Model-independent analysis of quark mass matrices

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In view of the apparent inconsistency of the Stech, Fritzsch-Stech, and Fritzsch-Shin models and only marginal agreement of the Fritzsch and modified Fritzsch-Stech models with recent data on B_d^0-$\bar{B}_d^0$ mixing, we analyze the general quark mass matrices for three generations. Phenomenological considerations restrict the range of parameters involved to different sectors. In the present framework, the constraints corresponding to various *Ansätze* have been discussed.

I. INTRODUCTION

The quark and lepton masses as well as the charge-current mixing angles and the Kobayashi-Maskawa (KM) phase, which is responsible for *CP* violation, are the free parameters within the standard model. The experimental knowledge on these parameters has recently been strengthened following a recent discovery of a relatively large B^0-$\bar{B}^0$ mixing[1] and the observation of charmless b decays involving baryons.[2] These results leave very little arbitrariness in the parameters of the quark sector of the standard model with three fermion generations.

In recent years a lot of effort has been directed towards understanding the values that the ten independent parameters characterizing the quark sector of the three-generation standard model (namely, the six quark masses, the three mixing angles, and the KM phase) assume. Attempts were made to make some *Ansätze* about the up- and down-quark mass matrices to obtain relations between the ten parameters. The idea is to start with some lesser numbers of parameters, so that some of the parameters come out as predictions of the model. The most prominent of these phenomenological models are the eight-parameter Fritzsch[3] mass matrices and the seven-parameter Stech[4] mass matrices. Several modifications of these two types have also been considered.[5-7]

Except for the Fritzsch *Ansatz*, the other models[4-7] mentioned here predict maximal *CP* violation, that is the KM phase assumes a value $\pi/2$. When the predictions of the Stech *Ansatz*[4] and the Fritzsch-Stech *Ansatz*[5] for the mixing angles are confronted with the experimental values, one obtains a limit on the top-quark mass $m_t^{\text{phys}} \leq 46$ GeV, whereas for other models[3,6,7] the limit is $m_t^{\text{phys}} \leq 85$ GeV.

A reanalysis[8] of these models[3-7] in the light of the recent experiments[1,2] has been done. It has been demonstrated that the limit on the top-quark mass $m_t^{\text{phys}} \leq 46$ GeV and the maximal *CP* violation (Stech model[4] and Fritzsch-Stech model[5]) is in contradiction to the value of $\epsilon = 2.3 \times 10^{-3}$ in the K^0-$\bar{K}^0$ system and the new measurement[1] of the B_d^0-$\bar{B}_d^0$ mixing parameter $r_d = 0.21 \pm 0.08$. It has also been pointed out that the Fritzsch-Shin *Ansatz*[6] cannot undergo all the confrontations with experiments successfully, whereas the Fritzsch model[3] and the modified Fritzsch-Stech[7] model can agree with all available information on KM parameters only for m_t^{phys} around 85 GeV and for specific values of other parameters. Any improvement on the experimental limits on the mixing angles or if m_t^{phys} is other than 85 GeV can rule out these two models also.

All these prompt a model-independent study of the problem.[9] In this paper we make such an attempt. We start with the most general form of the quark mass matrices. We then go to a basis where the up-quark mass matrix is diagonal. Then by some unitary transformation on the right-handed down-quark field we make the down-quark mass matrix Hermitian. The three diagonal terms of the up-quark mass matrix and the seven parameters of the Hermitian down-quark mass matrix are then related to the six quark masses, three mixing angles, and the KM phase. We determine the allowed values of the parameters of this model from experimental results. We can then write down any quark mass matrices in this basis and compare the parameters with the allowed values directly.

In Sec. II we list all the experimental inputs. We present our model in Sec. III. We write down other models in this basis in Sec. IV and compare. We summarize our result and conclude in the last section.

II. THE QUARK MASSES AND THE WEAK MIXING MATRIX

In this section we discuss the various parameters the relations between which one seeks to explain in the framework of various models. While seven of the ten parameters, namely, five quark masses and two mixing angles s_{12} and s_{23}, are experimentally determined to some degree of accuracy, only very weak limits exist for the other three, viz., m_t, s_{13}, and the KM phase δ. An exact determination of these quantities would constrain the possible choices of the mass matrices and hopefully be a pointer to a possible structure explaining these very values.

In QCD, the coupling constant and the quark masses are "running" parameters; i.e., they depend on the renormalization point at which they are computed. The renormalization-group equations for them are

$$\mu\frac{d}{d\mu}g(\mu)=\beta[g(\mu)]\ ,$$

$$\mu\frac{d}{d\mu}m_i(\mu)=-\gamma[g(\mu)]m_i(\mu)$$

with the boundary conditions that $g(\lambda),m_i(\lambda)$ coincide with the bare constants g^0,m^0 as the value of the cutoff $\lambda\to\infty$. In the modified minimal subtraction ($\overline{\text{MS}}$) scheme the β and γ functions are given by

$$\beta(g)=-\beta_0\frac{g^3}{16\pi^2}-\beta_1\frac{g^5}{(16\pi^2)^2}+O(g^7)\ ,$$

$$\gamma(g)=\gamma_0\frac{g^2}{4\pi^2}+\gamma_1\frac{g^4}{(4\pi^2)^2}+O(g^6)\ ,$$

$$\beta_0=11-\tfrac{2}{3}N_f,\quad \beta_1=102-\tfrac{38}{3}N_f,\quad \gamma_0=2\ ,$$

$$\gamma_1=\tfrac{101}{12}-\tfrac{5}{18}N_f,\quad N_f=\text{number of flavors}\ .$$

The renormalized coupling constant and quark masses given by the solutions to the differential equations are

$$\alpha_s(\mu)\equiv\frac{g^2(\mu)}{4\pi}$$

$$=\frac{4\pi}{\beta_0 L}\left[1-\frac{\beta_1}{\beta_0}\frac{\ln L}{L}+O\left[\left[\frac{\ln L}{L}\right]^2\right]\right]\ , \qquad (2.1)$$

$$m_i(\mu)=\overline{m}_i\left[\frac{L}{2}\right]^{-2\gamma_0/\beta_0}\left[1-\frac{2\beta_1\gamma_0}{\beta_0^3}\frac{\ln L+1}{L}+\frac{8\gamma_1}{\beta_0^2 L}+O\left[\left[\frac{\ln L}{L}\right]^2\right]\right]\ , \qquad (2.2)$$

$$L\equiv\ln(\mu^2/\Lambda^2)\ .$$

Λ and $\overline{m}_i$ are the renormalization-group-invariant scale and masses. The physical mass of an object is its value calculated at the same scale. Thus to one-loop order, the physical mass of the top quark would be

$$m_t^{\text{phys}}=m_t(m_t)\left[1+\frac{4}{3\pi}\alpha_s(m_t)\right]\ . \qquad (2.3)$$

While nonobservation of the top quark gives a lower limit to its mass[10]

$$m_t^{\text{phys}}\geq 45\ \text{GeV}\ , \qquad (2.4a)$$

experimental consistency of the radiative corrections in the standard model requires[11]

$$m_t^{\text{phys}}\leq 180\ \text{GeV}\ . \qquad (2.4b)$$

Substituting $N_f=6$ and $\Lambda_{\text{QCD}}=100$ MeV, we have for the above range of interest

$$m_t^{\text{phys}}\approx 0.6m_t(1\ \text{GeV})\ ,$$

which gives

$$75\ \text{GeV}\lesssim m_t(1\ \text{GeV})\lesssim 300\ \text{GeV}\ . \qquad (2.5)$$

The physical masses of charm and bottom quarks can be obtained from e^+e^- data by using QCD sum rules for the vacuum-polarization amplitude. The running masses at $\mu=1$ GeV and $\Lambda_{\text{QCD}}=100$ MeV are[12]

$$\begin{aligned}&m_c(1\ \text{GeV})=1.35\pm0.05\ \text{GeV}\ ,\\&m_b(1\ \text{GeV})=5.3\pm0.1\ \text{GeV}\ .\end{aligned} \qquad (2.6)$$

The determination of the light-quark masses involves larger errors. These are best evaluated using chiral QCD perturbation theory and meson and baryon spectroscopy.[12] Though the individual error bars are relatively large, restrictions on the ratio of the masses reduce the indeterminacy somewhat:

$$\begin{aligned}&m_u=5.1\pm1.5\ \text{MeV}\ ,\\&m_d=8.9\pm2.6\ \text{MeV}\ ,\\&m_s=175\pm55\ \text{MeV}\ ,\\&m_s/m_d=19.6\pm1.6\ ,\\&m_d/m_u=1.76\pm0.13\ ,\\&m_s/m_u=34.5\pm5.1\ ,\\&\frac{m_u-m_d}{m_u+m_d}=-0.28\pm0.03\ .\end{aligned} \qquad (2.7)$$

For the KM matrix we use the Maiani parametrization:[13]

$$K=\begin{bmatrix}c_{12}c_{13} & c_{13}s_{12} & s_{13}\\ -c_{23}s_{12}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23}e^{i\delta}\\ -c_{12}c_{23}s_{13}+s_{12}s_{23}e^{-i\delta} & -c_{23}s_{12}s_{13}-c_{12}s_{23}e^{-i\delta} & c_{23}c_{13}\end{bmatrix}\ , \qquad (2.8)$$

where $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$. While s_{12} is determined[14] to an accuracy of 1%

$$s_{12}=0.221\pm0.002\ , \qquad (2.9)$$

the value of s_{23} has a large error associated with it. It can be calculated from the semileptonic B-meson partial width assuming it to be given by W-mediated decay. This involves phase factors depending on m_b and m_c and thus the theoretical errors in their evaluation. Under the assumptions

$$B(b \to c l \bar{\nu}_l) = 0.121 \pm 0.008 \quad (\text{Ref. 15})$$

and

$$\tau_b = (1.16 \pm 0.16) \times 10^{-12} \text{ sec} \quad (\text{Ref. 10}) ,$$

one obtains[8]

$$s_{23} = 0.043^{+0.007}_{-0.009} . \tag{2.10}$$

The recent observation of charmless B decay $[\Gamma(b \to u l \bar{\nu}_l)/\Gamma(b \to c l \bar{\nu}_l)]$ puts a limit[15]

$$0.07 \le \left| \frac{K_{ub}}{K_{cb}} \right| \approx \left| \frac{s_{13}}{s_{23}} \right| \le 0.22 . \tag{2.11}$$

Since $|s_{23}| \le 0.05$, $|s_{13}| \le 0.011$.

III. THE GENERAL THREE-GENERATION MASS MATRIX

The fermion mass terms in the standard model arise from the Yukawa couplings due to the assumption of a nonzero vacuum expectation value by the Higgs field. Thus the mass term is not diagonal in the fermions and not even Hermitian.

So in the weak basis (denoted by prime) we have, for the charged current and the mass terms

$$\mathcal{L}_w = \bar{u}'_L \gamma_\mu d'_L W^\mu_+ + \text{H.c.} ,$$

$$\mathcal{L}_m = \bar{u}'_L M'_u u'_R + \bar{d}'_L M'_d d'_R + \text{H.c.}$$

As the left- and right-handed fields in the standard model are independent and can be rotated differently, one can diagonalize these matrices by multiplying with two different unitary matrices, one from the left the other from right: i.e.,

$$U_L^\dagger M'_u U_R = \hat{M}_u \text{ (real diagonal)} ,$$

where U_L and U_R diagonalize $M'_u M'^\dagger_u$ and $M'^\dagger_u M'_u$, respectively. Defining

$$u_L = U_L^\dagger u'_L, \quad u_R = U_R^\dagger u'_R, \quad \tilde{d}_R = d'_R ,$$

$$\tilde{d}_L = U_L^\dagger d'_L, \quad \text{and } \tilde{M}_d = U_L^\dagger M'_d ,$$

we have in the new basis

$$\mathcal{L}_m = \bar{u}_L \hat{M}_U u_R + \bar{\tilde{d}}_L \tilde{M}_d \tilde{d}_R + \text{H.c.}$$

and

$$\mathcal{L}_w = \bar{u}_L \gamma_\mu \tilde{d}_L W^\mu_+ + \text{H.c.} ,$$

where the up-quark fields are mass eigenstates. Though we can phase rotate u_R so that all eigenvalues of M_u are positive, for future convenience in discussing the Fritzsch matrix we shall not do so and fix only m_u and m_t to be positive and let m_c take either sign. In general, $\tilde{M}_d$ is not Hermitian but if we can find a unitary matrix U such that $M_d = \tilde{M}_d U$ is Hermitian, then we can diagonalize it by a unitary matrix K. Defining $d_L = K^\dagger \tilde{d}_L$ and $d_R = K^\dagger U^\dagger \tilde{d}_R$ we have

$$\mathcal{L}_m = \bar{u}'_L M'_u u'_R + \bar{d}'_L M'_d d'_R + \text{H.c.}$$

$$\mathcal{L}_w = \bar{u}_L \gamma_\mu K d_L W^\mu_+ + \text{H.c.} ,$$

where $\hat{M}_d = K^\dagger (M_d) K$ and $\hat{M}_u$ are diagonal and K is the KM matrix. Since none of the quarks are expected to be massless, $\tilde{M}_d$ is nonsingular and hence we can find a unitary matrix U such that $\tilde{M}_d U$ is Hermitian and positive definite and is diagonalized by the KM matrix. But choosing a particular form of the KM matrix as in Eq. (2.8) would take one away from a positive-definite M_d. In general, both M_u and M_d would have negative eigenvalues, which is not surprising as a sign of a mass term for fermions has no significance in the standard model.

In the basis in which M_u is diagonal, we have for three generations

$$\hat{M}_u = \begin{bmatrix} m_u & & \\ & m_c & \\ & & m_t \end{bmatrix} \tag{3.1}$$

and the most general Hermitian M_d is given by

$$M_d = \alpha \hat{M}_u + A ,$$

where

$$A = \begin{bmatrix} 0 & R_1 e^{i\rho_1} & R_2 e^{i\rho_2} \\ R_1 e^{-i\rho_1} & f & R_3 e^{i\rho_3} \\ R_2 e^{-i\rho_2} & R_3 e^{-i\rho_3} & d \end{bmatrix} . \tag{3.2}$$

Thus the mass matrices are a ten-parameter family determined by m_u, m_c, m_t, α, f, d, $R_{1,2,3}$, and the invariant phase $\rho_1 + \rho_3 - \rho_2$. Though on the face of it this parametrization has no predictive power as we are using ten parameters to relate ten others, in our analysis we would not be using all of them and most of our conclusions would be drawn from consideration of diagonal elements only.

On diagonalizing M_d we have

$$K \hat{M}_d K^\dagger = M_d = \alpha M_u + A ,$$

where

$$\hat{M}_d = \text{diag}(m_d, m_s, m_b) . \tag{3.3}$$

The diagonal elements of the matrix equation give three relations, of which one is the trace condition

$$\alpha = \frac{m_d + m_s + m_b - f - d}{m_u + m_c + m_t} \tag{3.4}$$

and

$$\alpha m_u = m_d + c_{13}^2 s_{12}^2 (m_s - m_d) + s_{13}^2 (m_b - m_d) , \tag{3.5}$$

$$\alpha m_c + f = m_d + |c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta}|^2 (m_s - m_d) + c_{13}^2 s_{23}^2 (m_b - m_d) . \tag{3.6}$$

A. The two-generation limit

As a first approximation we assume that the third generation essentially decouples from the first two—a not too strong assumption as experimentally s_{23} and s_{13} are small compared to s_{12}. In this limit (3.5) and (3.6) reduce to

$$\alpha m_u = m_d + s_{12}^2(m_s - m_d) ,$$
$$\alpha m_c + f = m_d + c_{12}^2(m_s - m_d) . \tag{3.7}$$

Eliminating α from above, we obtain

$$s_{12}^2 = \frac{\left[1-\frac{f}{m_s}\right]\frac{m_u}{m_c} - \frac{m_d}{m_s}}{\left[1+\frac{m_u}{m_c}\right]\left[1-\frac{m_d}{m_s}\right]} . \tag{3.8}$$

Using (2.7) and (2.9) in (3.8) gives an allowed range for f for a given m_d, which has been plotted in Fig. 1. It is seen that for $m_d/m_s<0$, f assumes small values irrespective of the sign of m_u/m_c, and is consistent with zero, while for $m_d/m_s>0$, f is comparatively larger and its sign opposite to that of m_u/m_c.

B. Back to three generations

Assuming the two-generation limit result for s_{12} and using it as an input in Eq. (3.6), we have, for $c_{13}\approx 1$,

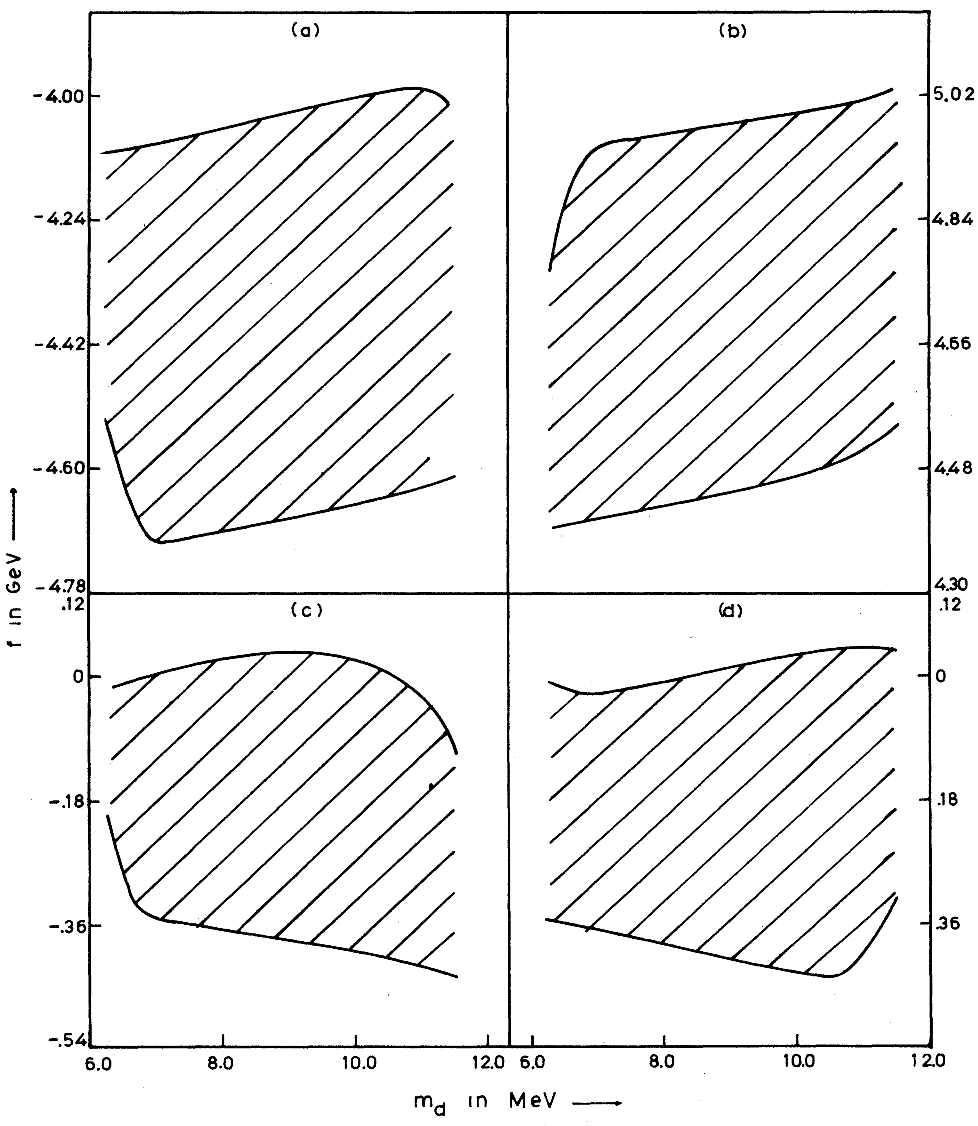

FIG. 1. The allowed range of f (shaded region) as a function of m_d [see Eq. (3.8)]. All values are calculated at $\mu = 1$ GeV. m_d has been assumed to be positive. For $m_d<0$, $f\to -f$. (a) $m_s/m_d>0$, $m_u/m_c>0$. (b) $m_s/m_d>0$, $m_u/m_c<0$. (c) $m_s/m_d<0$, $m_u/m_c>0$. (d) $m_s/m_d<0$, $m_u/m_c<0$.

$$s_{23}^2=\frac{m_c}{m_t+m_c+m_u}\,\frac{(m_b-d)(m_c+m_u)+m_t(f-m_s-m_d)}{m_b(m_c+m_u)-(m_d m_c+m_s m_c+f m_u)}\,, \tag{3.9}$$

i.e.,

$$\frac{d}{m_b}\left\{s_{23}^2\left[\frac{m_b}{m_d}\left[1+\frac{m_u}{m_c}\right]-\left[1+\frac{m_s}{m_d}+\frac{f m_u}{m_d m_c}\right]\right]+\left[1-\frac{f}{m_s}\right]\frac{m_s}{m_d}+1\right\}$$
$$=\left[1-\frac{m_t}{m_c}\right]\left[\left[1+\frac{m_u}{m_c}\right]\frac{m_b}{m_d}-s_{23}^2\left[1+\frac{m_u}{m_c}\right]\left[\frac{m_b}{m_d}\left[1+\frac{m_u}{m_c}\right]-\left[1+\frac{m_s}{m_d}+\frac{f m_u}{m_d m_c}\right]\right]\right].$$

Thus for a given s_{23} we have a linear relation between d and m_t with the slope and intercept depending on the signs of various mass ratios. In the Stech model, for example, d was required to be zero, thus fixing m_t up to error bars due to the experimental uncertainties. A nonzero value of d would unfreeze this restriction and allow for better agreement with experiments. The allowed region for d for a fixed m_t has been given in Table I. Similar to the case for f, d takes "small" values about zero for $m_d/m_s<0$ while for a positive value of the ratio it is considerably larger and a vanishing value is not consistent with observations.

Taking the two-generation result exactly and substituting in Eq. (3.5) one obtains

$$s_{13}^2=\frac{m_u}{m_t+m_c+m_u}\times\frac{(m_c+m_u)(m_b-d)-m_t(m_s+m_d-f)}{m_b(m_c+m_u)-m_u(m_s+m_d-f)}\approx\frac{m_u}{m_c}\left[1-\frac{m_s}{m_b}\right]s_{23}^2\,. \tag{3.10}$$

This implies that $m_u/m_c>0$. The analysis and the result are similar to Stech's. An attempt to obtain a better approximation by an iterative procedure [i.e., substituting current expressions for s_{13} and s_{23} in Eq. (3.7) and (3.8), instead of taking them to be zero and then redoing the same analysis] yields an extra term much smaller in magnitude.

TABLE I. Limits on d(1 GeV) in terms of m_t(1 GeV) as imposed by Eq. (3.9). The limits are calculated for positive m_u, m_t, and m_d. For $m_d<0$, $d\rightarrow -d$.

sgn $\left[\frac{m_b}{m_d},\frac{m_s}{m_d},m_c\right]$	Limits on d(1 GeV) (GeV)
$+++$	$-3.43m_t+5.19<d<-3.27m_t+5.39$
$++-$	$-3.47m_t+5.19<d<-3.29m_t+5.39$
$+-+$	$-0.15m_t+5.19<d<+0.19m_t+5.19$
$+--$	$-0.18m_t+5.19<d<+0.15m_t+5.19$
$-++$	$-3.46m_t+5.21<d<-3.35m_t+5.21$
$-+-$	$-3.44m_t+5.21<d<-3.26m_t+5.41$
$--+$	$-0.18m_t+5.21<d<+0.16m_t+5.21$
$---$	$-0.16m_t+5.21<d<+0.18m_t+5.21$

But this result is in direct contradiction to the Fritzsch model (Sec. IV B) wherein alternate generations have masses of opposite signs. Indeed in this scheme we have

$$s_{23}\approx\left|\left[-\frac{m_s}{m_b}\right]^{1/2}-e^{-i\phi_2}\left[-\frac{m_c}{m_t}\right]^{1/2}\right|\,,$$
$$s_{13}\approx\left|-\frac{m_s}{m_b}\left[\frac{m_d}{m_b}\right]^{1/2}+e^{-i\phi_1}\left[-\frac{m_u}{m_c}\right]^{1/2}\left[\left[-\frac{m_s}{m_b}\right]^{1/2}-e^{-i\phi_2}\left[-\frac{m_c}{m_t}\right]^{1/2}\right]\right|\,, \tag{3.11}$$

where ϕ_i are certain phases in the model. These expressions agree with the experimental limits. If Eqs. (3.11) are squared, one obtains an equation similar to (3.10) but with an extra term typically larger than the right-hand side.

The inconsistency lies in the analysis where one is aiming to solve for three angles from two equations. The relations (3.10) are thus shown not to be an outcome of the Stech *Ansatz* but rather arising from an overkill of Eqs. (3.7) and (3.8). The best one can achieve without using the off-diagonal terms is an expression for s_{13} in terms of three unknowns m_t, f, and d and the measured parameters s_{12} and the other quark masses:

$$s_{13}^2=\frac{\alpha m_u-[m_d+(m_s-m_d)s_{12}^2]}{m_b-[m_d+(m_s-m_d)s_{12}^2]}\,. \tag{3.12}$$

C. The off-diagonal terms

Until now we have used only the diagonal terms of the matrix equation (3.2) ignoring the off-diagonal terms, considering which would give exact results. We continue in the same vein but would like to look at these relations so as to get an idea of the relative magnitude of these terms. We have

$$R_1 e^{i\rho_1}=c_{12}c_{23}c_{13}s_{12}(m_s-m_d)+c_{13}s_{13}s_{23}(m_b-c_{12}^2c_{23}m_d-s_{12}^2m_s)e^{-i\delta}\ ,$$
$$R_2 e^{i\rho_2}=c_{23}c_{13}s_{13}(m_b-s_{12}^2m_s-c_{12}^2m_d)+c_{12}c_{13}s_{12}s_{23}(m_d-m_s)e^{i\delta}\ , \quad (3.13)$$
$$R_3 e^{i\rho_3}=c_{12}c_{23}^2s_{12}s_{13}(m_d-m_s)+c_{12}s_{12}s_{23}^2s_{13}(m_s-m_d)e^{2i\delta}+c_{23}s_{23}[c_{13}^2m_b+(c_{12}^2s_{13}^2-s_{12}^2)m_d+(s_{12}^2s_{13}^2-c_{12}^2)m_s]e^{i\delta}\ .$$

The complex phases ρ_1 and ρ_2 are relatively small and lie in the same quadrant as can be seen from the fact that $\tan\rho_1\tan\rho_2\approx s_{23}^2$. While $\sin\rho_1$ attains its maximum of 0.12 when m_s and s_{12} assume the lowest allowed values and s_{13}, s_{23}, m_b the highest and $\delta\approx 83°$, $\sin\rho_2$ is maximized to 0.15 by giving

$$\left|\frac{k_{ub}}{k_{cb}}\right|$$

and m_b their lowest values, m_s, s_{12} their highest and putting $\delta\approx 81.5°$. On the other hand $\rho_3\approx\delta$. Thus most of the *CP*-violating contribution comes from this term.

IV. MODELS AS SPECIAL CASES OF THE GENERAL FORM

In this section we discuss some of the better studied models for fermion mass generation. We demonstrate how these models could be obtained from the general mass matrix on imposing suitable constraints. This would exhibit the restrictions one is preimposing on the various parameters and hopefully afford a better understanding of the implications of an *Ansatz.*

A. Stech model

This model was motivated by grand unified theories where the gauge group has a SU(5) subgroup and the fermions of one generation are contained in an irreducible representation. The fermion masses arise from nonzero vacuum expectation values of Higgs fields transforming under different representations. The assumption was that the mass matrices belonging to the symmetric Higgs representations of SU(5) dominate and that the antisymmetric representations do not contribute to the up sector. A further choice of Hermiticity of the mass matrices restrict their form to

$$M_u=M_u^T=M_u^\dagger\ , \quad (4.1)$$
$$M_d=M_d^\dagger=\alpha M_u+A\ , \quad (4.2)$$

where A is an antisymmetric matrix.

M_u can be brought into a diagonal form by an orthogonal transformation. In this basis A is still Hermitian and antisymmetric. An analysis similar to that in Sec. III can now be effected. It should however be noticed that choosing a particular basis for the KM matrix would necessitate a unitary transformation by a phase matrix. While this would leave M_u invariant, A would lose its antisymmetry, and would be a Hermitian matrix with all diagonal elements zero.

Thus we see that the Stech *Ansatz* involves making the following choices for the three-generation case:

$$f=0\ , \quad (4.3)$$
$$d=0\ , \quad (4.4)$$
$$\rho_1-\rho_2+\rho_3=90°\ . \quad (4.5)$$

Using (4.5) in Eq. (3.13) one gets

$$\frac{s_{13}}{s_{23}}\cos\delta\approx 0\ .$$

This could be seen directly in the light of the discussion following Eq. (3.13). The Stech *Ansatz* thus restricts the mass matrices to a seven-parameter family which predicts near maximal *CP* violation.

The first two conditions obviously restrict the mass matrices to the negative m_s/m_d sector. Also a lower limit on the mass of the d quark is set:

$$\begin{aligned} &m_d(1\ \text{GeV})\gtrsim 7\ \text{MeV}\quad \text{for } m_u/m_c>0\ ,\\ &m_d(1\ \text{GeV})\gtrsim 8.5\ \text{MeV}\quad \text{for } m_u/m_c<0\ . \end{aligned} \quad (4.6)$$

Using (4.3) and (4.4) in (3.8) and (3.9) we get

$$s_{12}^2\approx\frac{\dfrac{m_u}{m_c}-\dfrac{m_d}{m_s}}{\left[1+\dfrac{m_u}{m_c}\right]\left[1-\dfrac{m_d}{m_s}\right]}\ , \quad (4.7)$$

$$s_{23}^2\approx\frac{m_bm_c-m_tm_s}{(m_t+m_c)(m_b-m_s)}\ . \quad (4.8)$$

Utilizing the entire allowed range of the measured parameters involved it is seen that for the above two relations to hold, $m_t^{\text{phys}}\lesssim 46$ GeV.

The derivation of the Stech prediction $s_{13}^2=(m_u/m_c)s_{23}^2$ is similar to that in Sec. III C. We have already shown that this result is not a consequence of the *Ansatz* but due to a flaw in the analysis in a recent analysis. Harari and Nir[8] claim that in the light of these restrictions, the predictions of the Stech model do not agree with the B_d^0-$\bar{B}_d^0$ data. But if one offers certain modifications of the Stech *Ansatz* as, for example, a nonzero d, or an invariant phase $(\rho_1-\rho_2+\rho_3)$ different from 90°, then this restriction can be circumvented. Furthermore, if Higgs-boson exchange is responsible for B_d^0-$\bar{B}_d^0$ mixing then these data cannot constrain the quark mass matrices.[17]

B. Fritzsch matrix

This model was first obtained for a field theory with $SU_L(2)\otimes SU_R(2)\otimes U(1)$ as the gauge group and two Higgs fields on imposition of a certain discrete symmetry. The cornerstone of this *Ansatz* is that, to start with, only the heaviest quarks are massive and the lighter quarks gain mass by weak-interaction mixing with the next higher generation. In a basis in which the up-quark matrix is real, one obtains

$$M_{u(F)}=\begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{pmatrix}, \quad (4.9)$$

$$M_{d(F)}=\begin{pmatrix} 0 & a_d e^{i\phi_1} & 0 \\ a_d e^{-i\phi_1} & 0 & b_d e^{i\phi_2} \\ 0 & b_d e^{-i\phi_2} & c_d \end{pmatrix}. \quad (4.10)$$

The quark sector is then characterized by eight parameters and thus predicts two relations between the masses and the KM matrix parameters. The expression in (4.10) implies that the middle eigenvalue of both $M_{u(F)}$ and $M_{d(F)}$ would have a sign opposite to that of the other two.

$M_{d(F)}$ can be brought into real form by performing a phase rotation on both the right- and left-handed down quark fields:

$$M_{d(F)}=P^{\dagger}M'_{d(F)}P \ ,$$

where

$$P=\begin{pmatrix} 1 & & \\ & e^{i\phi_1} & \\ & & e^{i(\phi_1+\phi_2)} \end{pmatrix} \text{ and } M'_{d(F)}=\begin{pmatrix} 0 & a_d & 0 \\ a_d & 0 & b_d \\ 0 & b_d & c_d \end{pmatrix}. \quad (4.11)$$

$M_{u(F)}$ and $M'_{d(F)}$ being real symmetric matrices can both be diagonalized by orthogonal transformations. For example,

$$O_u^T M_{u(F)} O_u = \hat{M}_u \equiv \text{diag}(m_u, m_c, m_t) \ , \quad (4.12)$$

$$O_u=\begin{pmatrix} \frac{1}{N_1} & \frac{1}{N_2} & \frac{1}{N_3} \\ \frac{m_u}{N_1 a_u} & \frac{m_c}{N_2 a_u} & \frac{m_t}{N_3 a_u} \\ \frac{-m_u b_u}{N_1 a_u(m_c+m_t)} & \frac{-m_c b_u}{N_2 a_u(m_u+m_t)} & \frac{-m_t b_u}{N_3 a_u(m_u+m_c)} \end{pmatrix}. \quad (4.13)$$

The eigenvalues m_i can be obtained by inverting the relations

$$\begin{aligned} a_u &= (-m_u m_c m_t / c_u)^{1/2} \ , \\ b_u &= [-(m_u+m_c)(m_u+m_t)(m_c+m_t)/c_u]^{1/2} \ , \\ c_u &= (m_u+m_c+m_t) \ , \end{aligned} \quad (4.14)$$

and N_i are the normalization for the eigenvectors of $M_{u(F)}$:

$$\begin{aligned} N_1^2 &= \frac{m_c-m_u}{m_c}\left[1+\frac{(m_c+m_u)m_u}{(m_t+m_c)m_c}\right] , \\ N_2^2 &= \frac{m_u-m_c}{m_u}\left[1+\frac{m_c}{m_t}\frac{m_u-m_c}{m_t+m_u}\right] , \\ N_3^2 &= \frac{m_t^3}{m_c m_u(m_c+m_u)} \\ &\quad \times \left[1-\frac{m_c^2+m_c m_u-m_u^2}{m_t^2}+\frac{m_c m_u(m_c+m_u)}{m_t^3}\right] . \end{aligned} \quad (4.15)$$

The weak mixing matrix being given by

$$K=O_u^T P^{\dagger} O_d \quad (4.16)$$

we have

$$\begin{aligned} s_{12} &\approx \left| \left[-\frac{m_d}{m_s}\right]^{1/2} - e^{-i\phi_1}\left[-\frac{m_u}{m_c}\right]^{1/2} \right| , \\ s_{23} &\approx \left| \left[-\frac{m_s}{m_b}\right]^{1/2} - e^{-i\phi_2}\left[-\frac{m_c}{m_t}\right]^{1/2} \right| , \\ s_{13} &\approx \left| \frac{m_s}{m_b}\left[\frac{m_d}{m_b}\right]^{1/2} - e^{-i\phi_1}\left[\left[-\frac{m_s}{m_b}\right]^{1/2} - e^{-i\phi_2}\left[-\frac{m_c}{m_t}\right]^{1/2}\right] \right| , \end{aligned} \quad (4.17)$$

$$\frac{\sin\delta}{\frac{s_{12}s_{23}}{s_{13}}-\cos\delta} \approx \frac{\sin\phi_1}{\cos\phi_1-\left[\frac{m_d m_c}{m_s m_u}\right]^{1/2}} \ .$$

The first two of these relations could be used to determine ϕ_1 and ϕ_2 and then the other two expressions are the predictions of the model.

The expression for s_{23} leads to

$$m_t \leq \frac{-m_c}{\left[\left[-\frac{m_s}{m_b}\right]^{1/2} - s_{23}\right]^2}$$

or $m_t^{\rm phys} \leq 88$ GeV, a condition that must be satisfied for Fritzsch form to hold good.

The simplest way to find the constraints to be imposed upon the general form to obtain the Fritzsch matrix is to rotate M_d with O_u and compare with $M_{d(F)}$:

$$M_{d(F)} = O_u M_d O_u^T$$

gives the two required constraints

$$\frac{\alpha m_u}{N_1^2} + \frac{\alpha m_c + f}{N_2^2} + \frac{\alpha m_t + d}{N_3^2} + \frac{2R_1 \cos\rho_1}{N_1 N_2} + \frac{2R_2 \cos\rho_2}{N_1 N_3} + \frac{2R_3 \cos\rho_3}{N_2 N_3} = 0 , \tag{4.18}$$

$$\frac{\alpha m_u^3}{N_1^2} + (\alpha m_c + f)\frac{m_c^2}{N_2^2} + (\alpha m_t + d)\frac{m_t^2}{N_3^2} + 2\frac{m_u m_c}{N_1 N_2} R_1 \cos\rho_1 + 2\frac{m_u m_t}{N_1 N_3} R_2 \cos\rho_2 + 2\frac{m_c m_t}{N_2 N_3} R_3 \cos\rho_3 = 0 . \tag{4.19}$$

Using Eqs. (3.13) and (4.15) in the above any two of the ten parameters can be eliminated. For example, if f and d are evaluated in terms of the masses and the KM parameters, then substituting the expressions for them in (3.8) and (3.9) would give us, say, s_{13} and m_t in terms of the others and these would be the predictions of the model.

Harari and Nir[8] have, on the basis of certain assumptions about some hadronic factors involved, shown that the Fritzsch scheme agrees with the present experimental data only if the following are satisfied simultaneously: (i) B_d^0-$\bar{B}_d^0$ mixing is at the lowest level allowed by the ARGUS group experiment at DESY; (ii) the B-meson decay constant assumes a value ~ 0.2 GeV (maximum of the "reasonable" range); (iii) the hadronic factor B_K in the expression for the CP-violating parameter ϵ in the K^0-$\bar{K}^0$ system (believed to be in the range $\frac{1}{3} \leq B_k \leq 1$) is about 1; (iv) the ratio $|m_s/m_b|$ is approximately 0.22 and this corresponds to m_s and m_b taking the lowest and highest allowed values, respectively; (v) the value of s_{23} assumes the maximum of the allowed range.

Also Oh, Rhee, and Kim[18] point out that while an attempt to explain the weak CP nonconservation by the KM matrix would demand $\phi_1 \approx 90°$, $\phi_2 \approx 0°$, to have $|\epsilon|$ for K^0-$\bar{K}^0$ system consistent with experiments one needs a small ϕ_1.

With only such tenuous agreement with the experiments, we feel that a more critical examination of the Fritzsch model and its possible extensions are due.

C. Fritzsch-Shin scheme

Shin's[6] parametrization of the Fritzsch matrix fixes the two hitherto arbitrary complex phases:

$$\phi_1 = 90°, \quad \phi_2 = 0° . \tag{4.20}$$

This reduces the variable number by two and now we have four predictions of the model.

This is equivalent to imposing two additional constraints on the general form over and above Eqs. (4.18) and (4.19):

$$\begin{aligned} &(m_c^2 - m_u^2)\frac{m_u m_c}{N_1 N_2} R_1 \sin\rho_1 + (m_t^2 - m_u^2)\frac{m_u m_t}{N_1 N_3} R_2 \sin\rho_2 + (m_t^2 - m_c^2)\frac{m_c m_t}{N_2 N_3} R_3 \sin\rho_3 = 0 , \\ &\frac{\alpha m_u^2}{N_1^2} + \frac{(\alpha m_c + f) m_c}{N_2^2} + \frac{(\alpha m_t + d) m_t}{N_3^2} + 2\frac{m_u + m_c}{N_1 N_2} R_1 \cos\rho_1 + 2\frac{m_u + m_t}{N_1 N_3} R_2 \cos\rho_2 + 2\frac{m_c + m_t}{N_2 N_3} R_3 \cos\rho_3 = 0 . \end{aligned} \tag{4.21a}$$

Proceeding in a manner similar to that for the general Fritzsch form, (4.21a) directly gives two more relations between the masses, the weak mixing angles and the CP-violating phase.

To be phenomenologically consistent one must apart from the conditions enumerated earlier also require that $s_{12} = 0.223$ (highest allowed value) and

$$\left|\frac{m_u}{m_c}\right| \quad \text{and} \quad \left|\frac{m_d}{m_s}\right|$$

have the lowest possible value.

D. Fritzsch-Stech matrices

The Fritzsch and Stech *Ansätze* are not inconsistent with each other. Gronau, Johnson, and Schechter[5] considered a model in which both these assumptions are incorporated. In a suitable basis the mass matrices are thus given by

$$M_u = \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{pmatrix}$$

and

$$M_d=\alpha M_u+\begin{bmatrix}0 & ia & 0\\ -ia & 0 & ib\\ 0 & ib & 0\end{bmatrix}\equiv\begin{bmatrix}0 & a_d & 0\\ a_d^* & 0 & b_d\\ 0 & b_d^* & c_d\end{bmatrix}. \quad (4.21b)$$

This gives a six-variable dependent model obtainable from the Fritzsch form by demanding that

$$\mathrm{Re}(a_d)=\frac{c_d}{c_u}a_u,\quad \mathrm{Re}(b_d)=\frac{c_d}{c_u}b_u \quad (4.22)$$

or in our language by requiring

$$\begin{aligned}&\left[\frac{\alpha m_u^3}{N_1^2(m_c+m_t)^2}+\frac{(\alpha m_c+f)m_c^2}{N_2^2(m_u+m_t)^2}+\frac{(\alpha m_t+d)m_t^2}{N_3^2(m_u+m_c)^2}+\frac{2m_um_cR_1\cos\rho_1}{N_1N_2(m_u+m_t)(m_c+m_t)}\right.\\&\left.+\frac{2m_um_tR_2\cos\rho_2}{N_1N_3(m_u+m_c)(m_c+m_t)}+\frac{2m_cm_tR_3\cos\rho_3}{N_2N_3(m_u+m_c)(m_c+m_t)}\right]\frac{(m_u+m_c)(m_u+m_t)(m_c+m_t)}{(m_u+m_c+m_t)^2}\\&=-\left[\frac{\alpha m_u^2}{N_1^2}+\frac{\alpha m_c+f}{N_2^2}m_c+\frac{\alpha m_t+d}{N_3^2}m_t+2\frac{m_u+m_c}{N_1N_2}R_1\cos\rho_1+2\frac{m_c+m_t}{N_2N_3}R_3\cos\rho_3+2\frac{m_u+m_t}{N_1N_3}R_2\cos\rho_2\right]\\&=\frac{\alpha m_u^3}{N_1^2(m_c+m_t)}+\frac{(\alpha m_c+f)m_c^2}{N_2^2(m_u+m_t)}+\frac{(\alpha m_t+d)m_t^2}{N_3^2(m_u+m_c)}\\&+\left[\frac{1}{m_u+m_t}+\frac{1}{m_c+m_t}\right]\frac{m_um_c}{N_1N_2}R_1\cos\rho_1+\left[\frac{1}{m_u+m_c}+\frac{1}{m_u+m_t}\right]\frac{m_cm_t}{N_2N_3}R_2\cos\rho_2\\&+\left[\frac{1}{m_u+m_c}+\frac{1}{m_c+m_t}\right]\frac{m_um_t}{N_1N_3}R_3\cos\rho_3\,. \end{aligned}\quad (4.23)$$

Imposing (4.23) over and above (4.19) and (4.20) is obviously equivalent to imposing (4.3) and (4.4) along with the second named. Equation (4.5), contrary to naive expectation, does not represent an extra constraint as it is trivially satisfied on imposition of the other four.

V. SUMMARY AND CONCLUSION

Our analysis has shown that the parameters involved in the general three-generation quark mass matrix are not fixed by the current experimental data but are allowed a continuous range. However, this range is limited to different sectors depending on the relative sign of the mass terms. Most of the large width of these sectors arises due to the large indeterminacy in the masses of the lighter quarks and only to a lesser degree from the inaccuracy of cb mixing strength.

From the expressions in Sec. III C we see that the off-diagonal terms in M_d are relatively small. In fact

$$\left|\frac{R_{1,2}}{m_s}\right|\lesssim 0.1 \quad\text{and}\quad \left|\frac{R_3}{m_s}\right|\lesssim 1\,.$$

In light of this if one demands that

$$\left|\frac{f}{m_s}\right| \quad\text{and}\quad \left|\frac{d}{m_b}\right|$$

not be too large either, then from Fig. 1 and Table I, we are limited to the $(m_d/m_s)<0$ sector. All the specific models that we have encountered so far lie in this category.

This implies that future model building could take two different courses. The more conservative course, given the relative success of the current models, would be to reexamine the present constraints and offer slight modifications that would alter or extend the models to a degree without drastically changing the basic structure. All these would be expected to lie in the $m_d/m_s<0$ sector. The other more radical approach would be to consider an entirely different class of models. This would entail f and d assuming much larger values compared to the other parameters in M_d and would demand a theoretical justification for such behavior.

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