Detailed estimations of the contributions to $\Delta B = 2$ effective Hamiltonian in supersymmetric models

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We estimate various contributions to the $\Delta B=2$ effective Hamiltonian in low-energy supergravity models. Estimations are made not only on the gluino contribution but also on the contributions by other supersymmetric particles and by the physical charged scalar, taking into account the external line momenta of b quarks. We have found that the charged-scalar contribution dominates other nonstandard contributions under the conditions of the radiative breakdown of $SU(2)_L \times U(1)_Y$ gauge symmetry, and that the effects of external momenta are very small.

I. INTRODUCTION

The discovery of large B^0-$\bar{B}^0$ mixing has brought valuable information in particle physics.[1] One of the most striking pieces of information is that the top-quark mass should be as large as 50 GeV or more in the standard model.[2] Some attempts have been made to obtain a light top quark which is interesting from the present experimental point of view.[3,4] We deal with supersymmetric models in this paper as one such attempt. Previous analyses have shown that the gluino-exchange contribution to the $\Delta B=2$ effective Hamiltonian can dominate the usual W-boson-exchange contribution for suitable values of parameters (gluino mass, scalar-quark mass, and so on).[5,6] In their analyses, masses of superparticles and the degree of flavor mixing in quark-scalar–quark-gluino couplings are taken as free parameters. Here, we make an analysis based on the unification scheme. That is, we deal with the so-called low-energy supergravity model, where the above-mentioned parameters are not necessarily free but have some relations among them. We further impose constraints on those parameters from phenomenology, e.g., the lower bound of scalar-quark masses. In this kind of model with constrained parameters, we estimate the contributions to the $\Delta B=2$ effective Hamiltonian not only by the gluino but also by other supersymmetric particles and by the physical charged scalar which necessarily enters in supersymmetric models. The contributions by those fields have been presumed to be negligible in comparison with that by the gluino in the previous analyses. But we show that this presumption is not necessarily right in the models discussed here. We take account of the mass of b quarks at the external lines in the calculations of box diagrams. This has never been done in the analyses of supersymmetric models although often done in those of the standard model.

The rest of this paper is organized as follows. In Sec. II we review the low-energy supergravity model. Constraints on the parameters are given in Sec. III. We estimate various contributions to the $\Delta B=2$ effective Hamiltonian in Sec. IV. A summary is given in Sec. V.

II. LOW-ENERGY SUPERGRAVITY MODEL

The low-energy supergravity model is a minimal supersymmetric extension of the standard model, where supersymmetry (SUSY) is broken through super-Higgs effects.[7] Chiral multiplets which appear in this model are shown in Table I with their transformation properties under $SU(3)\times SU(2)_L \times U(1)_Y$. The Lagrangian is given as

$$\mathcal{L}=\mathcal{L}_{SS}+\mathcal{L}_{SB} , \tag{2.1}$$

$$\begin{aligned}\mathcal{L}_{SS}=&(\text{kinetic terms})\\ &+(\text{gauge interaction terms})\\ &+(y_E^{mn}E_m^C L_n H + y_D^{mn} D_m^C Q_n H + y_U^{mn} U_m^C Q_n H'\\ &+\mu H H')_F + \text{H.c.} ,\end{aligned} \tag{2.2}$$

$$\begin{aligned}\mathcal{L}_{SB}=&(\xi_E^{mn}E_m^C L_n H + \xi_D^{mn} D_m^C Q_n H + \xi_U^{mn} U_m^C Q_n H'\\ &+\rho\mu H H')_A + \text{H.c.}\\ &-(m^2)_{ij}\phi_i^\dagger \phi_j - \sum_N (M_N/2)\lambda_N^T C \lambda_N ,\end{aligned} \tag{2.3}$$

where λ_N is a SU(N) [U(1) for $N=1$] gauge fermion. The SUSY-breaking part $\mathcal{L}_{SB}$ is supposed to be born at a very-high-energy scale near the Planck scale. Assuming grand unification of the gauge interactions, we have the following relations at the grand unification scale M_G:

TABLE I. Chiral superfields and their components. The index n represents generation.

Superfields	Components (fermion boson)	Under $SU(3)\times SU(2)_L\times U(1)_Y$
$Q_n=\begin{bmatrix}Q^u\\ Q^d\end{bmatrix}_n$	$\begin{bmatrix}u_L & \tilde{u}_L\\ d_L & \tilde{d}_L\end{bmatrix}_n$	$(3,2,\frac{1}{3})$
U_n^C	$(u_R^C \;\; \tilde{u}_R^*)_n$	$(3^*,1,-\frac{4}{3})$
D_n^C	$(d_R^C \;\; \tilde{d}_R^*)_n$	$(3^*,1,\frac{2}{3})$
$L_n=\begin{bmatrix}L^\nu\\ L^e\end{bmatrix}_n$	$\begin{bmatrix}\nu_L & \tilde{\nu}_L\\ e_L & \tilde{e}_L\end{bmatrix}_n$	$(1,2,-1)$
E_n^C	$(e_R^C \;\; \tilde{e}_R^*)_n$	$(1,1,2)$
$H=\begin{bmatrix}H_0\\ H_-\end{bmatrix}$	$\begin{bmatrix}\lambda_H^0 & h_0\\ \lambda_H^- & h_-\end{bmatrix}$	$(1,2,-1)$
$H'=\begin{bmatrix}H'_+\\ H'_0\end{bmatrix}$	$\begin{bmatrix}\lambda_{H'}^+ & h'_+\\ \lambda_{H'}^0 & h'_0\end{bmatrix}$	$(1,2,1)$

$$\xi_P^{mn}=Am_g y_P^{mn} \quad (P=E,D,U)\ , \tag{2.4}$$

$$\rho=Bm_g\ , \tag{2.5}$$

$$(m^2)_{ij}=m_g^2\delta_{ij}\ , \tag{2.6}$$

$$M_3=M_2=M_1=M_X\ , \tag{2.7}$$

$$g_3^2=g_2^2=\tfrac{5}{3}g_1^2=g_X^2\ , \tag{2.8}$$

where g_N is the gauge coupling constant of SU(N) [U(1) for $N=1$] gauge interaction. Though we assume grand unification, we do not necessarily require Yukawa couplings to obey the grand unification relations such as $y_D=y_E$. This is what usually happens in superstring-inspired models.[8] The constants A, B, m_g, and M_X in the above equations are the parameters which show SUSY breaking. The values of ξ, M_N, and the scalar masses at lower energy are given by solving renormalization-group equations (RGE's). The results are written in terms of the SUSY-breaking parameters, Yukawa couplings and some calculable constants:[9]

$$M_N=(g_N^2/g_X^2)M_X \quad (\times\tfrac{5}{3}\ \text{ for } N=1)\ , \tag{2.9}$$

$$\xi_D=Am_g y_D(w_D-\alpha_D y_D^\dagger y_D-\beta_D y_U^\dagger y_U)\ , \tag{2.10}$$

$$\xi_U=Am_g y_U(w_U-\alpha_U y_U^\dagger y_U-\beta_U y_D^\dagger y_D)\ , \tag{2.11}$$

$$m_Q^2=m_g^2(x_Q-\delta_D y_D^\dagger y_D-\delta_U y_U^\dagger y_U)\ , \tag{2.12}$$

$$m_D^2=m_g^2(x_D-2\delta_D y_D y_D^\dagger)\ , \tag{2.13}$$

$$m_U^2=m_g^2(x_U-2\delta_U y_U y_U^\dagger)\ , \tag{2.14}$$

where the Yukawa couplings are those at low energy not at M_G. For details please see the Appendix and Ref. 9. Other quantities such as ξ_E, m_L^2 are not important to our following discussions, so that we do not show them here.

The Higgs scalars h_0 and h_0' develop vacuum expectation values (VEV's) v and v', respectively, at the symmetry breaking of $SU(2)_L\times U(1)_Y$. The mass matrix of scalar d-type quarks is given as

$$(\tilde d_L^*,\tilde d_R^*)M_D^2(\tilde d_L,\tilde d_R)^T =(\tilde d_L^*,\tilde d_R^*)\begin{bmatrix}(L-L)_D & (L-R)_D^\dagger\\ (L-R)_D & (R-R)_D\end{bmatrix}\begin{bmatrix}\tilde d_L\\ \tilde d_R\end{bmatrix}\ , \tag{2.15}$$

where

$$(L-L)_D=\mu_L^2(D)+p_D\hat M_d^2-q_D V^\dagger \hat M_u^2 V\ , \tag{2.16}$$

$$(L-R)_D=A_D m_g\hat M_d-(r_D/m_g)\hat M_d^3-(s_D/m_g)\hat M_d V^\dagger \hat M_u^2 V\ , \tag{2.17}$$

$$(R-R)_D=\mu_R^2(D)+k_D\hat M_d^2\ . \tag{2.18}$$

In the above equations, V is the Kobayashi-Maskawa (KM) matrix, $\hat M_d=\text{diag}(m_d,m_s,m_b)$, $\hat M_u=\text{diag}(m_u, m_c,m_t)$ and the definitions of others are

$$\begin{aligned}
&\mu_L^2(D)=m_g^2-(g_2^2/4-g_1^2/12)(v^2-v'^2)+\sum_N(d_Q^N/2k_N)[1-(g_N/g_{NX})^2]M_X^2\ ,\\
&p_D=1-(m_g/v)^2\delta_D,\quad q_D=(m_g/v')^2\delta_U\ ,\\
&A_D=A+(\mu v'/m_g v)+\sum_N(c_D^N/2k_N)[1-(g_N/g_{NX})^2](M_X/m_g)\ ,\\
&r_D=A(m_g/v)^2\alpha_D,\quad s_D=A(m_g/v')^2\beta_D\ ,\\
&\mu_R^2(D)=m_g^2-(g_1^2/6)(v^2-v'^2)+\sum_N(d_D^N/2k_N)[1-(g_N/g_{NX})^2]M_X^2\ ,\\
&k_D=1-2(m_g/v)^2\delta_D\ ,
\end{aligned} \tag{2.19}$$

where $g_{NX}=g_N$ at M_G, $d_Q^N=(d_Q^3,d_Q^2,d_Q^1)=(\frac{16}{3},3,\frac{1}{9})$, $k_N=(-3,1,11)$, $c_D^N=(\frac{16}{3},3,\frac{7}{9})$, and $d_D^N=(\frac{16}{3},0,\frac{4}{9})$. Mass matrix for scalar u-type quarks are given by the following exchange of letters in the above equations:

$$D\leftrightarrow U,\quad d\leftrightarrow u,\quad V\leftrightarrow V^\dagger,\quad v\leftrightarrow v'\ . \tag{2.20}$$

and

$$g_1^2/6\to g_1^2/3 \ \text{ in } \mu_R^2(U)\ .$$

The constants are defined as $c_U^N=(\frac{16}{3},3,\frac{13}{9})$ and $d_U^N=(\frac{16}{3},0,\frac{16}{9})$. In the above mass matrices, scalar quarks are in the basis where corresponding quarks are in mass eigenstates. But these mass matrices are not generation diagonal, which gives flavor-changing interactions mediated by neutral gauginos or Higgsinos. This will be discussed in detail in Sec. IV.

III. CONSTRAINTS ON PARAMETERS

The parameters in our model have some relations among one another as described in the previous section. Still some parameters are left free. They may be fixed if we can put our model into the framework of a higher theory such as extended supergravity or superstring theory. Here we take another approach. We constrain the parameters from phenomenology. The following conditions are imposed: (1) $SU(2)_L\times U(1)_Y$ gauge symmetry breaks down through renormalization effects; (2) SU(3) gauge symmetry does not break down; (3) the mass of the lightest charge superparticle should not exceed experimental bound. Now let us examine each condition in detail.

A. Radiative breakdown of $SU(2)_L\times U(1)_Y$

The Higgs potential is given as

$$V_H=\tfrac{1}{8}(g_1^2+g_2^2)(|h_0|^2-|h_0'|^2)^2+m_1^2|h_0|^2+m_2^2|h_0'|^2-2m_3^2|h_0h_0'|\ , \tag{3.1}$$

where

$$m_1^2=|\mu|^2+m_H^2 \ , \tag{3.2}$$

$$m_2^2=|\mu|^2+m_{H'}^2 \ , \tag{3.3}$$

$$m_3^2=|\rho\mu| \ . \tag{3.4}$$

The conditions of the breakdown of $SU(2)_L \times U(1)_Y$ are given by Inoue, Kakuto, Komatsu, and Takeshita:[10]

$$m_1^2+m_2^2-2m_3^2>0 \ , \tag{3.5}$$

$$m_1^2 m_2^2-(m_3^2)^2<0 \ . \tag{3.6}$$

The first condition is necessary for the potential to be bounded from below. The second one means the existence of a negative direction in the quadratic terms of the potential. At the scale M_G, the second cannot be compatible with the first since we have, at that scale,

$$m_1^2=m_2^2=m_g^2+|\mu|^2, \quad m_3^2=|Bm_g\mu| \ . \tag{3.7}$$

At a lower scale, the values of m_1^2, m_2^2, and m_3^2 vary according to renormalization-group scaling, so that those two conditions can be simultaneously satisfied for some suitable values of parameters.[10] When the above conditions are satisfied, the Higgs potential takes its minimal value for

$$\frac{2vv'}{v^2+v'^2}=\frac{2m_3^2}{m_1^2+m_2^2} \ . \tag{3.8}$$

The solutions of RGE's for m_1^2, m_2^2, and m_3^2 are given by Komatsu under the approximation of neglecting Yukawa couplings except that for the top-quark mass.[11] In the previous analyses of this kind of breakdown of $SU(2)_L \times U(1)_Y$, authors calculated m_1^2, m_2^2, and m_3^2 with the aid of RGE's fixing some of the parameters such as A, m_g, and so on, and checked if the conditions (3.5) and (3.6) are satisfied. But in this work we take another approach.

First, we fix A, v/v', m_g, M_X, and the top-quark mass. The mass squared of the Higgs scalars, m_H^2, and $m_{H'}^2$, can be expressed in terms of these parameters by using the solution of Komatsu. Then the parameter $|\mu|^2$ can be determined through the following relation which can be obtained by exploring the Higgs potential:

$$M_Z^2=(m_2^2-m_1^2)\left[\frac{(v/v')^2+1}{(v/v')^2-1}\right]-(m_2^2+m_1^2) \ , \tag{3.9}$$

with Eqs. (3.2) and (3.3).

Next we determine m_3^2 from Eq. (3.8), which corresponds to take v/v' as a new parameter instead of B from the condition of electroweak breaking. Note that B is not fixed by a relation such as $B=A-1$ in this approach, but varies according to the values of other parameters.

Finally, we check the consistency. The first condition (3.5) is automatically satisfied. If the second condition (3.6) is also satisfied and $|\mu|^2$ is not negative, we adopt those values of parameters as the allowed ones. The allowed regions are shown in Figs. 1(a)–1(f). The region above the line α is allowed in each figure.

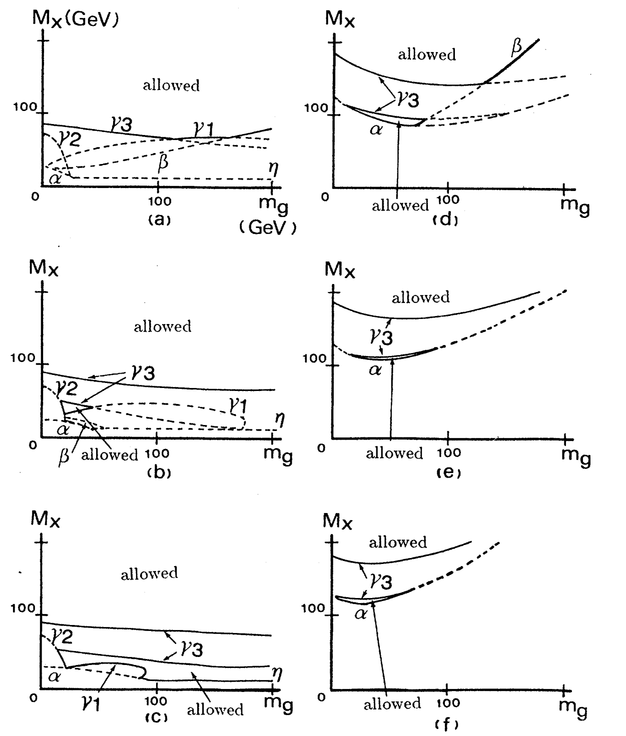

FIG. 1. Allowed region of M_X and m_g. Values of other parameters are given in the text. The regions above the solid line or enclosed by the solid line are allowed.

B. Unbroken SU(3)

We have assumed the Higgs scalars alone develop VEV's in the preceding discussion. Kounnas *et al.* have pointed out in Ref. 12 that there is another candidate of minimum which breaks color:

$$\langle h_0' \rangle=\langle \tilde{u}_R^* \rangle=\langle \tilde{u}_L \rangle\neq 0 \ , \tag{3.10}$$

which gives

$$V_{\min}=-y_0^2(y_0^2-a^2/3) \ , \tag{3.11}$$

where

$$y_0=\tfrac{1}{4}[-A_U-(A_U^2-8a^2/3)^{1/2}] \ , \tag{3.12}$$

$$a^2=(m_{Q3}^2+m_{U3}^2+m_2^2)/m_g^2 \ . \tag{3.13}$$

They have put the condition $V_{\min}\geq 0$. But in this work we require

$$V_H|_{\min}<V_{\min} \ . \tag{3.14}$$

The regions above the line β are allowed in Figs. 1(a)–1(f).

C. Mass bound on lightest charged superparticle

We put the lower bound on the masses of charged superparticles. The bound is taken to be 25 GeV from e^+e^- collider experiments. A candidate of the lightest one is one of scalar top quarks. The mass matrix of scalar quarks gives approximately

$$m_{\rm sq}^2 \sim m_g^2 + m_q^2 \pm A m_g m_q \ , \qquad (3.15)$$

so that one of the masses gets smaller as the quark mass gets larger. Other candidates are a scalar charged lepton and a chargino which is a linear combination of W-inos and Higgsinos. The mass matrix of the scalar charged lepton is obtained in a similar way as we have obtained the mass matrix of scalar d-type quarks. Mass matrix of charginos is given in Eq. (4.29) of Sec. IV E. We examine the eigenvalues of the mass matrices of these particles, and obtain the allowed regions of parameters which satisfy that bound. The lines $\gamma 1$ and $\gamma 2$ are the bounds from the scalar top quark and scalar charged lepton, respectively. Allowed regions are above these lines. The chargino mass bound is shown by the lines $\gamma 3$. The region below $\gamma 3$ in Fig. 1(a) and those between two $\gamma 3$ lines in Figs. 1(b)–1(f) are forbidden.

Although the gluino is not electrically charged, we set the lower bound of the gluino mass to be 50 GeV from $\bar{p}p$ collider experiments. This corresponds to the limit on M_X:

$$M_X \geq 15 \text{ GeV} \ . \qquad (3.16)$$

The bound is shown by the line η.

Finally, we combine these bounds, and allowed regions of parameters are obtained. These are shown in Figs. 1(a)–1(f). The top-quark mass is fixed to be 50 GeV. The values of other parameters in each figure are given as

(a) $A=3, \quad v/v'=0.9$,

(b) $A=2, \quad v/v'=0.9$,

(c) $A=3-\sqrt{3}, \quad v/v'=0.9$,

(d) $A=3, \quad v/v'=0.5$,

(e) $A=2, \quad v/v'=0.5$,

(f) $A=3-\sqrt{3}, \quad v/v'=0.5$,

We have also explored the cases $A=3,2,3-\sqrt{3}$ and $v/v'=0.1$, but no region m_g, $M_X \leq 200$ GeV is allowed by the electroweak breaking condition.

In the next section we estimate the contributions to the $\Delta B=2$ effective Hamiltonian with thus constrained parameters.

IV. CONTRIBUTIONS TO THE $\Delta B=2$ EFFECTIVE HAMILTONIAN

Now we estimate various contributions to the $\Delta B=2$ effective Hamiltonian.

A. *W* boson

We first give here the usual W-boson-exchange contribution. Feynman diagrams are shown in Fig. 2:[13]

$$\mathcal{H}_W = \frac{G_F^2 M_W^2}{4\pi^2} \sum_{i,j=u}^{t} \frac{2\lambda_i\lambda_j}{(1-x_i)(1-x_j)} \times \int_0^1 ds \left\{ \left[\left[1+\frac{x_i x_j}{4} \right] F_1 - \left[x_i x_j - \frac{x_b}{2}[x_i s + x_j(1-s)] \right] F_0 \right] (\bar{b}_L \gamma_\mu d_L)^2 + 2s(1-s)x_b \left[1+\frac{x_i x_j}{4} \right] F_0 (\bar{b}_R d_L)^2 \right\} + \text{H.c.} \ , \qquad (4.1)$$

where $\lambda_i = V_{ib}^* V_{id}$ and $x_i = (m_i/M_W)^2$. The functions are defined as

$$F_1 = F_1(x_i, x_j; s) = \sum_{a=1}^{2} (\Lambda_a \ln\Lambda_a - \Lambda_{a+2}\ln\Lambda_{a+2}) \ , \qquad (4.2)$$

$$F_0 = F_0(x_i, x_j; s) = \sum_{a=1}^{2} (\ln\Lambda_a - \ln\Lambda_{a+2}) \ , \qquad (4.3)$$

where

$$\Lambda_1 = x_i(1-s) + x_j s - x_b s(1-s) \ , \qquad (4.4)$$

$$\Lambda_2 = 1 - x_b s(1-s) \ , \qquad (4.5)$$

$$\Lambda_3 = x_i(1-s) + s - x_b s(1-s) \ , \qquad (4.6)$$

$$\Lambda_4 = (1-s) + x_j s - x_b s(1-s) \ . \qquad (4.7)$$

Calculations have been made in the 't Hooft–Feynman gauge.

B. Physical charged Higgs scalar

Because two Higgs doublets are present in the low-energy supergravity model, there exists one physical charged scalar φ^+. The interaction Lagrangian is given as[14]

$$\mathcal{L} = \frac{g_2}{\sqrt{2}M_W} \bar{u}[(v/v')\hat{M}_u V P_L + (v'/v) V \hat{M}_d P_R] d\varphi^+ + \text{H.c.} \ , \qquad (4.8)$$

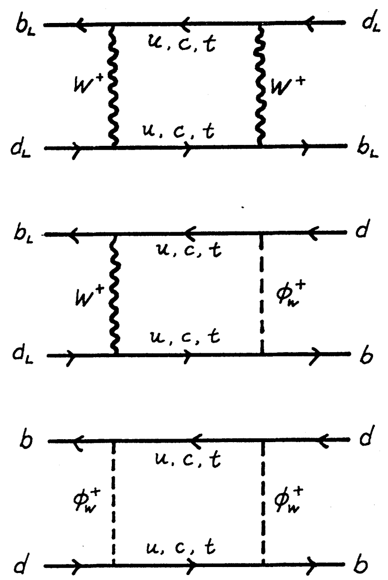

FIG. 2. Box diagrams of W-boson exchange. The dashed lines express the Goldstone boson ϕ_W^+. There are also crossed diagrams which are not shown.

where $P_{L(R)}=[1-(+)\gamma_5]/2$. The contribution shown in Fig. 3 is estimated as

$$\mathcal{H}_{\rm cH} \simeq \frac{G_F^2 M_W^2}{4\pi^2} \sum_{i,j=u}^{t} \lambda_i \lambda_j [A_{ij}(\bar b_L \gamma_\mu d_L)^2 + B_{ij}(\bar b_R d_L)^2] + \text{H.c.} \tag{4.9}$$

The coefficients A_{ij} and B_{ij} are defined as

$$A_{ij} = \frac{1}{2}\frac{M_H^2}{M_W^2}\frac{x_i' x_j'}{(1-x_i')(1-x_j')}\rho^2 \int_0^1 ds\, F_1 + \frac{2}{(x_W'-x_i')(1-x_j')}\int_0^1 ds \left[\frac{1}{2}\frac{M_H^2}{M_W^2} x_i' x_j' \rho \hat F_1 - (x_i' x_j' \rho + x_b' x_j' s)\hat F_0\right], \tag{4.10}$$

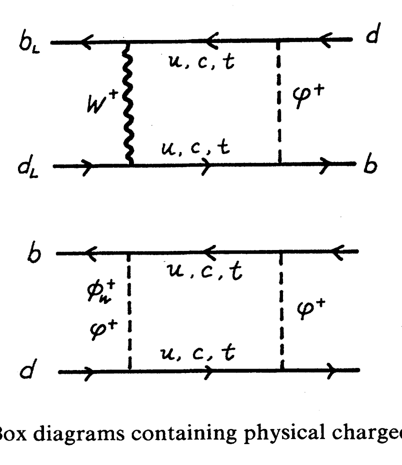

FIG. 3. Box diagrams containing physical charged scalar.

$$B_{ij} = \frac{1}{2}\frac{M_H^2}{M_W^2} x_i' x_j' x_b' \left[\frac{1+\rho}{(1-x_i')(1-x_j')}\int_0^1 ds [2+s(1-s)]F_0 + \frac{4}{(x_W'-x_i')(1-x_j')} \times \int_0^1 ds \left[\rho s(1-s)+1 - 2s - \frac{1}{\rho}\right]\hat F_0\right], \tag{4.11}$$

where $x_i'=(m_i/M_H)^2$, $\rho=(v/v')^2$, and the definitions of the functions $\hat F_1, \hat F_0$ are

$$\hat F_1 = \hat F_1(x_i', x_j'; s) = \sum_{a=1}^{2} (\Gamma_a \ln\Gamma_a - \Gamma_{a+2}\ln\Gamma_{a+2}), \tag{4.12}$$

$$\hat F_0 = \hat F_0(x_i', x_j'; s) = \sum_{a=1}^{2} (\ln\Gamma_a - \ln\Gamma_{a+2}), \tag{4.13}$$

with

$$\Gamma_1 = x_i'(1-s) + x_j' s - x_b' s(1-s), \tag{4.14}$$

$$\Gamma_2 = x_W'(1-s) + s - x_b'(1-s), \tag{4.15}$$

$$\Gamma_3 = x_i'(1-s) + s - x_b' s(1-s), \tag{4.16}$$

$$\Gamma_4 = x_W'(1-s) + x_j' s - x_b' s(1-s). \tag{4.17}$$

The mass of charged Higgs scalar M_H is given in our model as

$$M_H^2 = M_W^2 + m_1^2 + m_2^2. \tag{4.18}$$

C. Gluino

Quark-scalar–quark-gluino couplings are written as

$$\mathcal{L}_{\rm gl} = i\sqrt{2} g_3 (\tilde u_L^* \bar\lambda_3 u_L + \tilde d_L^* \bar\lambda_3 d_L - \bar u_R \tilde u_R \lambda_3 - \bar d_R \tilde d_R \lambda_3) + \text{H.c.} \tag{4.19}$$

Scalar quarks are not in mass eigenstates in the above Lagrangian, so that here we diagonalize the mass matrix of scalar d-type quarks (2.15), and obtain the interaction Lagrangian among physical fields.

As an approximation we drop the minor terms in Eq. (2.15) keeping in mind that $m_g, M_X \gg$ quark masses except those of third-generation quarks:

$$M_D^2 \simeq \begin{bmatrix} \mu_L^2(D) - q_D V^\dagger \hat M_t^2 V & *^\dagger \\ A_D m_g \hat M_b - [(s_D/m_g)\hat M_b V^\dagger \hat M_t^2 V] & \mu_R^2(D) \end{bmatrix}, \tag{4.20}$$

where $\hat M_t = \text{diag}(0,0,m_t)$ and $\hat M_b = \text{diag}(0,0,m_b)$. The term in the square brackets in the above equation gives flavor mixings of left-right scalar quarks and is important for the processes such as electric dipole moment, but not for the $\Delta B=2$ effective Hamiltonian so that we omit it here.[9] This M_D^2 is diagonalized in a very good approximation by

$$\Omega_D = V\Theta \,, \tag{4.21}$$

with

$$V=\begin{bmatrix} V^\dagger & \\ & \mathbb{1}_3 \end{bmatrix}, \quad \Theta=\begin{bmatrix} \mathbb{1}_2 & & & \\ & c & & s \\ & & \mathbb{1}_2 & \\ & -s & & c \end{bmatrix}, \tag{4.22}$$

where $\mathbb{1}_n$ is a $n \times n$ unit matrix. The matrix

$$\begin{bmatrix} c & s \\ -s & c \end{bmatrix} \tag{4.23}$$

diagonalizes the third-generation part of the quark mass matrix. The interaction Lagrangian is rewritten in terms of mass-eigenstate scalar d-type quarks S_d:

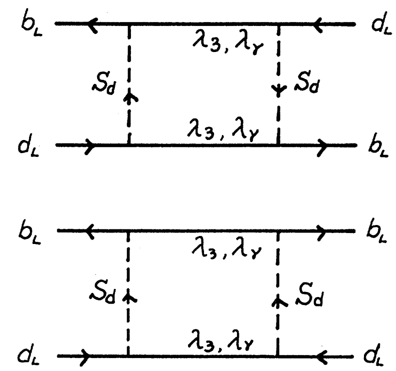

FIG. 4. Box diagrams of the gluino exchange. Note that the gluino is a Majorana fermion.

$$\mathcal{L}_{\rm gl} = i\sqrt{2} g_3 [(\bar{d}_L)_n \lambda_3 (\Omega_D)_{nk} (S_d)_k + \cdots] + \text{H.c.} \tag{4.24}$$

Feynman diagrams are shown in Fig. 4. The contribution is estimated to be

$$\mathcal{H}_{\rm gl} = \frac{\alpha_3^2}{M_3^2} \sum_{k,l} (\Omega_D)_{3k} (\Omega_D^\dagger)_{k1} (\Omega_D)_{3l} (\Omega_D^\dagger)_{l1}$$
$$\times \left[\frac{11}{18} \frac{1}{(1-y_k)(1-y_l)} \int_0^1 ds [F_1(y_k, y_l; s)(\bar{b}_L \gamma_\mu d_L)^2 + 2(m_b/M_3)^2 s(1-s) F_0(y_k, y_l; s)(\bar{b}_R d_L)^2] \right.$$
$$\left. - \frac{1}{9} \frac{1}{(y_k - y_l)} \int_0^1 ds [G(y_k; s) - G(y_l; s)](\bar{b}_L \gamma_\mu d_L)^2 \right] + \text{H.c.} \,, \tag{4.25}$$

where $y_p = (m_p/M_3)^2$. The functions F_1 and F_0 are already defined in Eqs. (4.2) and (4.3), respectively, while the definition of G is

$$G(y; s) = \frac{1}{1-y} \ln \left| \frac{1-s+ys-y_b s(1-s)}{1-y_b s(1-s)} \right| . \tag{4.26}$$

D. Neutral gauginos and Higgsinos

In this case mass eigenstates are linear combinations of neutral gauginos $(\lambda_\gamma, \lambda_Z)$ and Higgsinos $(\lambda_H^0, \lambda_{H'}^0)$, contrary to the case of the gluino which is an eigenstate of both mass and interaction. The mass matrix is given as

$$\begin{array}{c} \\ \lambda_\gamma \\ \lambda_H^0 \\ \lambda_{H'}^0 \\ \lambda_Z \end{array} \begin{array}{c} \begin{array}{cccc} \lambda_\gamma & \lambda_H^0 & \lambda_{H'}^0 & \lambda_Z \end{array} \\ \begin{bmatrix} \dfrac{g_2^2 M_1 + g_1^2 M_2}{g_1^2+g_2^2} & 0 & 0 & \dfrac{g_1 g_2 (M_2 - M_1)}{g_1^2+g_2^2} \\ 0 & -\sin 2\theta_H \mu & -\cos 2\theta_H \mu & 0 \\ 0 & -\cos 2\theta_H \mu & \sin 2\theta_H \mu & M_Z \\ \dfrac{g_1 g_2 (M_2 - M_1)}{g_1^2+g_2^2} & 0 & M_Z & \dfrac{g_2^2 M_2 + g_1^2 M_1}{g_1^2+g_2^2} \end{bmatrix} \end{array}, \tag{4.27}$$

where $\sin\theta_H = v/(v^2+v'^2)^{1/2}$, $\cos\theta_H = v'/(v^2+v'^2)^{1/2}$. Diagonalization of this matrix is complicated, so that we give here the contribution by λ_γ neglecting mixing among others for order estimation. Feynman diagrams are the same as those of the gluino:

$$\mathcal{H}_{\tilde{\gamma}}=\frac{\alpha^2}{81M_{\tilde{\gamma}}^2}\sum_{k,l}(\Omega_D)_{3k}(\Omega_D^\dagger)_{k1}(\Omega_D)_{3l}(\Omega_D^\dagger)_{l1}$$
$$\times\left[\frac{1}{(1-y_k')(1-y_l')}\int_0^1 ds[F_1(y_k',y_l';s)(\bar{b}_L\gamma_\mu d_L)^2+2(m_b/M_{\tilde{\gamma}})^2 s(1-s)F_0(y_k',y_l';s)(\bar{b}_R d_L)^2]\right.$$
$$\left.-\frac{2}{y_k'-y_l'}\int_0^1 ds[G(y_k';s)-G(y_l';s)](\bar{b}_L\gamma_\mu d_L)^2\right]+\text{H.c.}\ , \tag{4.28}$$

where $y_p'=(m_p/M_{\tilde{\gamma}})^2$, and $M_{\tilde{\gamma}}$ is the mass of λ_γ. Note that the differences between that of the gluino are mass, coupling constant, and color factor.

E. Charged gauginos and Higgsinos

Mass matrix of charged gauginos $(\lambda_W^+,\lambda_W^-)$ and Higgsinos $(\lambda_{H'}^+,\lambda_H^-)$ are

$$\begin{array}{c} \\ \lambda_W^+ \\ \lambda_{H'}^+ \end{array}\begin{array}{c}\begin{array}{cc}\lambda_W^- & \lambda_H^-\end{array}\\ \begin{bmatrix} M_2 & -g_2 v\\ -g_2 v' & \mu\end{bmatrix}\end{array}\equiv U_L^\dagger\begin{bmatrix}M_A & 0\\ 0 & M_B\end{bmatrix}U_R\ . \tag{4.29}$$

This matrix gives two Dirac fermions: say ψ_A^+ and ψ_B^+. The relevant part of the interaction Lagrangian is

$$\mathcal{L}_{\tilde{W}}=ig_2(S_u^*)_j(\Omega_U)_{jn}V_{nm}(d_L)_m$$
$$\times[\psi_A^+(U_L)_{AW}+\psi_B^+(U_L)_{BW}]+\text{H.c.}\ , \tag{4.30}$$

where summations are taken for $j=1-6$ and $n,m=1-3$. The unitary matrix Ω_U diagonalizes the mass matrix of scalar u-type quarks:

$$M_U^2\simeq\begin{bmatrix}\mu_L^2(U)+p_U\hat{M}_t^2 & A_U m_g\hat{M}_t-(r_U/m_g)\hat{M}_t^3\\ A_U m_g\hat{M}_t-(r_U/m_g)\hat{M}_t^3 & \mu_R^2(U)+k_U\hat{M}_t^2\end{bmatrix}. \tag{4.31}$$

This matrix is obtained from Eqs. (2.15)–(2.18) with the replacements (2.20) and the same procedure to obtain the approximate form of M_D^2 in (4.20). This is already generation diagonal and off-diagonal elements lie in the third generation only, so that we have

$$\Omega_U=\Theta'\ , \tag{4.32}$$

where Θ' is defined in the same way as that of Θ in (4.22). Finally, the contribution shown in Fig. 5 is estimated as

$$\mathcal{H}_{\tilde{W}}\simeq\frac{\alpha_2^2}{4M_A^2}|(U_L)_{AW}|^4\sum\frac{\xi_i\xi_j}{(1-z_i)(1-z_j)}\int_0^1 ds[F_1(z_i,z_j;s)(\bar{b}_L\gamma_\mu d_L)^2+2z_b s(1-s)F_0(z_i,z_j;s)(\bar{b}_R d_L)^2]+\text{H.c.}\ , \tag{4.33}$$

where $z_p=(m_p/M_A)^2$ and $\xi_i=\sum_{k,l}V_{3k}^\dagger(\Omega_U)_{ki}(\Omega_U^\dagger)_{il}V_{l1}$. To obtain the above equation we estimate only the contribution by ψ_A^+ for the simplicity of calculation. There also exist contributions by ψ_B^+ and a mixed one. We take ψ_A^+ to be the lighter of the two which gives the major contribution. Thus for order estimation the above expression is sufficient.

F. Numerical analyses

Let us make a comparison among the contributions and see which one dominates. The following points are important for this comparison. The usual W-boson contribution (4.1) is dominated by top-quark exchange; then we have, in a very good approximation,

$$M_{12}^W=(\lambda_t)^2\times(\text{real factor})\ , \tag{4.34}$$

where M_{12}^W is the dispersive part of $\langle\bar{B}^0|\mathcal{H}_W|B^0\rangle$. The charged-Higgs-scalar contribution (4.9) is also dominated by top-quark exchange since it is from Yukawa interaction, so we also have

$$M_{12}^{\text{cH}}\propto(\lambda_t)^2\ . \tag{4.35}$$

For gaugino contributions (4.25) and (4.28), we can show by using Eqs. (4.21) and (4.22) that

$$\sum_{k,l}(\Omega_D)_{3k}(\Omega_D^\dagger)_{kl}(\Omega_D)_{3l}(\Omega_D^\dagger)_{l1}f(x_k,x_l)=\sum_{m,n=1}^{5}\lambda_m\lambda_n f(x_m,x_n)-2s^2\lambda_t\sum_n\lambda_n[f(x_n,x_3)-f(x_n,x_6)]$$
$$+s^4(\lambda_t)^2[f(x_3,x_3)-2f(x_3,x_6)+f(x_6,x_6)]$$
$$=(\lambda_t)^2\{[f(x_3,x_3)+f(x_1,x_1)-2f(x_1,x_3)]$$
$$-2s^2[f(x_3,x_3)-f(x_1,x_1)-f(x_3,x_6)+f(x_1,x_6)]$$
$$+s^4[f(x_3,x_3)-2f(x_3,x_6)+f(x_6,x_6)]\}\ , \tag{4.36}$$

where $f(x,y)$ is a function which satisfies $f(x,y)=f(y,x)$, and the last equality is obtained under the condition $x_1=x_2$, which holds for y_p and y_p' in Eqs. (4.25) and (4.28). Therefore the contributions $M_{12}^{\rm gl}$ and $M_{12}^{\tilde{\gamma}}$ are proportional to $(\lambda_t)^2$, too. In the same way we can show, for $M_{12}^{\tilde{W}}$ from (4.33),

$$\sum_{i,j}\xi_i\xi_j f(x_i,x_j)=(\lambda_t)^2\{[f(x_3,x_3)+f(x_1,x_1)-2f(x_1,x_3)]-2s^2[f(x_3,x_3)-f(x_1,x_1)-f(x_3,x_6)+f(x_1,x_6)]$$
$$+s^4[f(x_3,x_3)-2f(x_3,x_6)+f(x_6,x_6)]\}\ . \tag{4.37}$$

Thus all the contributions we have so far estimated are written as $(\lambda_t)^2\times$(real factor). We have only to compare the real factors for the comparison.

As for the absorptive part Γ_{12}, supersymmetric particles cannot contribute since they are all heavier than the B meson. Thus the value of Γ_{12} in the supersymmetric models remains the same as in the case of the standard model.

Now we make numerical estimations of the contributions. The top-quark mass has been fixed to be 50 GeV. This is just the bound for the standard model to explain the large B^0-$\bar{B}^0$ mixing. If supersymmetric contributions are significant for this value of top-quark mass in comparison with the standard one, we can anticipate that the existence of the light top quark will be proven at KEK TRISTAN, the SLAC Linear Collider (SLC), or CERN LEP. For other parameters, we have adopted the values allowed in Fig. 1 in Sec. III. We have found that nonstandard contributions are much smaller than the standard one for most of the allowed values of parameters. The ratio $M_{12}^{\rm cH}/M_{12}^{W}$ is shown in Fig. 6(a), where the values of A and v/v' are chosen to be the same as the case of Fig. 1(a). The shaded region is not allowed from the constraints given in Sec. III. In the black-painted region electroweak breaking cannot take place, so that we cannot estimate M_{12}. But we have estimated M_{12} even if some superparticle masses get lower than the bounds to see how much the constraints restrict the value of M_{12}. In the same manner we have made estimations in the cases of Figs. 1(b)–1(f). The results for cases (b) and (c) are shown in Figs. 6(b) and 6(c), respectively. Minor

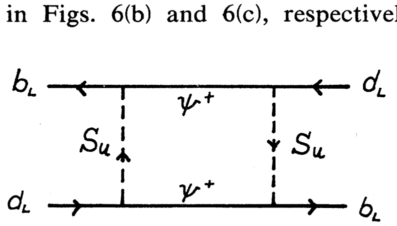

FIG. 5. Box diagram for the charged gaugino and the Higgsino exchange.

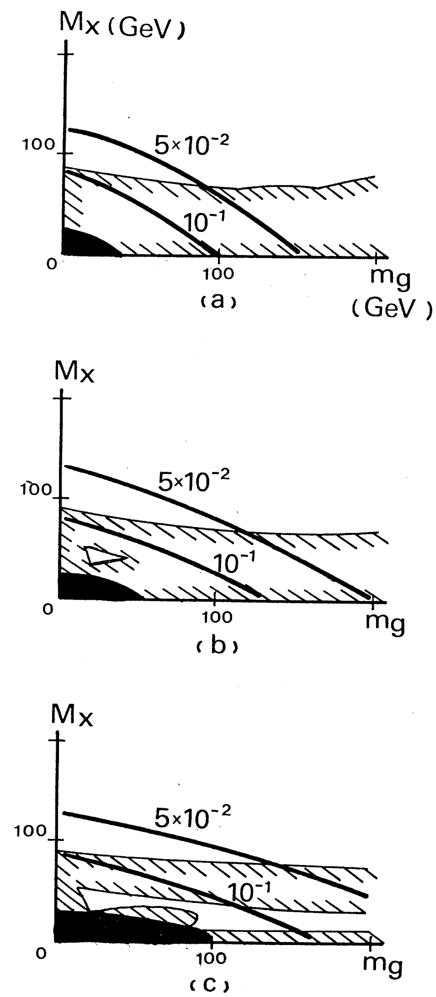

FIG. 6. The ratio M_{12}^{cH}/M_{12}^{W} shown by the bold solid lines. The values of A are $3,2,3-\sqrt{3}$ for the cases (a), (b), (c), and $v/v'=0.9$. The shaded region is not allowed from the constraints given in Sec. III.

values of the ratio (10^{-2}–10^{-1}) are obtained for cases (d)–(f).

In the same manner, we have obtained the ratios $M_{12}^{\rm gl}/M_{12}^{W}$ and $M_{12}^{\tilde{W}}/M_{12}^{W}$ which are shown Figs. 7(a)–7(c) and 8(a)–8(c), respectively. These ratios are smaller than 10^{-4} for $v/v'=0.5$ in all the allowed regions of m_g and M_X. Figure 7 shows that the large contribution of the gluino to M_{12} is inconsistent with the constraints given in Sec. III. We have also found that the photino contribution is very small ($M_{12}^{\tilde{\gamma}}/M_{12}^{W}<10^{-5}$) in every allowed region.

Here we comment on the top-quark-mass dependence of our estimations. Roughly speaking, the charged-Higgs-scalar contribution varies as $m_t^6/(m_H^4)$, which can be seen from Eq. (4.10). The solutions of the RGE's and Eqs. (3.9) and (4.18) show that M_H^2 varies approximately as m_t^2. Therefore, we approximately have $M_{12}^{\rm cH}\propto m_t^2$, which is the same dependence of M_{12}^{W}. [Note that this is because we decided M_H from Eq. (4.18). If we fix M_H, then $M_{12}^{\rm cH}\propto m_t^6$ as in the case of other multi-Higgs-scalar model.] The gluino and photino contributions are approximately proportional to m_t^4 since the mass-squared differences among scalar d-type quarks are proportional to m_t^2 as shown in Eq. (4.20). The chargino contribution complicatedly depends on m_t because of the mixing between gauginos and Higgsinos, so we cannot give a simple explanation. We have made numerical estimations of those contributions for $m_t=60$ GeV and have found that

$$M_{12}^{\rm cH}(m_t=60\ {\rm GeV})/M_{12}^{\rm cH}(m_t=50\ {\rm GeV})\sim 1.4\simeq(60/50)^2\ ,$$

$$M_{12}^{\rm gl}(m_t=60\ {\rm GeV})/M_{12}^{\rm gl}(m_t=50\ {\rm GeV})\sim 2\simeq(60/50)^4\ ,$$

$$M_{12}^{\tilde{W}}(m_t=60\ {\rm GeV})/M_{12}^{\tilde{W}}(m_t=50\ {\rm GeV})=3\text{–}6\ .$$

The first two results support our preceding discussions. Thus the contributions to the $\Delta B=2$ effective Hamiltonian get larger as the top quark gets heavier. The W-boson contribution is enough to explain the observed B^0-$\bar{B}^0$ mixing for $m_t>50$ GeV, so that if the top quark is found to be heavy the masses of superparticles have to be large enough not to give a too large mixing. On the other hand, if the top-quark mass is as small as about 30 GeV, we cannot explain the mixing even including the contri-

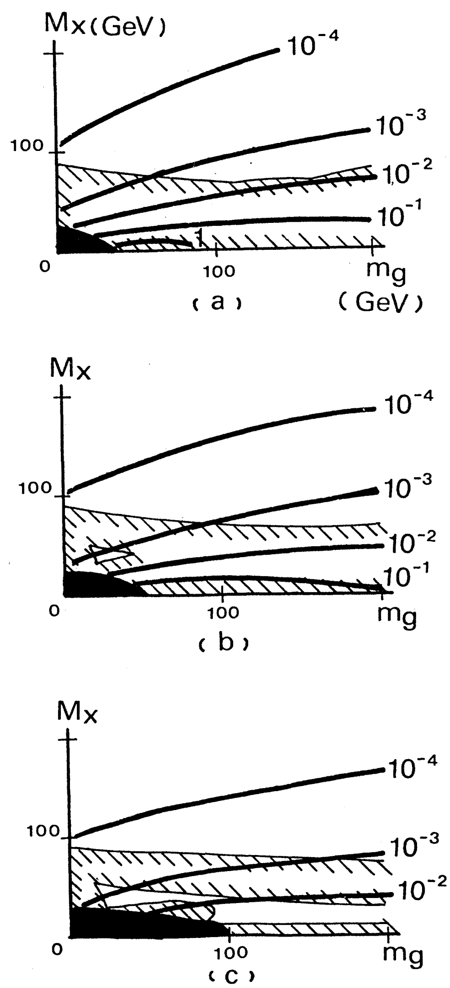

FIG. 7. The ratio $M_{12}^{\rm gl}/M_{12}^{W}$.

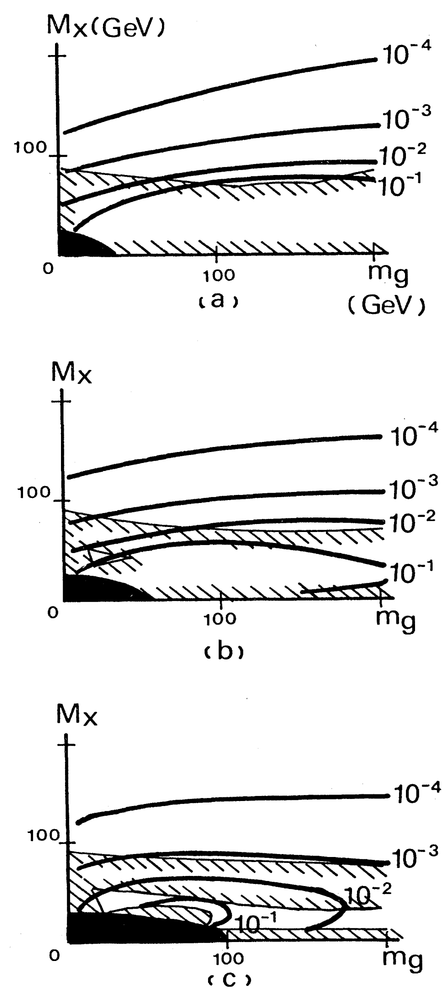

FIG. 8. The ratio $M_{12}^{\tilde{W}}/M_{12}^{W}$.

TABLE II. The ratios $(f-f_0)/f$, where f (f_0) is the value of a function $f(x,x)$ evaluated with (without) b-quark momenta. The b-quark mass is taken to be 4.6 GeV.

		W boson ($M_W=81$ GeV)					
	m_t (GeV)	50	60	70	80	90	100
	$\frac{f-f_0}{f}$ (%)	3.8	3.0	2.4	2.0	1.7	1.5
		Charged Higgs boson					
	m_t (GeV)	50	60	70	80	90	100
$\frac{f-f_0}{f}$ (%)	$M_H=40$ GeV	0.14	0.13	0.13	0.12	0.11	0.10
	$M_H=85$ GeV	0.030	0.035	0.039	0.040	0.040	0.03
		Gluino					
	M_{sq} (GeV)	50	60	70	80	90	100
$\frac{f-f_0}{f}$ (%)	$M_3=50$ GeV	0.41	0.33	0.28	0.24	0.21	0.18
	$M_3=100$ GeV	0.22	0.18	0.15	0.13	0.11	0.10
		Charged gaugino and Higgsino					
	M_{sq} (GeV)	50	60	70	80	90	100
$\frac{f-f_0}{f}$ (%)	$M_{\tilde{W}}=25$ GeV	0.72	0.58	0.48	0.40	0.35	0.30
	$M_{\tilde{W}}=50$ GeV	0.38	0.32	0.27	0.23	0.20	0.18

butions of supersymmetric particles and that of charged scalars in our model.

How large are the effects of the momenta of b quarks at external lines? To answer this question we have evaluated the functions obtained by calculating diagrams with and without b-quark momenta. We have evaluated the functions $f(x,x)$ in Eqs. (4.36) and (4.37), but not M_{12} because Glashow-Iliopoulos-Maiani (GIM) cancellation obscures the effects by external momenta. The results are shown in Table II for various values of particle masses. It shows that the effects of b-quark momenta are very small. However, they cannot be neglected if particle masses are almost degenerate and the super-GIM cancellation occurs to 0.1% order, which is often the case in the gluino and the photino contributions in the low-energy supergravity models.

V. SUMMARY

We have found that the typical order of the ratio among various contributions to the $\Delta B=2$ effective Hamiltonian in the low-energy supergravity models is

$$M_{12}^{W}:M_{12}^{\text{cH}}:M_{12}^{\tilde{W}}:M_{12}^{\text{gl}}:M_{12}^{\tilde{\gamma}} = 1:10^{-2}\text{–}0.5:\lesssim 10^{-1}:\lesssim 10^{-2}:<10^{-5}\,.$$

This shows that the contributions by SUSY particles are minor under the constraints given in Sec. III. It is because the super-GIM mechanism works very well in the low-energy supergravity models where flavor mixings among scalar quarks in their mass matrix come from radiative corrections. We have also found that the charged gaugino contribution can be as large as or larger than the gluino contribution, which has never been pointed out in the previous works. This is because one scalar top quark can be much lighter than other scalar quarks so that the super-GIM mechanism works less effectively than in the case of the gluino contribution where scalar d-type quarks are exchanged.

The effects of the external line momenta of b quarks have been found to be very small. They are less than 1%. But they can be significant in the case where scalar-quark masses are nearly degenerate in the super-GIM cancellation.

Including the contribution by the physical charged scalar, the sum of nonstandard contributions to the $\Delta B=2$ effective Hamiltonian in the low-energy supergravity models discussed here is at most about 50% of the standard one, so they cannot dominate the W contribution. However, if there is this extra 50% contribution, the lower bound of the top-quark mass can be lowered to about 40 GeV. The difference between 50 and 40 GeV is very significant for LEP and SLC whose beam energies are 100 GeV.

APPENDIX

Here we give solutions of renormalization-group equations (RGE's) adopted in this work. The RGE's are given in Refs. 10 and 15. For example,

$$\dot{y}_D=(y_D/2)[c_D^N g_N^2-\text{Tr}(y_E^\dagger y_E+3y_D^\dagger y_D)-3y_D^\dagger y_D - y_U^\dagger y_U]\,, \tag{A1}$$

$$\dot{\xi}_D=(\xi_D/2)[c_D^N g_N^2-\text{Tr}(y_E^\dagger y_E+3y_D^\dagger y_D)-5y_D^\dagger y_D-y_U^\dagger y_U] + y_D[c_D^N g_N^2 M_N-\text{Tr}(y_E^\dagger \xi_E+3y_D^\dagger \xi_D)-2y_D^\dagger \xi_D - y_U^\dagger \xi_U]\,, \tag{A2}$$

where $\dot{f}=(d/dt)f$ with $t=(1/8\pi^2)\ln(M_G/s)$ (s is the renormalization point). The constant c_D^N is defined below Eq. (2.19). To obtain exact analytic solutions is very difficult in practice, so that we solve the RGE perturbatively in terms of Yukawa coupling constants at the grand-unified-theory (GUT) scale M_G. To be concrete we give the solution for y_D:

$$y_D = z_D y_{DX}\{1-\tfrac{3}{2}I[z_D^2;w]y_{DX}^\dagger y_{DX} - \tfrac{1}{2}I[z_U^2;w]y_{UX}^\dagger y_{UX}\} , \tag{A3}$$

where y_{DX}, y_{UX} are the Yukawa coupling constants at M_G,

$$z_D = (g_3^2/g_X^2)^{8/9}(g_2^2/g_X^2)^{-3/2}(5g_1^2/3g_X^2)^{-7/198} , \tag{A4}$$

$$z_U = z_D(5g_1^2/3g_X^2)^{-3/99} , \tag{A5}$$

$$I[f;w] = \int_0^w f(t)dt \quad [w = (1/8\pi^2)\ln(M_G/M_W)] . \tag{A6}$$

The RGE's for other quantities are solved in the same way, and then we rewrite them in terms of the Yukawa couplings at M_W using Eq. (A3) and the similar expressions for y_U and y_E. This is because the experimentally known values of quark masses and KM matrix elements are related to the Yukawa coupling constants at low energy. After tedious calculations the parameters in Eqs. (2.10)–(2.14) are written as

$$w_D = 1 + (c_D^N/k_N)[1-(g_N^2/g_{NX}^2)](M_X/m_g) , \tag{A7}$$

$$\alpha_D = \alpha_{D1} + \alpha_{D2}(M_X/Am_g) , \tag{A8}$$

$$\alpha_{D1} = z_D^{-2}\{\tfrac{9}{2}I[z_D^2 w_D;w] - (3w_D/2)I[z_D^2;w]\} , \tag{A9}$$

$$\alpha_{D2} = z_D^{-2}\tfrac{3}{2}c_D^N\int_0^w dt\{(g_N^4/g_{NX}^2)I[z_D^2;t]\} , \tag{A10}$$

$$\beta_D \doteq \beta_{D1} + \beta_{D2}(M_X/Am_g) , \tag{A11}$$

$$\beta_{D1} = z_U^{-2}\{\tfrac{9}{2}I[z_U^2(w_U+w_D/2);w] - (w_D/2)I[z_U^2;w]\} , \tag{A12}$$

$$\beta_{D2} = z_U^{-2}\tfrac{1}{2}c_D^N\int_0^w dt\{(g_N^4/g_{NX}^2)I[z_U^2;t]\} . \tag{A13}$$

Expressions for w_U, α_U, and β_U are obtained by replacing the subscript D by U in the above equations. And we have, for the rest of the parameters,

$$x_F = 1 + (d_F^N/2k_N)[1-(g_N^4/g_{NX}^4)](M_X/m_g)^2 \quad (F=Q,D,U) , \tag{A14}$$

$$\delta_D = z_D^{-2}\int_0^w dt\, z_D^2\{3 + w_D^2 A^2 + (c_D^N/k_N)[1-(g_N^4/g_{NX}^4)](M_X/m_g)^2\} , \tag{A15}$$

$$\delta_U = z_U^{-2}\int_0^w dt\, z_U^2\{3 + w_U^2 A^2 + (c_U^N/k_N)[1-(g_N^4/g_{NX}^4)](M_X/m_g)^2\} , \tag{A16}$$

where constants were already given below Eq. (2.19). The results of our numerical calculations are given for convenience. We take $\alpha^{-1}(M_W)=128$ and $\sin^2\theta_W(M_W) = 0.225$ following Ref. 11, which gives $g_X^2 = 0.539$:[9]

$$\xi_D = Am_g y_D\{1 + 4.88(M_X/m_g) - [0.354 + 0.957(M_X/m_g)]y_D^\dagger y_D - [0.116 + 0.320(M_X/m_g)]y_U^\dagger y_U\} , \tag{A17}$$

$$\xi_U = Am_g y_U\{1 + 4.92(M_X/m_g) - [0.118 + 0.319(M_X/m_g)]y_D^\dagger y_D - [0.349 + 0.961(M_X/m_g)]y_U^\dagger y_U\} , \tag{A18}$$

$$x_Q = 1 + 9.56(M_X/m_g)^2 , \tag{A19}$$

$$x_D = 1 + 9.40(M_X/m_g)^2 , \tag{A20}$$

$$x_U = 1 + 9.46(M_X/m_g)^2 , \tag{A21}$$

$$\delta_D = 0.118(3 + A^2) + 0.638A(M_X/m_g) + 2.13(M_X/m_g)^2 , \tag{A22}$$

$$\delta_U = 0.116(3 + A^2) + 0.640A(M_X/m_g) + 2.15(M_x/m_g)^2 . \tag{A23}$$

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