rm inv}$ turns out to be

$$\mathscr{L}_{\rm Higgs}=i\overline{w_L}\gamma^\mu(\partial_\mu - if\mathcal{V}_\mu - if^*\mathcal{V}_\mu^*)w_L + \tfrac{1}{2}(g^2+g_s^2)\Lambda_{V^*}^2(\mathcal{V}_\mu^{*(a)})^2 , \tag{2.5}$$

with $f=g_s\sin\theta^*=g\cos\theta^*$ and $f^*=g_s\cos\theta^*$ for $\sin\theta^*=g/(g_s^2+g^2)^{1/2}$, where

$$\mathcal{V}_\mu=\sin\theta^*\mathscr{G}_\mu+\cos\theta^*V_\mu , \tag{2.6a}$$

$$\mathcal{V}_\mu^*=\cos\theta^*\mathscr{G}_\mu-\sin\theta^*V_\mu . \tag{2.6b}$$

The massive gauge bosons $\mathcal{V}_\mu^*$ are related to V_μ^*:$(g^2+g_s^2)^{1/2}\mathcal{V}_\mu^*=g^*V_\mu^*$ for $\xi=\Lambda_{V^*}$ and the new gauge bosons $\mathcal{V}_\mu^{(a)}$ are associated with the unbroken gauge symmetry $SU(2)_D^{loc}$.

To see the equivalence between $\mathscr{L}_{\rm conf}$ of Eq. (2.1) and $\mathscr{L}_{\rm Higgs}$ of Eq. (2.5) calls for the dynamical shift of V_μ and V_μ^* into the physical fields $\mathcal{V}_\mu$ and $\mathcal{V}_\mu^*$. In $\mathscr{L}_{\rm conf}$, $\mathcal{V}_\mu=V_\mu+(\lambda/\sqrt{1-\lambda^2})\mathcal{V}_\mu^*$ and $\mathcal{V}_\mu^*=\sqrt{1-\lambda^2}V_\mu^*$ are induced by the kinetic mixing between V and V^*, where λ is the mixing parameter [see Eq. (2.16)] and, in $\mathscr{L}_{\rm Higgs}$, the mass mixing as in Eqs. (2.6a) and (2.6b) occurs. It can then be demonstrated that, for the coupling to w_L as in Eqs. (2.2) and (2.5),

$$gV_\mu+g^*V_\mu^*=g\mathcal{V}_\mu+\frac{g^*}{\sqrt{1-\lambda^2}}\left[1-\frac{g\lambda}{g^*}\right]\mathcal{V}_\mu^* , \tag{2.7a}$$

$$f\mathcal{V}_\mu+f^*\mathcal{V}_\mu=f\mathcal{V}_\mu+\frac{g_s}{\cos\theta^*}(1-\sin^2\theta^*)\mathcal{V}_\mu^* , \tag{2.7b}$$

and also, for the possible gauge coupling to other fields, φ_i, for w_L appearing through the covariant derivative $D_\mu=\partial_\mu - igV_\mu$ as $\mathscr{L}(D_\mu\varphi_i)$,

$$gV_\mu=g\left[\mathcal{V}_\mu-\frac{\lambda}{\sqrt{1-\lambda^2}}\mathcal{V}_\mu^*\right]=f(\mathcal{V}_\mu-\tan\theta^*\mathcal{V}_\mu^*) . \tag{2.8}$$

Since g [of the $SU(2)_L^{loc}$ coupling] and g^* can be translated into f [of the $SU(2)_D^{loc}$ coupling] and g_s, respectively, the equivalence arises if

$$\lambda = g/g^* = \sin\theta^* (\equiv f/g_s) , \tag{2.9}$$

which turns out to be the case [see Eq. (2.17a)]. Thus, it establishes the physical equivalence between $\mathcal{L}_{conf}+\mathcal{L}(D_\mu\varphi_i)$ and $\mathcal{L}_{Higgs}+\mathcal{L}(\underline{D_\mu\varphi_i})$ (Ref. 26). The mass for $\mathcal{V}^*$ becomes $g^*\Lambda_{V^*}/\sqrt{1-\lambda^2}$ in $\mathcal{L}_{conf}$ and $g_s\Lambda_{V^*}/\cos\theta^*$ in $\mathcal{L}_{Higgs}$ that are the same.

The advantage of regarding vector bosons as massive gauge particles lies in the possibility that the broken gauge symmetry serves as a mass-protection symmetry for the vector bosons, which naturally allows light masses ($\sim g^*\Lambda_{V^*}$) of the vector bosons as far as their coupling g^* is small enough. If it is really what happens in the composite W^* and Z^*, they turn out to be light particles with $m^2_{W^*,Z^*} \ll \Lambda^2_{comp}$ (for $g^{*2} \ll 1$). Of course, a universality of the V^* coupling is also a natural consequence.[25]

B. Basic Lagrangian

As a realistic model, we adopt the fermion-boson symmetric model that contains scalar subquarks $\tilde{s}=(\tilde{c}_\alpha,\tilde{w}_i)$, as well as spinor subquarks $s=(c_\alpha,w_i)$ where $\tilde{c}_\alpha$ and c_α carry the lepton number ($\alpha=0$) and three colors ($\alpha=1,2,3$) (Ref. 16) and $\tilde{w}_i$ and w_i carry the two weak charges ($i=1,2$) (Ref. 7). Quarks and leptons, $f_{a,i}$, are expressed as $f_{a,i}=\tilde{c}_\alpha w_i+\tilde{w}_i c_\alpha$ for $f_{0,1}=\nu_e$, $f_{0,2}=e$, $f_{a,1}=u_a$, and $f_{a,2}=d_a$ ($a=1,2,3$), where $\tilde{w}=\tilde{w}_L L+\tilde{w}_R R$ and $\tilde{c}=\tilde{c}_L L+\tilde{c}_R R$ with $L=(1-\gamma_5)/2$ and $R=(1+\gamma_5)/2$. The electric charges Q_{em} of the subquarks are given as $Q_{em}=(-\frac{1}{2},\frac{1}{6},\frac{1}{6},\frac{1}{6})$ for $\alpha=(0,1,2,3)$ and $Q_{em}=(\frac{1}{2},-\frac{1}{2})$ for $i=(1,2)$.[1] The lightness of quarks and leptons can be ascribed to the global $SU(4)_L\times SU(4)_R$ symmetry[27] [or $SU(3)_L\times SU(3)_R$ symmetry],[28] under which $(\tilde{c}_{L\alpha},\tilde{c}_{R\alpha})$, $(c_{L\alpha},c_{R\alpha})$, and $(f_{Li,\alpha},f_{Ri,\alpha})$ transform as (4,4).

The interactions respect the local $SU(2)_L^{loc}\times U(1)_Y^{loc}$ symmetry for the gauge bosons and the global $SU(4)_L\times SU(4)_R$ symmetry for the quarks and leptons. The underlying dynamics is assumed to be described by $\mathcal{L}=\mathcal{L}_{conf}+\mathcal{L}_0+\mathcal{L}_{mass}$:

$$\mathcal{L}_{conf}=i\overline{w_L}\gamma^\mu(\partial_\mu-igV_\mu)w_L - \frac{1}{8\Lambda^2_{V^*}}(\overline{w_L}\gamma^\mu\tau^{(a)}w_L)^2 , \tag{2.10a}$$

$$\mathcal{L}_0=\left|\left[\partial_\mu-ig'\frac{Y}{2}B_\mu\right]\tilde{s}\right|^2+|(\partial_\mu-igV_\mu)\tilde{w}_L|^2 + i\bar{s}\gamma^\mu\left[\partial_\mu-ig'\frac{Y}{2}B_\mu\right]s , \tag{2.10b}$$

$$\mathcal{L}_{mass}=-[\mu_c^2(|\tilde{c}_L|^2+|\tilde{c}_R|^2)+\mu_W^2(|\tilde{w}_L|^2+|\tilde{w}_R|^2)] - m_C[\bar{c}c+\Lambda_{\tilde{c}}(\tilde{c}^\dagger_L\tilde{c}_R+\tilde{c}^\dagger_R\tilde{c}_L)] , \tag{2.10c}$$

for $\tilde{s}(s)\neq\tilde{w}_L(w_L)$, where $\Lambda_{\tilde{c}}\sim\Lambda_{comp}$; $Y=0$ for $\tilde{w}_L$ and w_L and $=2Q_{em}$ for others. The inclusion of the kinetic terms for the gauge fields is understood. The mass parameter m_C represents the feeble breaking of $SU(4)_L\times SU(4)_R$ and is expected to provide tiny masses for quarks and leptons. Requiring $m_C\ll\Lambda_{comp}$ can be considered as being natural.[29] On the other hand, requiring $\mu_{w,c}\ll\Lambda_{comp}$ is known to be unnatural unless $\tilde{w}_{L,R}$ are the Nambu-Goldstone particles[30] or supersymmetric partners. The fermions will be produced as light particles if the scalar subquarks contained are kept light; otherwise the fermions will acquire masses of the order of Λ_{comp}. In the following, we assume that all subquark masses are subject to $m^2_{subquark}\ll\Lambda^2_{comp}$.

For consistency, quarks ($f_{a,i}$), leptons ($f_{0,i}$), and a Higgs scalar (ϕ) are also introduced as auxiliary fields[31]

$$\begin{aligned}\mathcal{L}_1=\frac{1}{\Lambda_f}[&\bar{f}i(D_\mu\tilde{c})\gamma^\mu w-\bar{f}i(D_\mu\tilde{w})\gamma^\mu c\\ &-\bar{w}i\gamma^\mu(D_\mu\tilde{c})^\dagger f+\bar{c}i\gamma^\mu(D_\mu\tilde{w})^\dagger f]\\ &-[\bar{w}(\Phi^\dagger L+\Phi R)w+\xi_w|\phi|^2(|\tilde{w}_L|^2+|\tilde{w}_R|^2)]\\ &-\Lambda^2_\phi|\phi|^2-\frac{m_C}{\Lambda_F}\bar{f}(\Phi^\dagger L+\Phi R)f ,\end{aligned} \tag{2.11}$$

with D_μ being the appropriate covariant derivative containing $V^{(a)}_\mu$ or B_μ, where $\Lambda_{f,\phi}\sim\Lambda_{comp}$; $\xi_w\sim 1$; $\Phi=(\phi^G,\phi)$; $\phi=(\phi_1,\phi_2)^T$; and $\phi^G=(\phi_2^*,-\phi_1^*)^T$ with $Q_{em}=(1,0)$ for (ϕ_1,ϕ_2). It should be noted that the interactions for composite fermions are of the chirality-conserving type, which realizes the naive expectation that w_L (and $\tilde{w}_L$) assigned to the **2** of $SU(2)_L^{loc}$ leads to f_L being **2**. Since $\bar{f}\Phi f$ breaks $SU(4)_L\times SU(4)_R$, the factor m_C is placed in its coupling. The compositeness of f and Φ is described by

$$f_{Li,\alpha}=i\gamma_\mu(w_{Rj}\partial_\mu\tilde{c}_{R\alpha}-c_{R\alpha}\partial_\mu\tilde{w}_{Rj})\Phi_{ij}/m_C|\phi|^2\mathcal{M}^2+(\cdots) , \tag{2.12a}$$

$$f_{Ri,\alpha}=i\gamma_\mu(w_{Lj}\partial_\mu\tilde{c}_{L\alpha}-c_{L\alpha}\partial_\mu\tilde{w}_{Lj})(\Phi^\dagger)_{ij}/m_C|\phi|^2\mathcal{M}^2+(\cdots) , \tag{2.12b}$$

$$\Phi_{ij}=-\overline{w_{Rj}}w_{Li}/\mathcal{M}^2+(\cdots) , \tag{2.12c}$$

where $\mathcal{M}^2=\Lambda^2_\phi+\xi_w(|\tilde{w}_L|^2+|\tilde{w}_R|^2)$. The total Lagrangian $\mathcal{L}_{tot}=\mathcal{L}_{conf}+\mathcal{L}_0+\mathcal{L}_1+\mathcal{L}_{mass}$, together with the kinetic terms of the gauge fields, finally prescribes subquark dynamics.

C. Effective Lagrangian

Owing to quantum corrections, the composite auxiliary fields become propagating fields. The effective Lagrangian for composites is defined by

$$\exp\left[i\int d^4x\,\mathcal{L}_{eff}\right]=\int[d\tilde{s}][d\tilde{s}^\dagger][ds][d\bar{s}] \times\exp\left[i\int d^4x\,\mathcal{L}_{tot}\right] . \tag{2.13}$$

After integrating the subquark fields and rescaling ϕ according to $\phi\to g_\phi\phi$, one finds at the leading order the compositeness conditions

$$g^{*2}N_wJ_0/3=1\ , \tag{2.14a}$$

$$2g_\phi^2N_wJ_0=1\ , \tag{2.14b}$$

$$5J_2/2\Lambda_f^2=1\ , \tag{2.14c}$$

which yield canonical kinetic terms, respectively, for the excited gauge bosons, Higgs scalar, and quarks/leptons, where N_w is the number of the copies of $\bar{w}$ and w. The divergent integrals $J_{2,0}$ are defined

$$J_{2n}=(-1)^{n+1}\int\frac{d^4k}{(2\pi)^4}\frac{i}{(k^2-\bar{m}^2)^{2-n}}\ , \tag{2.15}$$

where $\bar{m}$ ($\ll\Lambda_{\rm comp}$) stands for an average mass of $\bar{s}$ and s. The divergent integrals $J_{0,2}$ are regulated to give $J_2=\Lambda^2/(4\pi)^2$ and $J_0=\ln(\Lambda/\bar{m})^2/(4\pi)^2$ for $\Lambda\sim\Lambda_{\rm comp}$ (Ref. 32).

The effective Lagrangian $\mathcal{L}_{\rm eff}$ is now written as

$$\begin{aligned}\mathcal{L}_{\rm eff}=&-\tfrac{1}{4}\{V^{(a)}_{\mu\nu}V^{(a)\mu\nu}+V^{*(a)}_{\mu\nu}V^{*(a)\mu\nu}+B_{\mu\nu}B^{\mu\nu}+2\lambda V^{(a)}_{\mu\nu}V^{*(a)\mu\nu}\\&+2(gV^{(a)}_{\mu\nu}+g^*V^{*(a)}_{\mu\nu})(i[V^{*\mu},V^{*\nu}])^{(a)}+g^{*2}(i[V^*_\mu,V^*_\nu])^{(a)}(i[V^{*\mu},V^{*\nu}])^{(a)}\}\\&+\tfrac{1}{2}\mu^2_{V^*}V^{*(a)}_\mu V^{*(a)\mu}+i\bar{f}_L\gamma_\mu\left[\partial_\mu-igV_\mu-ig^*V^*_\mu-ig'\frac{Y}{2}B_\mu\right]f_L\\&+i\bar{f}_R\gamma_\mu\left[\partial_\mu-ig'\frac{Y}{2}B_\mu\right]f_R+h\bar{f}(\Phi^\dagger L+\Phi R)f+\left|\left[\partial_\mu-igV_\mu-ig^*V^*_\mu-ig'\frac{Y}{2}B_\mu\right]\phi\right|^2-\mu_\phi^2|\phi|^2-\lambda_\phi|\phi|^4\ ,\end{aligned} \tag{2.16}$$

where

$$\lambda=g/g^*\ , \tag{2.17a}$$

$$h=g_\phi m_C(\Lambda_f+\Lambda_{\bar{c}}J_0)/\Lambda_f^2\ , \tag{2.17b}$$

$$\lambda_\phi=g_\phi^2(1-\xi_w^2)\ , \tag{2.17c}$$

$$\mu_\phi^2=g_\phi^2[\Lambda_\phi^2-(8\Lambda_f^2/5)(1-\xi_w)N_w]-2\xi_w\mu_w^2\ . \tag{2.17d}$$

The field strengths are defined by

$$V_{\mu\nu}\equiv\frac{\tau^{(a)}}{2}V^{(a)}_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu+ig[V_\mu,V_\nu]\ , \tag{2.18a}$$

$$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\mu B_\nu\ , \tag{2.18b}$$

$$\begin{aligned}V^*_{\mu\nu}&\equiv\frac{\tau^{(a)}}{2}V^{*(a)}_{\mu\nu}\\&=\partial_\mu V^*_\nu-\partial_\nu V^*_\mu+ig[V_\mu,V^*_\nu]-ig[V_\nu,V^*_\mu]\ ,\end{aligned} \tag{2.18c}$$

To provide the spontaneous breaking, $\mu_\phi^2<0$ and $\lambda_\phi>0$ are imposed so that $\langle\phi\rangle=(0,v/\sqrt{2})$ is generated.

Let us touch on the lightness of quarks and leptons. The mass of f is calculated to be $m_f=m_{W0}m_C(\Lambda_f+\Lambda_{\bar{c}}J_0)/\sqrt{2}\Lambda_f^2$. The suppression factor represented by m_C/Λ_f arises because of the explicit breaking of $SU(4)_L\times SU(4)_R$. The anomaly-matching constraints[29] on the chiral SU(4) symmetry may not be required if we take the nonlinear interactions as a basic dynamic since subquarks are not confined. However, if one insists that it is regarded as an approximation for deriving an effective theory as low-energy manifestation of a confining subcolor theory,[33] the anomaly matching should be respected and is readily satisfied by imposing $N_{\rm sc}=2N_g$ (Ref. 27), where $N_{\rm sc}$ (N_g) is the number of the subcolors (generations).

Finally, we make a few comments on the possible compositeness of the weak gauge bosons. Within the same approach based on the nonlinear interactions, the gauge bosons can be constructed as composites including the scalar subquarks. It is well known that composite vector bosons made of scalars behave like gauge particles.[34] The compositeness can be expressed by

$$V^{(a)}_\mu=-(i\bar{w}^\dagger_L\overleftrightarrow{\partial}_\mu\tau^{(a)}\bar{w}_L+\overline{w_L}\gamma_\mu t a^{(a)}w_L)/g\bar{w}^\dagger_L\bar{w}_L\ , \tag{2.19a}$$

$$B_\mu=-(i\bar{s}^\dagger\overleftrightarrow{\partial}_\mu Y\bar{s}+\bar{s}\gamma_\mu Ys)/g'\bar{s}^\dagger Y^2\bar{s}\ , \tag{2.19b}$$

for $(\bar{s},s)\neq(\bar{w}_L,w_L)$. The compositeness conditions for V_μ and B_μ are given by $g^2N_wJ_0/2=1$ and $g'^2(3N_w+4N_c)J_0/6=1$ (Ref. 35). These conditions together with Eq. (2.14a) yield the useful relations

$$\sin^2\theta=3N_w/2(3N_w+2N_c)\ , \tag{2.20a}$$

$$g^*=\sqrt{3/2}g\ , \tag{2.20b}$$

where N_c counts the number of the copies of $\bar{c}$ and c. The typical values of $\sin^2\theta$ are calculated to be $\sin^2\theta=\frac{3}{10}$ $(x=1)$, $\frac{3}{14}$ $(x=2)$, $\frac{3}{8}$ $(x=\frac{1}{2})$ for $x=N_c/N_W$. For later qualitative discussions, the value of $g^*=\sqrt{3/2}g$ that corresponds to a certain unification condition on g^* in $SU(2)_L^{\rm loc}\times U(1)_Y^{\rm loc}\times SU(2)_{\mathscr{C}}^{\rm loc}$ will be used as our reference value.

III. EFFECTIVE INTERACTIONS FOR W^* AND Z^*

The Lagrangian for W^* and Z^*, Eq. (2.16), contains the kinetic mixing of the gauge fields with the matter fields. It can be removed by the redefinition of the fields as

$$\mathcal{V}_\mu\equiv\frac{\tau^{(a)}}{2}\mathcal{V}^{(a)}_\mu=V_\mu+(\lambda/\sqrt{1-\lambda^2})\mathcal{V}^*_\mu\ , \tag{3.1a}$$

$$\mathcal{V}^*_\mu\equiv\frac{\tau^{(a)}}{2}\mathcal{V}^{(a)*}_\mu=\sqrt{1-\lambda^2}V^*_\mu\ , \tag{3.1b}$$

where λ is the coefficient of the kinetic mixing of $V_{\mu\nu}V^{*\mu\nu}$, i.e., $\lambda = g/g^* \; (=\sqrt{2/3})$. After the redefinition, the Lagrangian for vector bosons, $\mathcal{L}_{J=1}$, is transformed into

$$\mathcal{L}_{J=1} = -\tfrac{1}{4}\{\mathcal{V}^{(a)}_{\mu\nu}\mathcal{V}^{(a)\mu\nu} + \mathcal{V}^{*(a)}_{\mu\nu}\mathcal{V}^{*(a)\mu\nu} + 2(\lambda_V \mathcal{V}^{(a)}_{\mu\nu} + \lambda_{V^*}\mathcal{V}^{*(a)}_{\mu\nu})(i[\mathcal{V}^{*\mu},\mathcal{V}^{*\nu}])^{(a)} + \lambda_{V^*V^*}(i[\mathcal{V}^*_\mu,\mathcal{V}^*_\nu])^{(a)}(i[\mathcal{V}^{*\mu},\mathcal{V}^{*\nu}])^{(a)}\} + \tfrac{1}{2}m^2_{\mathcal{V}^*}\mathcal{V}^{*(a)}_\mu \mathcal{V}^{*(a)\mu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}, \quad (3.2)$$

with $\mathcal{V}_{\mu\nu} \equiv (\tau^{(a)}/2)\mathcal{V}^{(a)}_{\mu\nu} = \partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu + ig[\mathcal{V}_\mu, \mathcal{V}_\nu]$ and $\mathcal{V}_{\mu\nu} \equiv (\tau^{(a)}/2)\mathcal{V}^{*(a)}_{\mu\nu} = \partial_\mu \mathcal{V}^*_\nu - \partial_\nu \mathcal{V}^*_\mu + ig[\mathcal{V}_\mu, \mathcal{V}^*_\nu] - ig[\mathcal{V}_\nu, \mathcal{V}^*_\mu]$, where $\lambda_V = g$, $\lambda_{V^*} = [g^* + g\lambda(2\lambda^2-3)]/(1-\lambda^2)^{3/2}$, $\lambda_{V^*V^*} = [g^{*2} - 4gg^*\lambda - 3g^2\lambda^2(\lambda^2-2)]/(1-\lambda^2)^2$, and $m_{\mathcal{V}^*} = \mu_{V^*}/\sqrt{1-\lambda^2}$ (with $\mu_{V^*} = g^*\Lambda_{V^*}$).

The vector fields $\mathcal{V}^{(a)}_\mu$, B_μ, and $\mathcal{V}^{*(a)}_\mu$ are mixed to yield $A_\mu = s_\theta \mathcal{V}^{(3)}_\mu + c_\theta B_\mu$,

$$\begin{pmatrix} W^\pm_\mu \\ W^{*\pm}_\mu \end{pmatrix} = \begin{pmatrix} c_\delta & s_\delta \\ -s_\delta & c_\delta \end{pmatrix} \begin{pmatrix} V^\pm_\mu \\ V^{*\pm}_\mu \end{pmatrix}, \quad (3.3a)$$

$$\begin{pmatrix} Z_\mu \\ Z^*_\mu \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} Z^0_\mu \\ V^{*(3)}_\mu \end{pmatrix}, \quad (3.3b)$$

where $Z^0_\mu = c_\theta \mathcal{V}^{(3)}_\mu - s_\theta B_\mu$, with $c_\theta = \cos\theta$, $s_\theta = \sin\theta$, etc. The mixing with the photon A_μ is forbidden owing to $U(1)^{\rm loc}_{\rm em}$. The mixings of the remaining fields are determined by the mass terms. The spontaneously generated masses are obtained from $|[gV_\mu + g^*V^*_\mu + (g'/2)B_\mu]\phi|^2$ with ϕ replaced by $\langle\phi\rangle = (0, v/\sqrt{2})^T$. Adding the hard mass $m_{\mathcal{V}^*}$ gives mass matrices $M^{\rm ch}$ on the $(\mathcal{V}^{(\pm)}_\mu, \mathcal{V}^{*(\pm)}_\mu)$ basis and $M^{\rm n}$ on the $(Z^0_\mu, \mathcal{V}^{*(3)}_\mu)$ basis:

$$M^{\rm ch} = \begin{pmatrix} m^2_{W^0} & \epsilon m^2_{W^0} \\ \epsilon m^2_{W^0} & m^2_{\mathcal{V}^*} + \epsilon^2 m^2_{W^0} \end{pmatrix}, \quad (3.4a)$$

$$M^{\rm n} = \begin{pmatrix} m^2_{Z^0} & \epsilon c_\theta m^2_{Z^0} \\ \epsilon c_\theta m^2_{Z^0} & m^2_{\mathcal{V}^*} + \epsilon^2 m^2_{W^0} \end{pmatrix}, \quad (3.4b)$$

where $m_{W^0} = c_\theta m_{Z^0} = gv/2$ and $\epsilon = [(g^*/g) - \lambda]/\sqrt{1-\lambda^2} (=1/\sqrt{2})$. The masses of W, Z, W^*, and Z^* are calculated to be

$$m^2_W = (c_\theta + \epsilon s_\delta)^2 m^2_{W^0} + s^2_\delta m^2_{\mathcal{V}^*}, \quad (3.5a)$$

$$m^2_Z = (c_\alpha + \epsilon c_\theta s_\alpha)^2 m^2_{Z^0} + s^2_\alpha m^2_{\mathcal{V}^*}, \quad (3.5b)$$

$$m^2_{W^*} = (s_\delta - \epsilon c_\delta)^2 m^2_{W^0} + c^2_\delta m^2_{\mathcal{V}^*}, \quad (3.5c)$$

$$m^2_{Z^*} = (s_\alpha - \epsilon c_\theta c_\alpha)^2 m^2_{Z^0} + c^2_\alpha m^2_{\mathcal{V}^*}. \quad (3.5d)$$

The mixing angles are given by, for $s_{\alpha,\delta} < 0$,

$$s^2_\alpha = [\sqrt{b} - |c^2_\theta(\epsilon^2 + r^2_{\mathcal{V}^*}) - 1|]/2\sqrt{b}, \quad (3.6a)$$

$$s^2_\delta = (\sqrt{a} - |\epsilon^2 + r^2_{\mathcal{V}^*} - 1|)/2\sqrt{a}, \quad (3.6b)$$

where $a = (\epsilon^2 + r^2_{\mathcal{V}^*} - 1)^2 + 4\epsilon^2$ and $b = [c^2_\theta(\epsilon^2 + r^2_{\mathcal{V}^*}) - 1]^2 + 4c^2_\theta\epsilon^2$, with $r_{\mathcal{V}^*} = m_{\mathcal{V}^*}/m_{W^0}$.

Only demanding that the breaking of $SU(2)^{\rm loc}_L \times U(1)^{\rm loc}_Y$ to $U(1)^{\rm loc}_{\rm em}$ be generated by the interactions with the weak isospin $I \le 1$ as those from the Higgs scalar ϕ one can still find useful relations.[17] The mixing angles and masses satisfy

$$c^2_\delta m^2_W + s^2_\delta m^2_{W^*} = c^2_\theta(c^2_\alpha m^2_Z + s^2_\alpha m^2_{Z^*}), \quad (3.7a)$$

$$s^2_\delta m^2_W + c^2_\delta m^2_{W^*} = s^2_\alpha m^2_Z + c^2_\alpha m^2_{Z^*}, \quad (3.7b)$$

$$c_\delta s_\delta(m^2_{W^*} - m^2_W) = c_\theta c_\alpha s_\alpha(m^2_{Z^*} - m^2_Z), \quad (3.7c)$$

from which the simple relation

$$m_W m_{W^*} = \cos\theta\, m_Z m_{Z^*} \quad (3.8)$$

is derived. The limit of $\alpha, \delta \to 0$ leads to $m_W = c_\theta m_Z$ (and $m_{Z^*} = m_{W^*}$), which is specific to the $I=1$ interactions. The mixing angles can be expressed as functions of the masses: for $m_{Z^*} > m_{W^*}$,

$$s^2_\alpha = (m^2_{Z^*} - m^2_W)(m^2_{Z^*} - m^2_{W^*})/s^2_\theta m^2_{Z^*}(m^2_{Z^*} - m^2_Z), \quad (3.9a)$$

TABLE I. Triple-boson couplings involving at most one W^* and Z^*. The momentum carried by each bosons is denoted by k, p, or q.

k,p,q	g_1	g_2	g_3
$\gamma W^- W^+$	$s_\theta(gc^2_\delta + \lambda_V s^2_\delta)$	0	0
ZW^-W^+	$(gc^2_\delta + \lambda_V s^2_\delta)c_\theta c_\alpha + (gc_\delta + \lambda_{V^*}s_\delta)s_\delta s_\alpha$	$gc_\theta c_\alpha + [(\lambda_V + g)c_\delta + \lambda_{V^*}s_\delta]s_\delta s_\alpha$	Same as g_2
$W^{*+}W^-\gamma$	0	0	$(g-\lambda_V)_\theta s_\delta c_\delta$
$Z^*W^-W^+$	$-(gc^2_\delta + \lambda_V s^2_\delta)c_\theta s_\alpha + (gc_\delta + \lambda_{V^*}s_\delta)s_\delta c_\alpha$	$-gc_\theta s_\alpha + [(\lambda_V + g)c_\delta + \lambda_{V^*}s_\delta]s_\delta c_\alpha$	Same as g_2
$W^{*-}W^+Z$	$(gc^2_\delta - \lambda_V s^2_\delta)S_\alpha + \lambda_{V^*}s_\delta c_\delta s_\alpha$	$(\lambda_V c^2_\delta - gs^2_\delta)s_\alpha + \lambda_{V^*}s_\delta c_\delta s_\alpha$	$g(c^2_\delta - s^2_\delta)s_\alpha + [(\lambda_V - g)c_\alpha c_\theta + \lambda_{V^*}s_\alpha]s_\delta c_\delta$

$$s_\delta^2=(c_\theta^2 m_{Z^*}^2-m_W^2)(m_{Z^*}^2-m_{W^*}^2)/s_\theta^2 m_{Z^*}^2(m_{W^*}^2-m_W^2)\ . \tag{3.9b}$$

The decays of W^* and Z^* into γ, W, and Z proceed mainly via three-boson couplings, which are calculated and listed in Table I. The vertices of $V_\alpha(k)\to V_{1\beta}(p)+V_{2\gamma}(q)$ with $k=p+q$ are parametrized as $g_1(k_\gamma\eta_{\alpha\beta}-k_\beta\eta_{\alpha\gamma})+g_2(p_\gamma\eta_{\alpha\beta}-p_\alpha\eta_{\beta\gamma})+g_3(q_\alpha\eta_{\beta\gamma}-q_\beta\eta_{\alpha\gamma})$. In the present approach, $\lambda_V=g$ and $\lambda_{V^*}=[g^*+g\lambda(2\lambda^2-3)]/(1-\lambda^2)^{3/2}$, with $\lambda=g/g^*(=\sqrt{2}/3)$. The absence of $Z^*\to Z\gamma$ is due to $[I^{(3)}, I^{(3)}$ or $Y]=0$ (Ref. 36). Since $\lambda_V=g$, one finds the vanishing coupling of $W^*W\gamma$ and the standard coupling of $WW\gamma$ as well as the vanishing anomalous magnetic moment of W, $\delta\kappa=[(\lambda_V/g)-1]s_\delta^2=0$ and of W^*, $\delta\kappa^*=[(\lambda_V/g)-1]c_\delta^2=0$. The leading-order contribution thus disappears for the $W^*W\gamma$ coupling. The next-to-leading-order contribution should be included.

The additional $W^*W\gamma$ coupling is supplied by

$$-ig'[\alpha(\mathcal{D}_\mu\phi)^\dagger YB^{\mu\nu}\mathcal{D}_\nu\phi+\beta(D_\mu\phi)^\dagger YB^{\mu\nu}D_\nu\phi]\ , \tag{3.10}$$

with $\mathcal{D}_\mu=\partial_\mu-ig(\mathcal{V}_\mu+\epsilon\mathcal{V}_\mu^*)-ig'(Y/2)B_\mu$ and $D_\mu=\partial_\mu-ig(\mathcal{V}_\mu-\epsilon'\mathcal{V}_\mu^*)-ig'(Y/2)B_\mu$. The parameters are given by $\epsilon'=\lambda/\sqrt{1-\lambda^2}$, $\alpha=2J_{-2}/3J_0$, and $\beta=J_{-4}\Delta\mu_w^2/2J_0$ for $J_{-2}=1/[(4\pi)^2 2m_w^2]$ and $J_{-4}=1/[(4\pi)^2 6(\mu_w^2+\xi_w m_w^2)^2]$, where $m_w=g_\phi v/\sqrt{2}$. The first term in Eq. (3.10) is generated by the quantum corrections due to the spinor subquarks $w_{Li,Ri}$ and the second one by those due to the scalar subquarks $\tilde{w}_{Li,Ri}$. The anomalous couplings to the photon are then given by

$$g_{\gamma WW}(=\delta\kappa)=(m_{W^0}^2/2)[\alpha(c_\delta+\epsilon s_\delta)^2+\beta(c_\delta-\epsilon' s_\delta)^2]\ , \tag{3.11a}$$

$$g_{\gamma WW^*}=(m_{W^0}^2/2)[\alpha(c_\delta+\epsilon s_\delta)(s_\delta-\epsilon c_\delta)+\beta(c_\delta-\epsilon' s_\delta)(s_\delta+\epsilon' c_\delta)]\ , \tag{3.11b}$$

$$g_{\gamma W^*W^*}(=\delta\kappa^*)=(m_{W^0}^2/2)[\alpha(s_\delta-\epsilon c_\delta)^2+\beta(s_\delta+\epsilon' c_\delta)^2]\ , \tag{3.11c}$$

where $g_{\gamma XX'}$ is defined by $\mathcal{L}_{\gamma XX'}=-ieg_{\gamma XX'}A^{\mu\nu}X_\mu^+X_\nu'^-$. Roughly speaking, these induced couplings are at least suppressed by $N_w(m_{W^0}/4\pi v)^2\sim e^2$ (for $N_w\sim 100$ for $\Lambda\sim 1$ TeV as in the case of $g^*=\sqrt{3/2}g$) (Ref. 35).

IV. PHENOMENOLOGY OF W^* AND Z^*

Experiments probing W^* and Z^* depend on the couplings to quarks and leptons, which are specified by

$$\mathcal{L}_{\rm int}=\frac{g}{\sqrt{2}}[J_{L\mu}^{(+)}\mathcal{V}^{(-)\mu}+J_{L\mu}^{(-)}\mathcal{V}^{(+)\mu}+\epsilon(J_{L\mu}^{(+)}\mathcal{V}^{*(-)\mu}+J_{L\mu}^{(-)}\mathcal{V}^{*(+)\mu})]+g_Z J_\mu^Z Z_0^\mu+eJ_\mu^{\rm em}A^\mu+g\epsilon J_{L\mu}^{(3)}\mathcal{V}^{*(3)\mu} \tag{4.1}$$

with $J_{L\mu}^{(a)}=\bar{f}_L\gamma_\mu(\tau^{(a)}/2)f_L$ and $J_\mu^Z=J_{L\mu}^{(3)}-s_\theta^2 J_\mu^{\rm em}$, where $e=g\sin\theta$ and $g_Z=g/\sqrt{g^2+g'^2}$. The W, Z, W^*, and Z^* couplings parametrized by $V-A\gamma_5$ for W, $V_i-A_i\gamma_5$ for Z, $V^*-A^*\gamma_5$ for W^*, and $V_i^*-A_i^*\gamma_5$ for Z^* are expressed by

$$V=A=g(c_\delta+\epsilon s)\ , \tag{4.2a}$$

$$V^*=A^*=g(\epsilon c_\delta-s_\delta)\ , \tag{4.2b}$$

$$V_i=g_Z\eta(I^{(3)}-2\text{“}\sin^2\theta_w\text{”}Q_{\rm em})\ , \tag{4.2c}$$

$$A_i=g_Z\eta I^{(3)}\ , \tag{4.2d}$$

$$V_i^*=g_Z\eta^*(I^{(3)}-2\text{“}\sin^2\theta_w^*\text{”}Q_{\rm em})\ , \tag{4.2e}$$

$$A_i^*=g_Z\eta^* I^{(3)}\ , \tag{4.2f}$$

where $\eta=c_\alpha+\epsilon s_\alpha c_\theta$, “$\sin^2\theta_w$”$=c_\alpha s_\theta^2/\eta$, $\eta^*=\epsilon c_\alpha c_\theta-s_\alpha$, and “$\sin^2\theta_w^*$”$=-s_\alpha s_\theta^2/\eta^*$ (>0).

The low-energy limit of $\mathcal{L}_{\rm int}$ is described by $\mathcal{L}_{\rm eff}^{\rm ch}$ for the W and W^* exchanges and $\mathcal{L}_{\rm eff}^{\rm n}$ for the Z and Z^* exchanges, giving rise to[17]

$$\mathcal{L}_{\rm eff}^{\rm ch}=2\sqrt{2}G_F J_L^{(+)}J_L^{(-)}\ , \tag{4.3a}$$

$$\mathcal{L}_{\rm eff}^{\rm n}=4\sqrt{2}G_F\xi[(J_L^{(3)}-\sin^2\theta_w J^{\rm em})^2+C_{\rm em}(J^{\rm em})^2]\quad(\xi=1)\ , \tag{4.3b}$$

with

$$4\sqrt{2}G_F=\rho g^2/m_W^2\ ,$$

$$\rho=C+2\epsilon\Delta+\epsilon^2 S=(c_\delta+\epsilon s_\delta)^2+(m_W/m_{W^*})^2(s_\delta-\epsilon c_\delta)^2\ ,$$

and

$$\sin^2\theta_w=\sin^2\theta(C+\epsilon\Delta)/\rho\ , \tag{4.4a}$$

$$C_{\rm em}=\sin^4\theta\epsilon^2(CS-\Delta^2)/\rho^2\ , \tag{4.4b}$$

where the parameters of C, S, and Δ are defined by

$$C=c_\delta^2+s_\delta^2(m_W/m_{W^*})^2=1+(m_W^2-m_{W^0}^2)/m_{W^*}^2\ , \tag{4.5a}$$

$$S=s_\delta^2+c_\delta^2(m_W/m_{W^*})^2=m_{W^0}^2/m_{W^*}^2\ , \tag{4.5b}$$

$$\Delta=c_\delta s_\delta[1-(m_W/m_{W^*})^2]\ . \tag{4.5c}$$

The restriction from $I\le 1$ ensures $\xi=1$ in Eq. (4.3b). The standard mixing angle $\sin\theta_{WS}$ ($\equiv s_{WS}$), can be introduced by $s_{WS}^2=e^2/(4\sqrt{2}G_F m_W^2)$ and ρ is related to s_{WS}^2 as

$$\rho=\sin^2\theta/\sin^2\theta_{WS}\ge[1-(m_W/m_Z)^2]/\sin^2\theta_{WS}\ , \tag{4.6}$$

from $4\sqrt{2}G_F m_W^*(=e^2/s_{WS}^2)=\rho e^2/s_\theta^2$. Then, $\sin^2\theta_w$ and $C_{\rm em}$ are reduced to

$$\sin^2\theta_w=\sin^2\theta_{WS}(C+\epsilon\Delta)\ , \tag{4.7a}$$

$$C_{\rm em}=\sin^4\theta_{WS}\epsilon^2(m_W/m_{W^*})^2\ . \tag{4.7b}$$

In our specific model, one can show that $\Delta+\epsilon S=0$ by the use of Eq. (3.6b), leading to $\rho=C+\epsilon\Delta$. Thus, $\sin^2\theta_w$ is further reduced to

$$\sin^2\theta_w=\sin^2\theta\ . \tag{4.8}$$

The measured angle $\sin^2\theta_{\rm expt}$ is precisely $\sin^2\theta_w$ (up to radiative corrections) in low-energy neutrino-induced reactions since $J^{\rm em}=0$ for the neutrinos. The present phenomenology requires $m_W=80.76\pm1.72$ GeV, $m_Z=91.59\pm2.14$ GeV, $G_F=(1.166\,38\pm0.000\,02)\times10^{-5}$ GeV^{-2}, and $\alpha_{\rm em}=e^2/4\pi=(137.036)^{-1}$ as well as

$$\sin^2\theta_{\rm expt}=0.2283\pm0.0048(\nu_\mu q)\ (\text{Ref. 3}) \tag{4.9a}$$

$$C_{\rm em}\le 0.01(e^+e^-)\ (\text{Ref. 37})\ . \tag{4.9b}$$

The radiative corrections may not much deviate from the standard ones since the model discussed is essentially the same as the standard model except for heavy W^* and Z^*. The mixing angle θ turns out to be $\sin^2\theta(=\rho\sin^2\theta_{WS})=(37.281\ \text{GeV})^2\rho/m_W^2(1-\Delta r)$, where Δr represents the radiative-correction factor, which will be set at $\Delta r=0.0713$. Since $\sin^2\theta_w(=\sin^2\theta)=\sin^2\theta_{\rm expt}$, the allowed values of $\sin^2\theta$ lie from 0.2187 to 0.2379 within two standard deviations. For later discussions, we choose $\sin^2\theta=0.22$, 0.23, and 0.24 as representative values. The relation (3.8) now gives

$$m_{W^*}\simeq m_{Z^*}\ , \tag{4.10}$$

since $\cos\theta\simeq m_W/m_Z$. The constraint on $C_{\rm em}$ is satisfied for $m_{W^*}\gtrsim 1.6m_W$ because of $\sin^4\theta_{WS}$ ($\sim\sin^4\theta$) $\sim\frac{1}{20}$ and $\epsilon=1/\sqrt{2}$.

A comment on the suggested value of $\sin^2\theta$, $\sin^2\theta=3/[2(3+2x)]$ ($x=N_c/N_w$), is in order. It is reasonable to consider that the value is the one defined at $Q=\Lambda$ and that the effects from the light composite particles are included in renormalization of g and g'. For W^* and Z^*, since the triple couplings to W and Z in $\mathcal{V}^{(a)}_{\mu\nu}(i[\mathcal{V}^{*\mu},\mathcal{V}^{*\nu}])^{(a)}$ and $\mathcal{V}^{*(a)}_{\mu\nu}i([\mathcal{V}^\mu,\mathcal{V}^{*\mu}]-[\mathcal{V}^\nu,\mathcal{V}^{*\mu}])^{(d)}$ are all equal to g (i.e., $\lambda_V=g$), the contributions from W^* and Z^* are the same as those from W and Z (except for their mass differences). Including the excited weak gauge bosons W^* and Z^* as well as the Higgs scalar H and assuming $m_{W^*,Z^*}=G_F^{-1/2}$ and $m_H=m_W$, we find that $s_\theta^2=0.22$–0.24 at $Q=m_W$ is recovered by $s_\theta^2(\Lambda)=\frac{3}{10}$ ($x=1$) with $\Lambda\sim(1$–$2)$ TeV and by $s_\theta^2(\Lambda)=\frac{1}{4}$ ($x=\frac{3}{2}$) with $\Lambda\sim0.5$ TeV, which can be read off from Fig. 1.

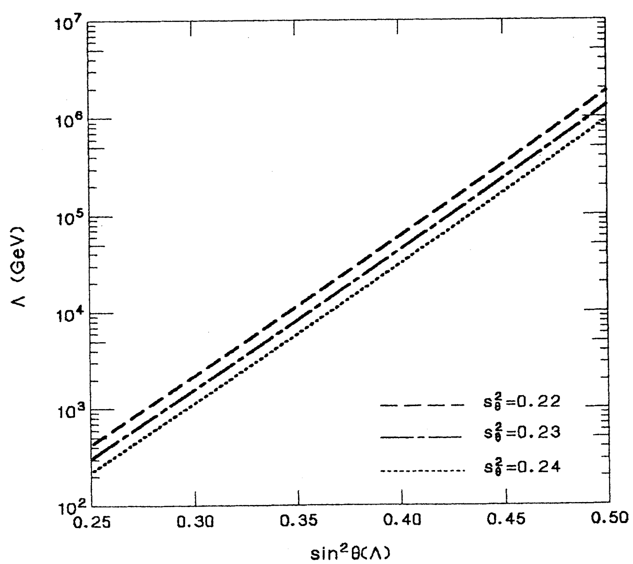

FIG. 1. The scale Λ vs $\sin^2\theta(\Lambda)$ for $\sin^2\theta=0.22$, 0.23, and 0.24.

Now, let us proceed to give qualitative discussions. Free parameters of the model are $\langle\phi\rangle$ and $m_{\mathcal{V}^*}$, which can be transferred to $\sin^2\theta$ and m_{Z^*} (or m_{W^*}) with $g^*=\sqrt{3/2}g$. The m_W and m_Z as functions of m_{Z^*} are plotted in Figs. 2(a) and 2(b) for $\sin^2\theta=0.22$–0.24 together with the experimental bounds. The lower bounds on m_{Z^*} are illustrated in Fig. 2(c), from which we find that, for $\sin^2\theta=(0.22,0.23,0.24)$,

$$m_{W^*,Z^*}\ge(220,297,-)\ \text{GeV (within } 1\sigma \text{ of } m_W)\ , \tag{4.11a}$$

$$\ge(223,263,365)\ \text{GeV (within } 1\sigma \text{ of } m_Z)\ , \tag{4.11b}$$

$$\ge(187,216,286)\ \text{GeV (within } 2\sigma \text{ of } m_W)\ , \tag{4.11c}$$

$$\ge(189,204,228)\ \text{GeV (within } 2\sigma \text{ of } m_Z)\ . \tag{4.11d}$$

The m_W-m_Z relation is also shown in Fig. 2(d).

For these sets of s_θ and m_{Z^*}, the deviations of the W and Z couplings from the standard ones are found to be not yet so significant thanks to the $SU(2)_L^{\rm loc}\times U(1)_Y^{\rm loc}$ symmetry. In fact, these couplings are explicitly calculated to be

$$V=A=\frac{4\sqrt{2}G_F m_W^2[1-\rho^{-1}(1+\epsilon^2)(m_W/m_{W^*})^2]^{1/2}}{[1-(m_W/m_{W^*})^2]^{1/2}}\ , \tag{4.12a}$$

$$g_Z\eta=\frac{4\sqrt{2}G_F m_Z^2[1-\rho^{-1}(1+\epsilon^2c_\theta^2)(m_Z/m_{Z^*})^2]^{1/2}}{[1-(m_Z/m_{Z^*})^2]^{1/2}}\ , \tag{4.12b}$$

$$\text{“}\sin^2\theta_w\text{”}=\frac{s_{WS}s_{\theta0}[1-(m_Z/m_{W^*})^2]^{1/2}}{[1-\rho^{-1}(1+\epsilon^2c_\theta^2)(m_Z/m_{W^*})^2]^{1/2}}\ , \tag{4.12c}$$

when $m_W m_{W^*}/m_Z m_{Z^*}=c_\theta$ of Eq. (3.8) and $s_{\alpha,\delta}$ of Eqs. (3.9a) and (3.9b) have been used. The gauge couplings of g and g_Z are related to g_0 and g_{Z0} of the standard model: $g=g_0/\sqrt{\rho}$ and $g_Z=g_{Z0}c_{WS}/\sqrt{\rho}c_\theta$. Numerically, $V/g_0\gtrsim0.94$, $g_Z\eta/g_{Z0}\gtrsim0.93$, and $0.25\gtrsim$ “$\sin^2\theta_w$” $\gtrsim0.22$ (for $m_{Z^*}\gtrsim200$ GeV).

The possible deviations of the Z coupling from the standard one can be reflected in $\Gamma(Z\to e^+e^-)$ and $\Gamma(Z\to\text{all})$ as shown in Figs. 3(a) and 3(b) (Ref. 38), which are to be measured at the SLAC Linear Collider (SLC) and CERN LEP. Also displayed is $\Gamma(W\to\text{all})$ as Fig. 3(c). The W^* and Z^* couplings are estimated to be for W^*, $V^*=A^*\simeq(0.9\text{–}0.7)g_0$ and for Z^*, $g_Z\eta^*\simeq(0.8\text{–}0.6)g_{Z0}$ and "$\sin^2\theta_w^*$" $\lesssim 0.05$. The branching ratios for the W^* and Z^* decays are calculated to be

$$B(W^*\to e\nu_e)\simeq 0.08\ , \tag{4.13a}$$

$$B(Z^*\to e^+e^-)\simeq 0.04\ , \tag{4.13b}$$

$$B(W^*\to jj)\simeq 0.73\ , \tag{4.13c}$$

$$B(Z^*\to jj)\simeq 0.63\ , \tag{4.13d}$$

$$B(W^*\to WZ)\simeq 0.03\ , \tag{4.13e}$$

$$B(Z^*\to WW)\simeq 7\times 10^{-3}\ , \tag{4.13f}$$

and $B(W^*\to\gamma W)\lesssim 10^{-3}$. The total widths of W^* and Z^* satisfy

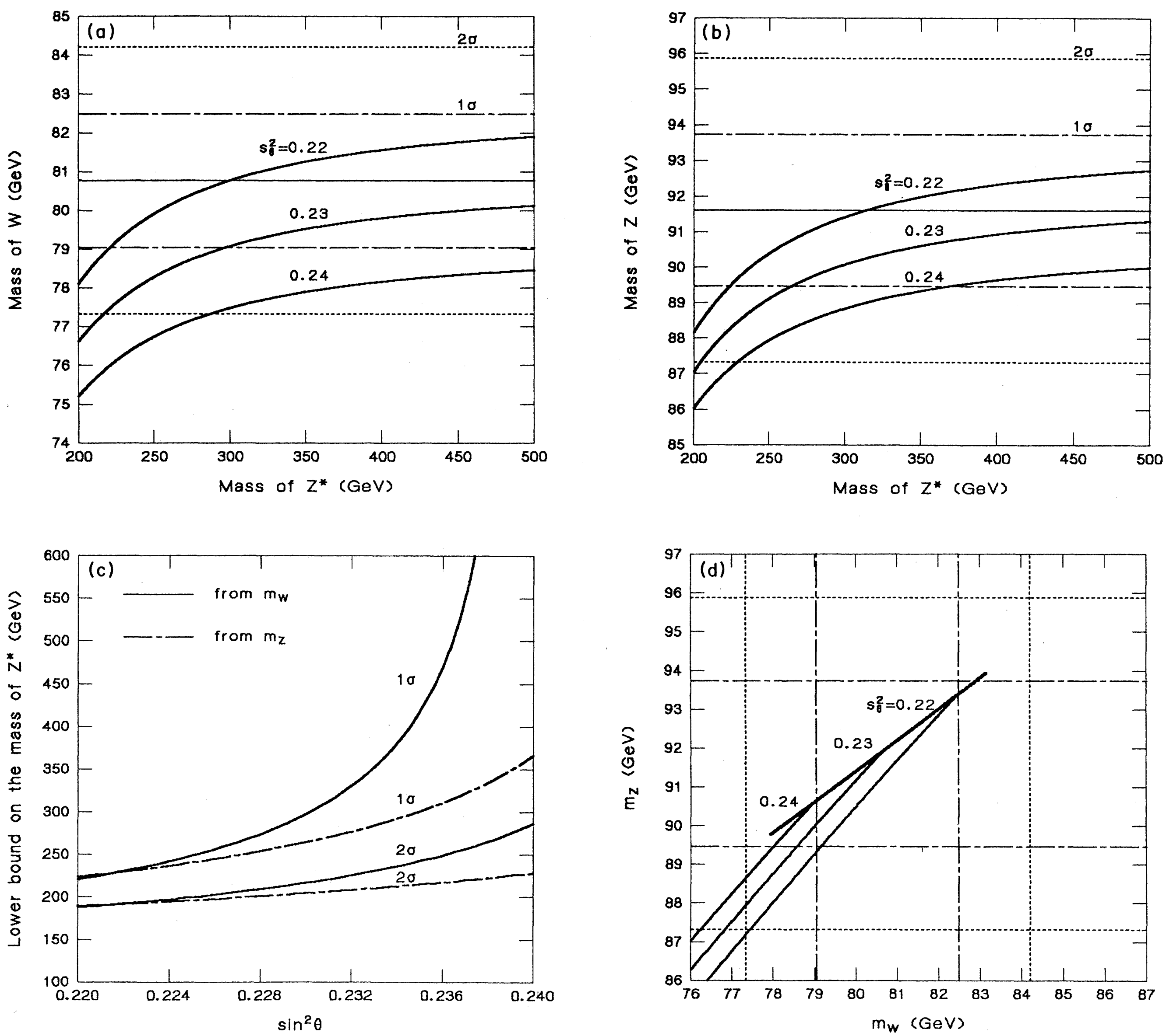

FIG. 2. (a) The mass of W vs the mass of Z^* for $\sin^2\theta=0.22$, 0.23, and 0.24. The observed values of m_W are plotted as 1σ (2σ) for one (two) standard deviation(s); (b) the same as in (a) but for Z; (c) lower bounds on m_{Z^*} set by the observed values of m_W (solid curves) and m_Z (dotted-dashed curves) within 1σ and 2σ; (d) the m_W-m_Z relation for $\sin^2\theta=0.22$, 0.23, and 0.24 with the experimental limits indicated by dotted-dashed lines (1σ) and dotted lines (2σ). The thicker line represents the values for $\alpha,\delta\to 0$: namely, for the standard model.

$$\Gamma(W^* \to \text{all}) \simeq \Gamma(Z^* \to \text{all}) \cong (g^{*2} m_{W^*} / 2g^2 m_W)\Gamma(W,Z \to \text{all}) \; . \tag{4.14}$$

In the following, we focus our attention to the asymmetries of $e^+e^- \to \mu^+\mu^-$ and the productions of W^* and Z^* at CERN and Fermilab $p\bar{p}$ colliders.

A. Asymmetries of $e^+e^- \to \mu^+\mu^-$

The pair production of μ in e^+e^- proceeds via γ, Z, and Z^* exchanges. The differential cross sections is given by, for γ ($i=0$), Z ($i=1$), and Z^* ($i=2$),

$$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\cos\theta_{e\mu}} = \frac{1}{32\pi s} \sum_{i,j=0}^{2} A_{ij}[B_{ij}(1+\cos^2\theta_{e\mu}) + 2C_{ij}\cos\theta_{e\mu}] \; , \tag{4.15}$$

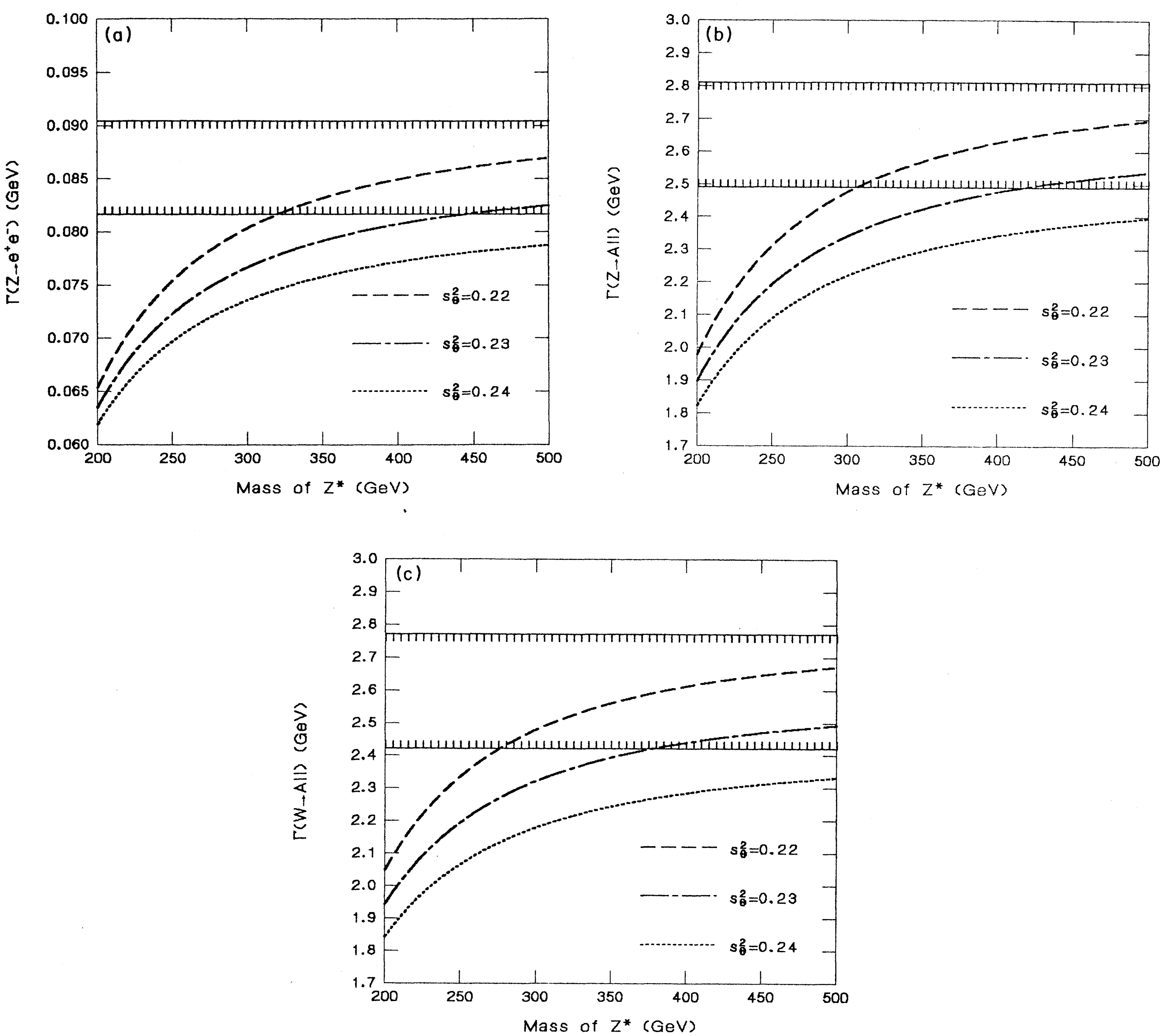

FIG. 3. (a) $\Gamma(Z \to e^+e^-)$ vs m_{Z^*} for $\sin^2\theta = 0.22$, 0.23, and 0.24. The hatched area represents the standard-model predictions for $m_W = c_{WS} m_Z = 80.76 \pm 1.72$; (b) the same as in (a) but for $\Gamma(Z \to \text{all})$; (c) the same as in (a) but for $\Gamma(W \to \text{all})$.

where

$$A_{ij}=s^2|\chi_i\chi_j^*| \ , \tag{4.16a}$$

$$B_{ij}=(v_i^e v_j^e+a_i^e a_j^e)(v_i^\mu v_j^\mu+a_i^\mu a_j^\mu) \ , \tag{4.16b}$$

$$C_{ij}=(v_i^e a_j^e+a_i^e v_j^e)(v_i^\mu a_j^\mu+a_i^\mu v_j^\mu) \ , \tag{4.16c}$$

with $\chi_i=(s-M_i^2+iM_i\Gamma_i)^{-1}$ for $(M_0,M_1,M_2)=(0,m_Z,m_{Z^*})$, $(v_0^{e,\mu},a_0^{e,\mu})=(-e,0)$, $(v_1^{e,\mu},a_1^{e,\mu})=(V_e/2,A_e/2)$, and $(v_2^{e,\mu},a_2^{e,\mu})=(V_e^*/2,A_e^*/2)$. In the limit of $\alpha,\delta\to 0$, V_e and A_e coincide with those of the standard model, i.e., $V_e=-g_{Z0}(1-4s_{WS}^2)/2$ and $A_e=-g_{Z0}/2$. The asymmetries are then calculated as the forward-backward asymmetry A_{FB} and the left-right asymmetry A_{LR}:

$$A_{FB}=\left[\int_{z=0}^{z=1}d\sigma-\int_{z=-1}^{z=0}d\sigma\right]\Big/\sigma \ , \tag{4.17a}$$

$$A_{LR}=(\sigma_L-\sigma_R)/\sigma \ , \tag{4.17b}$$

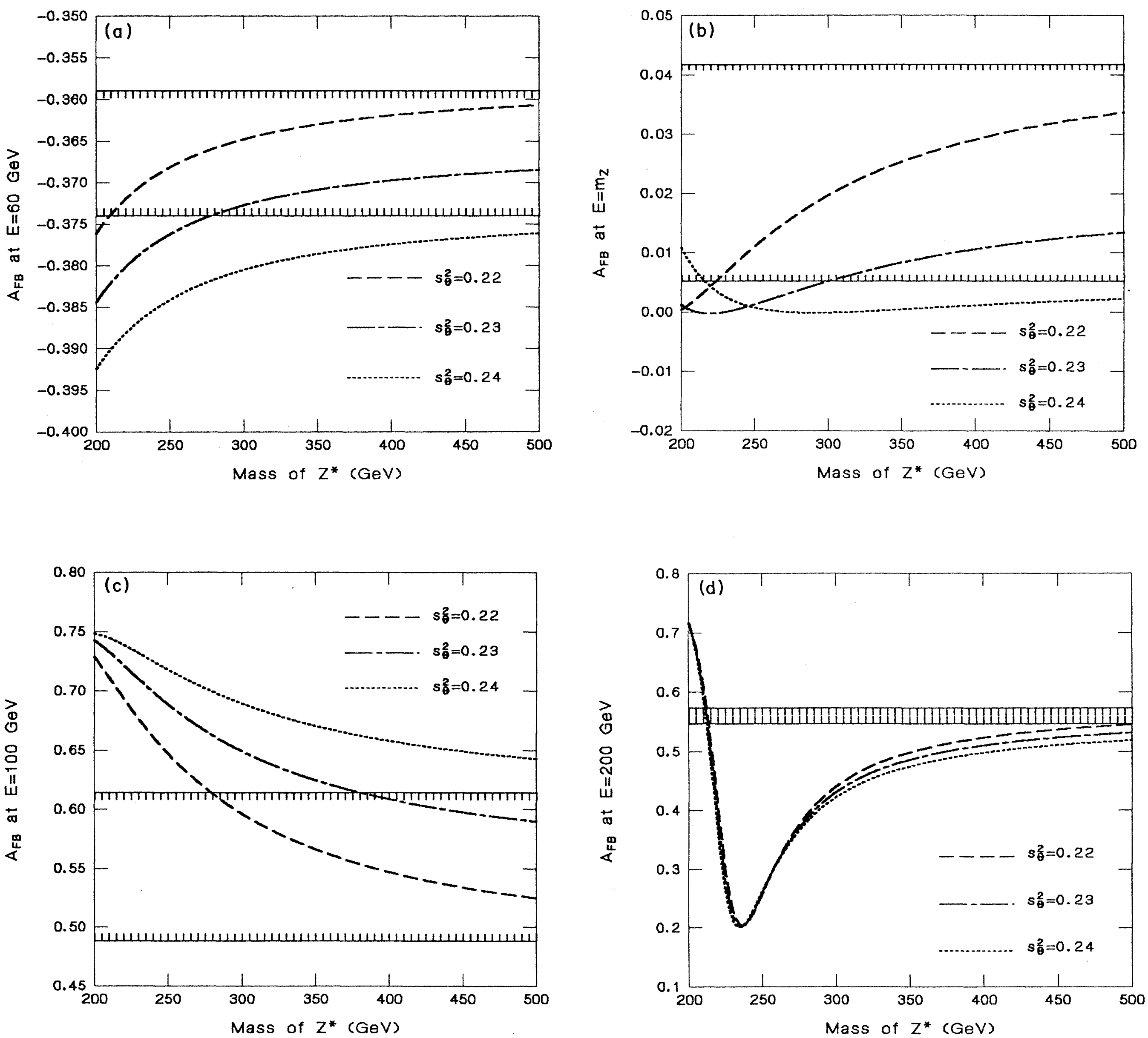

FIG. 4. The forward-backward asymmetry A_{FB} at $\sqrt{s}\,(=E)$ equal (a) 60 GeV, (b) m_Z, (c) 100 GeV, and (d) 200 GeV for $\sin^2\theta=0.22$, 0.23, and 0.24. The hatched area represents the standard-model predictions for $m_W=c_{WS}m_Z=80.76\pm1.72$.

where $z = \cos\theta_{e\mu}$; $\sigma_{L(R)}$ is the total cross section for left-(right-)handed initially polarized e^- beams obtained by the replacement of $v_i^e \to (v_i^e + \kappa a_i^e)/2$ and $a_i^e \to (a_i^e + \kappa v_i^e)/2$ with $\kappa = 1$ for σ_L and $\kappa = -1$ for σ_R.

Plotted in Figs. 4(a)–4(d) and Figs. 5(a)–5(d) are the asymmetries of A_{FB} and A_{LR} at $\sqrt{s} = 60$ GeV, m_Z, 100 GeV, and 200 GeV. The standard-model predictions are also plotted for $m_W = c_{WS} m_Z$ with $m_W = 80.76 \pm 1.72$. The following features can be found.

(1) At $\sqrt{s} = 60$ GeV to be reached at KEK TRISTAN, it is predicted that $A_{FB} = -0.39$–-0.36 and $A_{LR} = -0.02$–0 depending on m_{Z^*} compared with the standard-model predictions $A_{FB}^{\rm STD} = -0.37$–-0.36 and $A_{LR}^{\rm STD} = -0.02$–-0.01, depending on m_Z. For $s_\theta^2 \simeq 0.24$, $A_{FB} = -0.39$–-0.38 ($A_{LR} = -0.006$–0), which are smaller (larger) than the standard ones.

(2) At $\sqrt{s} = m_Z$, we find that $A_{FB} = 0$–0.034 and $A_{LR} = -0.12$–0.21 while $A_{FB}^{\rm STD} = 0.005$–0.04 and $A_{LR}^{\rm STD} = 0.08$–0.23. If the asymmetries (especially, A_{LR}) are found to be $\lesssim 0$, it gives a strong support for the existence of Z^* [with $m_{Z^*} \lesssim 300$ GeV, which can be read off from Fig. 5(b)]. However, the standard-model predictions are almost covered by our predictions for the smaller values of s_θ^2 (near $s_\theta^2 = 0.22$). For s_θ^2 near 0.24, the deviations are significant and possible to be detected (also at $\sqrt{s} = 100$ GeV).

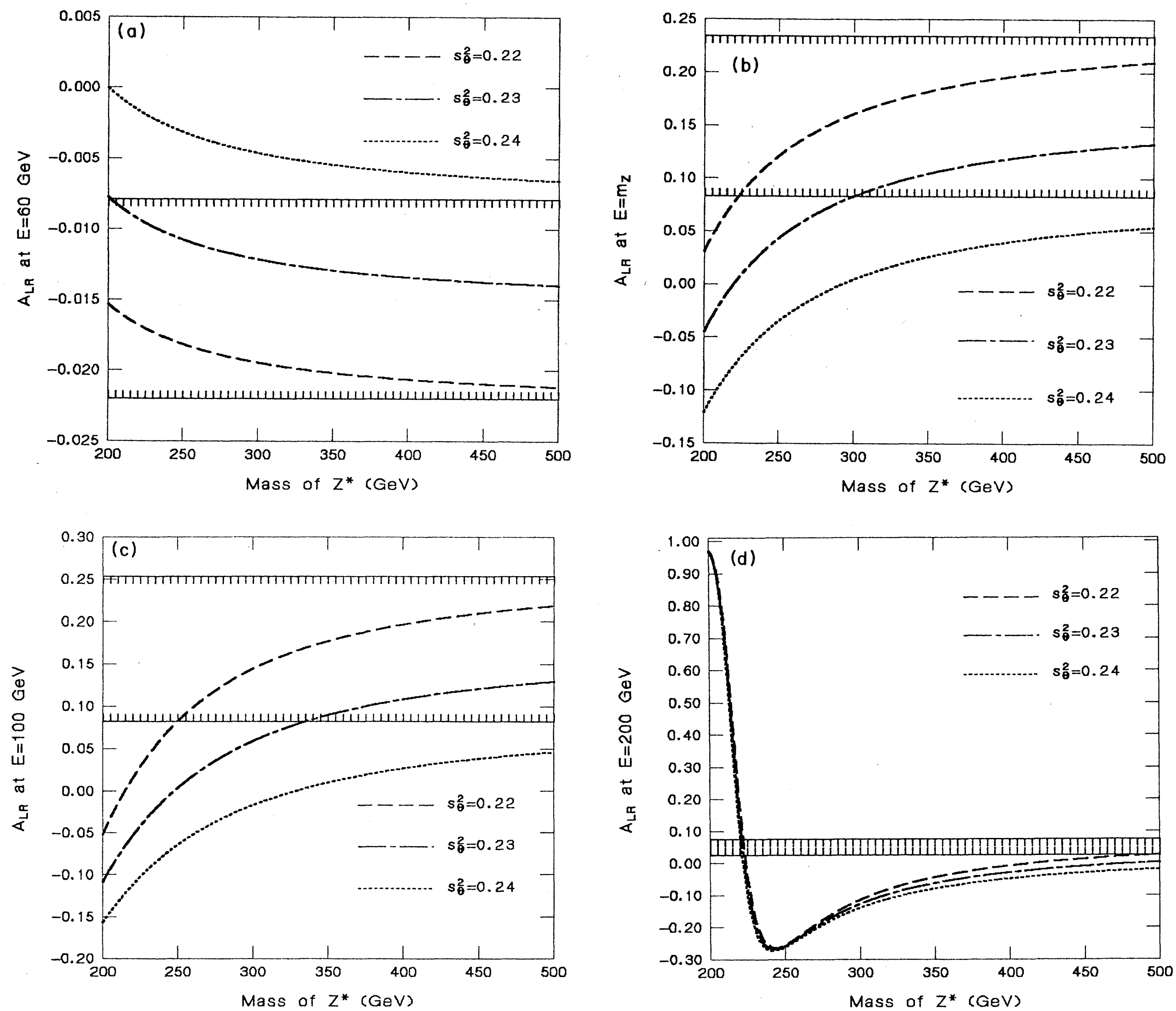

FIG. 5. The same as in Fig. 4 but for the left-right asymmetry A_{LR}.

(3) The deviations are enhanced at larger values of $\sqrt{s}$. At $\sqrt{s}=200$ GeV, to be reached by LEP II, our predictions yield $A_{FB}\simeq 0.2$–0.7 and $A_{LR}\simeq -0.25$–1 while the standard ones are very restricted, i.e., $A_{FB}^{\rm STD}=0.55$–0.57 and $A_{LR}^{\rm STD}=0.02$–0.07.

B. Production of W^* and Z^*

The production of W^* and Z^* at $p\bar{p}$ colliders depends on u $(\bar{u})$ and d $(\bar{d})$ quarks inside p $(\bar{p})$. The quark-structure functions $F_a(x)$, with the subscript a denoting the quark species, are taken to be the ones parametrized by Eichten-Hinchliffe-Lane-Quigg[39] (EHLQ) for $\Lambda_{\rm QCD}=0.2$ GeV. The cross sections for $p\bar{p}\to V+$spectators ($V=W$, Z, W^*, and Z^*) are calculated in terms of parton-parton scattering amplitudes and given by

$$\sigma(V)=\frac{16\pi^2 N_V}{sN_i}\frac{1}{m_V}\sum_{a,b}\Gamma(V\to ab)\frac{dL_{ab}}{d\tau}\ , \qquad (4.18)$$

where $(N_i,N_V)=(36,3)$ count the degrees of freedom of the initial partons (N_i) and the bosons V, (N_V); $dL_{ab}/d\tau$ is the parton-luminosity function defined by

$$\frac{dL_{ab}}{d\tau}=\int_\tau^1\frac{dx}{x}F_a(x)F_b(x/\tau)\Big|_{\tau=m_V^2/s}\ . \qquad (4.19)$$

Presented in Figs. 6(a) and 6(b) are the cross sections $\sigma(W^{*\pm})$ and $\sigma(Z^*)$, at $\sqrt{s}=630$ GeV (CERN) and 1800 GeV (Fermilab), whose values are listed in Table II for $\sin^2\theta=0.23$ and $m_{W^*,Z^*}=250,350,450$ GeV (Ref. 40). The decays of W^* and Z^* are described by $\mathcal{L}_{J=1}$ [Eq. (3.2)] and $\mathcal{L}_{\rm int}$ [Eq. (4.1)]: $W^*\to l\nu$, 2 jets, WZ and $W\gamma$ and $Z^*\to l^+l^-$, 2 jets, WW. The absence of $Z^*\to Z\gamma$ was due to $[I^{(3)},I^{(3)}]=[I^{(3)},Y]=0$. The coupling for $W^*\to W\gamma$ arises as next-to-leading-order effects, which are further suppressed by $\sim(m_{W0}/v)^2$ $(\simeq e^2)$ as shown in Eq. (3.11b). In fact, $B(W^*\to W\gamma)$ was calculated to be $<10^{-3}$. The branching ratios computed are also included in Figs. 6(a) and 6(b) (Ref. 41). The CERN SPS results indicate[19] $\sigma(W^{*\pm})B(W^*\to e\nu_e)<4.6$ pb and $\sigma(Z^*)B(Z^*\to e^+e^-)<4.7$ pb, which are satisfied by $m_{W^*}\geq(204,203,201)$ GeV and $m_{Z^*}\geq(177,176,173)$ GeV for $s_\theta^2=(0.22,0.23,0.24)$. Since $m_{W^*}\simeq m_{Z^*}$, roughly speaking, we get $m_{W^*,Z^*}\gtrsim 200$ GeV.

The expected numbers depend on the luminosity, which has reached 1.6 pb^{-1} at CERN SPS and will reach 10 pb^{-1} at CERN SPS ACOL and 5 pb^{-1} at Fermilab

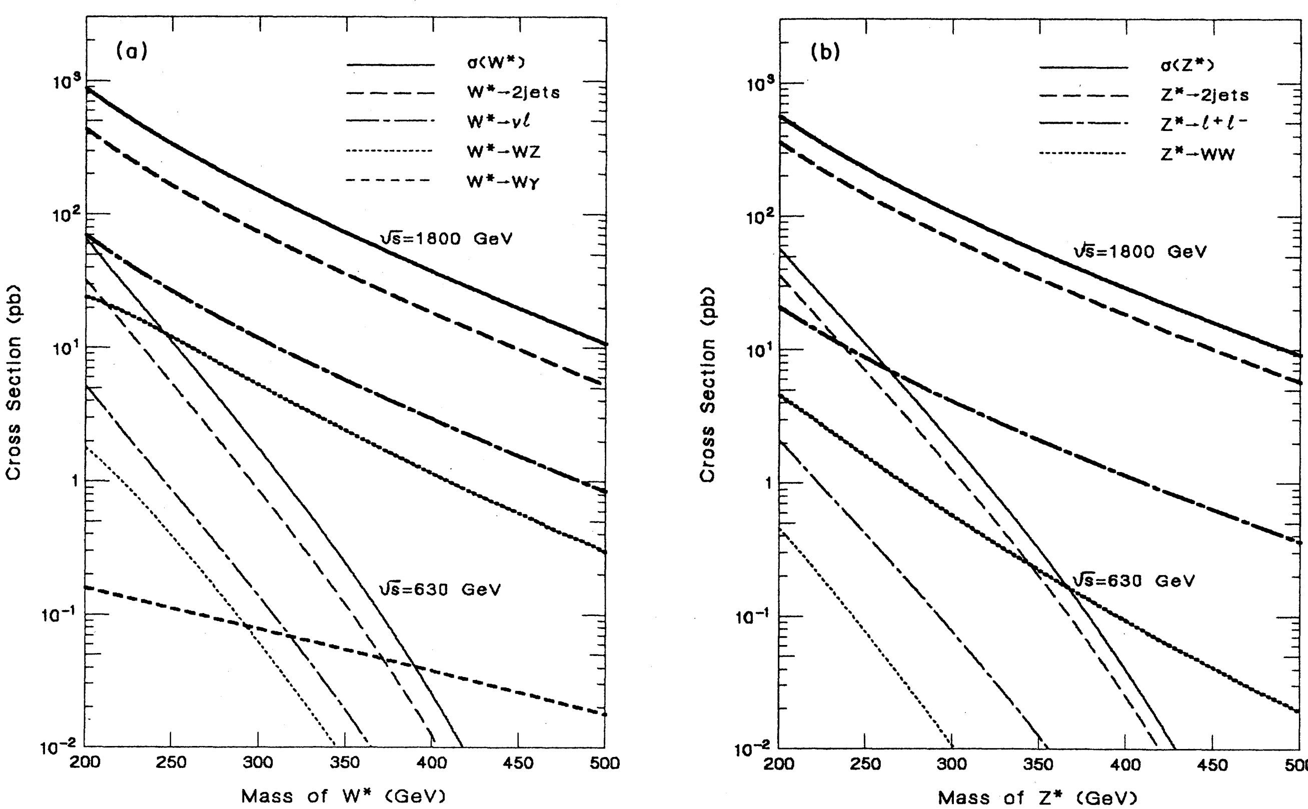

FIG. 6. (a) The cross sections for $p\bar{p}\to W^{*\pm}\to 2$ jets, $\nu l^\pm$, $W^\pm Z$, and $W^\pm\gamma$ for $\sin^2\theta=0.23$ vs m_{W^*} at $\sqrt{s}=630$ and 1800 GeV (thicker curves); (b) the same as in (a) but for $p\bar{p}\to Z^*\to 2$ jets, l^+l^-, and WW vs m_{Z^*}.

TABLE II. The production cross sections (pb), $\sigma(W^{*\pm},Z^*)$ for m_{W^*,Z^*}=250,350,450 GeV at $\sqrt{s}$=630 and 1800 GeV. The angle s_θ^2 is set to be 0.23. The listed values vary within ±10% (for W^*) and ±30% (for Z^*) for $0.22 \le s_\theta^2 \le 0.24$

$\sqrt{s}$	630 GeV			1800 GeV		
m_{W^*,Z^*}	250 GeV	350 GeV	450 GeV	250 GeV	350 GeV	450 GeV
$\sigma(W^*)$	11	0.2	0.002	330	72	19
$\sigma(Z^*)$	11	0.3	0.003	224	53	16

Tevatron. The calculated numbers of events (for m_{W^*,Z^*}=250 GeV and $\sin^2\theta$=0.23) are given as follows: at CERN SPS/CERN SPS ACOL/Fermilab Tevatron,

$p\bar{p}$	$\to W^{*\pm}$	$\to l^\pm\nu$		1.4/9/131
		$\to$2 jets		8/53/815
		$\to WZ$	$\to$4 jets	0.2/1.5/23
			$\to l^\pm\nu$+2 jets	0.02/0.1/2.4
			$\to l^+l^-$+2 jets	0.01/0.1/1
		$\to W\gamma$	$\to\gamma$+2 jets	–/0.03/0.4
	$\to Z^*$	$\to l^+l^-$		0.6/4/43
		$\to$2 jets		10/67/710
		$\to WW$	$\to$4 jets	0.04/0.4/2.5
			$\to l^\pm\nu$+2 jets	0.01/0.1/0.8

The signals of $W^*\to l\nu$ and $Z^*\to l^+l^-$ will be detectable both at CERN and at Fermilab.

V. SUMMARY AND DISCUSSIONS

The excited weak gauge bosons W^* and Z^* are assumed to be supplied by the composites of the left-handed spinor subquarks carrying the weak charge $V_\mu^{*(a)} \sim \overline{w}_L\gamma_\mu\tau^{(a)}w_L$, which are generated by four-Fermi interactions. We have, then, shown that such W^* and Z^* can be transmuted from the gauge bosons $\mathcal{G}^{(a)}$ of the "color"-$SU(2)_{\mathcal{C}}^{loc}$ symmetry. The "flavor"-$SU(2)_L^{loc}\times U(1)_Y^{loc}$ model with W^* and Z^* turns out to be equivalent to the $SU(2)_L^{loc}\times U(1)_Y^{loc}\times SU(2)_{\mathcal{C}}^{loc}$ model with $\mathcal{G}^{(a)}$. From complementarity, one observes that $SU(2)_L^{loc}\times SU(2)_{\mathcal{C}}^{loc}$ is reduced to $SU(2)_L^{loc}$ with W^* and Z^* in the confining phase of $SU(2)_{\mathcal{C}}^{loc}$ or broken to the diagonal subgroup $SU(2)_D^{loc}$ with massive gauge bosons of $SU(2)_{\mathcal{C}}^{loc}$ in the Higgs phase. Once the gauge couplings and physical fields are properly defined, the Lagrangians in the both phases are identical to each other and thus provide a concrete example of complementarity. The natural consequences of W^* and Z^* as gauge particles include (1) the light W^* and Z^* as far as the couplings are sufficiently small and (2) the universality of the W^* and Z^* couplings.

If the weak gauge bosons of $SU(2)_L^{loc}$ $[\times U(1)_Y^{loc}]$ are also composites of subquarks, they should contain scalar subquarks ϖ_L. The isotriplet gauge bosons $V^{(a)}$ are assumed to be $V_\mu^{(a)} \sim i\varpi_L^\dagger\overleftrightarrow{\partial}_\mu\tau^{(a)}\varpi_L + \overline{w}_L\gamma_\mu\tau^{(a)}w_L$. Under $\varpi_L\to U\varpi_L$ and $w_L\to Uw_L$, they transform as $V_\mu\to UV_\mu U^{-1} - iU\partial_\mu U^{-1}$. The compositeness of W and Z leads to $g^*=\sqrt{3/2}g$ and $\sin^2\theta=3x/[2(3+2x)]$ for $x=N_c/N_w$. It is then demonstrated that s_θ^2=0.22–0.24 at $Q=m_W$ is recovered by $s_\theta^2(\Lambda)=\frac{3}{10}$ (x=1) with $\Lambda\sim$(1–2) TeV and by $s_\theta^2(\Lambda)=\frac{1}{4}$ ($x=\frac{3}{2}$) with $\Lambda\sim$0.5 TeV.

One can observe in the long run that it is not necessary to stick to the compositeness of W^* and Z^*. The point is to start with the gauge model of $SU(2)_L^{loc}\times U(1)_Y^{loc}\times SU(2)_{\mathcal{C}}^{loc}$ realized in the Higgs phase of $SU(2)_{\mathcal{C}}^{loc}$. Then, the presence of our W^* and Z^* (which lie in the confining phase) is guaranteed by the complementarity principle. It should be noted that the extra W boson is now possible to exist in the models of (1) $SU(2)_L^{loc}\times U(1)_{B-L}^{loc}\times SU(2)_R^{loc}$ for W_R with the $V+A$ (dominant) coupling and (2) $SU(2)_L^{loc}\times U(1)_Y^{loc}\times SU(2)_{\mathcal{C}}^{loc}$ for W^* with the $V-A$ coupling. As long as the leading-order contributions are concerned, our results follow if $g^*=\sqrt{3/2}g$, which corresponds to a certain unification condition such as $g_R=g_L$ in (1), is maintained.

The vector bosons satisfy the mass relation of $m_{Z^*}=m_Z=c_\theta m_W m_{W^*}$, leading to $m_{W^*}\simeq m_{Z^*}$ since $c_\theta\sim m_W/m_Z$. The lower limits on m_{W^*,Z^*} set by the UA1 and UA2 results dictate $m_{W^*}\simeq m_{Z^*}\gtrsim 200$ GeV. Other bounds derived from the observed values of m_W and m_Z are given as $m_{W^*,Z^*}\gtrsim$(187,216,286) GeV (within

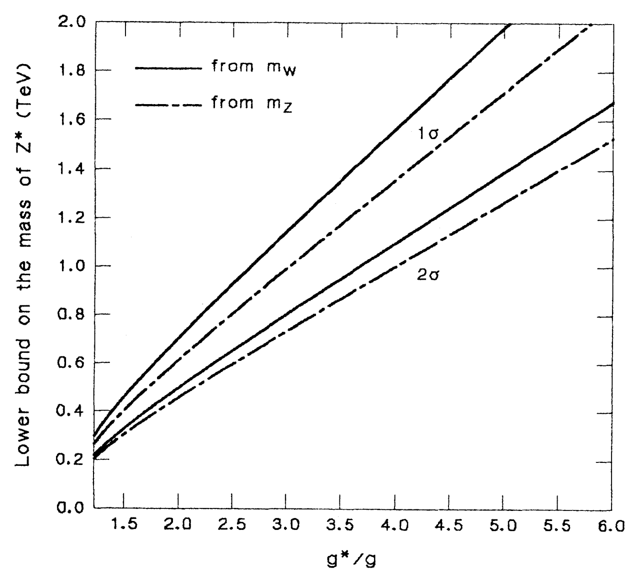

FIG. 7. The lower bounds on m_{Z^*} vs g^*/g at $\sin^2\theta$=0.23 given by the observed values of m_W (solid curves) and m_Z (dot-dash curves) within 1σ and 2σ.

2σ of m_W) and $m_{W^*,Z^*} \geq (189,204,228)$ GeV (within 2σ of m_Z) for $s_\theta^2=(0.22,0.23,0.24)$. [See Figs. 2(a) and 2(b).] All the analyses made so far are based on $g^*=\sqrt{3/2}g$. What happens if g^* departs from $\sqrt{3/2}g$ and gets larger? To see this, we plot, in Fig. 7, lower bounds on m_{Z^*} vs g^*/g at $\sin^2\theta=0.23$ that come from the experimental values of $m_{W,Z}$ within one (or two) standard deviation(s). For $m_{Z^*}(\simeq m_{W^*}) \leq 1$ TeV, it is found that $g^*/g \lesssim 2.8$ (3.8) within one (two) standard deviation(s) of $m_{W,Z}$. To be phenomenologically consistent, the larger g^* calls for heavier W^* and Z^*, which are expected by the relation $m_{W^*,Z^*} \sim g^* \Lambda_{\text{comp}}$.

The deviations of the asymmetries of $e^+e^- \to \mu^+\mu^-$ from the standard-model predictions (for $m_Z=c_{WS}m_W$ with $m_W=80.76\pm1.72$) get maximized at $s_\theta^2=0.24$. If $s_\theta^2=0.24$, we find that, for SLC and LEP, $A_{FB}=0$–0.01 (0.005–0.042 for the standard model) and $A_{LR}=-0.12$–0.06 (0.082–0.23) at $\sqrt{s}=m_Z$ and, at $\sqrt{s}=100$ GeV, $A_{FB}=0.65$–0.75 (0.48–0.61) and $A_{LR}=-0.16$–0.05 (0.08–0.25). The production cross sections for W^* and Z^* with $m_{W^*,Z^*}=250$ GeV at CERN-SPS-ACOL (Fermilab Tevatron) are given by, for $s_\theta^2=0.23$, $\sigma(W^{*\pm})=11$ pb (330 pb) and $\sigma(Z^*)=11$ pb (224 pb) that corresponds to 120 (1650) as the number of $W^{*\pm}$ and to 110 (1120) of Z^*. The recent $W+2$ jets events observed by UA1 (Ref. 42) and UA2 (Ref. 43) may have come from[44] $W^* \to WZ$ with $Z \to 2$ jets and/or $Z^* \to WW$ with $W \to 2$ jets (which are not the dominant decay modes for our W^* and Z^*). Future accumulated data on $p\bar{p}$ and e^+e^- experiments may disclose various signals of the existence of W^* and Z^*.

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[1]H. Terazawa, Phys. Rev. D **22**, 184 (1980); in *Proceedings of the Meeting on Physics at TeV Energy Scale,* KEK, Tsukuba, Ibaraki, 1988, edited by K. Hidaka and K. Hikasa (KEK Report No. 87-20, Ibaraki-ken, 1988), p. 131; M. E. Peskin, in *Proceedings of the 1985 International Symposium on Lepton and Photon Interactions at High Energies,* Kyoto, Japan, 1985, edited by M. Konuma and K. Takahashi (Research Institute for Fundamental Physics, Kyoto, 1986), p. 714; J. C. Pati, in *Superstrings, Supergravity and Unified Theories,* proceedings of the ICTP High Energy Physics and Cosmology Workshop, Trieste, Italy, 1985, edited by G. Furlan *et al.* (ICTP Series in Theoretical Physics, Vol. 2) (World Scientific, Singapore, 1986), p. 377.

[2]H. Terazawa, Phys. Rev. D **7**, 3663 (1973); in *Proceedings of the XIX International Conference on High Energy Physics,* Tokyo, 1978, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, Tokyo, 1979), p. 617; J. D. Bjorken, in *Proceedings of the Ben Lee Memorial International Conference on Parity Nonconservation, Weak Neutral Currents and Gauge Theories,* Fermilab, 1977, edited by D. B. Cline and F. E. Mills (Harwood Academic, New York, 1979), p. 701; Phys. Rev. D **19**, 335 (1979).

[3]G. Costa *et al.*, Nucl. Phys. **B297**, 224 (1988).

[4]M. Yasuè and S. Oneda, Phys. Rev. D **32**, 317 (1985); **32**, 3066 (1985); **37**, 2499 (1988); R. Casalbuoni, S. de Curtis, D. Dominici, and R. Gatto, Phys. Lett. **155B**, 95 (1985); Nucl. Phys. **B282**, 235 (1987); R. Casalbuoni, D. Dominici, F. Feruglio, and R. Gatto, Phys. Lett. B **200**, 495 (1988).

[5]Bjorken, in *Proceedings of the Ben Lee Memorial International Conference on Parity Nonconservation, Weak Neutral Currents and Gauge Theories* (Ref. 2); P. Q. Hung and J. J. Sakurai, Nucl. Phys. **B143**, 81 (1978).

[6]M. Kuroda and D. Schildknecht, Phys. Lett. **121B**, 173 (1983); U. Baur, H. Fritzsch, and H. Faissner, *ibid.* **135B**, 313 (1984); U. Baur and K. H. Schwarzer, Phys. Lett. B **180**, 163 (1986); C. Korpa and Z. Ryzak, Phys. Rev. D **34**, 2139 (1986); U. Baur, D. Schildknecht, and K. H. G. Schwarzer, *ibid.* **35**, 297 (1986); U. Baur, M. Lindner, and K. H. Schwarzer, Phys. Lett. B **193**, 110 (1987); Nucl. Phys. **B291**, 1 (1987).

[7]K. Akama and H. Terazawa, University of Tokyo, Institute for Nuclear Study Report No. 257, 1976 (unpublished).

[8]Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); J. D. Bjorken, Ann. Phys. (N.Y.) **24**, 174 (1963).

[9]H. Terazawa, Y. Chikashige, and K. Akama, Phys. Rev. D **15**, 480 (1977); T. Saito and K. Sigemoto, Prog. Theor. Phys. **57**, 242 (1977).

[10]M. Suzuki, Phys. Rev. D **37**, 210 (1988); A. Cohen, H. Georgi, and E. H. Simmons, *ibid.* **38**, 405 (1988).

[11]K. Akama, in *Proceedings of the Meeting on Physics at TeV Energy Scale* (Ref. 1); K. Akama and T. Hattori (in preparation).

[12]M. Yasuè, University of Tokyo, Institute for Nuclear Study Report No. 712, 1988 (unpublished). See also L. Epele *et al.*, Phys. Rev. D **38**, 2129 (1988), for W^* and Z^* of $SU(2)_{\mathcal{C}}^{\text{loc}}$ as an extra "flavor" symmetry operating on a new set of fermions other than quarks and leptons. For a gauge model of $SU(2)_L^{\text{loc}} \times U(1)_Y^{\text{loc}} \times SU(2)'$, see V. Barger, W. Y. Keung, and E. Ma, *ibid.* **22**, 727 (1980).

[13]G. 't Hooft, in *Recent Developments in Gauge Theories,* proceedings of the Cargese Summer Institute, 1979, edited by G. 't Hooft *et al.* (NATO Advanced Study Institute Series—Series B: Physics, Vol. 59) (Plenum, New York, 1980), p. 135; S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. **B173**, 208 (1980); T. Matsumoto, Phys. Lett. **97B**, 131 (1980); R. Casalbuoni and R. Gatto, *ibid.* **103B**, 113 (1981).

[14]H. Terazawa, Prog. Theor. Phys. **79**, 734 (1988).

[15]J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* **11**, 566 (1974); **11**, 2588 (1974); G. Senjanović and R. N. Mohapatra, *ibid.* **12**, 1502 (1975).

[16]Pati and Salam (Ref. 15).

[17]Yasué and Oneda (Ref. 4); Barger, Keung, and Ma (Ref. 12).

[18]M. Yasué, Prog. Theor. Phys. (to be published).

[19]T. Müller, Report No. CERN-EP/88-48, 1988 (unpublished).

[20]T. Kugo, Soryushiron Kenkyu (Kyoto) **71**, E78 (1985) (in Japanese); in *Proceedings of the International Workshop on Low Energy Effective Theory of QCD,* Nagoya, Japan, 1987, edited by S. Saito and K. Yamawaki (Nagoya University,

Nagoya, 1987), p. 40.

[21]For "W^*" and "Z^*" being hidden $SU(2)^{loc}_{L+R}$-triplet gauge fields in the nonlinear sigma model of $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ as well as their phenomenology, see Casalbuoni, de Curtis, Dominici, and Gatto (Ref. 4). See also M. Kobayashi and T. Matsuki, in *High Energy Physics—1980*, proceedings of the XX International Conference, Madison, Wisconsin, 1980, edited by L. Durand and L. Pondrom (AIP Conf. Proc. No. 68) (AIP, New York, 1981), p. I-440; R. Rosenfeld and J. L. Rosner, Phys. Rev. D **38**, 1530 (1988).

[22]It is even possible to generate W and Z based on $U(1)^{loc}_{em} \times SU(2)^{loc}_{\mathscr{C}}$. See M. Yasuè, Mod. Phys. Lett. A (to be published). See also T. Kugo, S. Uehara, and T. Yanagida, Phys. Lett. **147B**, 321 (1984); S. Uehara and T. Yanagida, *ibid.* **165B**, 94 (1985), for W and Z being hidden $SU(2)^{loc}_{\mathscr{C}}$-triplet gauge fields in the nonlinear σ model of $SU(6)_L/SU(4)_L \times SU(2)_L$.

[23]E. Fradkin and S. H. Shenker, Phys. Rev. D **19**, 3682 (1979); T. Banks and E. Rabinovici, Nucl. Phys. **B160**, 349 (1979).

[24]L. F. Abbott and E. Farhi, Phys. Lett. **101B**, 69 (1981); Nucl. Phys. **B189**, 547 (1981). See also Ref. 13.

[25]For a review, see M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. **164**, 217 (1988). See also the references on W and Z in the nonlinear σ model cited in Ref. 22.

[26]For nonlinear σ models, there are "hidden" and "external" bases, which are equivalent to each other. See, for example, U.-G. Meissner and I. Zahed, Z. Phys. A **237**, 5 (1987); K. Yamawaki, Phys. Rev. D **35**, 412 (1987).

[27]O. W. Greenberg, R. N. Mohapatra, and M. Yasuè, Phys. Rev. Lett. **51**, 1737 (1983).

[28]M. Kobayashi, Prog. Theor. Phys. **68**, 694 (1982); M. Yasuè, Nuovo Cimento A **85**, 229 (1985).

[29]'t Hooft, in *Recent Developments in Gauge Theories* (Ref. 13).

[30]N. Marinescu and M. G. Schmidt, Phys. Lett. **105B**, 347 (1981). See also B. Schrempp and F. Schrempp, Nucl. Phys. **B231**, 109 (1984).

[31]H. Terazawa, Prog. Theor. Phys. **64**, 1388 (1980).

[32]The evaluation is done in a gauge-invariant way for the vector sector with all insertions of the subquark masses m^2_{subquark} included in the propagators of subquarks.

[33]G. 't Hooft, in *Recent Developments in Gauge Theories* (Ref. 13); R. Casalbuoni and R. Gatto, Phys. Lett. **93B**, 47 (1980); Terazawa (Ref. 13); O. W. Greenberg and J. Sucher, Phys. Lett. **99B**, 339 (1981).

[34]It should be noted that the other possibility of $i\overline{w}_L \overleftrightarrow{\partial}_\mu I^{(a)} w_L$ does not exist for $SU(2)^{loc}_L$ since $V^{(a)}_\mu \propto i\overline{w}_L \overleftrightarrow{\partial}_\mu I^{(a)} w_L = 0$. See H. Sugawara, in *Proceedings of the XIX International Conference on High Energy Physics* (Ref. 2), p. 581; D. Amati, R. Barbieri, A. C. Davis, and G. Veneziano, Phys. Lett. **102B**, 408 (1981).

[35]The weak coupling g $[=2(\sqrt{2})^{1/2}G_F^{1/2}m_W]$ is known to be recovered, for $\overline{m} \sim 100$ GeV, by $N_{c,w} \sim 100$ with $\Lambda \sim 1$ TeV and $N_{c,w} \sim 1$ with $\Lambda \sim 10^{19}$ GeV.

[36]It is not true if the (low-energy) theory possesses gauge anomalies that provide $[I^{(3)}, I^{(3)}]$, etc. See K. Akama (private communication); D. Chang, W.-Y. Keung, and S.-C Lee, Phys. Rev. D **38**, 850 (1988).

[37]S. L. Wu, Phys. Rep. **107**, 60 (1984).

[38]The $O(\alpha_s)$ corrections are made in $\Gamma(W, Z, W^*, Z^* \rightarrow q\overline{q})$ for three generations with $m_t = 40$ GeV.

[39]E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, Rev. Mod. Phys. **56**, 579 (1984).

[40]The correction due to the K factor is included in the calculation as $K = 1 + \ln(10^2/\Lambda^2_{\text{QCD}})/\ln(Q^2/\Lambda^2_{\text{QCD}})$. Our estimation gives $\sigma(W^{\pm}) \simeq 6000$–$7000$ pb and $\sigma(Z) \simeq 2000$–2200 pb at $\sqrt{s} = 630$ GeV.

[41]For $W^* \rightarrow W\gamma$, we have set $m^2_{W0}\alpha = g^2/3$ [i.e., $N_w = (4\pi)^2$] with $\alpha \gg \beta$ (i.e., $m^2_w \ll \mu^2_w \ll \Lambda^2_{\text{comp}}$) in Eq. (3.10).

[42]UA1 Collaboration, C. Albajar *et al.*, Phys. Lett. B **193**, 389 (1987).

[43]UA2 Collaboration, P. Bagnaia *et al.*, Phys. Lett. **139B**, 105 (1984); J. Appel *et al.*, Z. Phys. C **30**, 1 (1986).

[44]R. Kleiss and W. J. Stirling, Phys. Lett. B **180**, 171 (1986); T. G. Rizzo, *ibid.* **187**, 169 (1987).