Structure of exotic generations

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Theoretical and phenomenological aspects of nonstandard multiplets of anomaly-free fermions (exotic generations) are examined. The standard quark-lepton generation is shown to be the simplest nontrivial member of a class of fermion multiplet structures allowed by the gauge and Higgs structure of the minimal standard model. The next simplest member includes weak-isospin-triplet quarks and leptons, whose structure and phenomenology is investigated. A feature of some of these models is the presence of small flavor-changing neutral currents at the tree level; however, these currents are shown to be within existing experimental bounds. It is shown that it is possible that some of the known members of the incomplete third generation (b quark, τ, and ν_τ) may actually be part of such a triplet family, rather than a standard doublet family. Upper bounds on the masses of the new particles are obtained from data on the ρ parameter.

I. INTRODUCTION

Given the results of the concerted effort made to test aspects of the standard model[1] (SM) in the last few years, one can be fairly confident that the essential framework of a theory of quark and lepton interactions is known. While the SM represents the simplest embodiment of this general structure, it is also possible that nature utilizes a nonminimal manifestation. Indeed, in the next several years, the minimal SM will be tested ever more thoroughly as new accelerators such as the CERN LEP, SLAC Linear Collider (SLC), and (possibly) the Superconducting Super Collider (SSC) come on line. Thus it is imperative that all reasonable extensions of the minimal SM be studied, so that rival theories can be efficiently sifted through once the data are available, and in fact also to guide the analysis of experimental results.

One may class extensions of the SM into roughly two groups. The first group selects certain phenomena of known physics that the minimal SM perhaps does not explain in a sufficiently compelling way and then tries to provide a more satisfactory explanation. Elements of this set include, to list a few, left-right-symmetric models, models of spontaneous *CP* violation, Majoron models, and axion models. These theories tackle the issues of parity violation, *CP* violation, lepton-number symmetry together with neutrino masses, and the strong *CP* problem, respectively. The second group simply studies the concept that "nature often offers a surprise." A historical example of such a surprise is the discovery of the muon. The muon, at the time, just did not seem to have a purpose, by contrast with, for example, the pion. Indeed we still do not understand today why the muon, and higher generations in general, are there. Perhaps the best-known example of a theory in this second group is the much-studied *CP*-conserving two-Higgs-doublet model with an explicitly broken Peccei-Quinn symmetry. Here the chief surprise is a physical charged Higgs particle. Although such a particle is not, as far as we know, strictly necessary, it does have very interesting phenomenology and is worthy of considerable attention.

Most of the extensions of the SM studied so far have involved an increase in the Higgs sector and/or an increase in the gauge group. However, it is also possible that history will repeat itself and that an apparently useless new fermion will be discovered. By this we do not mean a conventional fourth generation, which would not be such a surprise (although it would still be inexplicable). Instead, in a series of papers,[2,3] the physics of weak-isospin-triplet leptons has been recently studied. The minimal extension of this type hypothesizes a weak hypercharge neutral triplet of either left- or right-handed color-singlet fermions. If one demands, for example, that the SM pattern of $SU(2)_L$ interacting left-handed leptons and $SU(2)_L$-singlet right-handed leptons continues, then it is necessary to also introduce a triplet of Higgs bosons.[2] Weak-isospin-triplet leptons can also be used to generalize the conventional seesaw mechanism[4,5] for generating small neutrino masses.[3] The purpose of this paper is to again investigate the possibility of fermions in higher-dimensional representations of weak isospin, but this time in theories where the requirement that the right-handed components be in $SU(2)_L$ singlets is relaxed. In particular we focus on multiplets which require only the standard Higgs doublet for the particles to gain mass.

Two scenarios are considered in this paper. Leptonlike generations, which assume that the $SU(2)_L \otimes U(1)_Y$ anomalies cancel within the lepton spectrum are considered. Alternatively, one can have exotic generations which have both quarks and leptons with the $SU(2)_L \otimes U(1)_Y$ anomalies of the quarks canceling with the anomalies of the leptons. This latter case generalizes the standard-model generation.

The plan of this paper is as follows. In Sec. II we discuss the systematics of anomaly cancellation within the framework of the one-Higgs-doublet model. In Sec. III we investigate the simplest leptonlike generation. Section IV is then devoted to an elucidation of the simplest quark-lepton-like generation. From this analysis we show in Sec. V that the b quark may be a member of an exotic triplet generation, rather than the usual doublet generation. In Sec. VI constraints from the W-Z boson mass difference (note that $\rho=1$ at the tree level) are used to indicate upper bounds on the masses of the exotic fermions. We then conclude in Sec. VII with some phenomenological comments.

II. EXOTIC GENERATION STRUCTURES

There are a multitude of ways of extending the SM. In this paper we will consider one generic way of extending the fermion sector. Our investigation is defined by the following assumptions.

(i) There is only one Higgs doublet in nature. (We certainly know that this is enough to account for the masses of the two complete fermion generations that have been discovered, together with the electroweak boson masses.)

(ii) Fermions come in units called "generations" or "families" which are defined as $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ representations which are free of gauge anomalies (and which cannot be broken down into smaller such units). The two complete generations we know are also clearly separated in a mass hierarchy (except for possibly the neutrinos). The fragments of the third generation are again higher in mass, with the as yet hypothetical top quark required to be even more massive. However, if there is, for example, a fourth generation, then there is no reason why this pattern cannot be broken—there may be mass level mixing. We will *not* assume that the generational mass hierarchy is an inviolate principle.

(iii) New generations can either be leptonlike, quarklike or have both quarks and leptons. The last case is distinguished from the sum of the first two by the requirement that the $SU(2)_L \otimes U(1)_Y$ anomalies of the leptons cancel with those of the quarks, rather than having individual cancellation within the leptons and within the quarks. A standard generation is, of course, of the last type.

(iv) There should be no charged particles which are necessarily massless.

(v) Fermions are chiral. One can simply introduce nonchiral fermions which have arbitrary masses into the SM. Although such mass terms may have a role to play in nature,[6,7] they will not be central to the present analysis.

Since we are hypothesizing the standard $Y=1$ Higgs doublet

$$\phi=\begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix},\quad \tilde{\phi}=\begin{bmatrix}\phi^{0*}\\ -\phi^-\end{bmatrix} \tag{2.1}$$

[where electric charge is given by $Q=T_3+(Y/2)$], we require that the dimension of fermion $SU(2)_L$ representations form a sequence of *consecutive* integers. The generic structure of a generation is then

$$R_f=\underline{N}_R\oplus n_{N-1}(\underline{N-1})_L\oplus n_{N-2}(\underline{N-2})_R\oplus\cdots\oplus n_2\underline{2}_{L,R}\oplus n_1\underline{1}_{R,L}\ . \tag{2.2}$$

The small subscripted n's are the multiplicities of the $SU(2)_L$ multiplets. We have defined the sequence as alternating in chirality. This has no absolute meaning (until one assigns hypercharges) since one can simply take charge-conjugate representations and hence define equivalent sequences with any assignment of chiralities. The choice above is, however, an obvious one in that it emphasizes that adjacent multiplets mix with each other after spontaneous symmetry breaking to form massive Dirac particles. That is to say, Yukawa coupling terms such as $\overline{N}_R(N-1)_L\phi$, $\overline{N}_R(N-1)_L\tilde{\phi}$, $(\overline{N-1})_L(N-2)_R\phi$, $(\overline{N-1})_L(N-2)_R\tilde{\phi}$, and so on, exist. Since only those representations whose dimensions differ by one can have the doublet representation in the decomposition of their Kronecker product, this also means that Yukawa terms involving nonadjacent multiplets are not allowed. We may allow the series (2.2) to terminate before the singlets are reached (i.e., $n_M=0$ for all M less than some integer), but we will not consider a series with a representation missing in the middle.

Note that for simplicity we have put the multiplicity of the highest-dimensional representation equal to one. The unit Eq. (2.2) will be a leptonlike or quarklike generation if $SU(2)_L\otimes U(1)_Y$ anomalies cancel within itself. [For a quarklike generation $SU(3)_c$ anomalies must of course also vanish.] We will construct lepton-quarklike generations by making copies of Eq. (2.2). So if we have one lepton unit plus a color triplet of quarks, the number of units will equal four.

What are the hypercharges of these multiplets? Let the hypercharge of $\underline{N}_R$ be y. Then since $\underline{N}_R$ can couple to $\underline{N-1}_L$ through both a ϕ and a $\tilde{\phi}$, it follows that there are in general two types of $\underline{N-1}_L$'s: one type has $Y_{N-1}=y+1$ and the other $Y'_{N-1}=y-1$. It is clear then that the $n_{N-i}(N-i)$ multiplet breaks up into members with hypercharges given by $y+i, y+i-2, \ldots, y-i$. We will denote the multiplicities of these representations by $n^{i}_{N-i}, n^{+(i-2)}_{N-i}, \ldots, n^{-i}_{N-i}$, etc., respectively. Clearly,

$$n_{N-i}=(n^{+i}_{N-i}+n^{-i}_{N-i})+(n^{+(i-2)}_{N-i}+n^{-(i-2)}_{N-i})+\cdots\ . \tag{2.3}$$

An important constraint on the representation Eq. (2.2) is that the triangle anomaly[8] vanishes. Of course, for any representation R_f, one can always add a mirror representation[7] (which has the chiralities interchanged) which leads to trivial anomaly cancellation. However, here we are interested in the problem of nontrivial anomaly cancellation as is indicated by the generations of the SM. The anomaly coefficient has the form $\mathrm{Tr}(T^a\{T^b,T^c\})$ where T^a,T^b,T^c are generators in the appropriate representation of the Lie algebra of $SU(2)_L\times U(1)_Y$. Consider now the contribution to the $[SU(2)_L]^2U(1)_Y$ anomaly from the multiplet $\underline{N-i}$. It is given by

$$\begin{aligned}A_{N-i}=(-1)^{i+1}q(\underline{N-i})(&[n^{+i}_{N-i}(y+i)+n^{-i}_{N-i}(y-i)]\\&+\{n^{+(i-2)}_{N-i}(y+i-2)\\&\quad+n^{-(i-2)}_{N-i}[y-(i-2)]\}\\&+\cdots)\ ,\end{aligned} \tag{2.4}$$

where $q(\underline{N-i})$ is an SU(2) factor whose explicit value will not be required. A_{N-i} breaks up into a piece proportional to y and a piece independent of y:

$$A_{N-i}=(-1)^{i+1}q(\underline{N-i})[n_{N-i}y+i(n_{N-i}^{+i}-n_{N-i}^{-i})+\cdots] . \tag{2.5}$$

We now demand that

$$n_{N-i}^{+i}=n_{N-i}^{-i},\quad n_{N-i}^{+(i-2)}=n_{N-i}^{-(i-2)},\ldots , \tag{2.6}$$

so that the complete $[\mathrm{SU}(2)_L]^2\mathrm{U}(1)_Y$ anomaly is given by

$$A_2=y\left[\sum_i(-1)^{i+1}q(\underline{N-i})n_{N-i}\right] , \tag{2.7}$$

that is, it is proportional to the hypercharge of the representation $\underline{N}_R$. As well as dramatically simplifying the equations, we will show that the usual SM generation structure follows from the requirement Eq. (2.6). This will allow us to easily build structures which generalize the standard family in a systematic way.

Under the assumption Eq. (2.6) and under the requirement that there be equal numbers of left- and right-handed particles [that is $\sum_i(-1)^{i+1}n_{N-i}=0$] the $[\mathrm{U}(1)_Y]^3$ anomaly is also proportional to y:

$$A_Y\propto y . \tag{2.8}$$

This additional requirement follows as a necessary condition for allowing all particles to gain mass. The importance of the assumption Eq. (2.6) lies in the result that both the $[\mathrm{SU}(2)_L]^2\mathrm{U}(1)_Y$ and the $[\mathrm{U}(1)_Y]^3$ anomaly turn out to be proportional to y.

Representations of the group SU(2) may also possess global anomalies.[9] The inconsistent theories arise depending on whether $\mathrm{tr}T_3^2$, summed over all of the chiral fermions is an integer or half-integer. For instance, a standard lepton doublet *or* a standard quark doublet is anomalous; however, a standard generation with a quark doublet and a lepton doublet is consistent. In particular, note that for any quark-lepton representation [i.e., with quarks and leptons in identical SU(2) representations] the global anomaly trivially vanishes [as $(3+1)\mathrm{tr}T_3^2$ is always an integer].

We can now readily construct anomaly-free structures. Leptonlike generations are defined by requiring that all fermions be color singlets and have $y=0$. The simplest nontrivial example is derived by taking $N=3$,

$$\underline{3}_R(0)\oplus\underline{2}_L(-1)\oplus\underline{2}_L(+1)\oplus\underline{1}_R(0) , \tag{2.9}$$

where we have implemented all the conditions. (Note that the $N=2$ case is ruled out because of the global anomaly.) This is the exotic lepton family we will study in Sec. III. Actually, there is a series defined by

$$\underline{N}_R(y)\oplus\underline{N-1}_L(y-1)\oplus\underline{N-1}_L(y+1)\oplus\underline{N-2}_R(y) , \tag{2.10}$$

which features Eq. (2.9) as a member. Observe that the series Eq. (2.10) satisfies Eq. (2.6), so that the contribution of the triangle anomaly is proportional to y. Therefore, quark-leptonlike generations can be obtained by simply imposing that the hypercharges of the leptons and quarks satisfy $Y_l+3Y_q=0$. The representation Eq. (2.10) with $N=3$ is the largest such representation which has as many independent Yukawa coupling constants as there are Dirac fermions, so that all the fermion masses are unrelated. When $N>3$, there will necessarily be relations among the masses of a generation having the structure of Eq. (2.10), which would represent a striking manifestation of $\mathrm{SU}(2)_L$ symmetry if discovered. It is also amusing to note that Eq. (2.9) may be obtained from the adjoint representation of SU(3) (which of course is anomaly-free) by taking its decomposition under the $\mathrm{SU}(2)\otimes\mathrm{U}(1)$ subgroup.

The simplest quark-leptonlike generation consistent with our assumptions is essentially just a standard generation together with a right-handed neutrino:

$$[(\underline{2}_R(Y_l)\oplus\underline{1}_L(Y_l-1)\oplus\underline{1}_L(Y_l+1)]\oplus\{3\times[\underline{2}_R(Y_q)\oplus\underline{1}_L(Y_q-1)\oplus\underline{1}_L(Y_q+1)]\} . \tag{2.11}$$

Y_l and Y_q are the hypercharges of the lepton and quark doublet, respectively. The SU(2) triangle anomaly is proportional to Y_l+3Y_q so we must demand that

$$Y_l+3Y_q=0 . \tag{2.12}$$

If one takes $Y_l=+1$ then Eqs. (2.11) and (2.12) define a standard generation of quarks and leptons (charge conjugated) together with a left-handed antineutrino. Of course the left-handed antineutrino does not contribute to the anomaly and may be removed, as can any neutral SU(2)-singlet field.

Having now studied the structure of exotic generations in a wider framework, we can gain further insight into multiplets such as Eq. (2.10) and indeed other multiplets (see below). Before continuing this general discussion, we will present the exotic generation we will study in Secs. IV–VI of this paper: the quark-lepton version of Eq. (2.9). It is given by

$$\underline{3}_R(Y_l)\oplus\underline{2}_L(Y_l-1)\oplus\underline{2}_L(Y_l+1)\oplus\underline{1}_R(Y_l)\oplus3\times[\underline{3}_R(-\tfrac{1}{3}Y_l)\oplus\underline{2}_L(-\tfrac{1}{3}Y_l-1)\oplus\underline{2}_L(-\tfrac{1}{3}Y_l+1)\oplus\underline{1}_R(-\tfrac{1}{3}Y_l)] , \tag{2.13}$$

where Y_l is arbitrary. In later sections we will choose to study the case where $Y_l=-2$.

The structure used in Eq. (2.13) is of fundamental interest because it is the simplest nontrivial generalization of the standard family. It is instructive to consider the next most complicated triplet generation. In Eqs. (2.9) and (2.13) we demanded that $n_2^{-1}=n_2^{+1}=1$. If we instead require them to equal 2 we get three types of generations consistent with Eq. (2.6) and equal numbers of left- and right-handed states:

$$\underline{3}_R(y)\oplus 2\times\underline{2}_L(y-1)\oplus 2\times\underline{2}_L(y+1)\oplus 5\times\underline{1}_R(y)\ , \tag{2.14a}$$

$$\underline{3}_R(y)\oplus 2\times\underline{2}_L(y-1)\oplus 2\times\underline{2}_L(y+1)\oplus 3\times\underline{1}_R(y)\oplus\underline{1}_R(y-2)\oplus\underline{1}_R(y+2)\ , \tag{2.14b}$$

$$\underline{3}_R(y)\oplus 2\times\underline{2}_L(y-1)\oplus 2\times\underline{2}_L(y+1)\oplus\underline{1}_R(y)\oplus 2\times\underline{1}_R(y-2)\oplus 2\times\underline{1}_R(y+2)\ . \tag{2.14c}$$

However, it is only Eq. (2.14b) that has equal numbers of left- and right-handed states *for each electric charge quantum number;* Eqs. (2.14a) and (2.14c) necessarily have massless charged particles.

The representation Eq. (2.14b) proves to be, on closer inspection, an amalgamation of structures we have already met. It can be decomposed into a (2.9) multiplet (with $y\neq 0$) plus a mirror pair:

$$[(2.9)]\oplus[\underline{2}_L(y-1)\oplus\underline{1}_R(y-2)\oplus\underline{1}_R(y)]\oplus[\underline{2}_L(y+1)\oplus\underline{1}_R(y+2)\oplus\underline{1}_R(y)]\ . \tag{2.15}$$

This illustrates that Eqs. (2.9) and (2.13) are the fundamentally most interesting triplet generations.

It is amusing to consider the $N=4$ case (take a leptonlike scenario for concreteness). We begin with

$$\underline{4}_R(0)\oplus n_3^1[\underline{3}_L(+1)\oplus\underline{3}_L(-1)]\oplus n_2^2[\underline{2}_R(+2)\oplus\underline{2}_R(-2)]\oplus n_2^0\underline{2}_R(0)\oplus n_1^3[\underline{1}_L(+3)\oplus\underline{1}_L(-3)]\oplus n_1^1[\underline{1}_L(+1)\oplus\underline{1}_L(-1)]\ . \tag{2.16}$$

We equate the number of left- and right-handed states by requiring

$$4+4n_2^2+2n_2^0=6n_3^1+2(n_1^3+n_1^1)\ . \tag{2.17}$$

The simplest cases have $n_3^1=1$ and

$$n_2^2=0,\quad n_2^0=1,\quad n_1^3=n_1^1=0\ , \tag{2.18a}$$

$$n_2^2=1,\quad n_2^0=0,\quad n_1^3=1,\quad n_1^1=0\ , \tag{2.18b}$$

$$n_2^2=1,\quad n_2^0=0,\quad n_1^3=0,\quad n_1^1=1\ , \tag{2.18c}$$

and so on. Equation (2.18a) is just the quartet member of the (2.10) series. Equation (2.18c) necessarily has massless charged particles, while Eq. (2.18b) satisfies all our requirements. Let us rewrite it:

$$\underline{4}_R(0)\oplus\underline{3}_L(+1)\oplus\underline{3}_L(-1)\oplus\underline{2}_R(+2)\oplus\underline{2}_R(-2)\oplus\underline{1}_L(+3)\oplus\underline{1}_L(-3)\ . \tag{2.19}$$

The most interesting thing about this rather large exotic generation is that it is not a member of the (2.10) series. Thus in contrast to the doublet and triplet generations, there are two fundamentally interesting structures for the quartet case. Equation (2.19) therefore represents another type of generalization of the standard family.

Having said this, it should be noted that *in another sense* (2.10)-like multiplets may be regarded as building blocks for Eq. (2.19). If we add a $N=2$ (2.10) generation unit (with $y=2$) to Eq. (2.19) we get

$$[\underline{4}_R(0)\oplus\underline{3}_L(+1)\oplus\underline{3}_L(-1)\oplus\underline{2}_R(0)]\oplus[\underline{2}_R(+2)\oplus\underline{1}_L(+3)\oplus\underline{1}_L(+1)]\oplus[\underline{2}_R(-2)\oplus\underline{1}_L(-3)\oplus\underline{1}_L(-1)]\ , \tag{2.20}$$

which is just the sum of three (2.10)-like structures. Put in another way, the new structure Eq. (2.19) is obtained by taking three (2.10)-like generations, excising a particular anomaly-free piece, and pasting what is left together.

One could, of course, push this analysis further as a study in algebra by trying to prove general theorems. There seems to be no immediate need to do this, however, so we will be content with the above exposition. We hope that this section has been of interest in elucidating the way exotic weak isospin multiplets can be incorporated into the standard model.

III. THE TRIPLET-DOUBLET LEPTON GENERATION (TDLG)

In this section the simplest leptonlike generation is examined. This generation is given in Eq. (2.9), and will be denoted as the triplet-doublet lepton generation (TDLG). In this section we focus on the TDLG without the neutrino singlet, so that the multiplet contains a massless neutrino.

Consider then the lepton assignments

$$\psi_L=\begin{bmatrix} P' & \nu'/\sqrt{2} \\ \nu'/\sqrt{2} & N' \end{bmatrix}_L,\quad \mathcal{P}_R=\begin{bmatrix} P' \\ \nu_1' \end{bmatrix}_R,\quad \mathcal{N}_R=\begin{bmatrix} \nu_2' \\ N' \end{bmatrix}_R, \tag{3.1}$$

where (P',ν',N') form a $Y=0$ left-handed SU(2) triplet, and (P',ν_1') and (ν_2',N') form $Y=1,-1$ right-handed SU(2) doublets, respectively. The primes indicate that the fields are in the weak-eigenstate basis. The TDLG represents the simplest example of fermions in a higher-dimensional representation of SU(2) which retains only the usual Higgs doublet. The leptons are given mass via spontaneous symmetry breaking, through the Lagrangian

$$\mathcal{L}=\lambda_1\tilde{\phi}^T\overline{\psi}_L\mathcal{P}_R+\lambda_2\phi^T\overline{\psi}_L\mathcal{N}_R+\text{H.c.} \ , \tag{3.2}$$

where ϕ represents the $Y=1$ Higgs doublet defined in Eq. (2.1), with vacuum expectation value (VEV) given by

$$\langle\phi\rangle=\begin{bmatrix}0\\u\end{bmatrix}, \quad \langle\tilde{\phi}\rangle=\begin{bmatrix}u\\0\end{bmatrix} . \tag{3.3}$$

Note that we are considering here the exotic leptons in isolation from the leptons of the first three generations of the SM. An examination of the $SU(2)_L\otimes U(1)_Y$ quantum numbers of the exotic generation indicate that ordinary mixing (Higgs-boson fermion-fermion terms) as well as vectorlike mixing (fermion-fermion terms) are in general allowed. The theoretical lack of knowledge regarding the origin of the Yukawa couplings makes predictions difficult. Moreover, since the vectorlike couplings do not appear to have a natural scale (i.e., the mass terms are independent of the Higgs-boson VEV), we will, wherever they are possible exclude them by assuming that the Lagrangian possesses the extra U(1) symmetries associated with conserved lepton numbers. In any case, if lepton-number symmetry is not exact, but holds only approximately, then the mixing of the exotic leptons with the SM leptons will induce small flavor-changing neutral currents (FCNC's). The effects of these FCNC's can easily be made phenomenologically acceptable, with the most observable effect being an increase in the τ lepton lifetime[10] [we will examine the effects of small mixings when we examine the quark sector of the multiplet Eq. (2.13) in Sec. V]. In this regard it is perhaps significant to note that a small increase in the τ lifetime is not in disagreement with experiment.[11] If it is confirmed that the τ lifetime is longer than the SM prediction, then this can be regarded as evidence for an exotic generation of some sort.[12]

Evaluating the mass matrix of the leptons we find

$$\begin{gathered}M_P=\lambda_1 u, \quad M_N=\lambda_2 u \ , \\ M_{\nu_1}=[(\lambda_1^2+\lambda_2^2)/2]^{1/2}u, \quad M_{\nu_2}=0 \ ,\end{gathered} \tag{3.4}$$

where ν_1,ν_2 are the mass-eigenstate fields, with their left-handed components defined by $\nu_{1L}=\nu_L$ ($\nu_{2L}=0$), and their right-handed components given by

$$\nu_{1R}=\cos\alpha\nu'_{2R}+\sin\alpha\nu'_{1R} \ , \tag{3.5a}$$

$$\nu_{2R}=\sin\alpha\nu'_{2R}-\cos\alpha\nu'_{1R} \ . \tag{3.5b}$$

The mixing angle is given by

$$\tan\alpha=\lambda_1/\lambda_2 \ , \tag{3.6a}$$

or

$$\tan\alpha=M_P/M_N \ . \tag{3.6b}$$

Equation (3.4) can also be expressed as the mass constraint

$$2M_{\nu_1}^2=M_P^2+M_N^2 \ . \tag{3.7}$$

If one adds the $SU(2)_L\otimes U(1)_Y$-singlet fermion $\tilde{\nu}_L$ (which was previously omitted) then it couples to the new doublet fermions via

$$\mathcal{L}=\lambda'_1\overline{\mathcal{P}}_R\tilde{\nu}_L\phi+\lambda'_2\overline{\mathcal{N}}_R\tilde{\nu}_L\tilde{\phi}+\text{H.c.} \tag{3.8}$$

The addition of the singlet fermion $\tilde{\nu}_L$ allows all of the neutrinolike particles to gain mass.

The coupling of the TDLG to the gauge bosons is obtained from the fermion kinetic term:

$$\mathcal{L}=i\,\text{Tr}\overline{\psi}_L\not{D}\psi_L+i\overline{\mathcal{P}}_R\not{D}\mathcal{P}_R+i\overline{\mathcal{N}}_R\not{D}\mathcal{N}_R \ , \tag{3.9}$$

where D^μ is the covariant derivative,

$$D^\mu=\partial^\mu+ig\mathbf{W}^\mu\cdot\mathbf{t}+ig'B^\mu Y/2 \ , \tag{3.10}$$

and the SU(2) generators for the doublets and triplets are such that

$$\begin{gathered}t\mathcal{N}=\tfrac{1}{2}\tau\mathcal{N}, \quad t\mathcal{P}=\tfrac{1}{2}\tau\mathcal{P} \ , \\ t\psi=\tfrac{1}{2}\tau\psi+\tfrac{1}{2}\psi\tau^* \ ,\end{gathered} \tag{3.11}$$

where τ are the 2×2 Pauli matrices. The couplings of the gauge bosons are found to be

$$\mathcal{L}_{\text{int}}=-eJ^\mu_{\text{em}}A_\mu-(g/\cos\theta_W)J^\mu_N Z_\mu-(gJ^\mu_+W^+_\mu+\text{H.c.}) \ , \tag{3.12}$$

where

$$J^\mu_{\text{em}}=\bar{P}\gamma^\mu P-\bar{N}\gamma^\mu N \ , \tag{3.13a}$$

$$\begin{aligned}J^\mu_N=&\tfrac{1}{4}\cos2\alpha\bar{\nu}_1\gamma^\mu(1+\gamma_5)\nu_1-\tfrac{1}{4}\cos2\alpha\bar{\nu}_2\gamma^\mu(1+\gamma_5)\nu_2\\&+\tfrac{1}{4}[\sin2\alpha\bar{\nu}_1\gamma^\mu(1+\gamma_5)\nu_2+\text{H.c.}]\\&+\tfrac{1}{4}\bar{P}\gamma^\mu(4\cos^2\theta_W-1-\gamma_5)P\\&-\tfrac{1}{4}\bar{N}\gamma^\mu(4\cos^2\theta_W-1-\gamma_5)N \ ,\end{aligned} \tag{3.13b}$$

$$\begin{aligned}J^\mu_+=&\frac{1}{2\sqrt{2}}\bar{\nu}_1\gamma^\mu[\sqrt{2}+\cos\alpha+(\cos\alpha-\sqrt{2})\gamma_5]N\\&+\frac{1}{2\sqrt{2}}\bar{P}\gamma^\mu[\sqrt{2}+\sin\alpha+(\sin\alpha-\sqrt{2})\gamma_5]\nu_1\\&+\frac{\sin\alpha}{2\sqrt{2}}\bar{\nu}_2\gamma^\mu(1+\gamma_5)N-\frac{\cos\alpha}{2\sqrt{2}}\bar{P}\gamma^\mu(1+\gamma_5)\nu_2 \ .\end{aligned} \tag{3.13c}$$

A phenomenologically interesting possibility would be to consider charged leptons light enough to be produced via W- and Z-boson decay. An important parameter in this context is the ratio Γ_Z/Γ_W (Refs. 13–15). This parameter can be determined through the measurement of $R=N(W\to e\nu)/N(Z\to e^+e^-)$. We consider two scenarios: namely, $M_P\gg M_N$ and $M_N=M_P$. The ratio Γ_Z/Γ_W is given in Figs. 1 and 2. We have used the numerical results of Ref. 15 for the SM value, which is also

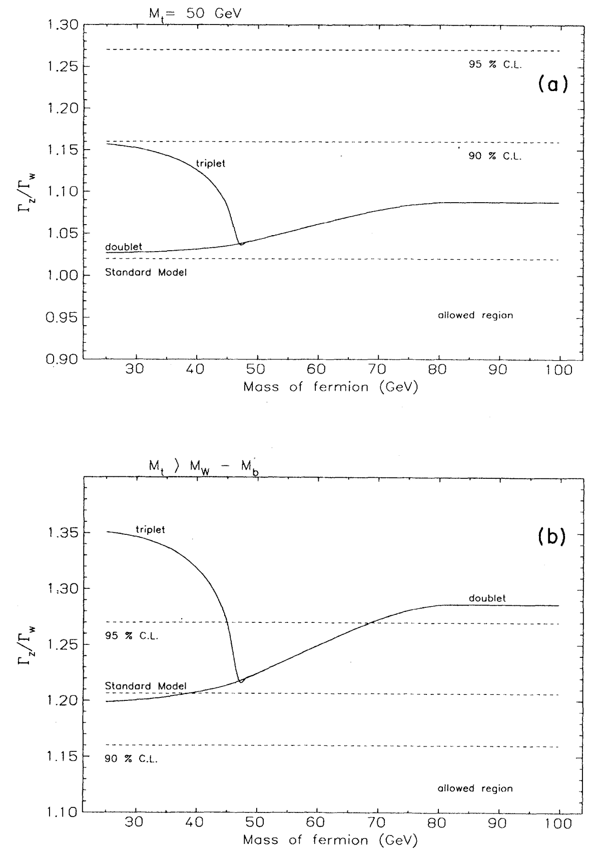

FIG. 1. (a) The ratio Γ_Z/Γ_W for the SM with $M_t=50$ GeV, and for the SM with an additional ordinary doublet or an additional exotic TDLG. In this figure, the fermion mass ordinate is either the charged doublet lepton or the charged N lepton mass, where the mass relation $M_N \ll M_P$ has been assumed. The broken lines indicate the UA1/UA2 upper limits. (b) Same as (a) except $M_t > M_W - M_b$.

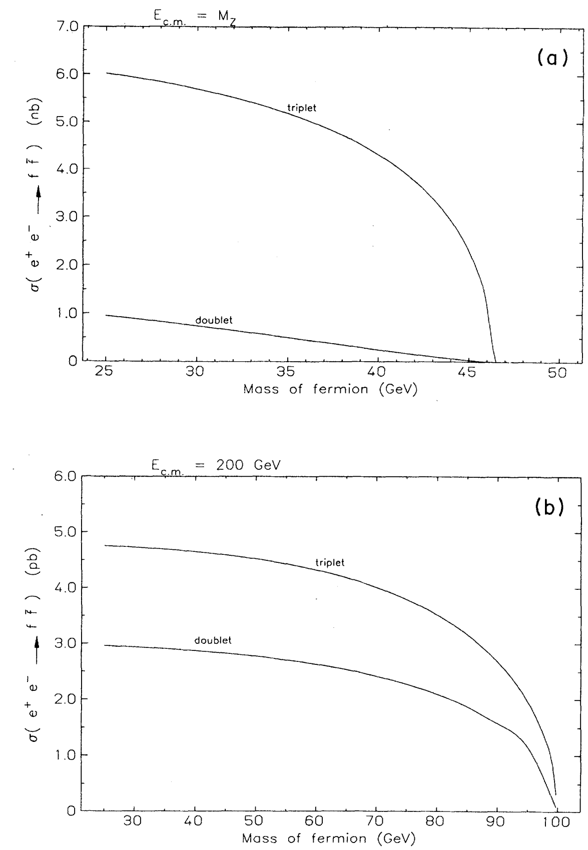

FIG. 3. (a) The cross section $\sigma(e^+e^- \to f\bar{f})$ for the production of the exotic charged N lepton of the TDLG, compared with ordinary doublet charged leptons at an e^+e^- collider at $\sqrt{s}=M_Z$ (note that the mass relation $M_N \ll M_P$ has been assumed). (b) Same as (a) except $\sqrt{s}=200$ GeV.

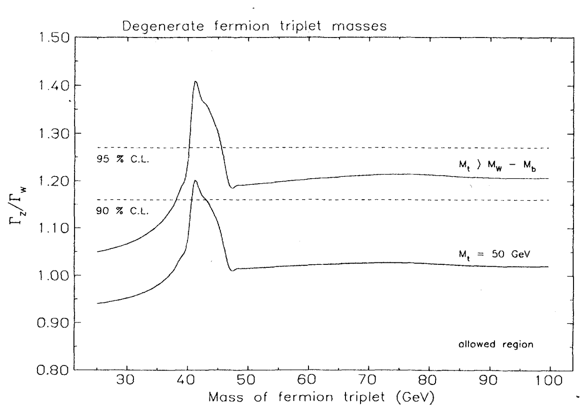

FIG. 2. Same as Fig. 1, except the mass relation $M_N = M_P$ has been assumed.

shown. Note that if the left- and right-handed representations are interchanged, that is if we start with a right-handed triplet, and two left-handed doublets, then for the scenario $M_P \gg M_N$ the N, ν_2 leptons will behave in a similar way to an ordinary doublet of leptons at low energies. Their charged current and the neutrino neutral current reduce to the $Y=1$ doublet case. A measurement of the Z-boson coupling to the N lepton should provide a means of distinguishing the two cases. The Z-boson coupling can be determined through the measurement of the total e^+e^- cross section, or through the forward-backward asymmetry. The cross section is given in Fig. 3, and the forward-backward asymmetry parameter A_{FB}, defined by $A_{FB}=(\sigma_F-\sigma_B)/(\sigma_F+\sigma_B)$, is given in Fig. 4.

To conclude this section, we have investigated an extension of the fermion sector involving a triplet of leptons, without any necessary extension to the Higgs sector. We will now investigate the simplest exotic generation which necessarily contains quarks and leptons—the triplet-doublet quark-lepton generation.

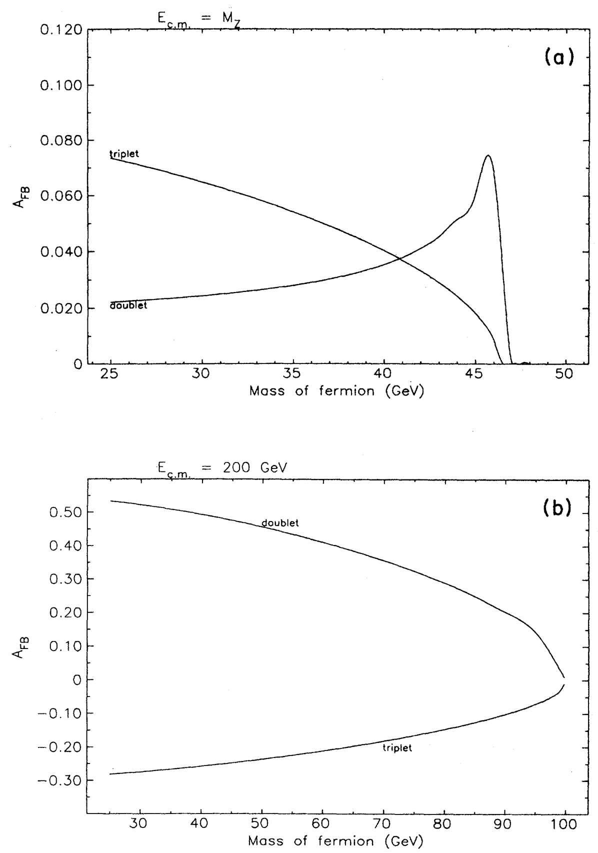

FIG. 4. (a) The forward-backward asymmetry parameter A_{FB} for the charged N lepton of the TDLG, compared with the ordinary doublet charged lepton, at an e^+e^- collider at $\sqrt{s}=M_Z$ (Note that the mass relation $M_N \ll M_P$ has been assumed). (b) Same as (a), except $\sqrt{s}=200$ GeV.

IV. THE TRIPLET-DOUBLET QUARK-LEPTON GENERATION (TDQLG)

In this section the simplest quark-leptonlike generation (beyond the familiar doublet-singlet structure) will be examined. This generation is given in Eq. (2.13) (with $Y_l=-2$) and will be hereafter denoted as the triplet-doublet quark-lepton generation (TDQLG). *A priori* two possibilities are envisaged. Either there exists such a multiplet with all of its particles hitherto unobserved (because they are too heavy for direct production and detection) or, perhaps some of the discovered quarks and leptons may be members of a TDQLG. Remarkably, the latter scenario is shown to be a possibility for either the b quark or the τ, ν_τ leptons (but not both b, τ, and ν_τ unless a TDQLG and its mirror generation is considered). The particle content of the TDQLG is given in Table I.

The left-handed lepton fields are given by a $Y=2$ triplet and a $Y=2$ singlet, while the right-handed fields are given by a $Y=3$ doublet and $Y=1$ doublet (note that we are considering charge conjugate fields relative to those used in Sec. II):

TABLE I. The particle content of the triplet-doublet quark-lepton generation.

Quarks	Charge	Leptons	Charge
t_1	$+\frac{2}{3}$	E	$+2$
b_1	$-\frac{1}{3}$	f	$+1$
$\tilde{b}_1$	$-\frac{1}{3}$	$\tilde{f}$	$+1$
χ	$-\frac{4}{3}$	ν	0

$$\psi_L = \begin{bmatrix} E' & \tilde{f}'/\sqrt{2} \\ \tilde{f}'/\sqrt{2} & \nu' \end{bmatrix}_L, \quad f'_L, \qquad \epsilon_R = \begin{bmatrix} E' \\ \tilde{f}' \end{bmatrix}_R, \quad \eta_R = \begin{bmatrix} f' \\ \nu' \end{bmatrix}_R, \tag{4.1}$$

where the primes indicate that the fields are in weak eigenstates. These fields gain mass via the Yukawa interaction Lagrangian

$$\mathcal{L} = \lambda_1 \tilde{\phi}^T \bar{\psi}_L \epsilon_R + \lambda_2 \phi^T \bar{\psi}_L \eta_R + \lambda_3 \bar{\epsilon}_R \phi f'_L + \lambda_4 \bar{\eta}_R f'_L \tilde{\phi} + \text{H.c.} \tag{4.2}$$

Recall that $\phi, \tilde{\phi}$ represents the $Y=1$ Higgs doublet and $Y=-1$ conjugate Higgs doublet defined in Eq. (2.1), with VEV's given by

$$\langle \phi \rangle = \begin{bmatrix} 0 \\ u \end{bmatrix}, \quad \langle \tilde{\phi} \rangle = \begin{bmatrix} u \\ 0 \end{bmatrix}. \tag{4.3}$$

Substituting $\langle \phi \rangle, \langle \tilde{\phi} \rangle$ for $\phi, \tilde{\phi}$ in Eq. (4.2) yields the mass terms

$$\mathcal{L}_{\text{mass}} = m_E \bar{E}_R E_L + m_\nu \bar{\nu}_R \nu_L + \bar{\mathbf{f}}'_R M \mathbf{f}'_L + \text{H.c.}, \tag{4.4}$$

where $m_E = \lambda_1 u$, $m_\nu = \lambda_2 u$, and M is the charge -1 mass matrix, and $\mathbf{f}'$ represents the 2×1 column vector given by

$$M = \frac{1}{\sqrt{2}} \begin{bmatrix} m_2 & m_1 \\ m_\nu & m_E \end{bmatrix}, \quad \mathbf{f}' = \begin{bmatrix} f' \\ \tilde{f}' \end{bmatrix}, \tag{4.5}$$

where $m_1 = \sqrt{2}\lambda_3 u$, $m_2 = \sqrt{2}\lambda_4 u$. The leptons E' and ν' are mass-eigenstate fields (so that the prime may be dropped as the mass eigenstates are the weak eigenstates for these fields). For the charged -1 sector, however, the weak-eigenstate fields are clearly not the same as the mass-eigenstate fields. The mass-eigenstate fields can be introduced by defining unitary matrices A, B by

$$\mathbf{f}_L = A^{-1}\mathbf{f}'_L, \quad \mathbf{f}_R = B^{-1}\mathbf{f}'_R, \tag{4.6}$$

where the unprimed fields on the left-hand side (LHS) of Eq. (4.6) are the mass-eigenstate fields. Then A, B satisfy

$$B^{-1} M A = D, \quad B^{-1} M M^\dagger B = A^{-1} M^\dagger M A = D^2, \tag{4.7}$$

where D is the diagonal mass matrix. The 2×2 matrices, A, B can be expressed as

$$A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix},\quad B=\begin{bmatrix}\cos\beta & -\sin\beta\\ \sin\beta & \cos\beta\end{bmatrix}, \tag{4.8}$$

where α,β are defined by

$$\tan2\alpha=\frac{2(m_1m_2+m_\nu m_E)}{m_E^2+m_1^2-m_2^2-m_\nu^2}, \tag{4.9a}$$

$$\tan2\beta=\frac{(2m_2m_\nu+m_1m_E)}{m_\nu^2+m_E^2-m_1^2-m_2^2}. \tag{4.9b}$$

The mass eigenvalues are $m_f^2=\lambda_-$, $m_{\bar{f}}^2=\lambda_+$, where

$$\lambda_\pm=\tfrac{1}{4}(m_E^2+m_\nu^2+m_1^2+m_2^2)\times\left[1\pm\left[1-\frac{4(m_1m_\nu-m_2m_E)^2}{(m_E^2+m_\nu^2+m_1^2+m_2^2)^2}\right]^{1/2}\right]. \tag{4.10}$$

The interaction Lagrangian of the lepton sector of the TDQLG can be obtained from the fermion kinetic term

$$L=i\,\mathrm{Tr}(\bar\psi_L \not{D}\psi_L)+i\bar{f}'_L\not{D}f'_L+i\bar\epsilon_R\not{D}\epsilon_R+i\bar\eta_R\not{D}\eta_R, \tag{4.11}$$

where D^μ is the covariant derivative defined in Eq. (3.10). The couplings of the gauge bosons are of the form Eq. (3.12), with the currents given by

$$J^\mu_{\rm em}=2\bar{E}\gamma^\mu E+\bar{f}\gamma^\mu f+\bar{\tilde{f}}\gamma^\mu\tilde{f}, \tag{4.12}$$

$$\begin{aligned}J^\mu_+=&\frac{1}{2\sqrt2}\bar{E}\gamma^\mu[\sqrt2\sin\alpha+\sin\beta+(\sin\beta-\sqrt2\sin\alpha)\gamma_5]f+\frac{1}{2\sqrt2}\bar{E}\gamma^\mu[\sqrt2\cos\alpha+\cos\beta+(\cos\beta-\sqrt2\cos\alpha)\gamma_5]\tilde{f}\\&+\frac{1}{2\sqrt2}\bar{\tilde{f}}\gamma^\mu[\sqrt2\cos\alpha-\sin\beta+(-\sin\beta-\sqrt2\cos\alpha)\gamma_5]\nu+\frac{1}{2\sqrt2}\bar{f}\gamma^\mu[\sqrt2\sin\alpha+\cos\beta+(\cos\beta-\sqrt2\sin\alpha)\gamma_5]\nu,\end{aligned} \tag{4.13}$$

$$\begin{aligned}J^\mu_N=&\bar{E}\gamma^\mu(\tfrac34-2\sin^2\theta_W-\gamma_5/4)E-\bar{f}\gamma^\mu(-\cos2\beta/4+\sin^2\theta_W-\gamma_5\cos2\beta/4)f\\&-\bar{\tilde{f}}\gamma^\mu(\cos2\beta/4+\sin^2\theta_W+\gamma_5\cos2\beta/4)\tilde{f}-[\bar{\tilde{f}}\gamma^\mu\sin2\beta/4(1+\gamma_5)f+\text{H.c.}]-\bar\nu\gamma^\mu(\tfrac34-\gamma_5/4)\nu.\end{aligned} \tag{4.14}$$

Observe that in the limit $\tan\alpha\approx\tan\beta\approx0$, the coupling of the charged $+1f$ lepton reduces to that of a familiar left-handed SU(2) doublet $Y=-1$ charged lepton. Consider for instance the possibility that the τ lepton is a member of this TDQLG, in which case we identify τ^c (where the superscript indicates charge-conjugate field) with f. In this interpretation, the (unmeasured) gauge coupling of the τ neutrino to the Z boson is different to the usual doublet case. Also the masses of the other members of the lepton sector of the TDQLG may gain a large value in a self-consistent way. For example, the constraint $\tan\alpha\approx\tan\beta\approx0$ may be implemented by assuming $m_E\gg(m_1,m_2)\gg m_\nu$, in which case the physical masses satisfy $m_E=m_{\tilde{f}}/\sqrt2\gg m_\tau\gg m_\nu$. However this interpretation appears unlikely to be realized for two reasons. First, note that in this interpretation of the τ and its neutrino, the Z-boson coupling to the neutrino is significantly larger than in the SM (and implies that the partial width of the Z-boson decay to the τ neutrino should be a factor of 5 times larger than in the SM). This provides a significant increase to the Z-boson width Γ_Z, while the W-boson width Γ_W does not get any significant compensating increase. Indirect determinations of the ratio Γ_Z/Γ_W through the measurements of $N(W\to\bar{e}\nu)/N(Z\to e^+e^-)$ (Refs. 13–15) certainly do not seem to favor this. Second, if the small values of the first three generation neutrino masses are due to the nonexistence of the right-handed singlet fields (in which case the Dirac mass vanishes), then it seems natural that the neutral member of the lepton triplet should not have a small mass as both its right- and left-handed components necessarily exist, as neither of them form singlets. However, if there exists a TDQLG in *addition* to the three generations of the SM, then the preceding argument indicates that it is natural that the triplet-doublet neutrino should be heavy (i.e., as heavy as the other particles in the multiplet). Therefore the constraints which apply to the usual fourth-generation sequential doublet from the measurements of the ratio Γ_Z/Γ_W (Refs. 13–15) will not apply. Furthermore, if lepton-number conservation is not assumed, then the triplet may mix with the usual doublets (note that there is no vectorlike mixing). Thus the mixing term

$$\mathcal{L}=\sum_{i=e\mu\tau}(\lambda'_{1i}\phi^T\bar\psi_L L^c_{iL}+\lambda'_{2i}\phi^T\bar\epsilon_R R^c_{iR}+\lambda'_{3i}\tilde\phi^T\bar\eta_R R^c_{iR}+\lambda'_{4i}\phi^T\bar{L}_{iL}f'^c_L+\text{H.c.}) \tag{4.15}$$

would be added to the Lagrangian Eq. (4.2) and to the SM Lagrangian. [Note that here L_{iL} denotes the three $(i=1,\ldots,3)$ SM lepton left-handed doublets and R_{iR} denotes the three $(i=1,\ldots,3)$ SM right-handed SU(2) singlets.] This mixing will give a Majorana mass to the τ neutrino (while the other two neutrinos remain massless), via a seesaw-type mechanism.[4,5] FCNC's will also be induced at the tree level because the singly charged leptons and neutrino associated with the lepton sector of the TDQLG will have a different Z-boson coupling to the SM singly charged leptons and neutrinos. However, these FCNC's will naturally be small enough to be within existing experimental constraints.[10]

The quark sector of the TDQLG will now be examined. The left-handed quark fields are given by a $Y=-\frac{2}{3}$ SU(2) triplet, and $Y=-\frac{2}{3}$ singlet, while the right-handed fields are given by a $Y=\frac{1}{3}$ doublet, and $Y=-\frac{5}{3}$ doublet:

$$\psi_L=\begin{bmatrix} t'_1 & \tilde{b}'_1/\sqrt{2} \\ \tilde{b}'_1/\sqrt{2} & \chi' \end{bmatrix}_L , \quad b'_{1L} ,$$
$$\mathcal{P}_R=\begin{bmatrix} t'_1 \\ b'_1 \end{bmatrix}_R , \quad \mathcal{N}_R=\begin{bmatrix} \tilde{b}'_1 \\ \chi' \end{bmatrix}_R . \tag{4.16}$$

These fields gain mass via the Yukawa interaction Lagrangian:

$$\mathcal{L}=\lambda_1\tilde{\phi}^T\bar{\psi}_L\mathcal{P}_R+\lambda_2\phi^T\bar{\psi}_L\mathcal{N}_R+\lambda_3\bar{\mathcal{P}}_R\phi b'_{1L}+\lambda_4\bar{\mathcal{N}}_R\tilde{\phi}b'_{1L} +\text{H.c.} \tag{4.17}$$

Substituting the VEV's $\langle\phi\rangle,\langle\tilde{\phi}\rangle$ for $\phi,\tilde{\phi}$ in Eq. (4.17) yields the mass terms

$$\mathcal{L}_{\text{mass}}=m_{t_1}\bar{t}_{1R}t_{1L}+m_\chi\bar{\chi}_R\chi_L+\bar{\mathbf{b}}'_R M\mathbf{b}'_L+\text{H.c.} , \tag{4.18}$$

where $m_{t_1}=\lambda_1 u$, $m_\chi=\lambda_2 u$, and M is the charge $-\frac{1}{3}$ mass matrix and $\mathbf{b}'$ represents to the 2×1 column vector, given by

$$M=\frac{1}{\sqrt{2}}\begin{bmatrix} m_1 & m_2 \\ m_{t_1} & m_\chi \end{bmatrix}, \quad \mathbf{b}'=\begin{bmatrix} b'_1 \\ \tilde{b}'_1 \end{bmatrix} , \tag{4.19}$$

where $m_1=\sqrt{2}\lambda_3 u$, $m_2=\sqrt{2}\lambda_4 u$. The t'_1 and χ' are mass-eigenstate fields (so that the prime may be dropped as the mass eigenstates are the weak eigenstates for these fields). For the charged $-\frac{1}{3}$ sector, the mass-eigenstate fields can be introduced by defining unitary matrices A,B by

$$\mathbf{b}_L=A^{-1}\mathbf{b}'_L, \quad \mathbf{b}_R=B^{-1}\mathbf{b}'_R , \tag{4.20}$$

where the unprimed fields on the LHS of Eq. (4.20) are the mass eigenstate fields. Then A,B satisfy Eq. (4.7), and the 2×2 matrices, A,B can be expressed as in Eq. (4.8) where α,β are defined by

$$\tan 2\alpha=\frac{2(m_1m_2+m_\chi m_{t_1})}{m_\chi^2+m_2^2-m_{t_1}^2-m_1^2} , \tag{4.21}$$

$$\tan 2\beta=\frac{2(m_1m_{t_1}+m_2m_\chi)}{m_\chi^2+m_{t_1}^2-m_1^2-m_2^2} . \tag{4.22}$$

The mass eigenvalues are $m_{b_1}^2=\lambda_-$, $m_{\tilde{b}_1}^2=\lambda_+$ where

$$\lambda_\pm=\tfrac{1}{4}(m_\chi^2+m_{t_1}^2+m_1^2+m_2^2) \times\left[1\pm\left[1-\frac{4(m_{t_1}m_2-m_\chi m_1)^2}{(m_\chi^2+m_{t_1}^2+m_1^2+m_2^2)^2}\right]^{1/2}\right] . \tag{4.23}$$

The interaction Lagrangian of the triplet-doublet quarks can be obtained from the fermion kinetic term (as in the case of the leptons). The couplings of the gauge bosons have the form given in Eq. (3.12), with the currents given by

$$J^\mu_{\text{em}}=\tfrac{2}{3}\bar{t}_1\gamma^\mu t_1-\tfrac{1}{3}\bar{b}_1\gamma^\mu b_1-\tfrac{1}{3}\bar{\tilde{b}}_1\gamma^\mu\tilde{b}_1-\tfrac{4}{3}\bar{\chi}\gamma^\mu\chi ,$$

$$\begin{aligned} J^\mu_+ =&\frac{1}{2\sqrt{2}}\bar{t}_1\gamma^\mu[\sqrt{2}\sin\alpha+\cos\beta+(\cos\beta-\sqrt{2}\sin\alpha)\gamma_5]b_1+\frac{1}{2\sqrt{2}}\bar{t}_1\gamma^\mu[\sqrt{2}\cos\alpha-\sin\beta+(-\sin\beta-\sqrt{2}\cos\alpha)\gamma_5]\tilde{b}_1 \\ &+\frac{1}{2\sqrt{2}}\bar{\tilde{b}}_1\gamma^\mu[\sqrt{2}\cos\alpha+\cos\beta+(\cos\beta-\sqrt{2}\cos\alpha)\gamma_5]\chi+\frac{1}{2\sqrt{2}}\bar{b}_1\gamma^\mu[\sqrt{2}\sin\alpha+\sin\beta+(\sin\beta-\sqrt{2}\sin\alpha)\gamma_5]\chi , \end{aligned}$$

$$\begin{aligned} J^\mu_N=&\bar{b}_1\gamma^\mu[\sin^2\theta_W/3-\cos 2\beta/4+(-\cos 2\beta/4)\gamma_5]b_1+\bar{\tilde{b}}_1\gamma^\mu[\sin^2\theta_W/3+\cos 2\beta/4+(\cos 2\beta/4)\gamma_5]\tilde{b}_1 \\ &+\{\bar{\tilde{b}}_1\gamma^\mu[(1+\gamma_5)\sin 2\beta/4]b_1+\text{H.c.}\}+\bar{t}_1\gamma^\mu(\tfrac{3}{4}-2\sin^2\theta_W/3-\gamma_5/4)t_1+\bar{\chi}\gamma^\mu(-\tfrac{3}{4}+4\sin^2\theta_W/3+\gamma_5/4)\chi . \end{aligned} \tag{4.24}$$

Hitherto, for simplicity, the quark sector of the TDQLG has been considered in isolation from the quark fields of the SM. Notice, however, that this restriction involves no loss of generality as the quantum numbers of the TDQLG quark sector are such that they allow no ordinary mixing (Higgs-quark-quark terms) with the usual doublet generations anyway. However they do allow for vectorlike (fermion-fermion terms) which we exclude by the imposition of an extra U(1) symmetry [cf. discussion following Eq. (3.3)]. Thus while in general there will be ordinary mixing in the lepton sector there will not be ordinary mixing in the quark sector. If, however, we consider a mirror TDQLG (i.e., a TDQLG which has left- and right-handed fields interchanged), then there will be no mixing in the lepton sector, but there will be mixing in the quark sector. This will be illustrated in the next section, where we consider a particularly interesting possibility, namely, that the bottom quark is a member of a mirror TDQLG.

V. IS THE BOTTOM QUARK A MEMBER OF A TRIPLET-DOUBLET QUARK-LEPTON GENERATION?

Consider again the gauge interactions of the quark sector of the TDQLG given by Eq. (4.24). Note that in the limit $\tan\alpha=0$ and $\tan\beta=0$, the gauge couplings of the b_1 quark reduce to the gauge couplings of an ordinary sequential SU(2)-doublet charge-$(-\frac{1}{3})$ quark, provided the L and R chiralities are flipped (i.e., we start with a mirror TDQLG). The t_1 quark coupling does not however reduce to that of an ordinary SU(2)-doublet charge-$\frac{2}{3}$ quark. Thus since the gauge couplings of the top quark have not been measured there is a possibility that the existing bottom quark may be one member of this triplet-doublet multiplet of quarks. The interpretation that the observed bottom quark is a member of a triplet-doublet multiplet of quarks is self-consistent because the limits $\tan\alpha=0$ and $\tan\beta=0$ [as required for the b_1 quark coupling to reduce to that of an ordinary sequential SU(2)-doublet charge $-\frac{1}{3}$ quark] may be realized by assuming the existence of the following mass relations:

(1) If $m_\chi \gg m_1, m_2, m_{t_1}$,

$$\text{then } m_{b_1} \ll m_{t_1} \ll m_{\tilde{b}_1} = m_\chi/\sqrt{2}\ , \tag{5.1a}$$

(2) If $m_{t_1} \gg m_1, m_2, m_\chi$,

$$\text{then } m_{b_1} \ll m_\chi \ll m_{\tilde{b}_1} = m_{t_1}/\sqrt{2}\ . \tag{5.1b}$$

Both of these possibilities lead to the b_1 quark being the lightest of the four quarks in the triplet-doublet multiplet. Of course, these limits need only be approximate, as the measured values of the gauge coupling of the bottom quark have rather large uncertainties.[16,17] In case (1), the charge $\frac{2}{3}$ quark is expected to be the lightest triplet-doublet quark after the b_1 quark. As can be seen from Eq. (4.24), this quark can be distinguished from a SU(2)-doublet charge $\frac{2}{3}$ quark, by the Z-boson gauge coupling. The second case defines a rather clear signature as the charge $-\frac{4}{3}$ quark is then the lightest triplet-doublet quark after the b_1 quark.

The lepton sector will be given by the fields in Eq. (4.1), with left- and right-handed fields interchanged. The gauge-boson interaction Lagrangian and currents are given by Eqs. (3.12) and (4.12)–(4.14) but with γ_5 replaced by $-\gamma_5$. This interchange of chiralities (which is necessary for the interpretation of the bottom quark as a member of the TDQLG) ensures that it is now no longer possible to interpret the τ lepton and the τ neutrino as members of the lepton sector, unless a TDQLG and its mirror generation is considered. In any case, it was argued in Sec. IV [see the discussion following Eq. (4.14)] that the possible interpretation of the τ lepton and the τ neutrino as members of a lepton sector of the TDQLG is unlikely. Thus we consider the possibility that the bottom quark is a member of the quark sector of a TDQLG, and, presumably,[18] the actual quark doublet partner of the third doublet of leptons required for anomaly cancellation is too heavy to be observed at present. For simplicity, we make the third doublet of quarks very heavy and degenerate (so that their contribution to $\delta\rho$ is zero). Note that we are invoking "mass level mixing" as discussed under assumption (ii) in Sec. II.

In the quark sector, if the triplet-doublet quark multiplet is added to the first two SU(2) doublets of quarks then there will be mixing allowed. The most general Yukawa interaction Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} &= \lambda_{1i}\tilde{\phi}^T\bar{\psi}_R L_i + \lambda_{2j}\bar{\mathcal{N}}_L D'_{Rj}\tilde{\phi} + \lambda_2\phi^T\bar{\psi}_R\mathcal{N}_L \\ &\quad + \Lambda_{ij}\bar{L}_i\phi D'_{Rj} + \Lambda'_{ik}\bar{L}_i U'_{Rk}\tilde{\phi} + \text{H.c.}\ ,\end{aligned} \tag{5.2}$$

where

$$i=1,\ldots,3,\ L_1=\begin{bmatrix} u' \\ d' \end{bmatrix}_L,\ L_2=\begin{bmatrix} c' \\ s' \end{bmatrix}_L,$$

$L_3=\mathcal{P}_L$, $j=1,\ldots,3$, $D'_{R1}=d'_R$, $D'_{R2}=s'_R$, $D'_{R3}=b'_{1R}$, $k=1,2$, $U'_{R1}=u'_R$, $U'_{R2}=c'_R$, and $\lambda_{1i},\lambda_{2i},\lambda_2,\Lambda_{ij},\Lambda'_{ik}$ are Yukawa couplings. Define weak-eigenstate fields U' and D' as

$$U'=(u',c',t'_1)^T,\quad D'=(d',s',b'_1,\tilde{b}'_1)^T\ . \tag{5.3}$$

The mass-eigenstate fields can be defined by introducing the 3×3 matrices A_U, B_U and the 4×4 matrices A_D, B_D as

$$\begin{aligned} U'_L &= A_U U_L,\quad D'_L = A_D D_L\ ,\\ U'_R &= B_U U_R,\quad D'_R = B_D D_R\ .\end{aligned} \tag{5.4}$$

The form of the gauge interactions is given by Eq. (3.12) with the gauge currents defined by

$$\begin{aligned} J^\mu_{\text{em}} &= \tfrac{2}{3}\bar{U}\gamma^\mu U - \tfrac{1}{3}\bar{D}\gamma^\mu D - \tfrac{4}{3}\bar{\chi}\gamma^\mu\chi\ ,\\ J^\mu_+ &= \bar{U}_L\gamma^\mu A_U^{-1} S A_D D_L + \bar{U}_R\gamma^\mu B_U^{-1} T B_D D_R \\ &\quad + \bar{D}_L\gamma^\mu A_D^{-1} C\chi_L + \bar{D}_R\gamma^\mu B_D^{-1} E\chi_R\ ,\\ J^\mu_N &= \bar{U}_L\gamma^\mu X U_L + \bar{U}_R\gamma^\mu B_U^{-1} Y B_U U_R + \bar{D}_L\gamma^\mu A_D^{-1} N A_D D_L \\ &\quad + \bar{D}_R\gamma^\mu Q D_R + \bar{\chi}\gamma^\mu(-\tfrac{3}{4} + 4\sin^2\theta_W/3 - \gamma_5/4)\chi\ ,\end{aligned} \tag{5.5}$$

where the matrices S,T,C,E,X,Y,N,Q are defined by

$$S=\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix},\quad T=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix},$$

$$C=\begin{bmatrix}0\\0\\0\\ \frac{1}{\sqrt{2}}\end{bmatrix},\quad E=\begin{bmatrix}0\\0\\0\\1\end{bmatrix},$$

$$\begin{aligned} X &= (\tfrac{1}{2}-\tfrac{2}{3}\sin^2\theta_W)I_3\ ,\\ Y &= \text{Diag}(-\tfrac{2}{3}\sin^2\theta_W, -\tfrac{2}{3}\sin^2\theta_W, 1-\tfrac{2}{3}\sin^2\theta_W)\ ,\\ N &= \text{Diag}(-\tfrac{1}{2}+\tfrac{1}{3}\sin^2\theta_W, -\tfrac{1}{2}+\tfrac{1}{3}\sin^2\theta_W\ ,\\ &\qquad -\tfrac{1}{2}+\tfrac{1}{3}\sin^2\theta_W, \tfrac{1}{2}+\tfrac{1}{3}\sin^2\theta_W)\ ,\\ Q &= (\tfrac{1}{3}\sin^2\theta_W)I_4\ ,\end{aligned} \tag{5.6}$$

where I_3 and I_4 represent the 3×3 and 4×4 identity matrix, respectively. Note that small right-handed charged currents have been induced at the tree level. These induced currents are expected to be small, with the largest induced right-handed charged current arising for b_1 quark decay. However, the gauge couplings of the bottom quark are only loosely measured, and while they are consistent with the usual doublet interpretation of the t,b quarks, they allow for the possibility of a considerable deviation from the sequential doublet couplings.[16,17] Perhaps a more important consequence of the mixing is that FCNC's have been induced at the tree level. This is basically due to the additional quarks which do not have the sequential SU(2) neutral-current couplings (the t_1 quark and the $\tilde{b}_1$ quark). However these neutral currents are expected to be small because there is a partial Glashow-Iliopoulos-Maiani (GIM) mechanism at work.[19] For instance, consider the induced $Z\bar{d}s$ coupling:

$$\mathcal{L}_{Z\bar{d}s}=(g/\cos\theta_W)\bar{d}Z(C_{ds}^V+C_{ds}^A\gamma_5)s+\text{H.c.} \tag{5.7}$$

Using Eqs. (5.5) and (5.6) the couplings C_{ds}^V and C_{ds}^A are found to be

$$C_{ds}^V=-C_{ds}^A=\tfrac{1}{2}A_{D_{41}}^*A_{D_{42}}\ . \tag{5.8}$$

The couplings C_{ds}^V and C_{ds}^A (and FCNC couplings in general) are expected to be very small since they are products of far off-diagonal matrix elements which also form the KM matrix $V_{UD}=A_U^{-1}SA_D$. From experiment the off-diagonal KM matrix elements are known to be small.

The form of the matrices A and B are *a priori* arbitrary (without any knowledge of the Yukawa couplings); however, the form of the Kobayashi-Maskawa matrix suggests the following ansatz for A and B (which we adopt for definiteness):

$$A_U\sim B_U\sim\begin{bmatrix}1 & \lambda/2 & 3\lambda^3/8\\ \lambda/2 & 1 & \lambda^2/2\\ 3\lambda^3/8 & \lambda^2/2 & 1\end{bmatrix},$$

$$A_D\sim\begin{bmatrix}1 & \lambda/2 & 3\lambda^3/8 & \lambda^5\\ \lambda/2 & 1 & \lambda^2/2 & \lambda^3\\ 3\lambda^3/8 & \lambda^2/2 & 1 & \lambda\\ \lambda^5 & \lambda^3 & \lambda & 1\end{bmatrix}, \tag{5.9}$$

$$B_D\sim\begin{bmatrix}1 & \lambda & \lambda^3 & \lambda^4\\ \lambda & 1 & \lambda^2 & \lambda^3\\ \lambda^3 & \lambda^2 & 1 & \lambda\\ \lambda^4 & \lambda^3 & \lambda & 1\end{bmatrix},$$

with $\lambda\sim0.2$ (and only magnitudes shown). The form of these matrices lead to Kobayashi-Maskawa matrices

$$V_{UD}=\begin{bmatrix}1 & \lambda & \lambda^3 & 7\lambda^4/8\\ \lambda & 1 & \lambda^2 & 3\lambda^3/2\\ \lambda^3 & \lambda^2 & 1 & \lambda\end{bmatrix},$$
$$\tilde{V}_{UD}=\begin{bmatrix}3\lambda^7/8 & 3\lambda^6/8 & 3\lambda^4/8 & 3\lambda^3/8\\ \lambda^6/2 & \lambda^5/2 & \lambda^3/2 & \lambda^2/2\\ \lambda^4 & \lambda^3 & \lambda & 1\end{bmatrix} \tag{5.10}$$

for the left- and right-handed fields, respectively. Of course it should be emphasized that the above ansatz (for A and B) may be completely wrong. However the limited information obtainable from the Kobayashi-Maskawa matrix makes the ansatz at least reasonable. We can now check whether the model, within the above assumptions, is consistent with the experimental constraints on the FCNC's. The interaction Lagrangian is given by Eq. (3.12) and the currents are defined in Eqs. (5.5), (5.6), and (5.9). Limits can be placed on the FCNC couplings by using experimental bounds from rare processes. For example, the mass difference from K^0-$\bar{K}^0$ mixing[20] (see Fig. 5) is given by

$$\Delta m\sim\frac{8\sqrt{2}}{3}G_Ff_k^2m_k(C_{ds}^A)^2\lesssim1.4\times10^{-17}\ \text{GeV}\ , \tag{5.11}$$

where f_k and m_k are the kaon decay constant and mass, respectively. The constant C_{ds}^A is the axial $Z\bar{d}s$ coupling defined by Eq. (5.7). This mass difference gives a bound on the axial $Z\bar{d}s$ coupling

$$C_{ds}^A\lesssim5\times10^{-6}\ . \tag{5.12}$$

Other rare processes such as $K^0\to\mu^+\mu^-$, $K^+\to\pi^+\bar{\nu}\nu$, etc., can also be used. Bounds on the other flavor-changing neutral gauge couplings can be calculated in a similar manner. It has been found that the best con-

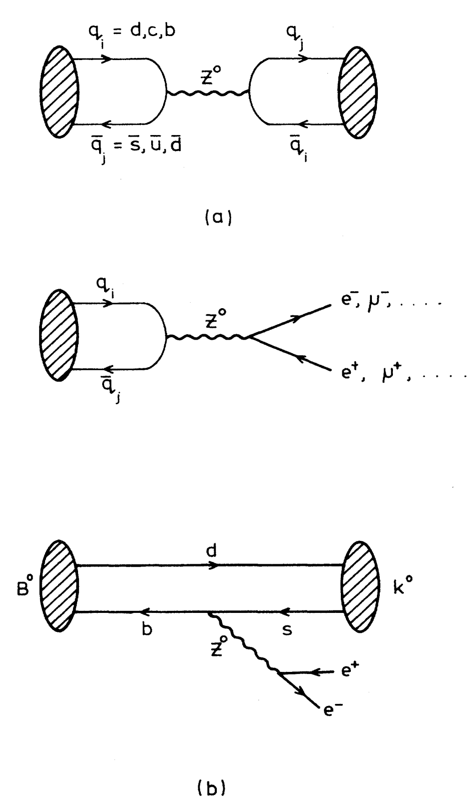

FIG. 5. Examples of rare processes induced by the tree level FCNC's: (a) Neutral meson mixing (T channel not shown). (b) Rare decays.

straints come from neutral meson mixing. The results are summarized below:

Process	Upper bound	From ansatz
$K^0 \leftrightarrow \bar{K}^0$ (Ref. 20)	$C^A_{ds} \lesssim 5\times 10^{-6}$	$\lambda^8/2 \sim 1.3\times 10^{-6}$
$D^0 \leftrightarrow \bar{D}^0$ (Ref. 21)	$C^A_{cu} \lesssim 3\times 10^{-4}$	$3\lambda^5/32 \sim 3\times 10^{-5}$
$B^0 \leftrightarrow \bar{B}^0$ (Ref. 22)	$C^A_{bd} \lesssim 4\times 10^{-4}$	$\lambda^6/2 \sim 3\times 10^{-5}$
$B^0 \to e^+e^-X$ (Ref. 23)	$C^A_{bs} \lesssim 9\times 10^{-4}$	$\lambda^2/2 \sim 8\times 10^{-4}$

The last column indicates the expected value for the couplings C^A obtained from the ansatz for A and B [see Eq. (5.9)].

VI. CONSTRAINTS FROM $\delta\rho$

In general, fermions which couple to the W and Z gauge bosons will contribute to their self-energies and hence contribute to $\delta\rho$, where ρ is defined by

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \; . \tag{6.1}$$

The calculation of $\delta\rho$ due to heavy fermions (or bosons) can be used to place constraints on their masses.[24] The contribution of heavy fermions with arbitrary couplings to M_W^2 and M_Z^2 is obtained by evaluating the fermion-loop contribution to the W and Z gauge-boson propagator. For a pair of fermions with general gauge couplings given in Fig. 6, we obtain

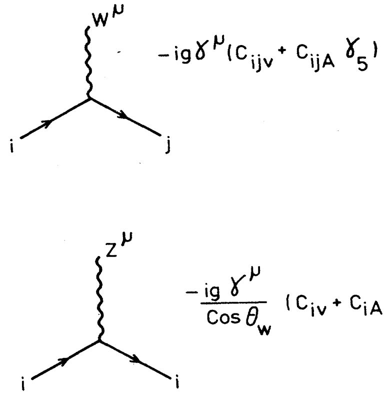

FIG. 6. The form of the fermion-gauge boson couplings used in the calculation of $\delta\rho$.

$$\delta M_Z^2 = \frac{g^2 m_i^2 C_{iA}^2}{2\pi^2\cos^2\theta_W}\left[\Delta - \ln\frac{m_i^2}{\mu^2}\right] , \tag{6.2}$$

$$\begin{aligned}\delta M_W^2 = \frac{-g^2}{8\pi^2}\Bigg[& -\Delta[(m_i^2+m_j^2)(C_{ijV}^2+C_{ijA}^2) - 2m_i m_j(C_{ijV}^2 - C_{ijA}^2)] \\ & +(C_{ijV}^2+C_{ijA}^2)\left[m_i^2\ln\frac{m_i^2}{\mu^2} + m_j^2\ln\frac{m_j^2}{\mu^2} + \frac{m_i^2 m_j^2}{m_i^2-m_j^2}\ln\frac{m_i^2}{m_j^2} - \tfrac{1}{2}(m_i^2+m_j^2)\right] \\ & +(C_{ijV}^2-C_{ijA}^2)m_i m_j\left[2-\ln\frac{m_i^2}{\mu^2} - \ln\frac{m_j^2}{\mu^2} - \frac{m_i^2+m_j^2}{m_i^2-m_j^2}\ln\frac{m_i^2}{m_j^2}\right]\Bigg] ,\end{aligned} \tag{6.3}$$

and

$$\Delta = 2/\epsilon - \gamma + \ln 4\pi \; , \tag{6.4}$$

where $\epsilon \to 0$ ($D = 4-\epsilon$), and γ is the Euler constant. To explain Eqs. (6.2) and (6.3) a few comments are in order. The integrals have been regularized using dimensional regularization (μ is the arbitrary parameter with dimensions of mass), the indices i and j are summed over all the particles in the multiplet and m_i, and m_j are the masses of the ith and jth particle, respectively.

To apply Eqs. (6.2) and (6.3) to a general TDQLG to obtain $\delta\rho$ would not be very helpful, because of the unknown parameters α,β. Qualitatively, we can say that in general, the masses of the particles in the triplet-doublet multiplet should be in the electroweak scale (<1 TeV). However, for the model discussed in Sec. V, we can be more definite, as the model assumes that $\tan\alpha \approx 0$ and $\tan\beta \approx 0$, for the quark sector. Recall, that the limits $\tan\alpha \approx 0$ and $\tan\beta \approx 0$, can be implemented by assuming the mass hierarchy given in Eq. (5.1). For case (1) [see Eq. (5.1)], we only need to consider the loops which contain either or both the χ and $\tilde{b}$ quarks. Neglecting the mixing angle λ, we obtain

$$\delta\rho = \frac{3g^2 m_\chi^2}{64\pi^2 m_W^2}(6\ln 2 - \tfrac{5}{2}) \; . \tag{6.5}$$

Imposing the constraint $\delta\rho < 0.02$ implies that $m_\chi < 200$ GeV. This scenario suggests that the mass of the new toplike quark should be less than about 100 GeV. For case (2) [see Eq. (5.1)], we obtain the same constraint as in the usual doublet interpretation of the t and b quarks:

$$\delta\rho = \frac{3g^2 m_{t_1}^2}{64\pi^2 m_W^2} \; . \tag{6.6}$$

The constraint $\delta\rho < 0.02$ implies that $m_{t_1} < 260$ GeV. Of course, it should be emphasized that these constraints only exist because of the large mass splitting (which is necessary for the model discussed in Sec. V). If a completely degenerate TDQLG is considered, then the contribution to $\delta\rho$ is zero (to one loop), as is also the case for a degenerate fermion doublet.[24]

VII. PHENOMENOLOGICAL COMMENTS AND CONCLUDING REMARKS

In summary, we have examined the structure of exotic generations which are specified from the constraints of anomaly cancellation, and that the members of an exotic generation be able to gain mass, assuming the existence of $Y=1$ Higgs doublets. The simplest extension involving leptons only, the TDLG, was investigated in Sec. III. We then investigated the simplest generation containing both quarks and leptons—the TDQLG. Models incorporating these exotic generations into the SM may feature FCNC's at the tree level. However, these FCNC's are typically within the experimental constraints. A particularly interesting possibility considered is that the bottom quark may actually be a member of a TDQLG. The phenomenological prospects for this interpretation are rather interesting. First, for the b quark to have the sequential doublet couplings implies the existence of certain mass relations. In the first case, [Eq. (5.1a)] there exists the mass relation $m_{b_1} \ll m_{t_1} \ll m_{\tilde{b}_1} = m_\chi/\sqrt{2}$. The constraints from the ρ parameter for this case indicate that $m_\chi < 200$ GeV. In the second case $m_{b_1} \ll m_\chi \ll m_{\tilde{b}_1} = m_{t_1}/\sqrt{2}$, the toplike quark is expected to be heavy and is limited by about 260 GeV. In this scenario the χ quark is the lightest of the exotic particles. In this case note that the UA1 lower bound[25] of 44 GeV on the mass of the top quark will not apply for χ, as its production at the CERN S$p\bar{p}$S collider is significantly smaller than a top quark of the same mass, because the $\chi b_1 W$ coupling is small. Thus, the only constraint on the χ quark mass appears to be from e^+e^- colliders which implies a lower limit of about 30 GeV for χ (Ref. 26). The existence of such a light χ quark would give a spectacular jump to the e^+e^- R ratio:

$$R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-) \ . \tag{7.1}$$

The ratio would rise by about a factor of 4 compared with that due to a toplike quark.

In conclusion then, we believe that some very interesting physics is associated with the incorporation of exotic generations into the SM. The quantum numbers of elementary particles is clearly a fundamental concern; it is wise to know what sort of physical signatures various types of exotic particles may have, and how they fit in with standard-model physics. There are, of course, many questions arising from this study. For example, the discovery of an exotic generation would represent a dramatic increase in the number of supposedly fundamental fields. In this regard, there has already been much speculation that quarks and leptons may be composite particles, and the discovery of unusual fermions may strengthen such suspicions. Such fermions will alter the renormalization-group evolution of gauge couplings. Can these particles be incorporated into a grand-unified-theory (GUT) scenario, or would their discovery be indirect evidence against at least simple GUT models? Can superstring models yield fundamental low-mass fermions in higher-dimensional representations? Perhaps the discovery of a triplet would be indicative of a direct product structure $SU(2)_1 \otimes SU(2)_2 \otimes SU(2)_3$ underlying the $SU(2)_L$ gauge group.[27] For example, if each SU(2) factor corresponded to a generation, then the so-called "intergenerational fermions" of Ma and Tuan[28] which transform as (2,2,1), (2,1,2), and (1,2,2) would yield triplets under the diagonal $SU(2)_L$ subgroup. Though it is important to know the immediate experimental consequences of exotic particles, it is perhaps premature to speculate too deeply about what their existence might imply; we will be content with the present analysis. We await with interest new experimental data, to see whether the minimal SM will survive as a successful theory to even higher-energy scales.

Note added. After the completion of this paper we became aware of the work of Fishbane *et al.*[29] These authors considered similar questions to those addressed in Sec. II of the present paper. The two analyses are complementary, though they differ a little in emphasis.

ACKNOWLEDGMENTS

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