Angular correlations in the decay $B \to VV$ and CP violation

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We study the exclusive decays of B mesons into a pair of spin-1 mesons. We look at the occurring asymmetries that could signal CP violation and estimate their relative size. We find that angular asymmetries are not significantly smaller than partial-rate asymmetries, and that their study can help disentangle the complicated dynamics of these decays.

I. INTRODUCTION

The topic of CP violation in the B-meson system has been the subject of extensive studies.[1] Most authors have been concerned with the study of partial-rate asymmetries occurring in different exclusive channels. Bigi and Sanda[2] have classified and estimated the different possibilities in the decay of a B meson into two pseudoscalars. In particular they have shown which are the channels where we expect largest and cleanest signals. These CP-odd observables can originate either through mixing or via interference of at least two amplitudes contributing to the same process. In the latter one requires, in addition to CP violation, the presence of unitarity phases which depend on hadron dynamics and are very difficult to estimate reliably.

In this paper we intend to look at the asymmetries that occur in the decay of a pseudoscalar B into two vector mesons.[3] What can be gained from this study is access to information on the unitarity phases, or alternatively one can obtain signals that do not depend on such phases, similar to the ones appearing in hyperon decays.[4] In Sec. II we will look at the kinematics of the reaction, and thus find which are the possible asymmetries. In Sec. III we will consider these asymmetries in the standard model to see how they can arise, and finally in Sec. IV we will attempt to estimate their relative sizes for specific examples.

The new type of signal that will emerge corresponds to triple-product correlations involving the momentum of one of the vector mesons and the two polarizations. They thus require reconstruction of the decays of both spin-1 mesons, and in practice one would then be looking at correlations between momenta of the final particles.

It was realized a long time ago that under time-reversal invariance both a particle's momentum and spin are reversed and hence that any triple scalar product of them is odd under the naive time-reversal operation. It was also noted that when one considers the full antilinear nature of the quantum-mechanical T operator, such correlations could appear even when all interactions conserved T. The are thus not very good indicators of T violation unless one can calculate and subtract the induced terms that mimic T nonconservation. This is very difficult to do. However, as was realized in the study of hyperon decays, this can be circumvented by comparing a pair of CP-conjugate processes to obtain a correlation that is truly CP odd.

II. KINEMATICS

Consider the weak decay of a B meson (containing a $\bar{b}$) into a pair of vector mesons, and let us define our notation as $B(p) \to V_1(k,\epsilon_1)V_2(q,\epsilon_2)$. We can write the most general invariant matrix element for this decay as a sum of three terms that we will call s, d, and p amplitudes in the form $M \equiv as + bd + icp$:

$$M = a\epsilon_1\cdot\epsilon_2 + \frac{b}{m_1 m_2}(p\cdot\epsilon_1)(p\cdot\epsilon_2) + i\frac{c}{m_1 m_2}\epsilon^{\alpha\beta\gamma\delta}\epsilon_{1\alpha}\epsilon_{2\beta}k_\gamma p_\delta \ , \tag{2.1}$$

where we denote the masses of particles V_1 and V_2 by m_1 and m_2, respectively. In general the scalars a, b, and c are complex, and can receive contributions from several amplitudes with different phases. Occasionally we shall loosely refer to them as isospin amplitudes. We separate the phases explicitly by writing

$$\begin{aligned} a &= \sum_j |a_j| e^{i\delta_{sj}} e^{i\phi_{sj}} \ , \\ b &= \sum_j |b_j| e^{i\delta_{dj}} e^{i\phi_{dj}} \ , \\ c &= \sum_j |c_j| e^{i\delta_{pj}} e^{i\phi_{pj}} \ , \end{aligned} \tag{2.2}$$

and identify them as one of two kinds. The ϕ_i are CP-violating phases in the weak Hamiltonian, and the factor of i in front of the p amplitude defines our convention in such a way that if all the ϕ_i are zero CP is conserved. The phases δ_i are the so-called "unitarity phases" which can arise since we have to consider all orders in the strong interaction. The sums extend over all amplitudes that can contribute to the decay, and may refer to, for example, different isospin configurations. One can then use CPT invariance to show that the corresponding matrix element for the antiparticle decay $\bar{B}(p) \to \bar{V}_1(k,\epsilon_1)\bar{V}_2(q,\epsilon_2)$ is given in terms of the quantities defined so far by

$$\bar{M} = \bar{a}\epsilon_1\cdot\epsilon_2 + \frac{\bar{b}}{m_1 m_2}(p\cdot\epsilon_1)(p\cdot\epsilon_2) - i\frac{\bar{c}}{m_1 m_2}\epsilon^{\alpha\beta\gamma\delta}\epsilon_{1\alpha}\epsilon_{2\beta}k_\gamma p_\delta \ , \tag{2.3}$$

where

$$\bar{a}=\sum_j |a_j|e^{i\delta_{sj}}e^{-i\phi_{sj}} ,$$
$$\bar{b}=\sum_j |b_j|e^{i\delta_{dj}}e^{-i\phi_{dj}} , \tag{2.4}$$
$$\bar{c}=\sum_j |c_j|e^{i\delta_{pj}}e^{-i\phi_{pj}} .$$

Note that *CPT* transforms each vector meson into its antiparticle with the same momentum and changes the sign of the polarization so that the three Lorentz scalars remain the same. Our task now is to find physical observables that are sensitive to the phases ϕ_i. To do so we first square the matrix elements to obtain

$$|M|^2= |a|^2|\epsilon_1\cdot\epsilon_2|^2+\frac{|b|^2}{m_1^2m_2^2}|(k\cdot\epsilon_2)(q\cdot\epsilon_1)|^2+\frac{|c|^2}{m_1^2m_2^2}|\epsilon^{\alpha\beta\mu\nu}\epsilon_{1\alpha}\epsilon_{2\beta}k_\mu p_\nu|^2$$
$$+2\frac{\mathrm{Re}ab^*}{m_1m_2}(\epsilon_1\cdot\epsilon_2)(k\cdot\epsilon_2)(q\cdot\epsilon_1)+2\frac{\mathrm{Im}ac^*}{m_1m_2}(\epsilon_1\cdot\epsilon_2)\epsilon^{\alpha\beta\mu\nu}\epsilon_{1\alpha}\epsilon_{2\beta}k_\mu p_\nu+2\frac{\mathrm{Im}bc^*}{m_1^2m_2^2}(k\cdot\epsilon_2)(q\cdot\epsilon_1)\epsilon^{\alpha\beta\mu\nu}\epsilon_{1\alpha}\epsilon_{2\beta}k_\mu p_\nu . \tag{2.5}$$

Summing over the polarization yields $\hat{\Gamma}\equiv\sum_{\lambda_1\lambda_2}|M|^2$:

$$\sum_{\lambda_1\lambda_2}|M|^2=|a|^2(2+x^2)+|b|^2(x^2-1)^2+|c|^2 2(x^2-1)+2\,\mathrm{Re}ab^*x(x^2-1) , \tag{2.6}$$

where the parameters x for a specific reaction is

$$x=\frac{k\cdot q}{m_1m_2}=\frac{m_B^2-m_1^2-m_2^2}{2m_1m_2} . \tag{2.7}$$

All the terms in Eq. (2.6) contain the *CP*-violating phases. The first three terms correspond to absolute squares of s, d, and p amplitudes and are sensitive to interference between the different terms that can contribute to each partial wave. They give rise to partial-rate asymmetries similar to the ones occurring in the exclusive decays of B mesons into two pseudoscalars.[2] The last term is an interference of s and d amplitudes and gives rise to partial-rate asymmetries from relative s-d phases.

These partial-rate asymmetries have a very complicated dependence on all the phases, and in general it will be very difficult to compare any result with a theoretical prediction. One obtains

$$\frac{\Delta\Gamma}{\Gamma}=\frac{4}{\hat{\Gamma}}\sum_{i>j}[a_ia_j\sin(\delta_{si}-\delta_{sj})\sin(\phi_{si}-\phi_{sj})(2+x^2)+b_ib_j\sin(\delta_{di}-\delta_{dj})\sin(\phi_{di}-\phi_{dj})(x^2-1)^2$$
$$+c_ic_j\sin(\delta_{pi}-\delta_{pj})\sin(\phi_{pi}-\phi_{pj})2(x^2-1)]+\frac{4}{\hat{\Gamma}}\sum_{ij}a_ib_j\sin(\delta_{si}-\delta_{dj})\sin(\phi_{si}-\phi_{dj})x(x^2-1) . \tag{2.8}$$

It appears that in general such a signal will be virtually impossible to analyze. There are, however, simple cases in which it might prove useful. In particular, if there is only one amplitude contributing to each partial wave then the only term that appears is that of s-d interference. To study isospin interference, the corresponding pseudoscalar channel is clearly better.

When one sums over polarizations there are not enough independent four-vectors left to form triple-product correlations. That is why interference terms with the p amplitude are not present in Eq. (2.6). The same would be true if we summed over only one of the polarizations. One can, however, define quantities sensitive to such interference terms. For that, one looks at configurations with a definite sign for the triple product.

One way to do so is to define the asymmetry:

$$A_B=\frac{N_{\text{events}}(\mathbf{k}\cdot\epsilon_1\times\epsilon_2>0)-N_{\text{events}}(\mathbf{k}\cdot\epsilon_1\times\epsilon_2<0)}{N_{\text{total}}} \tag{2.9}$$

and then to add the corresponding quantity for the antiparticles $A_{\bar{B}}$. In this case one is looking at

$$A_B\sim\mathrm{Im}ac^*\sim|ac|\sin(\delta+\phi) ,$$
$$A_B+A_{\bar{B}}\sim|ac|\cos(\delta)\sin(\phi) , \tag{2.10}$$
$$A_B-A_{\bar{B}}\sim|ac|\sin(\delta)\cos(\phi) .$$

Note that A_B can be generated by either *CP* violation or unitarity phases, but that $A_B+A_{\bar{B}}$ is a clean signal of *CP* nonconservation. It could also happen that one wants to determine experimentally what the unitarity phases are; in that case one looks at $A_B-A_{\bar{B}}$. We can now evaluate the kinematics of this asymmetry in the rest frame of the decaying B. A convenient way to do so is to parametrize the polarizations of the decay particles with angles defined in their rest frames as

$$\epsilon_1=\sin\theta_1\cos\phi_1\epsilon_1^{(1)}+\sin\theta_1\sin\phi_1\epsilon_1^{(2)}+\cos\theta_1\epsilon_1^{(3)} ,$$
$$\epsilon_2=\sin\theta_2\cos\phi_2\epsilon_2^{(1)}+\sin\theta_2\sin\phi_2\epsilon_2^{(2)}+\cos\theta_2\epsilon_2^{(3)} , \tag{2.11}$$

in terms of transverse and longitudinal polarization vectors that can be easily boosted to the B rest frame. With the momentum of V_1 defining the quantization axis they become

$$\epsilon_1^{(1)}=\epsilon_2^{(1)}=(0,1,0,0),\quad \epsilon_1^{(2)}=\epsilon_2^{(2)}=(0,0,1,0)\ ,$$
$$\epsilon_1^{(3)}=\left[\frac{|\mathbf{k}|}{m_1},0,0,\frac{E_k}{m_1}\right],\quad \epsilon_2^{(3)}=\left[\frac{|\mathbf{k}|}{m_2},0,0,\frac{-E_q}{m_2}\right]\ . \tag{2.12}$$

Summation over polarizations is now equivalent to integration over the solid angles $d\Omega_1 d\Omega_2$ up to a normalization factor:

$$\sum_{\lambda_1\lambda_2}|M|^2\rightarrow\frac{9}{16\pi^2}\int d\Omega_1 d\Omega_2|M|^2\ . \tag{2.13}$$

We proceed to find A_B by integrating over the regions where the triple product has a definite sign and normalizing both A_B, $A_{\bar{B}}$ to the total number of $B\rightarrow V_1V_2$ events we obtain ($A\equiv A_B+A_{\bar{B}}$)

$$A=\frac{\left[\frac{4}{\pi}\right]\sqrt{x^2-1}\sum_{ij}a_ic_j\cos(\delta_{si}-\delta_{pj})\sin(\phi_{si}-\phi_{pj})}{(2+x^2)|a|^2+(x^2-1)^2|b|^2+2(x^2-1)|c|^2+2x(x^2-1)\mathrm{Re}ab^*}\ . \tag{2.14}$$

When a full reconstruction is done one would have, for example, that V_1V_2 decay, respectively, into two pseudoscalars $\Phi_1\Phi_2$ and $\Phi_3\Phi_4$. In this case the correlation becomes $\mathbf{k}\cdot\mathbf{p}_1\times\mathbf{p}_3$.

More complicated correlations may turn out to be kinematically favored, in particular the following quantity will prove to be better:

$$\tilde{A}_B=\frac{N_{\text{events}}(\mathbf{k}\cdot\epsilon_2\mathbf{q}\cdot\epsilon_1\mathbf{k}\cdot\epsilon_1\times\epsilon_2>0)-N_{\text{events}}(\mathbf{k}\cdot\epsilon_2\mathbf{q}\cdot\epsilon_1\mathbf{k}\cdot\epsilon_1\times\epsilon_2<0)}{N_{\text{total}}}\ , \tag{2.15}$$

and in this case the result is ($\tilde{A}=\tilde{A}_B+\tilde{A}_{\bar{B}}$)

$$\tilde{A}=\left[\frac{4}{\pi}\right]\frac{\sqrt{x^2-1}}{\hat{\Gamma}}\left[x\sum_{ij}a_ic_j\cos(\delta_{si}-\delta_{pj})\sin(\phi_{si}-\phi_{pj})+(x^2-1)x\sum_{ij}a_ic_j\cos(\delta_{di}-\delta_{pj})\times\sin(\phi_{di}-\phi_{pj})\right]\ . \tag{2.16}$$

It is interesting to compare this system with others studied in the literature. The triple-product correlation is the equivalent of observing a net circular polarization in the case of the decay of a neutral kaon into two photons[5,6] (except that in this case our initial state does not have the CP properties of K_L so we have to add incoherently equal amounts of particle and antiparticle decays). In the study of hyperon decays there are both triple-product asymmetries (β-type terms), and partial-rate asymmetries; one also finds α-type scalar-product correlations.[7] These last ones have no counterpart here, where observation of just one polarization does not lead to new signals. They are also absent in the case where B decays into a vector and a pseudoscalar. This occurs because factors such as $(k\cdot\epsilon)$ are present in every term of the amplitude, and thus they always appear squared in the decay distributions. This seems to be a general feature of vector particles. One would find such correlations in B decays into hyperon. A particular example corresponds to the Λ polarization recently studied by Eilam and Soni.[8]

We can finish this section by comparing the relative sizes of all the signals as expected from purely kinematical considerations. For this we need, in addition to the parameter x, two more quantities. Let us denote by y a typical ratio of d or p to s amplitudes. Using the form-factor parametrization of Altomari and Wolfenstein or Bauer, Stech, and Wirbel[9–11] one finds

$$y\sim\frac{2m_1m_2}{(m_B+m_1)(m_B+m_2)}\ . \tag{2.17}$$

Finally one needs to know the relative sizes of interfering isospin amplitudes. Since we shall not attempt to calculate these let us just call such a ratio r. The pattern for the different signals is then

$$\begin{aligned}\frac{\Delta\Gamma}{\Gamma}&\sim r\sin(\delta_i)\sin(\phi_i)+\frac{x(x^2-1)}{2+x^2}y\sin(\delta_l)\sin(\phi_l)\ ,\\ A&\sim\frac{1}{\pi}\frac{\sqrt{(x^2-1)}}{2+x^2}y\cos(\delta_s-\delta_p)\sin(\phi_s-\phi_p)\ ,\\ \tilde{A}&\sim\frac{1}{\pi}\frac{\sqrt{(x^2-1)}}{2+x^2}[xy\cos(\delta_s-\delta_p)\sin(\phi_s-\phi_p)+(x^2-1)y^2\cos(\delta_s-\delta_p)\times\sin(\phi_s-\phi_p)]\ .\end{aligned} \tag{2.18}$$

We have at present no way to estimate the possible unitarity phases. It is known that in kaon physics those phases are generally small, and it has been argued in the literature that they are not necessarily small in B decays. We should, therefore, keep in mind that partial-rate asymmetries are proportional to such phases and that they are suppressed if the phases are small. On the contrary, the angular correlations defined above do not vanish with vanishing unitarity phases. Moreover, and perhaps most important, we saw before that one can use these asymmetries to gain experimental access to either the CP-odd phases or the unitarity phases.

TABLE I. Relative sizes of amplitudes. The relative sizes of the different terms as they appear from purely kinematical considerations.

Term	Relative size	$x\sim 20$ $y\sim 0.04$	$x\sim 2$ $y\sim 0.2$
s^2	1	1	1
d^2	$\frac{(x^2-1)^2}{2+x^2}y^2$	0.5	0.04
p^2	$\frac{2(x^2-1)}{2+x^2}y^2$	0.003	0.03
s-d	$\frac{x(x^2-1)}{2+x^2}y$	0.7	0.15
s-p	$\frac{1}{\pi}\frac{\sqrt{(x^2-1)}}{2+x^2}y$	$0.0005\rightarrow 0.002$	0.02
s-p	$\frac{1}{\pi}\frac{x\sqrt{(x^2-1)}}{2+x^2}y$	$0.01\rightarrow 0.05$	$0.03\rightarrow 0.06$
d-p	$\frac{1}{\pi}\frac{(x^2-1)^{3/2}}{2+x^2}y^2$	$0.01\rightarrow 0.05$	$0.01\rightarrow 0.05$

There are two different regimes that we can look at. When the decay products have masses between one-third and one-half of the B mass, x has a value around 2 and y a value around 0.2. It has also been argued in the literature that it is in this case when one can expect large final-state strong-interaction effects.[9] In the second regime the decay masses are small, the values of x are large, say around 20 and y is of order a few percent. One might expect final-state interactions to be less important, but we do not know much about possible real intermediate states. The expected kinematical factors are shown in Table I.

When we estimate the different asymmetries it is important to bear in mind that s-d interference can be very important for low-mass decay products, since it can substantially reduce the branching ratio. There is no agreement in the literature[10] as to whether or not this interference is destructive. This point, as well as the general form of the amplitudes is under further study. In any case if the interference is indeed destructive there will be an enhancement of the asymmetries relative to the total decay rate. From the numbers in Table I we can also see that $\tilde{A}$ is favored over A and thus we will adopt $\tilde{A}$ as the relevant observable. In both regimes we expect the signals to be around a few percent if there are no additional dynamical suppression factors. One can also see that for some channels the s-d contribution to the partial-rate asymmetry could be of order 1, such as the contribution from interference of amplitudes in the same partial wave (which should be analogous to the pseudoscalar case).

III. OBTAINING THE ASYMMETRY

We now turn to the question of how to generate the CP-odd phases within the standard model. For that we shall select decays in which the b quark turns into an s quark rather than a d quark, although the analysis for that case proceeds in an analogous manner. We are then interested in the effective weak Hamiltonian with $|\Delta B|=|\Delta S|=1$, which is given by[12,13]

$$\begin{aligned}
H_{\text{eff}} &= \frac{-G_F}{2\sqrt{2}}\left[A_c(c^+O_c^+ + c^-O_c^-) + A_t\sum_{i=1,6}c_iO_i\right],\\
O_1 &= (\bar{s}b)_L(\bar{u}u)_L,\quad O_2=(\bar{s}_\alpha b_\beta)_L(\bar{u}_\beta u_\alpha)_L,\\
O_3 &= (\bar{s}b)_L\sum_q(\bar{q}q)_L,\quad O_4=(\overline{s_\alpha}b_\beta)_L\sum_q(\overline{q_\beta}q_\alpha)_L,\\
O_5 &= (\bar{s}\lambda^a b)_L\sum_q(\bar{q}\lambda^a q)_R,\quad O_6=(\bar{s}b)_L\sum_q(\bar{q}q)_R,
\end{aligned}\tag{3.1}$$

with A_c, A_t standing for

$$\begin{aligned}
A_c &= (c_1^2c_2^2-s_2^2)c_3s_3+c_1c_2s_2c_\delta(c_3^2-s_3^2)+ic_1c_2s_2s_\delta,\\
A_t &= (c_1^2s_2^2-c_2^2)c_3s_3+c_1c_2s_2c_\delta(s_3^2-c_3^2)-ic_1c_2s_2s_\delta.
\end{aligned}\tag{3.2}$$

We shall use the coefficients including leading-log corrections with the numbers of Ponce:[13]

$$\begin{aligned}
&c^+=0.77,\quad c^-=1.67,\quad c_1=-0.44,\quad c_2=2.32,\\
&c_3=0.04,\quad c_4=-0.08,\quad c_5-0.05,\quad c_6=-0.01.
\end{aligned}\tag{3.3}$$

As is well known, a CP-odd asymmetry will arise only when there is a relative phase between two amplitudes. In our effective Hamiltonian there are only two complex constants: A_c and A_t. This means that one way to obtain an asymmetry is to look at a process that receives contributions from both types of diagrams, internal W emission and penguins, as sketched in Figs. 1 and 2. This already tells us something about the magnitude of the signal. To first order in Kobayashi-Maskawa (KM) angles $A_t=-A_c$ and there is no CP-violating phase. To find one we go to next order in KM angles, and write $A_t=-A_ce^{i\delta\phi}$, with the result

$$\delta\phi\approx\frac{c_1c_2c_3s_1^2s_2s_3s_\delta}{(s_3+s_2c_\delta)^2}.\tag{3.4}$$

It is interesting to see the rephasing-invariant combination of angles[14] appearing explicitly in Eq. (3.4). We can estimate from bounds on the KM angles $\delta\phi\leq 0.01$. This means that we can expect asymmetries as large as a few percent within the standard model. Also the penguin coefficients are only about 0.05 times the ones coming from internal W emission; an interference will, therefore, be further suppressed unless we look at b-u transitions. It is well known that one then pays the price of potentially large signals with small branching ratios.

A second way to obtain a relative phase is to look at

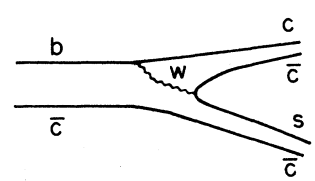

FIG. 1. Internal W emission.

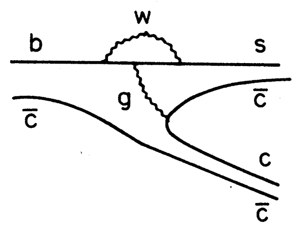

FIG. 2. Penguin contribution.

decays in which a quark-decay amplitude interferes with a weak annihilation. The latter is generally expected to be much smaller than the former[9] thus giving rise to very small asymmetries (we will briefly return to this point when discussing the vacuum-saturation estimate). It has been argued[2] that in special cases this need not be true. We shall not consider this case here.

Also, when one considers neutral B mesons, it is possible to obtain interference via mixing, by looking at final states that are common to B to $\bar{B}$ decays. The time evolution of the B mesons is generally written in the following way:[2]

$$\begin{aligned} |B^0(t)\rangle &= g_+(t)|B^0\rangle + \frac{q}{p} g_-(t)|\bar{B}^0\rangle \ , \\ \overline{|B}^0(t)\rangle &= g_+(t)\overline{|B}^0\rangle + \frac{p}{q} g_-(t)|B^0\rangle \ , \\ g_\pm(t) &= \tfrac{1}{2} e^{-\Gamma_1 t/2} e^{i m_1 t}(1 \pm e^{-\Delta\Gamma t/2} e^{i\Delta m t}) \ , \\ \frac{q}{p} &= \frac{1-\epsilon}{1+\epsilon} \ . \end{aligned} \tag{3.5}$$

For simplicity we will assume that the neutral B or $\bar{B}$ is produced along with a charged B that decays semileptonically, so that we know which one decayed into the pair of vector mesons by tagging the charged B. Writing

$$\begin{aligned} A(B \to VV) &= a_1 s + b_1 d + i c_1 p \ , \\ \bar{A}(\bar{B} \to VV) &= (a_2 s + b_2 d + i c_2 p) e^{i\phi} \ , \end{aligned} \tag{3.6}$$

where ϕ is chosen so that a_1, a_2 are relatively real. To be general we will allow a_i, b_i, c_i to have phases, noting that these will be there only if there are at least two amplitudes contributing to the decay. It is then standard procedure to write

$$\begin{aligned} M(t) \equiv A(B(t) \to VV) &= g_+(t)\left[A - \frac{i}{2}\frac{q}{p}\sin(\Delta m t)\bar{A} \right] \\ &\equiv g_+(t)(as + bd + icp) \ . \end{aligned} \tag{3.7}$$

Defining $\rho \equiv (q/p)e^{i\phi}$ we can directly apply our previous results by simply identifying

$$\begin{aligned} a &= a_1 - \frac{i}{2}\sin(\Delta m t)\rho a_2, \quad b = b_1 - \frac{i}{2}\sin(\Delta m t)\rho b_2 \ , \\ c &= c_1 - \frac{i}{2}\sin(\Delta m t)\rho c_2 \ . \end{aligned} \tag{3.8}$$

The resulting expression for the time-dependent asymmetry induced by mixing looks like the familiar result for pseudoscalar channels:

$$\begin{aligned} \frac{\Delta\Gamma}{\Gamma} = \frac{2\sin(\Delta m t)}{\hat{\Gamma}} [\, & |a_1^* a_2| \mathrm{Im}(\rho)(2+x^2) \\ & + \mathrm{Im}(\rho b_1^* b_2)(x^2-1)^2 \\ & + \mathrm{Im}(\rho c_1^* c_2) 2(x^2-1) \\ & + \mathrm{Im}\rho(a_2 b_1^* + a_1^* b_2) x(x^2-1)] \ . \end{aligned} \tag{3.9}$$

As in the latter, these asymmetries can be substantially large for specific channels. The analysis proceeds in very similar fashion and we shall not pursue it further. Let us instead turn our attention to the angular correlations. If, as usual one ignores the small $\mathrm{Re}\epsilon$:

$$\begin{aligned} \mathrm{Im}(ac^*) - \mathrm{Im}(\bar{a}\,\bar{c}^*) &= 2\,\mathrm{Im}(a_1 c_1^*) \\ &\quad + \tfrac{1}{2}\sin^2(\Delta m t)|\rho|^2 \mathrm{Im}(a_2 c_2^*) \ , \\ \mathrm{Im}(bc^*) - \mathrm{Im}(\bar{b}\,\bar{c}^*) &= 2\,\mathrm{Im}(b_1 c_1^*) \\ &\quad + \tfrac{1}{2}\sin^2(\Delta m t)|\rho|^2 \mathrm{Im}(b_2 c_2^*) \ . \end{aligned} \tag{3.10}$$

This means that up to small terms proportional to $\mathrm{Re}\epsilon$ mixing does not generate CP-odd triple-product correlations. If these are generated otherwise, say by W-exchange-penguin interference, mixing does modify the analysis as specified by Eq. (3.10). It is interesting to note that if one looks at the expression for $|M|^2$ there are triple-product correlations induced by the mixing that look like

$$\sim \sin(\Delta m t)\mathrm{Re}\rho(a_1^* c_2 - a_2 c_1^*)(\epsilon_1 \cdot \epsilon_2)\epsilon^{\alpha\beta\mu\nu}\epsilon_{1\alpha}\epsilon_{2\beta}k_\mu p_\nu \ . \tag{3.11}$$

They are not CP violating and cancel out when one subtracts the CP-conjugate reaction in the same manner as the terms induced by unitarity phases do.

A last way to obtain interference is to consider cascade processes.[15] This method is useful in channels with more than two particles in the final state which we will not consider.

IV. SOME EXAMPLES

There are many channels that can proceed via both internal W emission and penguin diagrams. These include neutral and charged modes such as

$$\begin{aligned} &\bar{B}_d \to \rho^{*+}K^{*-}, D^{*+}D_s^{*-}, \psi K^*, \rho K^* \ , \\ &\bar{B}_s \to K_s^{*+}K^{*-}, \psi\phi, D_s^{*+}D_s^{*-}, \rho\phi \ , \\ &B^- \to \rho K^{*-}, D^* D_s^{*-}, \psi K^{*-} \ , \\ &B_c^- \to D^* K^{*-}, \psi D_s^{*-}, \rho D_s^{*-} \ . \end{aligned} \tag{4.1}$$

To estimate the size of the asymmetries we use the vacuum-saturation approximation; we shall regard this method as an order-of-magnitude estimate to compare the different asymmetries, stressing that its predictions for a given observable are not very reliable. Within this scheme the amplitude for the decay will be a linear combination of the factors

$$
\begin{aligned}
F_1 &= \langle V_1 | J_\mu | B \rangle \langle V_2 | J^\mu | 0 \rangle , \\
F_2 &= \langle V_2 | J_\mu | B \rangle \langle V_1 | J^\mu | 0 \rangle , \\
F_3 &= \langle V_1 V_2 | J_\mu | 0 \rangle \langle 0 | J^\mu | B \rangle , \\
F_4 &= \langle V_1 V_2 | S | 0 \rangle \langle 0 | P | B \rangle ,
\end{aligned} \tag{4.2}
$$

where the last factor involving scalar and pseudoscalar densities can appear when one Fierz rearranges O_5 or O_6. One can argue (see, for example, Ref. 16) that the momentum dependence of the form factors suppresses terms F_3 and F_4 because they are dominated by a pole of mass smaller than the B meson and the form factors are evaluated at $q^2 \approx m_B^2$. A typical factor is, with $m_{\text{pole}} \approx 1$ GeV:

$$
\left[1 - \frac{m_B^2}{m_P^2} \right]^{-1} \approx 0.05 \tag{4.3}
$$

although one can find decays where the suppression is milder. Note that these terms correspond to the weak annihilation process. We will use as examples two charged B decays that illustrate the different regimes: $B^- \to \omega K^{*-}$ and $B_c^- \to \psi D_s^{*-}$. The first one should give large asymmetries since the internal W emission is suppressed by small KM angles but should have a small branching ratio. The second one is expected to have a larger branching ratio and smaller asymmetries. We use the notation of Eq. (4.2) with V_2 being the charged meson. A vacuum-saturation estimate of the matrix elements yields

$$
\begin{aligned}
\langle \omega K^{*-} | H_w | B^- \rangle = \frac{-G_F}{4} \{ & [(\tfrac{4}{3}c^+ + \tfrac{2}{3}c^-)A_c + (\tfrac{1}{3}c_1 + c_2 + \tfrac{1}{3}c_3 + c_4)A_t]F_1 \\
& + [(\tfrac{4}{3}c^+ - \tfrac{2}{3}c^-)A_c + (c_1 + \tfrac{1}{3}c_2 + c_3 + \tfrac{1}{3}c_4 + c_6)A_t]F_2 \\
& + [(\tfrac{4}{3}c^+ + \tfrac{2}{3}c^-)A_c + (\tfrac{1}{3}c_1 + c_2 + \tfrac{1}{3}c_3 + c_4)A_t]F_3 + (\tfrac{32}{9}c_5 + \tfrac{2}{3}c_6)A_t F_4 \} , \\
\langle \psi D_s^{*-} | H_w | B_c^- \rangle = \frac{G_F}{2\sqrt{2}} \{ & [(-\tfrac{4}{3}c^+ - \tfrac{2}{3}c^-)A_c + (\tfrac{1}{3}c_3 + c_4)A_t]F_1 + [(-\tfrac{4}{3}c^+ + \tfrac{2}{3}c^-)A_c + (c_3 + \tfrac{1}{3}c_4 + c_6)A_t]F_2 \\
& + [(-\tfrac{4}{3}c^+ - \tfrac{2}{3}c^-)A_c + (\tfrac{1}{3}c_3 + c_4)A_t]F_3 + (\tfrac{32}{9}c_5 + \tfrac{2}{3}c_6)A_t F_4 \} .
\end{aligned} \tag{4.4}
$$

We parametrize the different form factors following Bauer, Stech, and Wirbel,[9,11] obtaining

$$
\begin{aligned}
F_1 = & -\tilde{g}(m_B + m_1) A_1(m_2^2) \epsilon_1 \cdot \epsilon_2 \\
& + 2A_2(m_2^2) \frac{\tilde{g}}{m_B + m_1} (p \cdot \epsilon_1)(p \cdot \epsilon_2) \\
& + i \frac{2\tilde{g}}{m_B + m_1} V(m_2^2) \epsilon^{\alpha\beta\gamma\delta} \epsilon_{1\alpha} \epsilon_{2\beta} k_\gamma p_\delta , \\
F_2 = & -g(m_B + m_2) \tilde{A}_1(m_1^2) \epsilon_1 \cdot \epsilon_2 \\
& + 2\tilde{A}_2(m_1^2) \frac{g}{m_B + m_2} (p \cdot \epsilon_1)(p \cdot \epsilon_2) \\
& + i \frac{2g}{m_B + m_2} \tilde{V}(m_1^2) \epsilon^{\alpha\beta\gamma\delta} \epsilon_{1\alpha} \epsilon_{2\beta} k_\gamma p_\delta ,
\end{aligned} \tag{4.5}
$$

where the tilded quantities refer to V_2. We use the values of Ref. 9 for the form factors at zero-momentum transfer for the first decay and take them to be of order 1 for the second one. The momentum dependence of the form factors is found by assuming vector dominance of a pole with the relevant quantum numbers, and mass found in Refs. 9 and 11. At this point we do not know how to calculate F_3 and F_4, but in line with our previous arguments we will treat them as a 5–10 % correction to the numbers in F_1, F_2. For $B_c^- \to \psi D_s^{*-}$ naive use of this pole dominance model for the form factors as in Eq. (4.3) suppresses the weak annihilation process by about a factor of 10. With all this we write our numerical results for the asymmetries:

$$
\begin{aligned}
A &= a_1 \cos(\delta_s - \delta_p) , \\
\tilde{A} &= a_2 \cos(\delta_s - \delta_p) + a_3 \cos(\delta_d - \delta_p) , \\
\frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} &= a_4 \sin(\delta_d - \delta_s) .
\end{aligned} \tag{4.6}
$$

The constants a_i for the two decays considered are given in Table II. The numbers follow from using $\delta\phi = 0.01$, and scale linearly with $\delta\phi$ for smaller values. The asymmetries depend strongly on the values of the form factors,

TABLE II. Vacuum-saturation estimate of some asymmetries. The second row corresponds to the maximum values we could find by changing the form factors by 10% or less.

Channel	x	y	a_1	a_2	a_3	a_4
$B^- \to \omega K^{*-}$	19.2	0.037	0.0001	0.003	-0.001	0.05
			0.002	0.04	-0.03	0.23
$B_c^- \to \psi D_s^{*-}$	1.96	0.16	1.2×10^{-6}	2.4×10^{-6}	0.0	1.3×10^{-5}

changing the latter by a few percent changes the former by factors of 2. It is clear that these numbers cannot be taken too seriously. What we can say from this analysis is that the relative sizes of the angular and partial-rate asymmetries are consistent with the kinematical considerations of Sec. II. We can also see that A is indeed suppressed, and that this suppression is much larger for large values of x.

A detailed analysis remains to be done but it requires a better understanding of the hadronic matrix elements.

V. CONCLUSIONS

We have studied in some detail a type of CP asymmetry that has been overlooked in the past. We find that CP-odd observables can arise from interference among different isospin amplitudes, in a way comparable to the simpler case of pseudoscalar mesons. The large number of amplitudes present when we deal with vector particles makes it much more difficult to extract the CP phases. We also find asymmetries that originate in the interference of partial-wave amplitudes. In all cases the signals look reasonably large only in specific channels.

Angular correlations appear to be not much smaller than partial-rate asymmetries. They offer a way to study separately CP-odd and unitarity phases. These asymmetries are not generated via mixing, and a reliable estimate is not possible at present. A complete calculation would have to consider the decay of the spin-1 mesons and find the corresponding correlations involving only final-state momenta. We have discussed one case of angular correlations, but certainly not the only one. Other possibilities include the study of baryonic channels and of three or more decay products.

The B system seems to have a wide variety of places where searches for CP violation can be done. It is important to look at these signals with the understanding that it is not enough to see CP violation. It is equally important to be able to extract quantitative and precise information that can really give us some insight into the origin of the phenomenon. In this spirit it is, therefore, important to study all the possible signals that occur in different channels, with the aim of separating the information we want from the complicated and poorly understood hadron dynamics.

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