SU(3) predictions for nonleptonic B-meson decays

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The smallness of the up-, down-, and strange-quark masses compared with the QCD scale makes SU(3) flavor an approximate symmetry of the strong interactions. The B^-, B^0, and B_s^0 mesons form a $\overline{3}$ representation of SU(3). Using the SU(3) transformation properties of the effective Hamiltonian for weak nonleptonic B-meson decays, relations are derived between B-meson decay amplitudes. Some of these relations may provide information on the importance of various competing effects that can occur in nonleptonic B-meson decays.

I. INTRODUCTION

In the future it is likely that measurements of the branching ratios for many exclusive B-meson decays will be made. The B mesons come in three types B^-, B^0, and B_s^0 with flavor quantum numbers $b\overline{u}$, $b\overline{d}$, and $b\overline{s}$, respectively. In this paper we shall examine in detail the predictions of SU(3)-flavor symmetry for nonleptonic B-meson decays to two and three mesons.

In the standard six-quark model the couplings of the quarks to the charged W bosons are of the form

$$\mathcal{L}_I=\frac{g_2}{2\sqrt{2}}(\overline{u},\overline{c},\overline{t})\gamma^{\mu}(1-\gamma_5)V\begin{bmatrix}d\\s\\b\end{bmatrix}W_{\mu}^{+}+\text{H.c.} \qquad (1)$$

Here g_2 is the SU(2) gauge coupling and V is a 3×3 unitary matrix that is related to the transformations which diagonalize the quark mass matrices. It is possible to choose phases for the quark fields so that the Kobayashi-Maskawa matrix V is written as[1]

$$V=\begin{bmatrix}c_1 & -s_1c_3 & -s_1s_3\\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta}\\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta}\end{bmatrix}, \qquad (2)$$

where $c_i\equiv\cos\theta_i$ and $s_i\equiv\sin\theta_i$. The angles θ_1, θ_2, and θ_3 are chosen to lie in the first quadrant where their sines and cosines are positive. With this convention the quadrant of δ has physical significance and must be fixed by experiment.

Experimental information on nuclear β decay, semileptonic hyperon decays, and kaon decays gives that[2]

$$s_1\simeq0.22\ . \qquad (3a)$$

The angles θ_2 and θ_3 are also small. Experimental information on the B-meson lifetime and semileptonic B-meson decays implies (to leading order in small angles) that[2]

$$(s_2^2+s_3^2+2s_2s_3c_\delta)^{1/2}\simeq5\times10^{-2} \qquad (3b)$$

and

$$s_3\lesssim5\times10^{-2}\ . \qquad (3c)$$

It is the interaction Lagrangian density in Eq. (1) that determines the transformation properties of the effective Hamiltonian for nonleptonic B-meson decays under flavor SU(3). Noting that the charm quark, bottom quark, and top quark are SU(3) singlets, it is easy to see that the $\Delta b=-1$, $\Delta c=1$ part of the effective Hamiltonian transforms as an 8, the $\Delta b=-1$, $\Delta c=0$ part transforms as $\overline{3}\oplus6\oplus\overline{15}$, and the $\Delta b=-1$, $\Delta c=-1$ part transforms as $3\oplus\overline{6}$.

In low-energy physics SU(3) symmetry typically works at about the 30% level in decay amplitudes. For example, f_K/f_π which is unity in the limit of SU(3) symmetry, has the experimental value 1.28. In nonleptonic D-meson decay, the predictions of SU(3) symmetry[3] do not work well. A striking example is the SU(3) prediction

$$\frac{\Gamma(D^0\to K^+K^-)}{\Gamma(D^0\to\pi^+\pi^-)}=1 \qquad (4a)$$

for Cabibbo-suppressed nonleptonic D-meson decay. Experimentally,[2]

$$\frac{\Gamma(D^0\to K^+K^-)}{\Gamma(D^0\to\pi^+\pi^-)}=3.5\pm1.2\ . \qquad (4b)$$

Using, for example, the large-N_c limit,[4,5] it is possible to include some SU(3)-breaking effects into estimates for nonleptonic D-decay amplitudes.[6] In the large-N_c limit, amplitudes factorize and hence there is an enhancement of the K^+K^- rate over the $\pi^+\pi^-$ rate by roughly the factor $(f_K/f_\pi)^2$. This is not quite enough to explain the experimental value in Eq. (4b).

Despite the failure of SU(3) predictions for nonleptonic D-meson decay rates, we feel that the absence of rigorous methods for predicting exclusive nonleptonic B-meson decay rates makes a tabulation of SU(3) predictions for B-meson decays worthwhile. In the next section we consider two-body nonleptonic B decays to mesons and in Sec. III we examine some three-body decays. In Sec. IV concluding remarks, which contain a discussion of possible improvements and extensions of our work, are given.

II. TWO-BODY DECAYS

We begin by discussing decays with $\Delta b=-1$ and $\Delta c=1$. These arise from weak Hamiltonians with flavor quantum numbers $(b\bar{c})(u\bar{d})$ for the Cabibbo-allowed decays and $(b\bar{c})(u\bar{s})$ for the Cabibbo-suppressed decays. These two Hamiltonians are different components of the same **8** representation. Therefore, there will be SU(3) relations between the Cabibbo-allowed and Cabibbo-suppressed decays which have $\Delta c=1$. Denoting the **8** representation of the Hamiltonian by a 3×3 matrix with components H^i_j (the upper index labels rows and the lower columns) then for the Cabibbo-allowed decays

$$H=\begin{bmatrix}0 & 0 & 0\\ c_1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \tag{5a}$$

and for the Cabibbo-suppressed decays

$$H=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ -s_1c_3 & 0 & 0\end{bmatrix} . \tag{5b}$$

In Eq. (5b) the relative factor of $-s_1c_3/c_1$ arises because of the different weak mixing angles which occur in the two cases. We shall work to leading order in small weak mixing angles setting $c_1=c_2=c_3=1$.

We first consider $\Delta b=-1$, $\Delta c=1$ decays of the type $B\to DM$, where D denotes one of the D mesons D^0, D^+, and D_s^+ with flavor quantum numbers $c\bar{u}$, $c\bar{d}$, and $c\bar{s}$, respectively. M is one of the eight lowest-lying 0^- mesons $\pi,K,\bar{K},\eta$. Using the same notation as for the Hamiltonian, the B mesons and D mesons which transform as antitriplets under SU(3) are introduced as row vectors with components B_i and D_i, respectively. Explicitly,

$$B=(B^-,B^0,B_s^0) \tag{6}$$

and

$$D=(D^0,D^+,D_s^+) . \tag{7}$$

Finally the mesons M are in the octet representation which as a 3×3 matrix has elements

$$M=\begin{bmatrix}\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+\\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0\\ K^- & \bar{K}^0 & -\sqrt{2/3}\eta\end{bmatrix} . \tag{8}$$

We are interested in the transition amplitudes $A(B\to DM)=\langle DM|H_{\rm eff}|B\rangle$. As far as the group theory is concerned we can imagine these amplitudes arising from the effective Hamiltonian

$$H_{\rm eff}=a(B_i\bar{D}^i)(M^k_lH^l_k)+b(B_iM^i_kH^k_j\bar{D}^j) + c(B_iH^i_kM^k_j\bar{D}^j) \tag{9}$$

with H^l_k given by Eq. (5a) for the Cabibbo-allowed decays and Eq. (5b) for the Cabibbo-suppressed modes. Expanding out these three terms gives the results for the Cabibbo-allowed decays shown in Table I and for the Cabibbo-suppressed decays shown in Table II.

There is one SU(3) relation among the decay amplitudes $A(B\to DM)$ for the Cabibbo-allowed decays:

TABLE I. Rates for Cabibbo-allowed decays $B\to DM$ in terms of the three reduced matrix elements a, b, and c.

Process	Rate
$B^0\to D^+\pi^-$	$\|a+c\|^2$
$B^0\to D^0\pi^0$	$\frac{1}{2}\|b-c\|^2$
$B^0\to D^0\eta$	$\frac{1}{6}\|b+c\|^2$
$B^0\to D_s^+K^-$	$\|c\|^2$
$B^-\to D^0\pi^-$	$\|a+b\|^2$
$B_s^0\to D^0K^0$	$\|b\|^2$
$B_s^0\to D_s^+\pi^-$	$\|a\|^2$

$$|A(B^0\to D^0\pi^0)|^2+3|A(B^0\to D^0\eta)|^2 =|A(B^0\to K^-D_s^+)|^2+|A(B_s^0\to D^0K^0)|^2 . \tag{10}$$

There are several simple relations between the Cabibbo-allowed and the Cabibbo-suppressed decay rates. For example, SU(3) symmetry implies that

$$\frac{A(B^-\to D^0\pi^-)}{A(B^-\to D^0K^-)}=-1/s_1 . \tag{11}$$

The large value of the B-meson mass (compared with the QCD scale) suggests that relative complex phases between the reduced matrix elements a, b, and c, which can be generated by final-state strong interactions, are small. If this is the case then, up to sign ambiguities, measuring three of the Cabibbo-allowed decays determines a, b, and c. At the present time there are measurements of the branching ratios for two of the decays in Table I. Experimentally,[7,8]

$$B(B^-\to D^0\pi^-)=(3.0\pm 1.4)\times 10^{-3} , \tag{12a}$$

$$B(B^0\to D^+\pi^-)=(3.6\pm 1.4)\times 10^{-3} . \tag{12b}$$

Although we have focused on decays of the type $B\to DM$ our results can be trivially taken over for the decays of the form $B\to D^*M$. Also, for decays not involving the η we can use the results in Tables I and II for the corresponding decays $B\to DV$ and $B\to D^*V$, where V is one of the 1^- ρ or K^* vector mesons. [Since in the vector meson case there is a large mixing between SU(3)-octet and -singlet states, one cannot straightforwardly generalize the results involving the η, which neglect the small SU(3)-violating η-η' mixing.] So, for example, generalizations of Eq. (11) of the type

TABLE II. Rates for Cabibbo-suppressed decays $B\to DM$ in terms of the three reduced matrix elements a, b, and c.

Process	Rate
$B^0\to D^+K^-$	$s_1^2\|a\|^2$
$B^0\to D^0\bar{K}^0$	$s_1^2\|b\|^2$
$B^-\to D^0K^-$	$s_1^2\|a+b\|^2$
$B_s^0\to D^0\pi^0$	$s_1^2\frac{1}{2}\|c\|^2$
$B_s^0\to D^0\eta$	$s_1^2\frac{1}{6}\|c-2b\|^2$
$B_s^0\to D_s^+K^-$	$s_1^2\|a+c\|^2$
$B_s^0\to D^+\pi^-$	$s_1^2\|c\|^2$

$$\frac{A(B^-\to D^{*0}\pi^-)}{A(B^-\to D^{*0}K^-)}=\frac{A(B^-\to D^0\rho^-)}{A(B^-\to D^0K^{*-})}=\frac{A(B^-\to D^{*0}\rho^-)}{A(B^-\to D^{*0}K^{*-})}=-1/s_1 \quad (13)$$

hold. Experimentally,[7-11]

$$B(B^0\to D^*(2010)^+\pi^-)=(3.3^{+1.2}_{-1.0})\times 10^{-3}\ , \quad (14a)$$

$$B(B^0\to D^*(2010)^+\rho^-)=(8^{+7}_{-4})\times 10^{-2}\ , \quad (14b)$$

$$B(B^-\to D^*(2010)^0\pi^-)=(3\pm 4)\times 10^{-3}\ , \quad (14c)$$

$$B(B^-\to D^0\rho^-)=(2.1\pm 1.2)\times 10^{-2}\ , \quad (14d)$$

$$B(B^0\to D^+\rho^-)=(2.2\pm 1.5)\times 10^{-2}\ . \quad (14e)$$

Two-body decays of the type $B\to D\bar{D}$ arise from weak Hamiltonians with flavor quantum numbers $(b\bar{c})(c\bar{s})$ for the Cabibbo-allowed decays and $(b\bar{c})(c\bar{d})$ for the Cabibbo-suppressed decays. These Hamiltonians are different components of the same antitriplet representation. So, as far as group theory is concerned, the decays $B\to D\bar{D}$ can be thought of as arising from an effective Hamiltonian

$$H_{\rm eff}=\alpha(B_iH^i)(D_j\bar{D}^j)+\beta(B_i\bar{D}^i)(H^jD_j)\ , \quad (15)$$

where for the Cabibbo-allowed decays

$$H=\begin{bmatrix}0\\0\\1\end{bmatrix} \quad (16a)$$

and for the Cabibbo-suppressed decays

$$H=\begin{bmatrix}0\\s_1\\0\end{bmatrix}\ . \quad (16b)$$

Table III presents results for decays $B\to D\bar{D}$ which follow from Eq. (15). Some of the relations that follow from Table III are just consequences of SU(2)-isospin symmetry. They are, for the Cabibbo-allowed decays[12]

$$\Gamma(B^-\to D^0D_s^-)=\Gamma(B^0\to D^+D_s^-)\ , \quad (17a)$$

$$\Gamma(B_s^0\to D^0\bar{D}^0)=\Gamma(B_s^0\to D^+D^-)\ . \quad (17b)$$

For the Cabibbo-suppressed decays there are no isospin relations. Table III indicates that there are several SU(3) relations between the Cabibbo-allowed and the Cabibbo-suppressed $B\to D\bar{D}$ decays [e.g., $\Gamma(B^-\to D^0D^-)=s_1^2\Gamma(B^-\to D^0D_s^-)$]. The Cabibbo-suppressed $B\to D\bar{D}$ decays also get contributions from terms in the effective weak Hamiltonian that have no charm quarks. For example, the operator $(b\bar{u})(u\bar{d})$ which transforms under SU(3) as $\bar{3}\oplus 6\oplus \overline{15}$ contributes to the Cabibbo-suppressed $B\to D\bar{D}$ decays and if s_3 is near its experimental limit, this could alter the results in Table III.

The same Hamiltonians which give rise to the decays $B\to D\bar{D}$ also cause the decays $B\to J/\psi M$. Since there is only one way to combine the product of a triplet, antitriplet, and octet representations (J/ψ transforms as a singlet) into a singlet, these decays are characterized by a single reduced matrix element. SU(3) predictions for these decays are presented in Table IV. The contributions of operators without charm quarks [e.g., $(b\bar{u})(u\bar{d})$] to the Cabibbo-suppressed decays $B\to J/\psi M$ are negligible because they violate the Okubo-Zweig-Iizuka (OZI) rule. For the Cabibbo-allowed decays the relation

$$\Gamma(B^0\to J/\psi\bar{K}^0)=\Gamma(B^-\to J/\psi K^-) \quad (18)$$

is a consequence of isospin invariance. It has previously been noted that a comparison of branching ratios for these modes would determine the ratio of B^0 and B^- lifetimes.[13] At the present time it is known that[2] $0.4<\tau_{B^0}/\tau_{B^-}<2.1$. One of the relevant branching ratios has been measured:[8,14,15]

$$B(B^-\to J/\psi K^-)=(8.0\pm 2.8)\times 10^{-4}\ . \quad (19)$$

For the Cabibbo-suppressed decays the relation

$$\Gamma(B^0\to J/\psi\pi^0)=\tfrac{1}{2}(B^-\to J/\psi\pi^-) \quad (20)$$

is also a consequence of isospin invariance.

The results of Table IV generalize trivially to other $c\bar{c}$ resonances and also to decays $B\to J/\psi V$, where V is a 1^- ρ or K^* meson. There is also some experimental information on these decays:[8,14]

$$B(B^-\to\psi(2S)K^-)=(2.2\pm 1.7)\times 10^{-3}\ , \quad (21a)$$

$$B(B^0\to J/\psi\bar{K}^*(892)^0)=(3.7\pm 1.3)\times 10^{-3}\ . \quad (21b)$$

We consider next the SU(3) relations between the decay amplitudes which can arise from the $b\to uW^-$ weak cou-

TABLE III. Rates for B-meson decays of the type $B\to D\bar{D}$ in terms of the two reduced matrix elements α and β.

Process	Rate
$B^0\to D^+D_s^-$	$\lvert\beta\rvert^2$
$B^0\to D^0\bar{D}^0$	$s_1^2\lvert\alpha\rvert^2$
$B^0\to D^+D^-$	$s_1^2\lvert\alpha+\beta\rvert^2$
$B^0\to D_s^+D_s^-$	$s_1^2\lvert\alpha\rvert^2$
$B^-\to D^0D_s^-$	$\lvert\beta\rvert^2$
$B^-\to D^0D^-$	$s_1^2\lvert\beta\rvert^2$
$B_s^0\to D^0\bar{D}^0$	$\lvert\alpha\rvert^2$
$B_s^0\to D^+D^-$	$\lvert\alpha\rvert^2$
$B_s^0\to D_s^+D_s^-$	$\lvert\alpha+\beta\rvert^2$
$B_s^0\to D_s^+D^-$	$s_1^2\lvert\beta\rvert^2$

TABLE IV. SU(3) predictions for rates for $B\to J/\psi M$ normalized to the decay rate for $B^-\to J/\psi K^-$.

Process	Rate
$B^0\to J/\psi\bar{K}^0$	1
$B^0\to J/\psi\pi^0$	$s_1^2/2$
$B^0\to J/\psi\eta$	$s_1^2/6$
$B^-\to J/\psi K^-$	1
$B^-\to J/\psi\pi^-$	s_1^2
$B_s^0\to J/\psi\eta$	$\frac{2}{3}$
$B_s^0\to J/\psi K^0$	s_1^2

pling. For final states without charm, the effective Hamiltonian has the flavor quantum numbers of the operator $(b\bar{u})(u\bar{d})$ which transforms as $\overline{15}\oplus 6\oplus\bar{3}$. Explicitly, the decomposition of $(b\bar{u})(u\bar{d})$ into operators that are in irreducible SU(3) representations is

$$(b\bar{u})(u\bar{d})=\tfrac{1}{8}O_{(\overline{15})}+\tfrac{1}{4}O_{(6)}-\tfrac{1}{8}O_{(\bar{3})}+\tfrac{3}{8}O'_{(\bar{3})}\ , \qquad (22)$$

where

$$O_{(\overline{15})}=3(b\bar{u})(u\bar{d})+3(b\bar{d})(u\bar{u})-2(b\bar{d})(d\bar{d})-(b\bar{s})(s\bar{d})-(b\bar{d})(s\bar{s})\ , \qquad (23a)$$

$$O_{(6)}=(b\bar{u})(u\bar{d})-(b\bar{d})(u\bar{u})-(b\bar{s})(s\bar{d})+(b\bar{d})(s\bar{s})\ , \qquad (23b)$$

$$O_{(\bar{3})}=(b\bar{d})(u\bar{u})+(b\bar{d})(d\bar{d})+(b\bar{d})(s\bar{s})\ , \qquad (23c)$$

$$O'_{(\bar{3})}=(b\bar{u})(u\bar{d})+(b\bar{d})(d\bar{d})+(b\bar{s})(s\bar{d})\ . \qquad (23d)$$

In Eqs. (22) and (23) the subscripts on the operators denote the irreducible representation of SU(3) to which they belong. As far as group-theory factors are concerned we can take, as the effective Hamiltonian for B-meson decays $B\to MM$,

$$H_{\rm eff}=A_{(\bar{3})}B_iH(\bar{3})^i(M_l^kM_k^l)+C_{(\bar{3})}B_iM_k^iM_j^kH(\bar{3})^j+A_{(\overline{15})}B_iH(\overline{15})_k^{ij}M_j^lM_l^k+C_{(\overline{15})}B_iM_j^iH(\overline{15})_l^{jk}M_k^l+A_{(6)}B_iH(6)_k^{ij}M_j^lM_l^k\ . \qquad (24)$$

In Eq. (24) $H(\bar{3})$ is a vector with nonzero component

$$H(\bar{3})^2=1\ .$$

$H(\overline{15})$ is a traceless three-index tensor that is symmetric on its upper indices and has nonzero components

$$H(\overline{15})_1^{12}=3,\quad H(\overline{15})_1^{21}=3,\quad H(\overline{15})_2^{22}=-2\ ,$$
$$H(\overline{15})_3^{32}=-1,\quad H(\overline{15})_3^{23}=-1\ .$$

Finally, in Eq. (24), $H(6)$ is a traceless three-index tensor that is antisymmetric on its upper indices and has nonzero components

$$H(6)_1^{12}=1,\quad H(6)_1^{21}=-1,\quad H(6)_3^{32}=-1\ ,$$
$$H(6)_3^{23}=1\ .$$

The parameters $A_{(\bar{3})}$, $C_{(\bar{3})}$, $A_{(\overline{15})}$, $C_{(\overline{15})}$, and $A_{(6)}$ are the reduced matrix elements in terms of which the $B\to MM$ decay amplitudes are expressed. Note that since

$$B_iH(6)_k^{ij}M_j^lM_l^k+B_iM_l^iH(6)_k^{lj}M_j^k=0\ , \qquad (25)$$

there is only one reduced matrix element, $A_{(6)}$ parametrizing the contribution of the part of the Hamiltonian that transforms as a 6.

Table V summarizes the SU(3) predictions that follow from expanding the effective Hamiltonian in Eq. (24). There is only one simple relation

$$\frac{A(B_s^0\to K^0\pi^0)}{A(B_s^0\to K^0\eta)}=\sqrt{3}\ . \qquad (26)$$

There are no simple isospin relations between the $B\to MM$ decay rates in Table V. The effective Hamiltonian has both $I=\frac{1}{2}$ and $I=\frac{3}{2}$ pieces. The $I=\frac{3}{2}$ piece arises solely from the operator $O_{(\overline{15})}$. In the decays $B\to\pi\pi$ the two-pion final state is a linear combination of $I=0$ and $I=2$ states. The $I=2$ state can only be reached through the $I=\frac{3}{2}$ part of the effective Hamiltonian while the $I=0$ state gets contributions from both the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ parts. Since the $\pi^0\pi^-$ state is charged it is pure $I=2$, consequently the rate for $B^-\to\pi^0\pi^-$ originates only from the matrix element of $O_{(\overline{15})}$.

There are contributions to the decays $B\to MM$ listed in Table V, which survive in the limit $s_3\to 0$ (where the $b\to uW^-$ coupling is absent). They come from penguin-type Feynman diagrams with a charm or top quark in the loop (see Fig. 1). Writing

$$A_{(\bar{3})}=-s_1s_3\hat{A}_{(\bar{3})}-e^{i\delta}s_1s_2\hat{A}'_{(\bar{3})}\ , \qquad (27a)$$

$$C_{(\bar{3})}=-s_1s_3\hat{C}_{(\bar{3})}-e^{i\delta}s_1s_2\hat{C}'_{(\bar{3})}\ , \qquad (27b)$$

it is $\hat{A}'_{(\bar{3})}$ and $\hat{C}'_{(\bar{3})}$ that characterize these contributions, since the penguin-type diagrams only give rise to terms that transform as a $\bar{3}$ in the effective Hamiltonian of Eq. (24). The contribution of penguin-type Feynman diagrams is probably suppressed by $[\alpha_s(m_b)/\pi]$, so unless (s_3/s_2) is very small (a prospect that is unlikely if the standard six-quark model is to describe the CP violation observed in kaon decays)[16] $\hat{A}'_{(\bar{3})}$ and $\hat{C}'_{(\bar{3})}$ are unimpor-

TABLE V. SU(3) predictions for decays $B\to MM$ that do not change strangeness.

Process	Rate
$B^0\to\pi^+\pi^-$	$\lvert 2A_{(\bar{3})}+C_{(\bar{3})}+A_{(\overline{15})}+3C_{(\overline{15})}-A_{(6)}\rvert^2$
$B^0\to\pi^0\pi^0$	$\frac{1}{2}\lvert 2A_{(\bar{3})}+C_{(\bar{3})}+A_{(\overline{15})}-5C_{(\overline{15})}-A_{(6)}\rvert^2$
$B^0\to\eta\eta$	$\frac{1}{2}\lvert 2A_{(\bar{3})}+\frac{1}{3}C_{(\bar{3})}-A_{(\overline{15})}+C_{(\overline{15})}+A_{(6)}\rvert^2$
$B^0\to\pi^0\eta$	$\frac{1}{3}\lvert -C_{(\bar{3})}+5A_{(\overline{15})}+C_{(\overline{15})}-A_{(6)}\rvert^2$
$B^0\to K^0\bar{K}^0$	$\lvert 2A_{(\bar{3})}+C_{(\bar{3})}-3A_{(\overline{15})}+A_{(6)}\rvert^2$
$B^0\to K^+K^-$	$\lvert 2A_{(\bar{3})}+2A_{(\overline{15})}\rvert^2$
$B^-\to\pi^0\pi^-$	$32\lvert C_{(\overline{15})}\rvert^2$
$B^-\to\eta\pi^-$	$\frac{1}{6}\lvert 2C_{(\bar{3})}+6A_{(\overline{15})}+6C_{(\overline{15})}+2A_{(6)}\rvert^2$
$B^-\to K^0K^-$	$\lvert C_{(\bar{3})}+3A_{(\overline{15})}-C_{(\overline{15})}+A_{(6)}\rvert^2$
$B_s^0\to K^+\pi^-$	$\lvert C_{(\bar{3})}-A_{(\overline{15})}+3C_{(\overline{15})}-A_{(6)}\rvert^2$
$B_s^0\to K^0\pi^0$	$\frac{1}{2}\lvert -C_{(\bar{3})}+A_{(\overline{15})}+5C_{(\overline{15})}+A_{(6)}\rvert^2$
$B_s^0\to K^0\eta$	$\frac{1}{6}\lvert -C_{(\bar{3})}+A_{(\overline{15})}+5C_{(\overline{15})}+A_{(6)}\rvert^2$

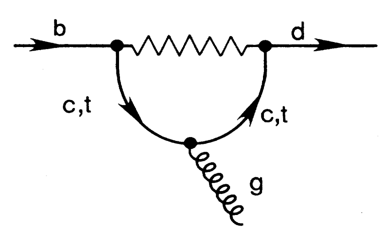

FIG. 1. Penguin-type Feynman diagram contributing to $B\to MM$ decays.

tant for the $B \to MM$ decays in Table V. However, if we examine $B \to MM$ decays that change strangeness by one unit, the situation is quite different. Here the penguin-type diagrams are again suppressed by $\alpha_s(m_b)/\pi$ but they are enhanced [over operators such as $(b\bar{u})(u\bar{s})$] by the ratio of weak mixing angles

$$\frac{(s_2^2+s_3^2+2s_2s_3c_\delta)^{1/2}}{s_1^2 s_3} . \tag{28}$$

These decays may be dominated by the penguin-type diagrams with a charm or top quark in the loop. Assuming this is the case we can use, as far as group-theory factors are concerned, the following effective Hamiltonian to describe the $\Delta s = -1$ $B \to MM$ decays:

$$H_{\text{eff}} = -(s_2 e^{i\delta}+s_3)[\hat{A}'_{(\bar{3})}(B_i H(\bar{3})^i)(M_l^k M_k^l) + \hat{C}'_{(\bar{3})} B_i M_k^i M_j^k H(\bar{3})^j] , \tag{29}$$

where now the nonzero component of $H(\bar{3})$ is

$$H(\bar{3})^3 = 1 . \tag{30}$$

Table VI gives the SU(3) predictions that follow from Eqs. (29) and (30). Note that since $\hat{A}'_{(\bar{3})}$ only effects the B_s^0 decays, the ratios of the various $\Delta s = -1$ $B^0 \to MM$ and $B^- \to MM$ decay rates are determined by SU(3).

Our assumption that penguin-type diagrams dominate the $\Delta s = -1$ $B \to MM$ decays implies that the effective Hamiltonian is $I=0$. Since there is only one way to combine two $I=\frac{1}{2}$ states into an $I=1$ state, all the relations between $B \to K\pi$ decays in Table VI are consequences of isospin. Similar isospin relations hold for decays of the type $B \to K\rho$, $B \to K^*\pi$, and $B \to K^*\rho$. The relations

$$\Gamma(B^0 \to \bar{K}^0\eta) = \Gamma(B^- \to K^-\eta) , \tag{31a}$$

$$\Gamma(B_s^0 \to K^+K^-) = \Gamma(B_s^0 \to K^0\bar{K}^0) , \tag{31b}$$

$$\Gamma(B_s^0 \to \pi^0\pi^0) = \tfrac{1}{2}\Gamma(B_s^0 \to \pi^+\pi^-) \tag{31c}$$

are also consequences of isospin symmetry. The $(b\bar{u})(u\bar{s})$ operator has both $I=0$ and $I=1$ pieces. Verifying some of the above isospin relations would provide strong evidence that penguin-type diagrams dominate the $\Delta s = -1$ $B \to MM$ decays.

The $b \to uW^-$ coupling also causes $\Delta b = -1$, $\Delta c = -1$ decays $B \to \bar{D}M$. To leading order in weak mixing angles the effective Hamiltonian for such decays has the flavor quantum numbers of $(b\bar{u})(c\bar{s})$. Under SU(3) this operator transforms as $3 \oplus \bar{6}$. Explicitly, the decomposition in terms of operators in irreducible representations is

$$(b\bar{u})(c\bar{s}) = O_{(3)} + O_{(\bar{6})} , \tag{32}$$

where

$$O_{(3)} = \tfrac{1}{2}[(b\bar{u})(c\bar{s}) - (b\bar{s})(c\bar{u})] , \tag{33a}$$

$$O_{(\bar{6})} = \tfrac{1}{2}[(b\bar{u})(c\bar{s}) + (b\bar{s})(c\bar{u})] . \tag{33b}$$

As far as group-theory factors are concerned we can take as the effective Hamiltonian for the $\Delta b = -1$, $\Delta c = -1$, decays $B \to \bar{D}M$:

$$H_{\text{eff}} = \alpha_{(\bar{6})} D_i H(\bar{6})^{ij} B_k M_j^k + \beta_{(\bar{6})} B_i H(\bar{6})^{ij} D_k M_j^k + \alpha_{(3)} D_i H(3)^{ij} B_k M_j^k + \beta_{(3)} B_i H(3)^{ij} D_k M_j^k . \tag{34}$$

Here $H(\bar{6})$ is a two-index symmetric tensor with nonzero components,

$$H(\bar{6})^{13} = 1, \quad H(\bar{6})^{31} = 1 \tag{35}$$

and $H(3)$ is a two-index antisymmetric tensor with nonzero components:

$$H(3)^{13} = 1, \quad H(3)^{31} = -1 . \tag{36}$$

Table VII shows the results which follow from the effective Hamiltonian in Eq. (34). There are two simple relations

$$\Gamma(B_s^0 \to D^-\pi^+) = 2\Gamma(B_s^0 \to \bar{D}^0\pi^0) , \tag{37a}$$

$$\Gamma(B^0 \to D_s^-\pi^+) = 2\Gamma(B^- \to D_s^-\pi^0) \tag{37b}$$

and they are a consequence of isospin invariance.

There is a small dynamical enhancement of the Wilson coefficient of $O_{(3)}$ over that of $O_{(\bar{6})}$ coming from pertur-

TABLE VI. SU(3) predictions for $B \to MM$ decays that change strangeness. Here it is assumed that penguin-type diagrams with a charm quark or top quark in the loop dominate. Entries in the second column should be multiplied by $|s_2 e^{i\delta}+s_3|^2$ if compared with Table V using Eq. (27).

Process	Rate (divided by $\lvert s_2 e^{i\delta}+s_3\rvert^2$)
$B^0 \to \pi^+K^-$	$\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B^0 \to \pi^0\bar{K}^0$	$\frac{1}{2}\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B^0 \to \bar{K}^0\eta$	$\frac{1}{6}\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B^- \to K^-\pi^0$	$\frac{1}{2}\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B^- \to K^-\eta$	$\frac{1}{6}\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B^- \to \bar{K}^0\pi^-$	$\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B_s^0 \to K^+K^-$	$\lvert 2\hat{A}'_{(\bar{3})}+\hat{C}'_{(\bar{3})}\rvert^2$
$B_s^0 \to K^0\bar{K}^0$	$\lvert 2\hat{A}'_{(\bar{3})}+\hat{C}'_{(\bar{3})}\rvert^2$
$B_s^0 \to \pi^0\pi^0$	$2\lvert\hat{A}'_{(\bar{3})}\rvert^2$
$B_s^0 \to \eta\eta$	$2\lvert\hat{A}'_{(\bar{3})}\rvert^2$
$B_s^0 \to \pi^+\pi^-$	$4\lvert\hat{A}'_{(\bar{3})}\rvert^2$

TABLE VII. SU(3) predictions for decays $B \to \bar{D}M$.

Process	Rate
$B^- \to \bar{D}^0K^-$	$\lvert\alpha_{(\bar{6})}+\alpha_{(3)}+\beta_{(\bar{6})}+\beta_{(3)}\rvert^2$
$B^- \to D_s^-\eta$	$\frac{1}{6}\lvert\alpha_{(\bar{6})}-\alpha_{(3)}-2\beta_{(\bar{6})}-2\beta_{(3)}\rvert^2$
$B^- \to D^-\bar{K}^0$	$\lvert\beta_{(\bar{6})}+\beta_{(3)}\rvert^2$
$B^- \to D_s^-\pi^0$	$\frac{1}{2}\lvert\alpha_{(\bar{6})}-\alpha_{(3)}\rvert^2$
$B^0 \to \bar{D}^0\bar{K}^0$	$\lvert\alpha_{(\bar{6})}+\alpha_{(3)}\rvert^2$
$B^0 \to D_s^-\pi^+$	$\lvert\alpha_{(\bar{6})}-\alpha_{(3)}\rvert^2$
$B_s^0 \to \bar{D}^0\pi^0$	$\frac{1}{2}\lvert\beta_{(\bar{6})}-\beta_{(3)}\rvert^2$
$B_s^0 \to D_s^-K^+$	$\lvert\alpha_{(\bar{6})}-\alpha_{(3)}+\beta_{(\bar{6})}-\beta_{(3)}\rvert^2$
$B_s^0 \to \bar{D}^0\eta$	$\frac{1}{6}\lvert -2\alpha_{(\bar{6})}-2\alpha_{(3)}+\beta_{(\bar{6})}-\beta_{(3)}\rvert^2$
$B_s^0 \to D^-\pi^+$	$\lvert\beta_{(\bar{6})}-\beta_{(3)}\rvert^2$

bative QCD corrections. In the effective Hamiltonian for $\Delta b=-1$, $\Delta c=-1$ decays the ratio of Wilson coefficients for $O_{(3)}$ and $O_{(\bar{6})}$ is[17] $[\alpha_s(m_b)/\alpha_s(m_W)]^{(18/23)}\approx 1.5$. If either the matrix elements of $O_{(3)}$ or $O_{(\bar{6})}$ dominate the $B\to\bar{D}M$ decays, then Table VII indicates that there would be some SU(3) relations. For example, either 3 or $\bar{6}$ dominances implies that

$$|A(B^0\to\bar{D}^0\bar{K}^0)|=|A(B^0\to D_s^-\pi^+)| \ . \tag{38}$$

Of course, generalizations of the results of Table VII to decays $B\to\bar{D}^*M$, $B\to\bar{D}V$, and $B\to\bar{D}^*V$ hold.

III. THREE-BODY DECAYS

The three-body decays, $B\to J/\psi MM$, can have the relative orbital angular momentum L of the two M mesons be either even or odd. For the case L even we can take, as far as group-theory factors are concerned,

$$H_{\rm eff}=[F(B_iH^i)(M_j^kM_k^j)+G(B_iM_l^iM_k^lH^k)](J/\psi) \ , \tag{39}$$

as the effective Hamiltonian [Lorentz indices are suppressed in Eq. (39)]. In Eq. (39),

$$H=\begin{bmatrix}0\\ s_1\\ 1\end{bmatrix} \ . \tag{40}$$

Table VIII presents the results that follow from the effective Hamiltonian in Eq. (39). For the case L odd, the effective Hamiltonian must change sign under interchange of the SU(3) quantum numbers of the two M mesons. Only the second term in Eq. (39) can be antisymmetrized and so the rates for $B\to J/\psi(MM)_{L=1,3,\ldots}$ are determined in terms of a single reduced matrix element. Table IX presents the relative $B\to J/\psi MM$ decay rates for odd L. With L odd the amplitudes for $B^-\to J/\psi\pi^-\eta$ and $B^0\to J/\psi\pi^0\eta$ vanish by SU(3) symmetry and therefore these processes do not appear in Table IX.

The Cabibbo-allowed $B\to J/\psi MM$ decays arise from an effective Hamiltonian that is an isosinglet. There are several isospin relations among the Cabibbo-allowed decays that hold independent of L. They are

$$\begin{aligned}\Gamma(B^0\to J/\psi\pi^+K^-)&=\Gamma(B^-\to J/\psi\pi^-\bar{K}^0)\\ &=2\Gamma(B^0\to J/\psi\pi^0\bar{K}^0)\\ &=2\Gamma(B^-\to J/\psi\pi^0K^-) \ , \end{aligned}\tag{41a}$$

$$\Gamma(B^0\to J/\psi\eta\bar{K}^0)=\Gamma(B^-\to J/\psi\eta K^-) \ , \tag{41b}$$

$$\Gamma(B_s^0\to J/\psi K^0\bar{K}^0)=\Gamma(B_s^0\to J/\psi K^+K^-) \ , \tag{41c}$$

and

$$\tfrac{1}{2}\Gamma(B_s^0\to J/\psi\pi^+\pi^-)=\Gamma(B_s^0\to J/\psi\pi^0\pi^0) \ . \tag{41d}$$

In the case $B_s^0\to J/\psi\pi\pi$ [Eq. (41d)] isospin invariance forces the two pions to be in an even L state. The effective Hamiltonian for Cabibbo-suppressed $B\to J/\psi MM$ decays is $I=\frac{1}{2}$. Again, there are isospin relations which are L independent. They are

$$\Gamma(B^0\to J/\psi\pi^0\eta)=\tfrac{1}{2}\Gamma(B^-\to J/\psi\pi^-\eta) \ , \tag{42a}$$

$$\Gamma(B_s^0\to J/\psi K^0\pi^0)=\tfrac{1}{2}\Gamma(B_s^0\to J/\psi K^+\pi^-) \ . \tag{42b}$$

Some isospin relations that hold only for L even are

TABLE VIII. SU(3) predictions for decays $B\to J/\psi MM$, when the relative angular momentum of the two pseudoscalar mesons is even.

Process	Rate
$B^0\to J/\psi\pi^+K^-$	$\|G\|^2$
$B^0\to J/\psi\pi^0\bar{K}^0$	$\frac{1}{2}\|G\|^2$
$B^0\to J/\psi\eta\bar{K}^0$	$\frac{1}{6}\|G\|^2$
$B^0\to J/\psi\pi^+\pi^-$	$s_1^2\|2F+G\|^2$
$B^0\to J/\psi\pi^0\pi^0$	$\frac{1}{2}s_1^2\|2F+G\|^2$
$B^0\to J/\psi\eta\eta$	$\frac{1}{2}s_1^2\|2F+\frac{1}{3}G\|^2$
$B^0\to J/\psi\pi^0\eta$	$\frac{1}{3}s_1^2\|G\|^2$
$B^0\to J/\psi K^+K^-$	$s_1^2\|2F\|^2$
$B^0\to J/\psi K^0\bar{K}^0$	$s_1^2\|2F+G\|^2$
$B^-\to J/\psi\pi^0K^-$	$\frac{1}{2}\|G\|^2$
$B^-\to J/\psi\eta K^-$	$\frac{1}{6}\|G\|^2$
$B^-\to J/\psi\pi^-\bar{K}^0$	$\|G\|^2$
$B^-\to J/\psi\pi^-\eta$	$\frac{2}{3}s_1^2\|G\|^2$
$B^-\to J/\psi K^-K^0$	$s_1^2\|G\|^2$
$B_s^0\to J/\psi\pi^0\pi^0$	$\frac{1}{2}\|2F\|^2$
$B_s^0\to J/\psi\eta\eta$	$\frac{1}{2}\|2F+\frac{4}{3}G\|^2$
$B_s^0\to J/\psi K^0\bar{K}^0$	$\|2F+G\|^2$
$B_s^0\to J/\psi K^+K^-$	$\|2F+G\|^2$
$B_s^0\to J/\psi\pi^+\pi^-$	$\|2F\|^2$
$B_s^0\to J/\psi K^+\pi^-$	$s_1^2\|G\|^2$
$B_s^0\to J/\psi K^0\pi^0$	$\frac{1}{2}s_1^2\|G\|^2$
$B_s^0\to J/\psi K^0\eta$	$\frac{1}{6}s_1^2\|G\|^2$

TABLE IX. SU(3) predictions for decays $B\to J/\psi MM$, when the relative orbital angular momentum of the two pseudoscalar mesons is odd. Rates are normalized to that for $B^0\to J/\psi\pi^+K^-$.

Process	Rate
$B^0\to J/\psi\pi^+K^-$	1
$B^0\to J/\psi\pi^0\bar{K}^0$	$\frac{1}{2}$
$B^0\to J/\psi\eta\bar{K}^0$	$\frac{3}{2}$
$B^0\to J/\psi\pi^+\pi^-$	s_1^2
$B^0\to J/\psi K^0\bar{K}^0$	s_1^2
$B^-\to J/\psi\pi^0K^-$	$\frac{1}{2}$
$B^-\to J/\psi\pi^-\bar{K}^0$	1
$B^-\to J/\psi\eta K^-$	$\frac{3}{2}$
$B^-\to J/\psi\pi^0\pi^-$	$2s_1^2$
$B^-\to J/\psi K^0K^-$	s_1^2
$B_s^0\to J/\psi K^+K^-$	1
$B_s^0\to J/\psi K^0\bar{K}^0$	1
$B_s^0\to J/\psi K^+\pi^-$	s_1^2
$B_s^0\to J/\psi K^0\eta$	$\frac{3}{2}s_1^2$
$B_s^0\to J/\psi K^0\pi^0$	$\frac{1}{2}s_1^2$

$$\Gamma(B^0\to J/\psi\,\pi^0\pi^0)=\tfrac{1}{2}\Gamma(B^0\to J/\psi(\pi^+\pi^-)_{L=0,2,\ldots})\,, \tag{43a}$$

$$\Gamma(B^0\to J/\psi(\pi^0\pi^-)_{L=0,2,\ldots})=0\,. \tag{43b}$$

There are also SU(3) relations between the Cabibbo-allowed and Cabibbo-suppressed decay amplitudes that are L independent. For example, two such relations are

$$s_1^2|A(B^-\to J/\psi\,\pi^-\bar{K}^0)|^2=|A(B^-\to J/\psi\,K^-K^0)|^2\,, \tag{44a}$$

$$s_1^2|A(B^0\to J/\psi\,\pi^+K^-)|^2=|A(B_s^0\to J/\psi\,K^+\pi^-)|^2\,. \tag{44b}$$

All the L-odd $B\to J/\psi MM$ decays are related by SU(3)-flavor symmetry. However, there is an important source of SU(3) violation for resonant MM pairs. There is significant mixing between the lowest-lying SU(3)-singlet and -octet 1^- mesons resulting in the ϕ and w mass eigenstates with flavor quantum numbers $s\bar{s}$ and $(1/\sqrt{2})(u\bar{u}+d\bar{d})$, respectively. This occurs not because of an anomalously large SU(3)-violating mass mixing element, but rather because of the near degeneracy of the SU(3)-singlet and -octet states. Nonetheless, because the decay of the w to $K\bar{K}$ is kinematically forbidden, this mass mixing can result in large violations of our SU(3) predictions for decays $B\to J/\psi(MM)_{L=1}$ when the MM pair is resonant.

Next we consider the implications of SU(3) symmetry for decays $B\to DMM$. As was noted in Sec. II, the effective Hamiltonian for these decays transforms as an octet under flavor SU(3). Again, we shall separately treat the cases where the relative orbital angular momentum L of the MM pair is even and odd. As far as group-theory factors are concerned, when L is even we can take, as our effective Hamiltonian,

$$\begin{aligned}H_{\text{eff}}=&\,aB_iM_j^iM_l^kH_k^j\bar{D}^{\,l}+bB_iM_j^iM_k^jH_l^k\bar{D}^{\,l}\\&+cB_iH_j^iM_k^jM_l^k\bar{D}^{\,l}+d(B_iM_j^i\bar{D}^{\,j})(M_l^kH_k^l)\\&+e(B_iH_j^i\bar{D}^{\,j})(M_l^kM_k^l)+f(B_i\bar{D}^{\,i})(M_k^jM_l^kH_j^l)\,.\end{aligned} \tag{45}$$

In Eq. (45) H_j^i are elements of the 3×3 matrix (upper index labeling rows and lower index labeling columns)

$$H=\begin{bmatrix}0&0&0\\1&0&0\\-s_1&0&0\end{bmatrix}. \tag{46}$$

Table X presents the results that follow from this effective Hamiltonian for the Cabibbo-allowed decays. Under isospin the effective Hamiltonian for the Cabibbo-allowed decays is $I=1$. When two pions possessing a net charge are in an even partial wave they form an $I=2$ state. This implies the isospin relation

$$\begin{aligned}\tfrac{1}{4}\Gamma(B^-\to D^+\pi^-\pi^-)&=\Gamma(B^-\to D^0(\pi^-\pi^0)_{L=0,2,\ldots})\\&=\Gamma(B^0\to D^+(\pi^-\pi^0)_{L=0,2,\ldots})\,.\end{aligned} \tag{47}$$

The first process that appears in Eq. (47) has been observed. Experimentally,[8]

$$B(B^-\to D^+\pi^-\pi^-)=(2.5^{+4.8}_{-2.4})\times 10^{-3}\,. \tag{48}$$

Since the amplitudes with L odd do not interfere with those with L even, we conclude that

$$4\Gamma(B^-\to D^0\pi^0\pi^-)>\Gamma(B^-\to D^+\pi^-\pi^-)\,, \tag{49a}$$

$$4\Gamma(B^0\to D^+\pi^-\pi^0)>\Gamma(B^-\to D^+\pi^-\pi^-)\,. \tag{49b}$$

Of course, the results in Table X generalize to decays involving a D^* instead of a D. Experimentally,[8,9]

$$B(B^-\to D^*(2010)^+\pi^-\pi^-)=(2.5^{+1.5}_{-1.3})\times 10^{-3}\,, \tag{50a}$$

$$B(B^0\to D^*(2010)^+\pi^-\pi^0)=(1.5\pm 1.1)\times 10^{-2}\,, \tag{50b}$$

which is consistent with the generalization of Eq. (49b) and indicates that the $B^0\to D^{*+}\pi^-\pi^0$ rate is dominated by L odd.

There are also some SU(3) relations between the Cabibbo-allowed amplitudes with L even. They are

$$|A(B^0\to D_s^+(K^-\pi^0)_{L=0,2,\ldots})|^2=3|(B^0\to D_s^+(K^-\eta)_{L=0,2})|^2\,, \tag{51a}$$

$$|A(B^-\to D_s^+(\pi^-K^-)_{L=0,2,\ldots})|^2=\tfrac{1}{2}|A(B^-\to D^+\pi^-\pi^-)|^2\,, \tag{51b}$$

TABLE X. Implications of SU(3) symmetry for Cabibbo-allowed decays $B\to DMM$, where the relative angular momentum of the M mesons is even.

Process	Rate
$B^0\to D^0\pi^0\pi^0$	$\frac{1}{2}\lvert 2e+c+b-a\rvert^2$
$B^0\to D^0\eta\eta$	$\frac{1}{2}\lvert 2e+\frac{1}{3}c+\frac{1}{3}b+\frac{1}{3}a\rvert^2$
$B^0\to D^0\eta\pi^0$	$\frac{1}{3}\lvert c-b\rvert^2$
$B^0\to D^0\pi^+\pi^-$	$\lvert 2e+d+c+b\rvert^2$
$B^0\to D^0K^0\bar{K}^0$	$\lvert 2e+b\rvert^2$
$B^0\to D^0K^+K^-$	$\lvert 2e+c\rvert^2$
$B^0\to D^+\eta\pi^-$	$\frac{1}{6}\lvert 2f+d+2c+a\rvert^2$
$B^0\to D^+\pi^-\pi^0$	$\frac{1}{2}\lvert d+a\rvert^2$
$B^0\to D^+K^-K^0$	$\lvert f+c\rvert^2$
$B^0\to D_s^+\bar{K}^0\pi^-$	$\lvert d+c\rvert^2$
$B^0\to D_s^+K^-\pi^0$	$\frac{1}{2}\lvert c-a\rvert^2$
$B^0\to D_s^+K^-\eta$	$\frac{1}{6}\lvert c-a\rvert^2$
$B^-\to D^0\pi^0\pi^-$	$\frac{1}{2}\lvert d+a\rvert^2$
$B^-\to D^0\eta\pi^-$	$\frac{1}{6}\lvert 2f+d+2b+a\rvert^2$
$B^-\to D^0K^-K^0$	$\lvert f+b\rvert^2$
$B^-\to D^+\pi^-\pi^-$	$\frac{1}{2}\lvert 2d+2a\rvert^2$
$B^-\to D_s^+\pi^-K^-$	$\lvert d+a\rvert^2$
$B_s^0\to D^0K^+\pi^-$	$\lvert d+b\rvert^2$
$B_s^0\to D^0K^0\pi^0$	$\frac{1}{2}\lvert b-a\rvert^2$
$B_s^0\to D^0K^0\eta$	$\frac{1}{6}\lvert b-a\rvert^2$
$B_s^0\to D^+K^0\pi^-$	$\lvert d+a\rvert^2$
$B_s^0\to D_s^+\eta\pi^-$	$\frac{2}{3}\lvert f-d\rvert^2$
$B_s^0\to D_s^+K^-K^0$	$\lvert f+a\rvert^2$

$$|A(B_s^0\to D^+(K^0\pi^-)_{L=0,2,\ldots})|^2 = \tfrac{1}{2}|A(B^-\to D^+\pi^-\pi^-)|^2\,, \qquad (51c)$$

$$|A(B_s^0\to D^0(K^0\eta)_{L=0,2,\ldots})|^2 = \tfrac{1}{3}|A(B_s^0\to D^0(K^0\pi^0)_{L=0,2,\ldots})|^2\,. \qquad (51d)$$

For the case L odd, the effective Hamiltonian must be antisymmetric under interchange of the flavor quantum numbers of the M mesons. For example, an antisymmetric version the term proportional to a in Eq. (45) is

$$(\partial^\mu B_i)[(\partial_\mu M_j^i)M_l^k - M_j^i(\partial_\mu M_l^k)]\bar{D}^{\,l}H_k^j\,. \qquad (52)$$

Only the term proportional to the reduced matrix element e has no antisymmetric analog. So the $B\to DMM$ decay amplitudes with L odd are parametrized by five reduced matrix elements which we denote by a', b', c', d', and f'. Table XI presents the implications of the SU(3)-flavor symmetry of the strong interactions for the Cabibbo-allowed decays $B\to D(MM)_{L=1,3,\ldots}$. There are several SU(3) relations. For example,

$$|A(B^-\to D^0(\eta\pi^-)_{L=1,3,\ldots})|^2 = \tfrac{1}{6}|A(B^-\to D_s^+(\pi^-K^-)_{L=1,3,\ldots})|^2\,. \qquad (53)$$

Tables XII and XIII present the predictions of SU(3)-flavor symmetry for the Cabibbo-suppressed $B\to DMM$ decays with L even and L odd, respectively. Since the Hamiltonian for the Cabibbo-suppressed decays is part of the same octet as the Hamiltonian for the Cabibbo-

TABLE XII. Implications of SU(3) symmetry for Cabibbo-suppressed $B\to DMM$ decays when the relative angular momentum of the M mesons is even.

Process	Rate (divided by s_1^2)
$B^0\to D^+\pi^0K^-$	$\frac{1}{2}\lvert f-d\rvert^2$
$B^0\to D^+\pi^-\bar{K}^0$	$\lvert f+a\rvert^2$
$B^0\to D^+\eta K^-$	$\frac{1}{6}\lvert -f+d\rvert^2$
$B^0\to D^0\pi^+K^-$	$\lvert d+b\rvert^2$
$B^0\to D_s^+\bar{K}^0K^-$	$\lvert d+a\rvert^2$
$B^0\to D^0\pi^0\bar{K}^0$	$\frac{1}{2}\lvert -b+a\rvert^2$
$B^0\to D^0\eta\bar{K}^0$	$\frac{1}{6}\lvert -b+a\rvert^2$
$B^-\to D^0\pi^0K^-$	$\frac{1}{2}\lvert f+d+b+a\rvert^2$
$B^-\to D^0\pi^-\bar{K}^0$	$\lvert f+b\rvert^2$
$B^-\to D^0\eta K^-$	$\frac{1}{6}\lvert -f+d-b+a\rvert^2$
$B^-\to D^+\pi^-K^-$	$\lvert d+a\rvert^2$
$B^-\to D_s^+K^-K^-$	$2\lvert d+a\rvert^2$
$B_s^0\to D_s^+\pi^0K^-$	$\frac{1}{2}\lvert f+c\rvert^2$
$B_s^0\to D_s^+\pi^0\bar{K}^0$	$\lvert f+c\rvert^2$
$B_s^0\to D_s^+\eta K^-$	$\frac{1}{6}\lvert f+2d+c+2a\rvert^2$
$B_s^0\to D^0\pi^0\pi^0$	$\frac{1}{2}\lvert 2e+c\rvert^2$
$B_s^0\to D^0\eta\eta$	$\frac{1}{2}\lvert 2e+c/3+4b/3-2a/3\rvert^2$
$B_s^0\to D^0\pi^+\pi^-$	$\lvert 2e+c\rvert^2$
$B_s^0\to D^0\bar{K}^0K^0$	$\lvert 2e+b\rvert^2$
$B_s^0\to D^0K^+K^-$	$\lvert 2e+d+c+b\rvert^2$
$B_s^0\to D^+K^0K^-$	$\lvert d+c\rvert^2$
$B_s^0\to D^0\eta\pi^0$	$\frac{1}{3}\lvert -a+c\rvert^2$
$B_s^0\to D^+\eta\pi^-$	$\frac{2}{3}\lvert c-a\rvert^2$

TABLE XI. Implications of SU(3) symmetry for Cabibbo-allowed decays $B\to DMM$, where the relative orbital angular momentum of the M mesons is odd. Rates are expressed in terms of five reduced matrix elements a', b', c', d', and f'.

Process	Rate
$B^0\to D^0\eta\pi^0$	$\frac{1}{3}\lvert a'\rvert^2$
$B^0\to D^0\pi^+\pi^-$	$\lvert d'-c'+b'\rvert^2$
$B^0\to D^0K^0\bar{K}^0$	$\lvert b'\rvert^2$
$B^0\to D^0K^+K^-$	$\lvert c'\rvert^2$
$B^0\to D^+\eta\pi^-$	$\frac{1}{6}\lvert d'+a'\rvert^2$
$B^0\to D^+\pi^-\pi^0$	$\frac{1}{2}\lvert -2f'+d'-2c'+a'\rvert^2$
$B^0\to D^+K^-K^0$	$\lvert f'+c'\rvert^2$
$B^0\to D_s^+\bar{K}^0\pi^-$	$\lvert d'-c'\rvert^2$
$B^0\to D_s^+K^-\pi^0$	$\frac{1}{2}\lvert -c'+a'\rvert^2$
$B^0\to D_s^+K^-\eta$	$\frac{1}{6}\lvert 3c'+a'\rvert^2$
$B^-\to D^0\pi^0\pi^-$	$\frac{1}{2}\lvert 2f'+d'+2b'-a'\rvert^2$
$B^-\to D^0\eta\pi^-$	$\frac{1}{6}\lvert d'-a'\rvert^2$
$B^-\to D^0K^-K^0$	$\lvert f'+b'\rvert^2$
$B^-\to D_s^+\pi^-K^-$	$\lvert d'-a'\rvert^2$
$B_s^0\to D^0K^+\pi^-$	$\lvert d'+b'\rvert^2$
$B_s^0\to D^0K^0\pi^0$	$\frac{1}{2}\lvert -b'+a'\rvert^2$
$B^0\to D^0K^0\eta$	$\frac{1}{6}\lvert 3b'+a'\rvert^2$
$B_s^0\to D^+K^0\pi^-$	$\lvert d'+a'\rvert^2$
$B_s^0\to D_s^+\eta\pi^-$	$\frac{2}{3}\lvert d'\rvert^2$
$B_s^0\to D_s^+K^-K^0$	$\lvert f'-a'\rvert^2$
$B_s^0\to D_s^+\pi^0\pi^-$	$2\lvert f'\rvert^2$

TABLE XIII. Implications of SU(3) symmetry for Cabibbo-suppressed decays $B\to DMM$, where the relative orbital angular momentum of the M mesons is odd. Rates are expressed in terms of the same five reduced matrix elements (a', b', c', d', and f') as the Cabibbo-allowed case.

Process	Rate (divided by s_1^2)
$B^0\to D^+\pi^0K^-$	$\frac{1}{2}\lvert f'-d'\rvert^2$
$B^0\to D^+\pi^-\bar{K}^0$	$\lvert f'-a'\rvert^2$
$B^0\to D^+\eta K^-$	$\frac{1}{6}\lvert 3f'+d'\rvert^2$
$B^0\to D^0\pi^+K^-$	$\lvert d'+b'\rvert^2$
$B^0\to D_s^+\bar{K}^0K^-$	$\lvert d'+a'\rvert^2$
$B^0\to D^0\pi^0\bar{K}^0$	$\frac{1}{2}\lvert b'+a'\rvert^2$
$B^0\to D^0\eta\bar{K}^0$	$\frac{1}{6}\lvert 3b'-a'\rvert^2$
$B^-\to D^0\pi^0K^-$	$\frac{1}{2}\lvert f'+d'+b'-a'\rvert^2$
$B^-\to D^0\pi^-\bar{K}^0$	$\lvert f'+b'\rvert^2$
$B^-\to D^0\eta K^-$	$\frac{1}{6}\lvert 3f'+d'+3b'-a'\rvert^2$
$B^-\to D^+\pi^-K^-$	$\lvert d'-a'\rvert^2$
$B_s^0\to D_s^+\pi^0K^-$	$\frac{1}{2}\lvert f'+c'\rvert^2$
$B_s^0\to D_s^+\pi^-\bar{K}^0$	$\lvert f'+c'\rvert^2$
$B_s^0\to D_s^+\eta K^-$	$\frac{1}{6}\lvert 3f'-2d'+3c'-2a'\rvert^2$
$B_s^0\to D^0\pi^+\pi^-$	$\lvert c'\rvert^2$
$B_s^0\to D^0\bar{K}^0K^0$	$\lvert b'\rvert^2$
$B_s^0\to D^0K^+K^-$	$\lvert d'-c'+b'\rvert^2$
$B_s^0\to D^+K^0K^-$	$\lvert d'-c'\rvert^2$
$B_s^0\to D^0\eta\pi^0$	$\frac{1}{3}\lvert a'\rvert^2$
$B_s^0\to D^+\pi^0\pi^-$	$2\lvert c'\rvert^2$
$B_s^0\to D^+\eta\pi^-$	$\frac{2}{3}\lvert a'\rvert^2$

allowed decays, we can express the Cabibbo-suppressed decay rates in terms of the same reduced matrix elements as were used for the Cabibbo-allowed decays. An inspection of Tables X–XIII reveals that there are simple SU(3) relations between the Cabibbo-allowed decays and the Cabibbo-suppressed decays which hold independent of the value of L. They are

$$|A(B^0\to D^+\pi^-\bar{K}^0)|^2=s_1^2|A(B_s^0\to D_s^+K^-K^0)|^2\ , \quad (54a)$$

$$|A(B^0\to D_s^+\bar{K}^0K^-)|^2=s_1^2|A(B_s^0\to D^+K^0\pi^-)|^2\ , \quad (54b)$$

$$|A(B^-\to D_s^+K^-K^-)|^2=s_1^2|A(B^-\to D^+\pi^-\pi^-)|^2\ , \quad (54c)$$

$$|A(B^-\to D^0\pi^-\bar{K}^0)|^2=s_1^2|A(B^-\to D^0K^-K^0)|^2\ , \quad (54d)$$

$$2|A(B_s^0\to D_s^+\pi^0K^-)|^2=s_1^2|A(B^0\to D^+K^-K^0)|^2\ , \quad (54e)$$

$$|A(B_s^0\to D^0\pi^+\pi^-)|^2=s_1^2|A(B^0\to D^0K^-K^+)|^2\ , \quad (54f)$$

$$|A(B_s^0\to D^0\bar{K}^0K^0)|^2=s_1^2|A(B^0\to D^0\bar{K}^0K^0)|^2\ , \quad (54g)$$

$$|A(B_s^0\to D^0K^+K^-)|^2=s_1^2|A(B^0\to D^0\pi^+\pi^-)|^2\ , \quad (54h)$$

$$|A(B_s^0\to D^+K^0K^-)|^2=s_1^2|A(B^0\to D_s^+\bar{K}^0\pi^-)|^2\ . \quad (54i)$$

There are important sources of SU(3) violation in the decays $B\to DMM$ which can occur when two of the final-state particles arise from the decay of a resonance. In addition to the consequences of the mixing of the SU(3)-singlet and SU(3)-octet 1^- vector mesons mentioned earlier, large SU(3) violations can arise because the D^* can decay to $D\pi$ while the D_s^* is kinematically forbidden from decaying to $D_s\pi$ or DK.

Finally, we consider the three-body decays $B\to D\bar{D}M$. As far as group-theory factors are concerned we can take as the effective Hamiltonian for these processes

$$H_{\rm eff}=\eta_1(B_iH^i)(D_kM_l^k\bar{D}^l)+\eta_2(B_i\bar{D}^i)(D_kM_l^kH^l)+\eta_3(B_iM_l^i\bar{D}^l)(D_kH^k)+\eta_4(B_iM_l^iH^l)(\bar{D}^kD_k)\ , \quad (55a)$$

where

$$H=\begin{bmatrix}0\\ s_1\\ 1\end{bmatrix}\ . \quad (55b)$$

Tables XIV and XV present the predictions that follow from the Hamiltonian for Cabibbo-allowed and Cabibbo-suppressed decays, respectively. Cautionary remarks, similar to those given in the case of $B\to DMM$ decays, concerning possible large SU(3) violations induced from resonance effects also apply here.

Recall that for the Cabibbo-suppressed decays the effective Hamiltonian in Eq. (55) neglects the contribution of operators such as $(b\bar{u})(u\bar{d})$, which arise from the $b\to uW^-$ coupling and transform as $\bar{3}\oplus 6\oplus\overline{15}$. Since the $\overline{15}$ representation contains an $I=\frac{3}{2}$ piece, isospin relations for the Cabibbo-suppressed modes which follow from the $I=\frac{1}{2}$ Hamiltonian in Eq. (55) are useful for testing the dominance of the operators with charm quarks. Table XV contains two isospin relations,

$$\Gamma(B^-\to D_s^-D_s^+\pi^-)=2\Gamma(B^0\to D_s^-D_s^+\pi^0)\ , \quad (56a)$$

$$\Gamma(B_s^0\to D_s^+\bar{D}^0\pi^-)=2\Gamma(B_s^0\to D_s^+D^-\pi^0)\ . \quad (56b)$$

Figure 2 shows quark-line diagrams which illustrate how the two operators $(b\bar{c})(c\bar{d})$ and $(b\bar{u})(u\bar{d})$ can contribute to the decay $B_s^0\to D_s^+\bar{D}^0\pi^-$.

For the Cabibbo-allowed decays, the effective Hamiltonian transforms as an isosinglet. The following relations in Table XIV follow from isospin symmetry:

$$\Gamma(B^-\to D^0D^-\bar{K}^0)=\Gamma(B^0\to D^+\bar{D}^0K^-)\ , \quad (57a)$$

$$\Gamma(B^0\to D^0\bar{D}^0\bar{K}^0)=\Gamma(B^-\to D^+D^-K^-)\ , \quad (57b)$$

$$2\Gamma(B^-\to D_s^-D^0\pi^0)=\Gamma(B^0\to D^0D_s^-\pi^+)=\Gamma(B^-\to D_s^-D^+\pi^-)=2\Gamma(B^0\to D^+D_s^-\pi^0)\ , \quad (57c)$$

$$\Gamma(B^-\to D_s^+D_s^-K^-)=\Gamma(B^0\to D_s^+D_s^-\bar{K}^0)\ , \quad (57d)$$

TABLE XIV. Implications of flavor SU(3) for Cabibbo-allowed decays $B\to D\bar{D}M$ assuming the effective Hamiltonian transforms as a $\bar{3}$.

Process	Rate
$B^-\to D^0\bar{D}^0K^-$	$\lvert\eta_2+\eta_4\rvert^2$
$B^-\to D^+D^-K^-$	$\lvert\eta_4\rvert^2$
$B^-\to D^0D^-\bar{K}^0$	$\lvert\eta_2\rvert^2$
$B^-\to D^0D_s^-\eta$	$\frac{1}{6}\lvert-2\eta_2+\eta_3\rvert^2$
$B^-\to D^0D_s^-\pi^0$	$\frac{1}{2}\lvert\eta_3\rvert^2$
$B^-\to D_s^+D_s^-K^-$	$\lvert\eta_3+\eta_4\rvert^2$
$B^-\to D_s^-D^+\pi^-$	$\lvert\eta_3\rvert^2$
$B^0\to\bar{D}^0D^+K^-$	$\lvert\eta_2\rvert^2$
$B^0\to D^+D^-\bar{K}^0$	$\lvert\eta_2+\eta_4\rvert^2$
$B^0\to D^+D_s^-\eta$	$\frac{1}{6}\lvert-2\eta_2+\eta_3\rvert^2$
$B^0\to D^0D_s^-\pi^+$	$\lvert\eta_3\rvert^2$
$B^0\to D^+D_s^-\pi^0$	$\frac{1}{2}\lvert\eta_3\rvert^2$
$B^0\to D_s^+D_s^-\bar{K}^0$	$\lvert\eta_3+\eta_4\rvert^2$
$B^0\to D^0\bar{D}^0\bar{K}^0$	$\lvert\eta_4\rvert^2$
$B_s^0\to D^0\bar{D}^0\pi^0$	$\frac{1}{2}\lvert\eta_1\rvert^2$
$B_s^0\to D^0\bar{D}^0\eta$	$\frac{1}{6}\lvert\eta_1-2\eta_4\rvert^2$
$B_s^0\to\bar{D}^0D^+\pi^-$	$\lvert\eta_1\rvert^2$
$B_s^0\to\bar{D}^0D_s^+K^-$	$\lvert\eta_1+\eta_2\rvert^2$
$B_s^0\to D^-D^0\pi^+$	$\lvert\eta_1\rvert^2$
$B_s^0\to D^+D^-\pi^0$	$\frac{1}{2}\lvert\eta_1\rvert^2$
$B_s^0\to D^+D^-\eta$	$\frac{1}{6}\lvert\eta_1-2\eta_4\rvert^2$
$B_s^0\to D^-D_s^+\bar{K}^0$	$\lvert\eta_1+\eta_2\rvert^2$
$B_s^0\to D_s^-D^0K^+$	$\lvert\eta_1+\eta_3\rvert^2$
$B_s^0\to D_s^-D^+K^0$	$\lvert\eta_1+\eta_3\rvert^2$
$B_s^0\to D_s^+D_s^-\eta$	$\frac{2}{3}\lvert\eta_1+\eta_2+\eta_3+\eta_4\rvert^2$

TABLE XV. Implications of flavor SU(3) for Cabibbo-suppressed decays $B \rightarrow D\bar{D}M$. The effective Hamiltonian is assumed to transform as a $\bar{3}$. Entries in the second column should be multiplied by s_1^2 when comparing with the results of Table XIV.

Process	Rate (divided by s_1^2)
$B^- \rightarrow D^0\bar{D}^0\pi^-$	$\lvert\eta_2+\eta_4\rvert^2$
$B^- \rightarrow D^0 D^-\pi^0$	$\frac{1}{2}\lvert-\eta_2+\eta_3\rvert^2$
$B^- \rightarrow D^+ D^-\pi^-$	$\lvert\eta_3+\eta_4\rvert^2$
$B^- \rightarrow D^0 D^-\eta$	$\frac{1}{6}\lvert\eta_2+\eta_3\rvert^2$
$B^- \rightarrow D^0 D_s^- K^0$	$\lvert\eta_2\rvert^2$
$B^- \rightarrow D_s^+ D^- K^-$	$\lvert\eta_3\rvert^2$
$B^- \rightarrow D_s^+ D_s^-\pi^-$	$\lvert\eta_4\rvert^2$
$B^0 \rightarrow D^0\bar{D}^0\pi^0$	$\frac{1}{2}\lvert\eta_1-\eta_4\rvert^2$
$B^0 \rightarrow D^0\bar{D}^0\eta$	$\frac{1}{6}\lvert\eta_1+\eta_4\rvert^2$
$B^0 \rightarrow D^- D^0\pi^+$	$\lvert\eta_1+\eta_3\rvert^2$
$B^0 \rightarrow \bar{D}^0 D_s^+ K^-$	$\lvert\eta_1\rvert^2$
$B^0 \rightarrow D^+\bar{D}^0\pi^-$	$\lvert\eta_1+\eta_2\rvert^2$
$B^0 \rightarrow D^+ D^-\pi^0$	$\frac{1}{2}\lvert\eta_1+\eta_2+\eta_3+\eta_4\rvert^2$
$B^0 \rightarrow D^+ D^-\eta$	$\frac{1}{6}\lvert\eta_1+\eta_2+\eta_3+\eta_4\rvert^2$
$B^0 \rightarrow D^- D_s^+\bar{K}^0$	$\lvert\eta_1+\eta_3\rvert^2$
$B^0 \rightarrow D_s^- D^0 K^+$	$\lvert\eta_1\rvert^2$
$B^0 \rightarrow D_s^- D^+ K^0$	$\lvert\eta_1+\eta_2\rvert^2$
$B^0 \rightarrow D_s^+ D_s^-\pi^0$	$\frac{1}{2}\lvert\eta_4\rvert^2$
$B^0 \rightarrow D_s^+ D_s^-\eta$	$\frac{1}{6}\lvert-2\eta_1+\eta_4\rvert^2$
$B_s^0 \rightarrow D_s^+\bar{D}^0\pi^-$	$\lvert\eta_2\rvert^2$
$B_s^0 \rightarrow D_s^+ D^-\pi^0$	$\frac{1}{2}\lvert\eta_2\rvert^2$
$B_s^0 \rightarrow D_s^+ D^-\eta$	$\frac{1}{6}\lvert\eta_2-2\eta_3\rvert^2$
$B_s^0 \rightarrow D_s^+ D_s^- K^0$	$\lvert\eta_2+\eta_4\rvert^2$
$B_s^0 \rightarrow D^0 D^- K^+$	$\lvert\eta_3\rvert^2$
$B_s^0 \rightarrow D^+ D^- K^0$	$\lvert\eta_3+\eta_4\rvert^2$
$B_s^0 \rightarrow D^0\bar{D}^0 K^0$	$\lvert\eta_4\rvert^2$

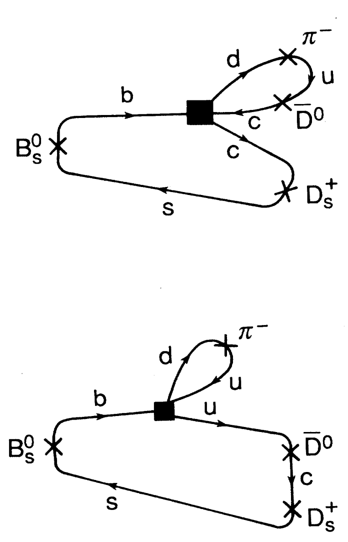

FIG. 2. Quark-line diagrams illustrating how the two operators $(b\bar{c})(c\bar{d})$ and $(b\bar{u})(u\bar{d})$ (denoted by shaded squares) contribute to the decay $B_s^0 \rightarrow D_s^+\bar{D}^0\pi^-$.

$$\Gamma(B_s^0 \rightarrow \bar{D}^0 D^+\pi^-) = \Gamma(B_s^0 \rightarrow D^- D^0\pi^+) = 2\Gamma(B_s^0 \rightarrow D^0\bar{D}^0\pi^0) = 2\Gamma(B_s^0 \rightarrow D^+ D^-\pi^0) , \quad (57e)$$

$$\Gamma(B_s^0 \rightarrow D_s^- D^0 K^+) = \Gamma(B_s^0 \rightarrow D_s^- D^+ K^0) , \quad (57f)$$

$$\Gamma(B_s^0 \rightarrow D^- D_s^+\bar{K}^0) = \Gamma(B_s^0 \rightarrow \bar{D}^0 D_s^+ K^-) , \quad (57g)$$

$$\Gamma(B_s^0 \rightarrow D^+ D^-\eta) = \Gamma(B_s^0 \rightarrow D^0\bar{D}^0\eta) . \quad (57h)$$

Comparison of Tables XIV and XV reveals that there are many simple SU(3) relations between the Cabibbo-allowed and the Cabibbo-suppressed $B \rightarrow D\bar{D}M$ decays. Some of them are

$$|A(B^- \rightarrow D^0 D_s^- K^0)|^2 = s_1^2 |A(B^- \rightarrow D^0 D^- \bar{K}^0)|^2 , \quad (58a)$$

$$|A(B^- \rightarrow D_s^+ D^- K^-)|^2 = s_1^2 |A(B^- \rightarrow D_s^- D^+ \pi^-)|^2 , \quad (58b)$$

$$|A(B^- \rightarrow D_s^+ D_s^- \pi^-)|^2 = s_1^2 |A(B^0 \rightarrow D^0\bar{D}^0\bar{K}^0)|^2 , \quad (58c)$$

$$|A(B^- \rightarrow D^+ D^- \pi^-)|^2 = s_1^2 |A(B^- \rightarrow D_s^+ D_s^- K^-)|^2 . \quad (58d)$$

IV. CONCLUSIONS

The smallness of the up-, down-, and strange-quark masses compared with the QCD scale implies that the strong-interaction Lagrangian possesses an approximate $SU(3)_L \times SU(3)_R$ symmetry. This chiral symmetry is spontaneously broken to a vectorlike SU(3)-flavor symmetry by the vacuum expectation value of quark bilinears. The smallness of the pion mass, compared with the kaon mass, indicates that the up- and down-quark masses are much smaller than the strange-quark mass. This is why the SU(2) isospin subgroup is a much better symmetry than the full SU(3)-flavor group. In this paper we have used the transformation properties of the weak Hamiltonian for nonleptonic B-meson decays to derive SU(3) relations among many of the possible two- and three-body B-meson decays. Since the SU(2)-isospin symmetry works much better than the full SU(3) we have emphasized which of our predictions follow from isospin. The isospin relations provide useful tools for discerning the importance of various competing effects that can occur in nonleptonic B-meson decays.

It is possible to include, in a phenomenological fashion, some SU(3)-breaking effects and hence improve upon the results of this paper. For example, in Sec. II it was noted that generalizing the predictions for $B \rightarrow DM$ to $B \rightarrow DV$, where V is one of the low-lying 1^- mesons is not straightforward because of mixing of the SU(3)-octet state $|V_8\rangle = (1/\sqrt{6})(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)$ with the SU(3) singlet $|V_1\rangle = (1/\sqrt{3})(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$. The mass eigenstates $|\phi\rangle = |s\bar{s}\rangle$ and $|w\rangle = (1/\sqrt{2})(|u\bar{u}\rangle + |d\bar{d}\rangle)$ are linear combinations of $|V_8\rangle$ and $|V_1\rangle$. Explicitly,

$$|V_8\rangle = \frac{1}{\sqrt{3}}(|w\rangle - \sqrt{2}|\phi\rangle) , \tag{59a}$$

$$|V_1\rangle = \frac{1}{\sqrt{3}}(\sqrt{2}|w\rangle + |\phi\rangle) . \tag{59b}$$

As far as group theory is concerned we can take as our effective Hamiltonian for the decays $B \to DV$:

$$H_{\text{eff}} = a'(B_i\bar{D}^{\,i})(V_l^k H_k^l) + b'(B_i V_k^i H_j^k \bar{D}^{\,j}) + c'(B_i H_k^i V_j^k \bar{D}^{\,j}) + e'(B_i H_j^i \bar{D}^{\,j})V_1 , \tag{60}$$

where V_j^k are elements of the 3×3 matrix (the upper index labels rows and the lower index columns)

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{V_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{V_8}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{2/3}V_8 \end{pmatrix} , \tag{61}$$

and H_k^l given by Eq. (5a) for the Cabibbo-allowed decays and Eq. (5b) for the Cabibbo-suppressed decays. The amplitude $A(B^0 \to D^0\phi)$ is expected to be very small since the decay $B^0 \to D^0\phi$ is forbidden by the Okubo-Zweig-Iizuka (OZI) rule. Setting this amplitude to zero implies the relation

$$e' = \frac{b'+c'}{\sqrt{3}} \tag{62}$$

between reduced matrix elements. So the $B \to DV$ decay amplitudes are expressible in terms of the three reduced matrix elements: a', b', and c'. Using these expressions we find that the generalization of Eq. (10) is

$$|A(B^0 \to D^0\rho^0)|^2 + |A(B^0 \to D^0 w)|^2 = |A(B^0 \to D_s^+ K^{*-})|^2 + |A(B_s^0 \to D^0 K^{*0})|^2 . \tag{63}$$

In the large-N_c limit matrix elements for nonleptonic B decays factorize. This provides a pattern of SU(3) breaking that might be used to improve some of our results. For example, factorization suggests that

$$\frac{A(B^- \to D^0K^-)}{A(B^- \to D^0\pi^-)} = -\left[\frac{f_K}{f_\pi}\right] s_1 , \tag{64}$$

would be an improvement over Eq. (11).

In this paper we have focused on SU(3) predictions for nonleptonic B-meson decays to final states with mesons. It is also possible to consider SU(3) predictions for B-meson decays to final states involving baryons. For example, there may be SU(3) relations between the Cabibbo-allowed and the Cabibbo-suppressed decays $B \to DN\bar{N}$, where N denotes a member of the lowest-lying baryon octet (consisting of the nucleons and hyperons). It is also possible to consider the consequences of SU(3)-flavor symmetry for semileptonic B-meson decays. For example, since the effective Hamiltonian for $\Delta c = 1$ $B \to De\bar{\nu}_e$ decays is an SU(3) singlet, it follows that

$$\Gamma(B^0 \to D^+ e\bar{\nu}_e) = \Gamma(B^- \to D^0 e\bar{\nu}_e) = \Gamma(B_s^0 \to D_s^+ e\bar{\nu}_e) . \tag{65}$$

The first equality in Eq. (65) follows from isospin. For semileptonic decays $B \to Me\bar{\nu}_e$ that do not change charm, the effective Hamiltonian transforms as an antitriplet. Since there is only one way to combine the product of a triplet, an antitriplet, and an octet into a singlet, these decays are also related by SU(3)-flavor symmetry. In this case,

$$\Gamma(B^0 \to \pi^+ e\bar{\nu}_e) = 2\Gamma(B^- \to \pi^0 e\bar{\nu}_e) = \Gamma(B_s^0 \to K^+ e\bar{\nu}_e) = 6\Gamma(B^- \to \eta e\bar{\nu}_e) . \tag{66}$$

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