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Electromagnetic charge radii of pseudoscalar mesons

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The charge radii of the pseudoscalar mesons π^+, K^+, and K^0 are analyzed in an extended vector-dominance framework and the results are compared with those which follow the nonrelativistic quark model and constituent-quark triangle loop approach. Further, estimates of the root-mean-square relative coordinate parameter are obtained which are in conformity with normal theoretical expectations.

I. INTRODUCTION

The meson summary table of the Particle Data Group classifies[1] the $\rho(1600)$ as a resonance with the quantum numbers of the $\rho(770)$. However, the $\rho(1250)$ is not identified as an established candidate for the radial excitation of the $\rho(770)$. Apparently, the evidence[2] for the $\rho(1250)$ is not quite compelling in the 2π annihilation of e^+e^- in which the heavier $\rho(1600)$ is cleanly detected. On the other hand, the $\rho(1250)$ and $\rho(1600)$ are clearly distinguished in the $e^+e^-\to 4\pi$ process.

Recently, the $e^+e^-\to 2\pi$ reaction has yielded[3] some precise information on pion form-factor behavior. The absence of the $\rho(1250)$ in the $e^+e^-\to 2\pi$ channel may therefore appear to be worrying since it is well known that the description of the pion form-factor data requires contributions from several ρ mesons. It is, however, true that one can account for these data by having a model with the $\rho(770)$ and $\rho(1600)$ only and using the inelastic contributions instead of the $\rho(1275)$. In this context, it is worthwhile to recall[4] an old paper of Renard in which it was claimed that one could not really decide between the inelastic effects and those coming from the excited vector mesons. Nevertheless, one must note that the inclusion of the $\rho(1250)$ does improve the $\chi^2/N_{\rm DF}$ value of the pion form-factor data.

From the point of view of model calculations of these data and the ones following from them, such as the pion and kaon charge radii, Kl_3 slope, etc., this effectively means imposing stronger bounds on the form factors. Thus if one works in a vector-dominance model (VDM), whether or not the $\rho(1250)$ exists, the corresponding dispersion integrals are required to be saturated by several vector mesons of a particular family. Such a problem was addressed by Zovko[5] in the context of the pion and kaon form factors several years ago. To estimate the impact of vector mesons on them, Zovko assumed analyticity along with the asymptotic bounds and considered feeding of the dispersion integral by the ρ, ω, Φ, and their excited partners. To get a satisfactory fit with the data, it was found that at least three isovector and three isoscalar vector mesons must exist. This was in tune with the solution obtained from the nucleon form factors. Zovko also calculated the pion and kaon charge radii and found that the pion charge radius was slightly bigger than the kaon.

The current status of the pion and kaon charge radii is

$$\langle r^2_{\pi^+}\rangle = 0.44\pm0.02 \text{ fm}^2 , \quad (1a)$$

$$\langle r^2_{K^+}\rangle = 0.34\pm0.05 \text{ fm}^2 , \quad (1b)$$

$$\langle r^2_{K^0}\rangle = -0.054\pm0.026 \text{ fm}^2 . \quad (1c)$$

The above value on $\langle r^2_{\pi^+}\rangle$ is taken from Ref. 6 and is estimated from the slope of the form factor at zero momentum transfer. Such a slope appears in the expression for the differential cross section of meson-electron scattering. The value in (1a) corresponds to the pole fit to the 250-GeV pion data and almost coincides with the result at 300 GeV. We do not enter into a discussion of the previous results, a brief survey of which may be found in Ref. 7(a). However, it may be noted that this value of $\langle r^2_{\pi^+}\rangle$ is in reasonable agreement with the earlier elastic-scattering measurements.

The result (1b) is obtained[6] after taking into account the estimated systematic error of 1% and happens to be insensitive to the functional form assumed for the kaon form factor. However, only a pole fit to the kaon data gives $\langle r^2_{K^+}\rangle = 0.40\pm0.11$ fm^2, which is appreciably larger than the one quoted in (1b). The experimental group of Ref. 6 have also obtained a difference between the mean-square charge radii of the pion and kaon which is free from common systematic errors. Their result is

$$\langle r^2_{\pi^+}\rangle - \langle r^2_{K^+}\rangle = 0.10\pm0.045 \text{ fm}^2 \quad (1d)$$

with a χ^2 probability of 50%. This value is considerably smaller than the previous one[7(a)] of 0.16 ± 0.06 fm^2. However, the uncertainty in (1d) is substantially large, thus calling for a more precise measurement.

The result (1c) on $\langle r^2_{K^0} \rangle$, which is taken from Refs. 6, 7(a), and 8, also shows a large uncertainty and so an independent check is necessary for this value too.

The purpose of this paper is to study the above results on the charge radius of the pion and kaon in an extended vector-dominance scheme and look for consistency with other models such as the nonrelativistic quark approach and the constituent-quark triangle loop scheme. In setting up our scenario, taking into account the effects of the radial vector-meson states, we have been guided by the analysis of Zovko in that we too have considered analyticity and imposed asymptotic bounds on the meson form factors. For the excited vector mesons we have included the effects of the $\rho(1250)$ and $\rho(1600)$ in the $I=1$ sector. For the isoscalar state, there is currently[9] only one candidate, viz., $\Phi(1634)$. So we have considered its effects along with the conventional $^3S_1$ $\omega(783)$ and $\Phi(1020)$ states. It may be remarked that the $^3D_1$ resonance at 1634 MeV is not the same one as considered by Zovko who studied the effects of a still higher state.

The plan of this work is as follows. In Sec. II we consider the charge radii of the pion and kaon in an extended vector-dominance scheme and compared our results with those following naively if the dispersion integrals are saturated by the low-lying ρ, ω, and Φ only. In Secs. III and IV we discuss the nonrelativistic quark model and the quark triangle loop approach to the problem of meson charge radii. We also estimate the unknown root-mean-square relative coordinate parameter which enters into the expressions of the various charge radii. Finally, in Sec. V, we give a summary of our paper.

II. CHARGE RADII IN AN EXTENDED VECTOR-DOMINANCE FRAMEWORK

The essence of the VDM lies in treating a photon of some given momentum and helicity as a superposition of the virtual vector mesons. Thus if X and Y are arbitrary states, one can relate the amplitude $A(\gamma+X\rightarrow Y)$ to $A(V+X\rightarrow Y)$ through

$$A(\gamma+X\rightarrow Y)=\sum \frac{e}{f_V}F_V(q^2)A(V+X\rightarrow Y) \ , \quad \text{(2a)}$$

where the summation may extend over vector mesons (V) with suitable quantum numbers, $F_V(q^2)$ is an appropriate form factor and e/f_V is the coupling of the electromagnetic current to the vector mesons measured in the e^+e^- annihilation

$$\Gamma(V\rightarrow e^+e^-)=\tfrac{1}{3}\alpha m_V \left[\frac{e}{f_V}\right]^2 . \quad \text{(2b)}$$

Consider the electron-proton scattering in which a virtual photon is exchanged. The appearance of a form factor such as in (2a) reflects the structure of the proton. In other words, the exchanged photon will be absorbed by a meson cloud at some distance from the proton, the latter being enclosed in a pion shell.

The charge distribution may be assumed to be spherically symmetric. The form factor may then be expressed as

$$F(q^2)=\int d^3r\, e^{-iqr}\rho(r)$$
$$=4\pi\int r^2dr\,\rho(r)\frac{\sin qr}{qr} \ . \quad \text{(3)}$$

Expanding $\sin qr$, one obtains, for small q^2,

$$F(q^2)=Q-\frac{q^2}{3!}\langle r^2_{\text{em}}\rangle \ , \quad \text{(4a)}$$

where

$$Q=\int \rho(r)4\pi r^2dr \quad \text{(4b)}$$

and $\langle r^2_{\text{em}}\rangle$ is the electromagnetic mean-square charge radius. A convenient formula is

$$\langle r^2_{\text{em}}\rangle=-6\frac{dF(q^2)}{dq^2} \quad \text{(5)}$$

which gives a measurement of the slope of the form factor as a function of q^2.

The asymptotic behavior of the electromagnetic form factors is subject to certain constraints. In this regard one generally assumes[10] analyticity to hold along with the asymptotic bounds

$$t^nF(t)\rightarrow 0 \quad \text{as } t\rightarrow\infty \ ;$$

i.e.,

$$\int t^{n-1}F(t)dt=0 \ . \quad \text{(6)}$$

In particular, for $n=1$, (6) implies simply $tF(t)\rightarrow 0$ as $t\rightarrow\infty$.

Within the VDM the above bound leads to relationships between the vector-pseudoscalar-pseudoscalar (VPP) coupling constants if we apply (5) to different low-lying pseudoscalar mesons such as the pion and kaon.

The pion form factor may be given by

$$F_\pi(t)=\frac{f_{\rho\pi\pi}}{f_\rho}\frac{m_\rho^2}{m_\rho^2-t}$$
$$+\text{higher excited states (HES) of } \rho \ . \quad \text{(7)}$$

The normalization condition $F_\pi(0)=1$, and the case $n=1$, lead to

$$\sum \frac{f_{\rho\pi\pi}}{f_\rho}=1 \ , \quad \text{(8a)}$$

$$\sum m_\rho^2\frac{f_{\rho\pi\pi}}{f_\rho}=0 \ . \quad \text{(8b)}$$

For the kaon, an isovector $[F_K^V(t)]$ as well as an isoscalar $[F_K^S(t)]$ form factor can be defined. These are

$$F_K^V(t) = \frac{f_{\rho K\bar{K}}}{f_\rho} \frac{m_\rho^2}{m_\rho^2 - t} + \text{HES of } \rho \,, \tag{9a}$$

$$F_K^S(t) = \frac{f_{\omega K\bar{K}}}{f_\omega} \frac{m_\omega^2}{m_\omega^2 - t} + \frac{f_{\Phi K\bar{K}}}{f_\Phi} \frac{m_\Phi^2}{m_\Phi^2 - t} + \text{HES of } \omega \text{ and } \Phi \,. \tag{9b}$$

The coupling constants in (9) are subject to the following restrictions on account of (6):

$$\sum_\rho \frac{f_{\rho K\bar{K}}}{f_\rho} = \frac{1}{2} \,, \tag{10a}$$

$$\sum_\rho m_\rho^2 \frac{f_{\rho K\bar{K}}}{f_\rho} = 0 \,, \tag{10b}$$

$$\sum_{\omega,\Phi} \frac{f_{\omega K\bar{K}}}{f_\omega} = \frac{1}{2} \,, \tag{10c}$$

$$\sum_{\omega,\Phi} m_\omega^2 \frac{f_{\omega K\bar{K}}}{f_\omega} = 0 \,, \tag{10d}$$

where $\sum_\rho$ runs over the $\rho(770)$, $\rho(1250)$, and $\rho(1600)$ while $\sum_{\omega,\Phi}$ runs over the $\omega(783)$, $\Phi(1020)$, and $\Phi(1634)$ states.

Using (5), (7), and (9) the formula for the charge radius of π^+, K^+, and K^0 mesons may be expressed as

$$\langle r_{K^+}^2 \rangle = 6 \left[\sum_\rho \frac{f_{\rho K\bar{K}}}{f_\rho m_\rho^2} + \sum_{\omega,\Phi} \frac{f_{\omega K\bar{K}}}{f_\omega m_\omega^2} \right] , \tag{11a}$$

$$\langle r_{K^0}^2 \rangle = 6 \left[-\sum_\rho \frac{f_{\rho K\bar{K}}}{f_\rho m_\rho^2} + \sum_{\omega,\Phi} \frac{f_{\omega K\bar{K}}}{f_\omega m_\omega^2} \right] , \tag{11b}$$

$$\langle r_{\pi^+}^2 \rangle = 6 \sum_\rho \frac{f_{\rho\pi\pi}}{f_\rho m_\rho^2} \,. \tag{11c}$$

A. Ground-state result (naive VDM)

If the right-hand side (RHS) of (11a)–(11c) are saturated by the low-lying ρ, ω, and Φ only, then these reduce to

$$\langle r_{K^+}^2 \rangle = 6 \left[\frac{f_{\rho K\bar{K}}}{f_\rho m_\rho^2} + \frac{f_{\omega K\bar{K}}}{f_\omega m_\omega^2} + \frac{f_{\Phi K\bar{K}}}{f_\Phi m_\Phi^2} \right] , \tag{12a}$$

$$\langle r_{K^0}^2 \rangle = 6 \left[-\frac{f_{\rho K\bar{K}}}{f_\rho m_\rho^2} + \frac{f_{\omega K\bar{K}}}{f_\Phi m_\Phi^2} + \frac{f_{\Phi K\bar{K}}}{f_\Phi m_\Phi^2} \right] , \tag{12b}$$

$$\langle r_{\pi^+}^2 \rangle = 6 \left[\frac{f_{\rho\pi\pi}}{f_\rho m_\rho^2} \right] . \tag{12c}$$

The exact SU(3) for the VPP vertices yields

$$\langle r_{K^+}^2 \rangle = \frac{3}{m_\rho^2} + \frac{2}{m_\Phi^2} + \frac{1}{m_\omega^2} \,, \tag{13a}$$

$$\langle r_{K^0}^2 \rangle = -\frac{3}{m_\rho^2} + \frac{2}{m_\Phi^2} + \frac{1}{m_\omega^2} \,, \tag{13b}$$

$$\langle r_{\pi^+}^2 \rangle = \frac{6}{m_\rho^2} \,, \tag{13c}$$

where an ideal SU(3) mixing for the ω-Φ pair has been assumed which causes the physical ω and Φ to have the structures $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$, respectively. As is well known, in reality the deviation from this mixing hypothesis is very slight.

From (13), one is led to the sum rule

$$\langle r_{K^+}^2 \rangle - \langle r_{\pi^+}^2 \rangle = \langle r_{K^0}^2 \rangle \,. \tag{14}$$

First let us see what the predictions of naive VDM are for the pion and kaon charge radii. We find

$$\langle r_{K^+}^2 \rangle = 0.33 \text{ fm}^2 \,, \tag{15a}$$

$$\langle r_{K^0}^2 \rangle = -0.055 \text{ fm}^2 \,, \tag{15b}$$

$$\langle r_{\pi^+}^2 \rangle = 0.39 \text{ fm}^2 \,. \tag{15c}$$

At first glance, the above predictions for the kaon (K^+ and K^0) charge radii may appear to be in almost perfect agreement with the experimental values (1b) and (1c). Unfortunately, not much trust can be placed on these estimates, the reason being that the VDM expressions for the kaon charge radii are sensitive to the contributions of the radial vector states. We shall return to this point later (Sec. II C) when we study the impact of such contributions on the $\langle r_{K^+}^2 \rangle$ and $\langle r_{K^0}^2 \rangle$. Further, the estimate (15c) of the pion mean-square charge radius is a good deal smaller than the experimental value. Since the inclusion of the excited states of the ρ does improve this value we are left with the following alternatives: either believe in naive VDM predictions and disregard the present experimental value on $\langle r_{\pi^+}^2 \rangle$ or accept this value and thereby move over from the naive VDM to the extended VDM philosophy (i.e., include the radial partners along with the ρ, ω, and Φ states). Since there is a general consensus on the current $\langle r_{\pi^+}^2 \rangle$ value in (1a) in that it agrees with most of the previous results, except for the one coming[7(b)] from $e\pi \to e\pi$ data, it seems that accepting the extended VDM should be the more reasonable step.

Turning to the sum rule (14), it may be seen that its agreement with the experimental values (1) is somewhat mixed. If we consider the results of Ref. 6, then the LHS which represents the difference between the kaon and pion mean-square radii is -0.10 ± 0.045 fm^2 while the RHS is -0.054 ± 0.026 fm^2. This is reasonable enough given the large uncertainty in the $\langle r_{K^0}^2 \rangle$ value. However, if we compare (14) with the results of Ref. 7(a) then the above value of $\langle r_{K^0}^2 \rangle$ is to be balanced against 0.16 ± 0.06 fm, which shows a clear mismatch of a factor of 2.

This discrepancy in the central values of the pion and kaon charge radii may be attributed to several factors. The most important of these are that (i) the uncertainty associated with the $\langle r_{K^0}^2 \rangle$ value is large, more than an order of 50%, and (ii) the present data on $\langle r_{K^+}^2 \rangle$ and $\langle r_{\pi^+}^2 \rangle$ are yet to be established accurately. It is relevant to take note of a prevailing view[11] that $\langle r_{K^+}^2 \rangle$ may be substantially larger than its present experimental value. An analysis[11(a)] within chiral perturbation theory does support this claim, as do the measurements of Ref. 6, albeit mildly.

On the other hand, if the experimental values of Ref. (7a) are trusted then one would not have any option but to disregard the sum rule (14) altogether. We would not, however, advocate this course of action. The chief reason is that if we consider contributions from the radial isovector mesons, viz., $\rho(1250)$ and $\rho(1600)$ along with the $\rho(770)$ in (11) and restrict SU(3) to the low-lying VPP couplings only, the ground-state result (14) remains unaffected. This is a subtle point and does not seem to have been recognized before. One has, therefore, to attach some degree of importance on the result (14) since SU(3) at the ground-state level is expected[12] to hold within 20% accuracy.

B. Inclusion of excited states (extended VDM)

Eliminating the ρ'' couplings from (8a), (8b), (10a), and (10b) it follows in a straightforward way that

$$\frac{f_{\rho'\pi\pi}}{f_{\rho'}}=\frac{m_{\rho''}^2}{m_{\rho''}^2-m_{\rho'}^2}\left[1-\left[1-\frac{m_\rho^2}{m_{\rho''}^2}\right]\frac{f_{\rho\pi\pi}}{f_\rho}-\text{contributions from HES}\right] \tag{16}$$

and

$$2\frac{f_{\rho'K\bar{K}}}{f_{\rho'}}=\frac{m_{\rho''}^2}{m_{\rho''}^2-m_{\rho'}^2}\left[1-2\left[1 \quad \frac{m_\rho^2}{m_{\rho''}^2}\right]\frac{f_{\rho K\bar{K}}}{f_\rho}-\text{contributions from HES}\right]. \tag{17}$$

Subtracting (17) from (16) we get

$$2f_{\rho'K\bar{K}}-f_{\rho'\pi\pi}=\frac{m_{\rho''}^2}{m_{\rho''}^2-m_{\rho'}^2}\left[\frac{f_{\rho'}}{f_\rho}\left[\frac{m_\rho^2}{m_{\rho''}^2}-1\right](2f_{\rho K\bar{K}}-f_{\rho\pi\pi})+\text{contributions from HES}\right]. \tag{18a}$$

If we consider in (8) and (10) the effects of two excited states of the ρ only, then it is obvious from (18a) that the relation $f_{\rho'K\bar{K}}=\frac{1}{2}f_{\rho'\pi\pi}$ is valid if the exact SU(3) relation $f_{\rho K\bar{K}}=\frac{1}{2}f_{\rho\pi\pi}$ is assumed since in this case the RHS of (18a) makes a null contribution. One can thus view $f_{\rho'K\bar{K}}=\frac{1}{2}f_{\rho'\pi\pi}$ and $f_{\rho''K\bar{K}}=\frac{1}{2}f_{\rho''\pi\pi}$ as simple consequences of the ground-state SU(3) relation $f_{\rho K\bar{K}}=\frac{1}{2}f_{\rho\pi\pi}$ provided one has imposed analyticity and asymptotic bounds on the form factors.

Inserting the above relations in (11) it is trivial to check the validity of the sum rule (14). Conversely, one might claim that the asymptotic bounds are consistent with the SU(3)-related VPP couplings and their excited analogs.

One should, however, be cautious in generalizing the SU(3) relation $f_{\rho K\bar{K}}=\frac{1}{2}f_{\rho\pi\pi}$ to an arbitrary number of radial states, for if more than two excited partners of the ρ contribute in (11) then the vanishing of the quantities $2f_{\rho^n K\bar{K}}-f_{\rho^n\pi\pi}$ (n is arbitrary) becomes dependent on the vanishing of the LHS of (18). In other words, SU(3) must at least hold for the ground and the first excited state of the ρ in order that it may remain valid for still higher states as well.

C. Evaluation of charge radii

We are now in a position to determine the charge radius of the pion and kaon from (10) and (11). This we do as follows. We first estimate $f_{\omega K\bar{K}}/f_\omega$ from (10c) and (10d):

$$\frac{f_{\omega K\bar{K}}}{f_\omega}=\frac{m_{\omega'}^2}{m_{\omega'}^2-m_\omega^2}\left[\frac{1}{2}-\left[1-\frac{m_\Phi^2}{m_{\omega'}^2}\right]\frac{f_{\Phi K\bar{K}}}{f_\Phi}\right]. \tag{18b}$$

For $m_\omega=783$ MeV, $m_\Phi=1020$ MeV, $m_\Phi=1634$ MeV, and $f_{\Phi K\bar{K}}/f_\Phi=0.35$, we get

$$\frac{f_{\omega K\bar{K}}}{f_\omega}=0.37 \tag{19a}$$

leading to

$$\frac{f_{\omega K\bar{K}}^2}{4\pi}=2.52\ . \tag{19b}$$

This corresponds to $f_\omega^2/4\pi=18.4$. The above value of $f_{\omega K\bar{K}}$ implies

$$\frac{f_{\omega'K\bar{K}}}{f_{\omega'}}=-0.22\ . \tag{20}$$

The coupling of the ρ' to $K\bar{K}$ may be estimated in a similar manner. We obtain

$$\frac{f_{\rho'K\bar{K}}}{f_{\rho'}}=\frac{m_{\rho''}^2}{m_{\rho''}^2-m_{\rho'}^2}\left[\frac{1}{2}-\left[1-\frac{m_\rho^2}{m_{\rho''}^2}\right]\frac{f_{\rho K\bar{K}}}{f_\rho}\right] \tag{21}$$

since $f_{\rho\pi\pi}=2f_{\rho K\bar{K}}$ and[13] $f_{\rho\pi\pi}=1.2f_\rho$, it follows that $f_{\rho K\bar{K}}=0.60f_\rho$ and hence, from (21),

$$\frac{f_{\rho'K\bar{K}}}{f_{\rho'}}=0.099\ . \tag{22}$$

Consequently,

$$\frac{f_{\rho''K\bar{K}}}{f_{\rho''}}=-0.20\ . \tag{23}$$

It should be noted that the corresponding estimates of $f_{\rho'\pi\pi}/f_{\rho'}$ and $f_{\rho''\pi\pi}/f_{\rho''}$ are twice those given by (22) and (23), respectively.

The predictions of the various couplings of the ρ, ω, and their radial partners as made in (19a), (20), (22), and

(23) enable us to evaluate the charge radii of the pion and kaon. We obtain, from (11),

$$\langle r_{K^+}^2 \rangle = 0.40 \text{ fm}^2 , \quad (24a)$$

$$\langle r_{K^0}^2 \rangle = -0.03 \text{ fm}^2 , \quad (24b)$$

$$\langle r_{\pi^+}^2 \rangle = 0.43 \text{ fm}^2 . \quad (24c)$$

The above estimates of $\langle r_{K^0}^2 \rangle$ and $\langle r_{\pi^+}^2 \rangle$ may be considered to be fairly satisfactory as these are well within the uncertainties of their corresponding experimental values. However, our result of $\langle r_{K^+}^2 \rangle$, although not irreconcilable with the timelike extrapolated value[14] of Blatnik *et al.* (see Table I), is certainly a few standard deviations away from its present experimental number. One can also see the sensitivity of the kaon mean-square charge radii to the effects of the excited states by comparing (24a) and (24b) with the corresponding naive VDM results (15a) and (15b). A comparison of the present estimate of $\langle r_{K^+}^2 \rangle$ with other determinations is given in Table I. It is quite gratifying to note that the present result of $\langle r_{K^+}^2 \rangle$ is very close to the estimate of Gasser and Leutwyler[11(a)] obtained by performing an expansion in powers of the light-quark-mass parameters in chiral perturbation theory. We thus see that the conventional VDM coupled with exact SU(3) for the VPP vertices lead to results which are at par with those obtained from a more sophisticated scenario, viz., that of the chiral perturbation theory. That these models require a large experimental value for $\langle r_{K^+}^2 \rangle$ should add further support to the need that the current experimental values of kaon mean-square charge radii be reassessed and a fresh best estimate be made.

Actually, the values determined in (24) call for slight modifications due to the finite-width corrections. To incorporate such corrections we replace (7) and (9) by

TABLE I. Comparison of the present estimate of $\langle r_{K^+}^2 \rangle$ with other determinations.

Group	$\langle r_{K^+}^2 \rangle$ (fm^2)
Present work (extended VDM)	0.40 (without narrow-width corrections)
	0.38 (with narrow width corrections)
Amendolia *et al.* (Ref. 6)	0.34±0.05
Dally *et al.* (Ref. 7)	0.28±0.05
Gasser and Leutwyler (Ref. 11)	0.38±0.03
Blatnik *et al.* (Ref. 14)	0.385±0.46
Micelmacher *et al.* (Ref. 15) (Kl_4 decay)	0.45±0.09
Godfrey and Isgur (Ref. 16) (potential model)	0.35
Cosmai *et al.* (Ref. 17)	0.45

$$F_\pi(t) = a_\rho \frac{f_{\rho\pi\pi}}{f_\rho} \frac{m_\rho^2}{m_\rho^2 - t} + \text{HES} , \quad (25a)$$

$$F_K^V(t) = a_\rho \frac{f_{\rho K\bar{K}}}{f_\rho} \frac{m_\rho^2}{m_\rho^2 - t} + \text{HES} , \quad (25b)$$

$$F_K^S(t) = a_\omega \frac{f_{\omega K\bar{K}}}{f_\omega} \frac{m_\omega^2}{m_\omega^2 - t} + a_\Phi \frac{f_{\Phi K\bar{K}}}{f_\Phi} \frac{m_\Phi^2}{m_\Phi^2 - t} + \text{HES} , \quad (25c)$$

where a_ρ, a_ω, and a_Φ stand for the finite-width correction factors. Performing calculations similar to what have been done to get (24), we obtain

$$\langle r_{K^+}^2 \rangle = 0.38 \text{ fm}^2 , \quad (26a)$$

$$\langle r_{K^0}^2 \rangle = -0.028 \text{ fm}^2 , \quad (26b)$$

$$\langle r_{\pi^+}^2 \rangle = 0.42 \text{ fm}^2 . \quad (26c)$$

In making the above estimates we have assumed[18] $a_\rho = 0.93$, $a_\omega = 1.00$, and $a_\Phi = 1.18$. Comparing (26) with (24) we find that the value of $\langle r_{K^+}^2 \rangle$ has been reduced by about 5% and so stands very close to the prediction of Gasser and Leutwyler, while $\langle r_{K^0}^2 \rangle$ and $\langle r_{\pi^+}^2 \rangle$ have remained more or less unaffected.

To conclude this section, it may be noted that an attractive feature of the VDM is that the amplitudes of the process $\rho \to e^+ e^-$ and $\rho \to \pi^+ \pi^-$ may be related, yielding the universality relation $f_{\rho\pi\pi} = f_\rho$. This coupled with SU(3) relates all other relevant coupling constants so that the expressions of different charge radii may be evaluated individually as in (24) or (26). This is generally not so for other schemes such as the nonrelativistic quark model[19] and quark-triangle loop approach.[20] In the former, an unknown mean-square relative coordinate enters while in the latter various parameters such as the pseudoscalar-meson coupling constants, charges, and the colors of the quarks are to be accounted for. In the following sections we consider each of these models in turn.

III. VDM AND NONRELATIVISTIC QUARK MODEL

Despite the reasonableness of predictions (24) or (26), the VDM does not throw much light on the question why $\langle r_{K^0}^2 \rangle$ must be negative. One of the advantages of the nonrelativistic quark-model approach is that it gives some explanation why the K^0 mean-square charge radius is negative. To inquire[19] more into this we note that in the nonrelativistic constituent-quark model, one has

$$\langle r_{\text{em}}^2 \rangle = \left\langle \sum e_i (\mathbf{r}_i - \mathbf{R})^2 \right\rangle , \quad (27)$$

where the ith constituent quark has been labeled by $\mathbf{r}_i$ and $\mathbf{R}$ stands for

$$\mathbf{R} = \frac{m_q \mathbf{r}_q + m_{\bar{q}} \mathbf{r}_{\bar{q}}}{m_q + m_{\bar{q}}} . \quad (28)$$

The mean-squared charge radius of the pion and kaon may be computed by setting

$$\boldsymbol{\rho}=\mathbf{r}_q-\mathbf{r}_{\bar{q}} \ . \tag{29}$$

One obtains

$$\langle r_{K^0}^2\rangle=\frac{m_d^2/m_s^2-1}{3(1+m_s/m_d)^2}\langle\rho_K^2\rangle \ , \tag{30a}$$

$$\langle r_{\pi^+}^2\rangle=\frac{2m_d^2/m_u^2+1}{3(1+m_d/m_u)^2}\langle\rho_\pi^2\rangle \tag{30b}$$

$$\langle r_{K^+}^2\rangle=\frac{2m_s^2/m_u^2+1}{3(1+m_s/m_u)^2}\langle\rho_K^2\rangle \ . \tag{30c}$$

Since, after all, $m_d^2/m_s^2 \ll 1$, we at once see from (30a) why $\langle r_{K^0}^2\rangle$ is negative: the negatively charged d quark orbits[9] the more massive anti-s-quark.

Nevertheless, a clear shortcoming of this scheme is that the relative coordinates $\langle\rho^2\rangle$ enter into the expressions for the charge radii and to estimate them requires a specific model to fall back on. It may be noted that a comparison of (30) with the corresponding VDM results does not take us far, except for yielding the trivial result that $\langle\rho^2\rangle_K \ll \langle\rho^2\rangle_\pi$. In the following we show how the quark-loop approach to the charge radii problem in QCD can be effectively used to pin down the ranges of $\langle\rho^2\rangle$ for the various mesons.

IV. CONSTITUENT-QUARK LOOP APPROACH TO CHARGE RADII

In QCD, the quark-model triangle diagrams contribute to the electromagnetic form factors in the lowest order. Employing the notation of Ref. 20, the form factor reads as

$$-(p+p')^\mu F(q^2)=3g^2\int\frac{d^4k}{(2\pi)^4}\left[Q_l\gamma^\mu\frac{i}{\not p+\not k-m_l}\gamma_5\frac{i}{\not k-m_j}\times\gamma_5\frac{i}{\not p+\not K-m_l}+(l\to j)\right], \tag{31}$$

where the number of colors has been set equal to 3, g stands for the pseudoscalar-meson coupling constant, Q represents the charge of the quark or antiquark with mass m and the labels l and j distinguish a quark from an antiquark.

Recently the ratios of the pion and kaon charge radii have been determined[20(b)] from (31) to the second order in the SU(3)-breaking parameter $\delta m/m$ where $\delta m=m_s-m$ and $m=m_u=m_d$. These have turned out to be

$$\frac{\langle r_{K^+}^2\rangle}{\langle r_{\pi^+}^2\rangle}=1-\frac{5}{6}\frac{\delta m}{m}+\frac{3}{5}\left[\frac{\delta m}{m}\right]^2+\cdots \ , \tag{32a}$$

$$\frac{\langle r_{K^0}^2\rangle}{\langle r_{\pi^+}^2\rangle}=-\frac{1}{3}\frac{\delta m}{m}+\frac{1}{2}\left[\frac{\delta m}{m}\right]^2+\cdots \ . \tag{32b}$$

An important outcome of the above result is the emergence of the sum rule

$$\langle r_{\pi^+}^2\rangle-\langle r_{K^+}^2\rangle=-\tfrac{5}{2}\langle r_{K^0}^2\rangle \tag{33}$$

to first order in $\delta m/m$. Indeed, using the standard nonrelativistic or constituent masses for m and m_s, the dimensionless quantity $\delta m/m$ turns out to be very small: $0<\delta m/m\le 1$, $\delta m/m\simeq 0.37$. An interesting feature of (33) is that its agreement with the corresponding experimental value of Ref. 7(a) is remarkably good; more so, since it is independent of the number of colors, the coupling constant g, and the actual values of the constituent masses. Further, by using (32) and keeping $(\delta m/m)^2$ terms to predict $\langle r_{K^+}^2\rangle$, the latter turns out to be smaller than other theoretical estimates and hence closer to the experimental value. Notice that the coefficient of $\langle r_{K^0}^2\rangle$ in the RHS of (33) has been modified by a factor of 2.5 in comparison with the VDM result (14).

All this, therefore, points to the fact that the constituent-quark model approach to the meson charge radii problem may be taken as a viable alternative to the vector-dominance scheme. However, despite the success of this model one can have two misgivings against this scheme. First, truncating the series in (32) to any desired[21] order of the parameter $\delta m/m$ is not a safe approximation, for the two series in the RHS of (32) are alternating and so the terms left out may well turn out to be of the same order as those retained. Second, the results (32) may be greatly affected by higher-order diagrams which have been neglected.

Nevertheless, if we do retain terms up to the order of $(\delta m/m)^2$, we can compare (32) with the corresponding ratios obtained from (30a)–(30c). Consider, say, the ratio of $\langle r_{K^+}^2\rangle$ and $\langle r_{\pi^+}^2\rangle$ in (30). We get

$$\frac{\langle r_{K^+}^2\rangle}{\langle r_{\pi^+}^2\rangle}=C\left[1+\frac{1}{3}\frac{\delta m}{m}+\frac{1}{12}\left[\frac{\delta m}{m}\right]^2+\cdots\right] , \tag{34}$$

where C stands for the quantity $\langle\rho_K^2\rangle/\langle\rho_\pi^2\rangle$. Comparing (34) with (32a) yields

$$C=0.61 \text{ to first order in } \delta m/m \ ,$$
$$=0.68 \text{ to second order in } \delta m/m \ . \tag{35}$$

The experimental values of the charge radii give $C=0.62$ using $\delta m/m=0.37$. The closeness of the estimates of C in (35) to the experimental value is not surprising since, as we have just now discussed, the series in (32) to the order $(\delta m/m)^2$ is compatible with the experimental data.

Corresponding to (24) [or (26)], the estimates of $\langle\rho^2\rangle^{1/2}$ are

$$\langle\rho_{K^+}^2\rangle^{1/2}=1.19\ (1.16)\text{ fm} \ , \tag{36a}$$

$$\langle\rho_{\pi^+}^2\rangle^{1/2}=1.31\ (1.33)\text{ fm} \ . \tag{36b}$$

These values of $\langle\rho^2\rangle^{1/2}$ which are expected to represent typical hadron size are in reasonable agreement with what one expects from a naive counting of hadronic dimensions.

Finally, it may be remarked that in essence the quark-triangle loop approach is an SU(3)-breaking mechanism, the expressions for the charge radii being expanded in terms of the SU(3)-breaking parameter $\delta m/m$ in the constituent masses. In contrast, the vector-dominance scheme relies on the exact SU(3) relations between the VPP coupling constants and the universality condition by which these are related to the electromagnetic couplings of the vector mesons.

V. SUMMARY

The present work may be summarized as follows.

(a) We have estimated the charge radii of the pseudoscalar mesons π^+, K^+, and K^0 in an extended vector-dominance scheme. Except for the K^+, our predictions match the corresponding experimental values reasonably well including the most recent measurement on the difference between the mean-square charge radii of the π^+ and K^+. For the K^+, our estimate is closer to the prediction of the chiral perturbation theory and gets slightly improved if corrections due to the narrow width approximation are accounted for. Since, as we have shown, our predictions are essentially SU(3) results with SU(3) restricted to ground-state VPP couplings only and since SU(3) is expected to be satisfied within 20%, a remeasurement of the K^+ charge radius may help one to understand the source of the discrepancy between the theoretical values and the experimental predictions.

(b) We have also considered the charge radii in other schemes and in the process have been able to relate the root-mean-square relative coordinate parameter of the nonrelativistic quark model to the $\delta m/m$ expanded expressions of the quark-triangle loop model. Our estimates for these parameters are in conformity with the hadronic dimensions which these coordinates are supposed to represent.

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