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Resonant pair production in strong electric fields

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We show that strong classical electric fields with certain kinds of space-time variation can create e^+e^- pairs which are strongly resonant in energy and momentum. These resonance effects are peculiar to spin $\frac{1}{2}$, and are allowed because of the unconventional structure of a "unitarity" relation governing the pair-production process. This relation is a physical expression of the Klein paradox. The general principles are illustrated with exact solutions of idealized examples.

This work was motivated by, but cannot (because of idealizations) claim to explain the recent experimental results[1] on e^+e^- production in heavy-nucleus collisions, showing multiple resonances in the pairs, which are characterized by equal scalar momenta (most likely oppositely directed), total pair energies of 1.6–1.8 MeV, and rather narrow widths[2] (<40 keV). Theoretical explanations have focused on (1) the decay of an elementary particle (e.g., the axion[3]), (2) a new QCD-like phase of QED,[4] and (3) interference effects among different amplitudes.[5] The first possibility is ruled out by the multiple-resonance structure and by the negative results of axion searches in e^+e^- colliders.[6] It is difficult for us to believe the second explanation because quantum-field-theoretic effects such as vacuum polarization are $O(\alpha \ln(eE/M^2))$ for electric fields $E \gg E_c \equiv M^2/e$, so these effects[7] are ~ 1 only when $E \simeq E_c \exp(\alpha^{-1})$. Interference effects are very interesting but difficult to compute.[5] We will study here some special interference effects which in principle could lead to sharp resonances in pair production, and illustrate them with soluble examples. The effects in question are closely related to the Klein paradox.

The exactly soluble examples are highly idealized; in particular, we can only deal analytically with time-varying but space-independent electric fields (at the end of the paper we comment on spatial-variation effects). We cannot argue from our results that the heavy-nucleus experiments are understood; nevertheless, the resonance phenomenon itself is curious and interesting in its own right, and worthy of discussion quite aside from the experiments which motivated us to begin this work. A major feature is that unitarity does not forbid the appearance of a physical-region pole in the *connected* S-matrix element for spin-$\frac{1}{2}$ pair production (unitarity is restored by disconnected graphs), and it is easy to find soluble examples with such poles. But we show that such poles, or in general sharp resonances, are forbidden in spin-0 pair production. Thus the coupling of the electric field to spin is essential for resonance.

Another way of describing the effect of spin is based on the mathematical analogy between the Dirac equation in a time-varying electric field and scattering off a potential which varies in space, not time. There is a "unitarity" relation for this analog scattering problem which relates reflection and transmission coefficients R^c, T^c which refer to reflection and transmission in time, not space; these describe connected S-matrix elements. (Pair production from the vacuum can be thought of as reflection of a positron in time and R^c is the pair-production amplitude.) This relation between R^c and T^c follows from charge conservation; because the charge current is free of time derivatives in the spin-$\frac{1}{2}$ case, there is an unexpected change of sign and we find

$$1+|R^c|^2=|T^c|^2 \tag{1}$$

instead of the usual unitarity relation

$$1=|R|^2+|T|^2 \tag{2}$$

which holds for the scattering problem in space. The strange "unitarity" relation (1) does not bound the magnitudes of R^c and T^c (which refer to connected amplitudes), as does the normal relation (2), which holds for spin-0 pair production. This is because the conserved spin-0 current has one time derivative in its time component. In the spin-$\frac{1}{2}$ case, physical unitarity is restored by multiplying the connected S-matrix elements by a factor, as we now show.

The connected S-matrix element $R^c(\mathbf{p}s\mathbf{p}'s')$ for production of a single e^+e^- pair is simply the Fourier transform of the connected electron propagator[8] S_{Ac}:

$$R^c(\mathbf{p}s\mathbf{p}'s')=2(\omega\omega')^{-1/2}\int d^4x\, d^4y\, e^{ip\cdot x+ip'\cdot y} \times \bar{u}_s(p)(\not{p}-M)S_{Ac}(x,y) \times(\not{p}'+M)v_{s'}(p') \ . \tag{3}$$

The complete S-matrix element $R(\mathbf{p}s,\mathbf{p}'s')$ is defined as in (3) by another propagator S_A, which differs from S_{Ac} by a factor related to connected vacuum graphs. The connected propagator S_A obeys

$$(i\partial\!\!\!/-eA\!\!\!/-M)S_{Ac}(x,y)=\delta(x-y) \tag{4}$$

or equivalently the integral equation

$$S_{Ac}=S_0+S_0eA\!\!\!/S_{Ac} \tag{5}$$

while the disconnected propagator S_A is defined by

$$iS_A(x,y)=\langle T[\psi(x)\bar{\psi}(y)]\rangle_A\ ; \tag{6}$$

in both of these equations $A_\mu(x)$ is a fixed c-number gauge potential which we assume approaches a pure gauge as $|x_0|\to\infty$. These propagators are related by

$$S_A=e^{iW}S_{Ac}\ , \tag{7}$$

where $W=\mathrm{Tr}\ln S_{Ac}$ is the sum of connected vacuum graphs; this is important for our case because W has an imaginary part, signifying instability of the vacuum. Similarly, R and R^c are related by $R=\exp(iW)R^c$. Thus the probability of producing one pair is

$$P_1=e^{-2\,\mathrm{Im}W}\sum|R^c|\equiv e^{-2\,\mathrm{Im}W}P_1^c\ . \tag{8}$$

Under the assumption that the number of pairs produced per unit volume is not large enough to lead to electron-degeneracy effects, the probability for producing N pairs is

$$P_N=\frac{1}{N!}(P_1^c)^N e^{-2\,\mathrm{Im}W} \tag{9}$$

(valid also for $N=0$) and the constraint of unitarity is

$$1=\sum P_N=\exp(P_1^c-2\,\mathrm{Im}W) \tag{10}$$

so $\mathrm{Im}W=\frac{1}{2}P_1^c$. An elementary calculation shows that the average number $\langle N\rangle$ of produced pairs is just

$$\langle N\rangle=\sum NP_N=P_1^c=\sum|R^c|^2\ . \tag{11}$$

Unitarity provides no limit on $\langle N\rangle$, and thus no limit on the size of R^c. Even a pole in R^c in the physical region is not forbidden by unitarity. Of course, other physical effects will prevent such a pole, notably the electromagnetic current of the produced pairs which will modify A_μ so as to limit $\langle N\rangle$. Accounting for this reaction current is really the same as imposing energy conservation, which is of course a consequence of Maxwell's equations. By comparing $\mathbf{J}$ to $\dot{E}$, with E of order $E_c=M^2/e$ and $\dot{E}\sim M^3/e$, we see that such reaction currents are important when $\langle N\rangle\gtrsim e^{-2}\simeq 10$, a number much too large to matter for the heavy-nucleus experiment, where $\langle N\rangle\le 1$. Ultimately, applications of our work are directed toward the case $\langle N\rangle\le 1$ so we will ignore the reaction currents in the idealized examples below. The electromagnetic fields of the examples are supported by fictitious currents whose physical interpretation we do not attempt.

The connected propagator $S_{Ac}(x,y)$ can be written in terms of two solutions of the Dirac equation χ_1,χ_2 whose free-particle versions are TCP conjugate to each other:

$$iS_{Ac}(x,y)=\sum\chi_1^{\mathbf{p}s}(x)\bar{\chi}_1^{\mathbf{p}s}(y)\theta(x_0-y_0) -\sum\chi_2^{\mathbf{p}s}(x)\bar{\chi}_2^{\mathbf{p}s}(y)\theta(y_0-x_0)\ , \tag{12}$$

where χ_j obeys the Dirac equation

$$(i\partial\!\!\!/-eA\!\!\!/-M)\chi_j=0\ . \tag{13}$$

The label j distinguishes particle from antiparticle (at any convenient fixed time, since a particle can evolve into an antiparticle), and the sum in (12) goes over all other labels (spin, momentum) necessary to identify a complete set of solutions; when these labels are superfluous we omit them. We choose the χ_j to form a complete orthonormal set; in particular,

$$\sum[\chi_1(x)\chi_1^\dagger(y)+\chi_2(x)\chi_2^\dagger(y)]|_{x_0=y_0}=\delta(\mathbf{x}-\mathbf{y})\ , \tag{14}$$

where time independence of the left-hand side (LHS) is guaranteed by current conservation. One could thus extract the connected amplitude R^c from solutions to the homogeneous Dirac equation by using them to form the propagator, as in (12). A more convenient approach is to define other homogeneous solutions ψ_j of (13) via

$$\psi_j^{\mathbf{p}s}(x)=\psi_{j0}^{\mathbf{p}s}(x)+e\int d^4z\,S_0(x-z)A\!\!\!/(z)\psi_j^{\mathbf{p}s}(z) \tag{15}$$

with the free solutions ψ_j given by the standard[8] forms

$$\psi_{10}^{\mathbf{p}s}(x)=\frac{U_S(p)e^{-ip\cdot x}}{(2\omega)^{1/2}}\ , \tag{16a}$$

$$\psi_{20}^{\mathbf{p}s}(x)=\frac{V_S(p)e^{+ip\cdot x}}{(2\omega)^{1/2}}\ . \tag{16b}$$

In (16), p is on shell: $p_0\equiv\omega=(\mathbf{p}^2+M^2)^{1/2}$. The asymptotic behavior as $|x_0|\to\infty$ of (15) is governed by a pair-production ("reflection") amplitude R, and a scattering ("transmission") amplitude T, as follows (*we drop temporarily the superscript c indicating connected amplitudes*):

$$\psi_2^{\mathbf{p}s}(x)\underset{x_0\to\infty}{\longrightarrow}\psi_{20}^{\mathbf{p}s}(x)+i\sum\psi_{10}^{\mathbf{p}'s'}(x)R(\mathbf{p}s,\mathbf{p}'s') \tag{17a}$$

$$\underset{x_0\to-\infty}{\longrightarrow}\psi_{20}^{\mathbf{p}s}(x)+i\sum\psi_{20}^{\mathbf{p}'s'}(x)T(\mathbf{p}s,\mathbf{p}'s')\ , \tag{17b}$$

where the sum is over the primed variables. One sees that R is the pair production of Eq. (3) and T is the S-matrix amplitude for scattering of a positron off A_μ. Analogous expressions to (17) hold for ψ_1, with T replaced by $\tilde{R}$ and $\tilde{R}$ by $\tilde{T}$, the electron-scattering amplitude; $\tilde{R}$ is the pair-annihilation amplitude.

Now R and T are related through a "unitarity" relation of a curious type. This relation can be derived by demanding the propagator S_{Ac} of (12) become the free propagator when its time arguments are large enough that $A_\mu=0$ and using (17), or more simply by invoking charge conservation:

$$\int d^3x\,\psi_j^\dagger\psi_j|_{x_0=\infty}=\int d^3x\,\psi_j^\dagger\psi_j|_{x_0=-\infty}\qquad (j\text{ not summed})\ ; \tag{18}$$

for $j=2$, (17) and (18) imply a sort of optical theorem:

$$\sum_{p's'} |R(\mathbf{p}s,\mathbf{p}'s')|^2 = -2\,\mathrm{Im}T(\mathbf{p}s,\mathbf{p}s) + \sum_{p's'} |T(\mathbf{p}s,\mathbf{p}'s'|^2 \,, \tag{19}$$

where the LHS of (19) corresponds to the LHS of (18), and similarly for the RHS. We can put this in the form of Eq. (1) by introducing operators $\mathcal{R},\mathcal{T}$ such that

$$R(\mathbf{p}s\mathbf{p}'s) = \langle \mathbf{p}s|\mathcal{R}|\mathbf{p}'s'\rangle \,, \tag{20}$$

etc., and then (19) is the matrix element of the operator equation

$$I + \mathcal{R}\mathcal{R}^\dagger = (I + i\mathcal{T})(I + i\mathcal{T})^\dagger \tag{21}$$

involving the total transmission operator $I + i\mathcal{T}$.

A similar calculation can be done for spin-0 pair production, but with a result analogous to the familiar one of Eq. (2). The reason for this difference is that the conserved current from which the charges of (18) are constructed contains one time derivative and an extra minus sign emerges on the LHS of (19) when antiparticles at $t=-\infty$ and particles at $t=\infty$ are compared.

We can illustrate these general principles with some soluble (but not necessarily realistic) examples. Let the only nonvanishing component of $A_\mu(x)$ be A_3, which is taken to be a function of $x_0 = t$ only,[9] vanishing at $|t| = \infty$. Then the solutions ψ_j of (15) can be Fourier-transformed in space:[10]

$$\psi_j \sim \chi_j(t) e^{i\mathbf{p}\cdot\mathbf{x}} \tag{22}$$

and in this simple case ψ_j and χ_j are related by a multiplicative factor. Consider first, instead of (13), the second-order Dirac equation

$$[(\partial_\mu + ieA_\mu)^2 + M^2 + \tfrac{1}{2}e\sigma^{\mu\nu}F_{\mu\nu}]\chi = 0 \,. \tag{23}$$

For a spinless particle ($\sigma^{\mu\nu}\to 0$) our choice of A_μ leads to

$$[-\partial_0^2 + p_3^2 - (p_3 + eA_3)^2]\chi = (\mathbf{p}^2 + M^2)\chi = \omega^2\chi \,, \tag{24}$$

which is a Schrödinger equation with time as the variable instead of space, a potential $V = p_3^2 - (p_3 + eA_3)^2$, and energy eigenvalue ω^2. Note that this always exceeds the maximum value of V by at least $p_\perp^2 + M^2[\mathbf{p}_\perp = (p_1, p_2)]$. Now pair production corresponds to a reflection process, in which there is no particle present at $t = -\infty$, but at $t = \infty$ there is a particle moving forward in time and an antiparticle moving backward in time. The reflection coefficient R for a smooth barrier lying below the particle energy has an exponentially small factor, in just the same way that the transmission coefficient has for a barrier higher than the energy.[11]

In consequence, one does not expect dramatic resonance effects for spin-0 pair production, consistent with the limitations imposed by (2). However, for a spin-$\frac{1}{2}$ particle in an electric field the non-Hermitian term $\sigma^{\mu\nu}F_{\mu\nu}$ plays an essential role.

Our first example involves a sum of step functions for A_μ leading to an electric field composed of δ functions. It should not be thought that these singularities are what drive the resonance phenomena; later, we will discuss resonance in the presence of smooth potentials. We write

$$eA_3(t) = \begin{cases} eA, & 0 \le t \le t_A \,, \\ 0 & \text{otherwise} \,, \end{cases} \tag{25}$$

with all other components of A_μ vanishing. The Dirac equation can easily be solved in either first- or second-order form; we recommend that the reader study the second-order form (23) with and without the spin term, which will show that no resonance can occur for spin-0 pair production. As for the Dirac equation (13), it reads (after multiplying by γ_0)

$$[i\partial_0 + \alpha_3(p_3 + eA_3) - \beta' m]\chi_j(t) = 0 \tag{26}$$

with

$$\alpha_i = \gamma_0\gamma_i, \quad \beta' m = \gamma_0 M - \boldsymbol{\alpha}_\perp\cdot\mathbf{p}_\perp \,, \qquad m \equiv (p_\perp^2 + M^2)^{1/2} \,. \tag{27}$$

This involves only two anticommuting Hermitian matrices, α_3 and β', and so can be written in 2×2 matrix form. It is convenient to choose

$$\beta' = \sigma_3, \quad \alpha_3 = -\sigma_1 \tag{28}$$

and the free orthonormal χ_j are

$$\chi_{10} = \begin{bmatrix} \cos\frac{1}{2}\theta \\ \sin\frac{1}{2}\theta \end{bmatrix} e^{-i\omega t} \,, \tag{29a}$$

$$\chi_{20} = \begin{bmatrix} -\sin\frac{1}{2}\theta \\ \cos\frac{1}{2}\theta \end{bmatrix} e^{i\omega t} \,, \tag{29b}$$

where

$$\cos\theta = m/\omega, \quad \sin\theta = p_3/\omega \tag{30}$$

with θ lying between 0 and $\pi/2$. According to (17), the solution to the Dirac equation (26) which approaches a multiple of χ_{20} as $t\to-\infty$ contains information on R, the pair-production amplitude, as $t\to+\infty$. Specifically, we require the evolution of the function ψ_2, subject to the boundary condition

$$\psi_2(t) = (1 + iT)\chi_{20}(t), \quad t < 0 \tag{31}$$

which is just (17b) under the condition that T and R are diagonal, with matrix elements T,R respectively (after factoring out a δ function[9]). R is identified from the behavior of ψ_2 for $t > t_A$, which is a unitary transformation (i.e., rotation) on (31):

$$\psi_2(t) = (1 + iT)e^{-iH_0 t}e^{-iH_A t_A}\chi_{20}(0) \tag{32}$$

$$= \chi_{20}(t) + iR\chi_{10}(t), \quad t > t_A \,, \tag{33}$$

where

$$H_0 = \sigma_1 p_3 + \sigma_3 m \,, \tag{34a}$$

$$H_A = \sigma_1(p_3 + eA) + \sigma_3 m \,, \tag{34b}$$

and (33) is a rewriting of (17a). Let us put (32) in the form

$$U(t)\chi_{20}(0)=(1+iT)^{-1}[\chi_{20}(t)+iR\chi_{10}(t)] , \tag{35}$$

where U is a unitary transformation; the orthonormality of the χ_{j0} leads to

$$1+|R|^2=|1+iT|^2 \tag{36}$$

as in (21).

Observe that if T (and hence also R) has a pole, then the χ_{20} term in (35) will be missing. This will in fact be the case if the rotation $\exp(-iH_A t_A)$ in (32) is a rotation through an odd multiple of π about an axis, defined in (34b), which is orthogonal to the axis of (34a). Such a rotation takes χ_{20} into χ_{10}. These conditions amount to

$$\omega_A t_A=(N+\tfrac{1}{2})\pi, \quad N=0,1,2,\ldots, \qquad \omega_A=[(p_3+eA)^2+m^2]^{1/2} , \tag{37}$$

$$p_3(p_3+eA)+m^2=0 \quad \text{or } p_3eA=-\omega^2 . \tag{38}$$

Note that (37) and (38), for a given A and t_A, uniquely determine p_3 and p. These conditions can only be satisfied for strong fields; the reality of p_3 implies

$$|eA|\geq 2m \tag{39}$$

while the reality of p requires

$$1-\left[1-\frac{4m^2}{e^2A^2}\right]^{1/2}<2\left[\frac{\pi(N+\frac{1}{2})}{eAt_A}\right]^2 <1+\left[1-\frac{4m^2}{e^2A^2}\right]^{1/2} . \tag{40}$$

As one might expect, near threshold ($|eA|\simeq 2m$), t_A must be $O(m^{-1})$:

$$mt_A\simeq 2^{-1/2}\pi(N+\tfrac{1}{2}) \quad \text{for } |eA|\simeq 2m . \tag{41}$$

For example, the $N=1$ resonance gives $t=3.3m^{-1}$. When the conditions (37) and (38) are not satisfied or the reality conditions (39) and (40) on p are violated, the physical-region pole becomes a resonance. We see this through the explicit form of R:

$$R^c=\frac{-\sin\omega_A t_A \sin(\theta_A-\theta)}{\cos\omega_A t_A+i\sin\omega_A t_A\cos(\theta_A-\theta)} , \qquad \sin\theta_A=\frac{p_3+eA}{\omega_A} . \tag{42}$$

Here we have reinstated the superscript c to remind the reader that so far we have only calculated the connected amplitude. To see what happens for the full amplitude R of Eq. (4), suppose that R^c has a simple Breit-Wigner form

$$R^c=\frac{Q}{\omega-\omega_0+i\Gamma} \tag{43}$$

and we form the connected probability P_1^c, as in (8), by integration over ω (for simplicity, ignoring the electron mass):

$$P_1^c=\int_0^\infty d\omega|R^c|^2=\frac{\pi}{2\Gamma}|Q|^2=2\,\mathrm{Im}W , \tag{44}$$

where the last equality follows from (10). From (9), the probability for production of a single pair is

$$P_1=\int d\omega\frac{|Q|^2}{(\omega-\omega_0)^2+\Gamma^2}\exp\left[-\frac{\pi|Q|^2}{2\Gamma}\right] =\frac{\pi|Q|^2}{2\Gamma}\exp\left[-\frac{\pi|Q|^2}{2\Gamma}\right] \tag{45}$$

which goes to zero as $\Gamma\rightarrow 0$, and has a maximum value of e^{-1} at $|Q|^2=\pi/2\Gamma$.

The reader may be concerned at this point that poles in R^c, T^c come from the singular (δ-function) electric fields associated with the choice of A_μ in (25). We will show that this worry is unfounded by working the problem backwards, finding a smooth rotation of antiparticle to particle and constructing from that rotation the required smooth vector potential. Before doing that we point out that there are smooth vector potentials for which the Dirac equation is exactly soluble. It is not easy, however, to find the corresponding reflection coefficient R^c except by detailed numerical work. The resemblance of these smooth potentials to that found by solving the problem backwards suggests that the soluble examples also show sharp resonances for a range of parameters.

In Table I we offer three vector potentials $A_3(t)$, each one corresponding to a different way of smoothing a step function, for which the Dirac equation is soluble. Our original example in Eq. (25) consists of two step functions, so it is necessary to patch together the solutions given in Table I in different time domains, with different parameter values, in order to complete the smoothed solution. Case 1 of Table I is just a constant electric field; it can be used to construct a trapezoidal $A_3(t)$ by patching together with the solution for constant $A_3(t)$. Cases 2 and 3 approach step functions as $\Gamma\rightarrow\infty$. Resonance phenomena are possible when $\Lambda>m$, $\Gamma>m$, that is, $E>m^2e^{-1}$.

The essence of the phenomenon of resonance is the rotation of a free particle solution to a free antiparticle solution (or vice versa), which is always a rotation by an odd multiple of π. Let χ denote any solution of the two-dimensional Dirac equation (26), normalized to unity, and define a current $\mathbf{J}$ by

$$\mathbf{J}=\chi^\dagger\sigma\chi, \quad \mathbf{J}^2=1 . \tag{46}$$

From the Dirac equation it follows that

$$\dot{\mathbf{J}}=2\mathbf{\Omega}\times\mathbf{J}, \quad \mathbf{\Omega}=(p_3+eA_3(t),0,m) . \tag{47}$$

In the notation of Eq. (29), a free particle corresponds to $\mathbf{J}=(\sin\theta,0,\cos\theta)$ and a free antiparticle to the negative of this. It is easy to find a smooth interpolation from $-\mathbf{J}$ to $\mathbf{J}$, and from this we can extract a smooth vector potential. Define

$$J_1=-\sin\Theta\cos\frac{\phi}{2}+\zeta\cos\Theta\sin\phi , \qquad J_2=-\sin\frac{\phi}{2}\left[1-4\zeta^2\cos^2\frac{\phi}{2}\right]^{1/2} , \qquad J_3=-\cos\Theta\cos\frac{\phi}{2}-\zeta\sin\Theta\sin\phi , \tag{48}$$

TABLE I. Potentials for which the Dirac equation is soluble and the solutions themselves. Notation for the special functions is standard.

$eA_3(t)$	Solutions to the (second-order) Dirac equations	Parameter values
(1) $\Lambda\Gamma t$	$\psi_\nu(z), \psi_{\nu-1}(z)$ where $\psi_\nu(z)=e^{-z^2/2}\frac{\Gamma(\nu+1)}{2\pi i}\int dt\, t^{-\nu-1}e^{-t^2+2tz}$ (Hermite)	$z=e^{-i\pi/4}(\Lambda\Gamma)^{1/2}[t+(\Lambda\Gamma)^{-1}p_3]$ $\nu=\frac{-im^2}{2\Lambda\Gamma}$
(2) $\frac{1}{2}\Lambda[1+\epsilon(t)(1-e^{-\Gamma\vert t\vert})]$	For $t>\theta$: $\rho^{\pm is}e^{-iq\rho}\,{}_1F_1(\alpha,\gamma;z)$ (Confluent hypergeometric)	$\rho=e^{-\Gamma t}$, $s=\frac{[(p_3+\Lambda)^2+m^2]^{1/2}}{\Gamma}$, $\alpha=i\left[s-\frac{p_3+\Lambda}{\Gamma}\right]$, $\gamma=1+2is$, $q=\frac{\Lambda}{2\Gamma}$, $z=2iq\rho$
(3) $\Lambda\tanh\Gamma t$	$P\begin{Bmatrix}\theta & \infty & 1\\ \alpha & \beta & \gamma\\ -\alpha & 1-\beta & -\gamma\end{Bmatrix} z$ [Riemann (essentially hypergeometric)]	$\alpha=\frac{1}{2\Gamma}[(p_3+\Lambda)^2+m^2]^{1/2}$, $\beta=\frac{1}{2}\{1+[1+4\Gamma^{-2}(\Lambda^2\pm i\Lambda\Gamma)]^{1/2}\}$, $\gamma=\frac{1}{2\Gamma}[(p_3-\Gamma)^2+m^2]$, $z=\frac{1}{2}(1-\tanh\Gamma\tau)$

where

$$0<2\zeta<\min(1,\tan\Theta)\ . \qquad (49)$$

Here ζ is a constant and $\phi(t)$ varies smoothly from 0 at $t=-\infty$ to 2π at $t=\infty$, which will be seen from (48) to change the sign of $\mathbf{J}$. Note that J_2 is always real if $|\zeta|<\frac{1}{2}$. By substitution in (47) we find the relations

$$\dot{\phi}=-4mJ_2K^{-1},\quad eA_3=-\omega K^{-1}\sin\phi/2\ , \qquad (50)$$

where

$$K=\sin\theta\sin\phi/2+2\zeta\cos\theta\cos\phi\ . \qquad (51)$$

Observe that K is always positive, given the inequality in (49). It is easy to solve the differential equation (50) numerically, and even easier to find $E_3=\dot{A}_3$ as a function of ϕ:

$$\frac{eE}{m^2}=-4\zeta J_2K^{-3}\cos\phi/2\ . \qquad (52)$$

In Fig. 1 we plot ϕ and E in units of m^2e^{-1} for $\zeta=0.25$, $\cos\theta=1/\sqrt{2}$; it will be seen that the maximum value of E is about 5 and the time scale is $O(m^{-1})$.

Let us conclude by discussing qualitatively what might happen for more complicated situations, where E varies both in space and time.[12] The propagator S_{Ac} can be written as a proper-time integral[13] of a certain kernel the essential part of which

$$K(x,y;s)=\langle x|e^{iHs}|y\rangle\ , \qquad (53)$$

where the proper-time Hamiltonian H is displayed in the second-order Dirac equation (23). We can write K as a path integral and look for dominant paths corresponding to reflection in time (that is, $dx^0/ds<0$) which signify the act of pair production.[14] As we have already mentioned, the reflection is by a barrier of height less than the

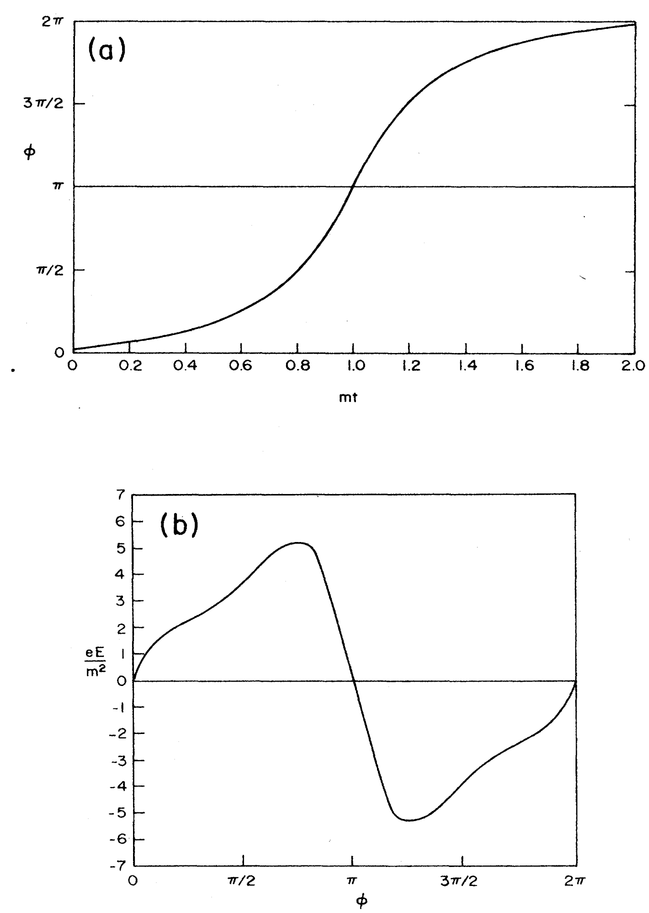

FIG. 1. (a) The function $\phi(t)$ vs t for $\zeta=0.25$, $\cos\theta=1/\sqrt{2}$. (b) The electric field in units of m^2/e as a function of ϕ, for the same parameter values.

particle energy, and the dominant paths are not classical, but instantonlike: they make an excursion into the complex plane. For example, in a constant field if we make the change $s \rightarrow is$, $x^0 \rightarrow ix^0$ the usual hyperbolic motion becomes simple harmonic motion, and reflection occurs after half a cycle. For spin-0 particles each such reflection introduces an exponentially damped factor into K, and resonance effects will not occur even if the electric field is constructed so that repeated reflections are possible (as in our soluble example). But for spin $\frac{1}{2}$ the spin-dependent term in H yields exponentially growing factors (such as[13] $\cosh Es/2$ for a constant electric field E) which can balance the exponential damping of the reflection instantons, allowing for the sum over paths to be resonant. Further analysis along these lines will be given in a subsequent publication.

What do our remarks here have to do with the nuclear-physics experiments which motivated this work? Clearly, the spatial dependence of the electric field is dominant for certain effects, such as the diving of the 1s level below $-2M$ when $Z\alpha$ exceeds a critical value somewhat above unity, which was the theoretical stimulus[15] for the present experiments, and we have nothing to say about that. Certain qualitative features of the electric fields produced by heavy-nucleus collisions do resemble the field of our idealized example in Fig. 1. Consider a spherical volume of radius M^{-1} centered on the point where the two nuclei would meet if there were no Coulomb forces; the region outside this sphere is relatively unimportant both because the electric field is subcritical and because the relevant matrix elements involve integrals of E multiplied by $\sin pr$, and $p \simeq M$. Inside this sphere we have $|E| \sim Ze/r^2 \geq (Z\alpha)E_c$ (see the figures in Ref. 7). Just as important as the magnitude of E is its time variations. At a point at radius $r \sim (2M)^{-1}$ along the initial line of approach of the two nuclei, the radial electric field is inward when the nuclei just cross the sphere of radius M^{-1}, but is directed outward as the nuclei cross $r=(2M)^{-1}$; along the line which the nuclei follow after the collision, the radial field is outward at first, becoming inward after the collision. In both cases, the radial electric (up to an overall sign) is similar to that plotted in Fig. 1, in both magnitude and time variation, and it may be reasonable to look for resonance effects such as we have. If our resonance mechanism is indeed acting in the heavy-nucleus collisions, it is clear that the momentum p of the electron or positron will be $O(M)$, and thus the total pair energy will be around $3M$, as observed. A crucial prediction of our scenario is that the position and width of the resonances will vary (but not dramatically, i.e., not like Z^{20}) as the nuclear-collision parameters change. It is not easy to say what governs the resonance width, but a reasonable lower limit comes from the inverse of the time it takes the nuclei to enter, then leave, a sphere of radius M^{-1}, which is $\beta M/2$ with β the nucleus velocity. Since $\beta=0.1$, this width is about 25 keV, as seen in the experiments. While our speculations so far are encouraging, we admit that we have no idea how the e^+-e^- pair manages to come out with equal and opposite momenta, once one goes beyond the spatially constant fields we explicitly discussed. We are presently engaged in computer experiments to see if these very qualitative remarks stand up to closer scrutiny; results will be published elsewhere.

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[8]Our spinors are normalized so that $\bar{u}_s(p)u_s(p) = -\bar{v}_s(p)v_s(p)=2M$.

[9]In this case S-matrix elements such as R of Eq. (3) have an overall factor of $(2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}')\delta_{ss}$ and the physically interpretable quantity is $V^{-1}\sum|R|^2$, that is, the number of particles produced per unit volume V.

[10]In (19) for $j=2$, p is the negative of the antiparticle momentum; cf. Eq. (12).

[11]It is easy, using the techniques of L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, New York, 1977), Sec. 52, to find this exponential factor for a field which is constant for $t<T$ and zero otherwise; as $T \rightarrow \infty$, one recovers the famous results of J. Schwinger, Phys. Rev. **82**, 664 (1951).

[12]One simple case which is exactly soluble is the longitudinal traveling wave $E=\hat{e}_3E(x^3-x^0)$ which, unlike the transverse traveling wave, does lead to pair production. We will analyze this case in detail elsewhere.

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