Effects of initial-state QCD interactions in the Drell-Yan process

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We develop a simple, intuitive picture of the effects, in the Drell-Yan process, of initial-state interactions between the projectile and the target. For purposes of illustration, we present specific analyses in terms of a nonrelativistic QED model. However, our principal conclusions are valid in QCD as well. We show that initial-state interactions lead to an increase in the average of the square of the transverse momentum of the lepton pair that is proportional to the length of the target, and we also demonstrate that initial-state interactions can invalidate the parton-model (factorized) form of the cross section unless the beam energy is greater than a scale that grows with the length of the target.

I. INTRODUCTION

The QCD-improved parton model is an extraordinarily powerful tool for analyzing high-energy hadronic processes. However, one must take care not to take parton-model pictures too literally. Consider, for example, the Drell-Yan process,[1] $p\bar{p}\to l+\bar{l}+X$ (Fig. 1), where $l\bar{l}$ is a lepton-antilepton pair and X represents any other final-state particles. Factorization theorems[2-5] tell us that, when the invariant mass of the lepton pair is large, the cross section for this process can be written in the parton-model form:

$$\frac{d\sigma}{dQ^2}=\sum_q \int_0^1 dx_1 dx_2 G_{q/p}(x_1,Q)G_{\bar{q}/\bar{p}}(x_2,Q) \times\delta(x_1x_2s-Q^2)\sigma_{q\bar{q}\to l\bar{l}+X}(x_1x_2s)\ , \quad (1.1)$$

where the sum is over all quarks and antiquarks, $G_{q/p}$ is the distribution function for finding parton q in hadron p, $\sigma_{q\bar{q}\to l\bar{l}+X}$ is the hard subprocess cross section (including QCD radiative corrections), and Q is the momentum of the lepton pair. At first sight the expression (1.1) seems highly intuitive; but taken literally it seems to imply, remarkably, that the $\bar{q}$ from the antiproton does not interact with spectator partons in the proton. If we replace the proton by a ^{238}U nucleus, this expression suggests that the $\bar{q}$ is unaffected by its passage through the nucleus en route to annihilation on, say, the back face of the nucleus. Consequently, all nucleons in the nucleus participate equally in the process. There is no nuclear-induced energy loss for the incoming parton, and there is no shadowing. Insofar as nucleon-nucleon interactions can be neglected, the cross section then must grow linearly with A, the number of nucleons in the target.

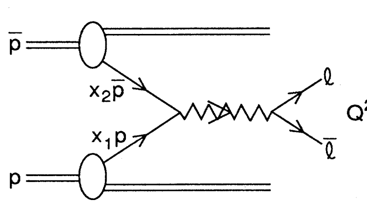

FIG. 1. The Drell-Yan process $p\bar{p}\to l+\bar{l}+X$. Solid lines denote leptons, the saw-toothed line denotes the Drell-Yan virtual photon, and the circles with solid lines emerging represent the proton and antiproton wave functions.

Is this still true if we replace the nucleus by a lead brick? What happends to the $\bar{q}$ as it passes through a large nucleus? Are there really no "initial-state interactions" between the beam partons and spectators in the target? In fact QCD implies that there are initial-state interactions in the Drell-Yan process, including active-spectator initial-state interactions. (Here, and throughout this paper, we use the term "active-spectator initial-state interactions" to refer to intial-state interactions between an active parton in one hadron and a spectator parton in the other hadron.) However, for purposes of computing $d\sigma/dQ^2$ or $d\sigma/dQ^2d\mathbf{Q}_1^2$, initial-state interactions involving target spectator partons are universal in that their contribution does not depend upon details of the beam particle's structure. Consequently, the effects of initial-state interactions involving target spectators may be absorbed into the target's distribution function, and so are usually not distinguished from other effects related to the target's structure. In this paper we isolate initial-state interactions involving an active parton from the beam and a spectator parton from the target from these other effects so as to explore the experimental consequences of such interactions. We show how these active-spectator initial-state interactions affect such things as the transverse momentum of the lepton pair. We also demonstrate that in very long targets, whose length is greater than a scale set by the energy of the incoming parton, initial-state interactions involving spectators in the target destroy the (factored) parton picture, thereby reducing the cross section at fixed Q^2. Such nuclear effects arise in virtually all inclusive hadronic processes.

Active-spectator initial-state interactions necessarily involve small transfers of momentum. Thus, in QCD their effects are nonperturbative and fall outside the calculational domain of perturbation theory. However, one can arrive at a qualitative understanding of these effects by studying simple, calculable models. To this end we have analyzed a QED analogue of the Drell-Yan process, namely, $\bar{e}+\mathrm{H}\to l\bar{l}+X$, the annihilation of the electron in a hydrogen atom. This QED process contains a good deal of the physics of the hadronic interactions and is quite convenient for studying initial-state interactions. It has a parton structure that is closely analogous to that of hadronic Drell-Yan reactions. Furthermore, the gauge-theory factorization theorems proven for QCD obviously are valid for QED as well. Hence, our model automatically contains all of the physical consequences of factorization. Since α is small, cross sections are calculable through weak-coupling techniques, and the analysis is greatly simplified by the nonrelativistic nature of QED atoms. Also, we can rely upon our considerable intuition about atoms and their structure. A further simplification is the Abelian nature of the QED gauge-theory interactions.

In Sec. II of this paper we present a qualitative analysis of the initial-state interactions that occur in QED Drell-Yan processes. We focus upon the underlying physics, thereby identifying generic features relevant to the hadronic case. In Sec. III we present a detailed analysis of a QED model, illustrating how the ideas developed in Sec. II are realized in perturbation theory. This section establishes the validity of our qualitative picture of initial-state interactions. Finally in Sec. IV we summarize our main results. There we also explore the implications of our analysis for the hadronic Drell-Yan process, and for a variety of other hadronic reactions; and we examine experimental evidence for active-spectator initial-state interactions.

In some instances the results we present have actually been derived more generally in the proofs of the factorization theorems. However, our purpose in this paper is not to offer a rigorous proof of factorization, but rather to develop a simple, intuitive picture of the physics of initial-state interactions. Although some of the physical consequences of initial-state interactions have been analyzed previously,[6] the earlier analyses are incomplete in that they focus on the so-called "Glauber" region. As a result, these previous analyses are inconsistent with the factorization theorems.[7] In this paper, we show that the physical consequences of initial-state interactions described in the earlier work survive in a complete analysis and, hence, fit within the framework provided by the factorization theorems.

II. QUALITATIVE ANALYSIS

A. Elastic interactions

The simplest QED analogue for the Drell-Yan process is the positron-hydrogen reaction:

$$\bar{e}\mathrm{H}\to l\bar{l}+X \ . \tag{2.1}$$

The lowest-order amplitude for this reaction is shown in Fig. 2(a). We work in the atom's rest frame, denoting the electron's mass and momentum by m and $\mathbf{k}=(k,\mathbf{k}_\perp)$, the proton's mass by M, and the positron's momentum by $\mathbf{P}=(P,\mathbf{0}_\perp)$. The invariant mass of the lepton pair is

$$Q^2\approx 2P(m-k)\equiv sx \ , \tag{2.2}$$

where the atomic binding energy ε (<0) and corrections of order $1/P$ have been neglected, and where we have used definitions suitable for an infinitely heavy nucleus:

$$s\equiv 2mP \gg m^2, \quad x\equiv 1-\frac{k}{m}\approx 1 \ . \tag{2.3}$$

[We do not use the conventional definitions $s=2(m+M+\varepsilon)P$ and $x=(m-k)/(m+M)$ since they are not well behaved in the limit $M\to\infty$.] The cross section is then

$$\frac{d\sigma}{dQ^2}=\int\frac{d^3\mathbf{k}}{(2\pi)^3}|\psi(\mathbf{k})|^2\delta(Q^2-xs)\sigma_H(e\bar{e}\to l\bar{l}) \ . \tag{2.4}$$

Here $\psi(\mathbf{k})$ is the ordinary Schrödinger wave function for the hydrogen atom. Since an understanding of the space-time structure of the reaction is critical to our analysis, we Fourier transform (2.4) to obtain

$$4\pi P\int\frac{d^3\mathbf{k}}{(2\pi)^3}|\psi(\mathbf{k})|^2\delta(Q^2-xs) \to \int dz_1 dz_2 d^2\mathbf{z}_\perp \psi^*(z_2,\mathbf{z}_\perp)e^{ik(z_2-z_1)} \times\psi(z_1,\mathbf{z}_\perp)|_{k=(1-x)m} \ . \tag{2.5}$$

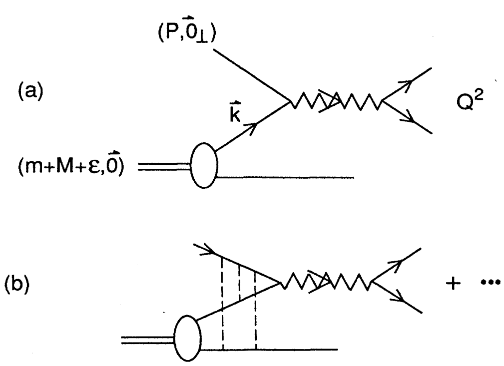

FIG. 2. The Drell-Yan process $\bar{e}+\mathrm{H}\to l\bar{l}+p$. (a) represents the basic "partonic" process. (b) represents a typical elastic initial-state interaction. Dashed lines denote Coulomb photons.

This expression demonstrates that the cross section $d\sigma/dQ^2$ involves interference between annihilations at the same impact parameter, but not necessarily at the same longitudinal position—i.e., although $\mathbf{z}_{1\perp}=\mathbf{z}_{2\perp}\equiv\mathbf{z}_\perp$, $z_1\neq z_2$. The impact parameters are the same because in computing the cross section one integrates over all $\mathbf{Q}_\perp$, the transverse momentum of the lepton pair.

Although (2.4) is consistent with the general factored form expected for the cross section [compare with (1.1)], it is far from being complete. As it stands, (2.4) is not even gauge invariant. This is because we have not yet included the effect upon the positron of the Coulomb fields generated by the electron and proton in the atom. As the positron penetrates the atom, it suffers multiple soft interactions with these Coulomb fields [Fig. 2(b)]. (Hard interactions occur as well, but we will ignore these for the moment.) Since these are soft, vector interactions, one can justify using eikonal methods, which amount to approximating the positron's Hamiltonian as follows:

$$\begin{aligned}\hat{H}&=(p^2+\mathbf{p}_\perp^2+m^2)^{1/2}+e\mathcal{A}^0(z,\mathbf{z}_\perp)\\&\approx p+\frac{\mathbf{p}_\perp^2+m^2}{2p}+e\mathcal{A}^0(z,\mathbf{z}_\perp)\\&\approx p+e\mathcal{A}^0(z,\mathbf{z}_\perp)\ .\end{aligned}\tag{2.6}$$

Here $\mathcal{A}^\mu$ is the Coulomb-gauge vector potential for the atom's electric field, and we have assumed that the longitudinal momentum p is large compared to $p_\perp$ and m. The kinetic energy term in this approximate Hamiltonian serves only to propagate the positron in the longitudinal direction at the speed of light, keeping its impact parameter fixed. The potential-energy term leads to a phase in the positron's wave function, each segment $dz\approx dt$ of the positron's trajectory contributing a factor

$$e^{ie\mathcal{A}^0(z,\mathbf{z}_\perp)dt}\approx e^{ie\mathcal{A}^0(z,\mathbf{z}_\perp)dz}\ .\tag{2.7}$$

If the positron annihilates at z_1 the total phase is

$$\exp\left[ie\int_{-\infty}^{z_1}\mathcal{A}^0(z,\mathbf{z}_\perp)dz\right]=\exp\left[ie\int_{-\infty}^{z_1}\mathcal{A}\cdot dz\right]\ ,\tag{2.8}$$

where the last integral is over the path followed by the positron on its way to annihilation. Thus the atomic wave functions in (2.5) are replaced by gauge-invariant wave functions:

$$\psi(z_1,\mathbf{z}_\perp)\to\exp\left[ie\int_{-\infty}^{z_1}\mathcal{A}\cdot dz\right]\psi(z_1,\mathbf{z}_\perp)\ .\tag{2.9}$$

By including eikonal phase factors for the positron, we have made the amplitude explicitly gauge invariant. Note that the eikonal factor depends on the nature of the projectile only through the charge of the active parton (positron). Hence, all of the effects of the soft interactions can be associated completely with the target. That is, one can absorb the eikonal phases into the definition of the target distribution function to obtain the factored form of (1.1). There are also eikonal phases associated with the projectile, which arise from interactions in which the exchanged photons are collinear to the projectile. Such collinear exchanges can occur only between the active target parton and the projectile—otherwise they are suppressed by powers of $1/Q^2$. For simplicity, we ignore them here.

Hard initial-state interactions cannot be analyzed using eikonal methods, since for such interactions one cannot ignore the transverse momentum of the exchanged photon relative to the longitudinal momentum of the positron. However, to leading order in $1/Q^2$, it can be shown that the hard interactions involve only the active partons and that, even in the presence of hard interactions, one can account for the soft interactions with the spectator partons through factors of the form given in (2.9) (Refs. 2 and 3). Then the hard interactions appear immediately before the annihilation, and they can be included as radiative corrections to the hard-scattering cross section σ_H in (2.4). For the most part we will not be concerned with these hard interactions in what follows.

Let us now consider replacing the atom by a molecule consisting of A atoms. This is the QED analogue of a nucleus. One might suspect that the eikonal phases introduce nontrivial A dependence. The positron samples the fields within every atom along its trajectory, and so its eikonal phase (2.8) depends upon A. However, this is not true for the cross section $d\sigma/dQ^2$. This is because the bulk of the phase factor associated with $\psi(z_1,\mathbf{z}_\perp)$ is canceled by that associated with $\psi^*(z_2,\mathbf{z}_\perp)$. The expression (2.5) is then replaced by

$$\int dz_1dz_2d^2\mathbf{z}_\perp\psi^*(z_2,\mathbf{z}_\perp)e^{ik(z_2-z_1)}\times\exp\left[ie\int_{z_1}^{z_2}\mathcal{A}\cdot dz\right]\psi(z_1,\mathbf{z}_\perp)|_{k=(1-x)m}\ ,\tag{2.10}$$

where now the integral in the eikonal phase is restricted to a line joining the (interfering) annihilation points z_1 and z_2. This cancellation occurs because only annihilations at the same impact parameter $\mathbf{z}_\perp$ can interfere in $d\sigma/dQ^2$. The annihilations must also occur within the same atom, since there is negligible overlap between the final states otherwise. Hence, the remaining part of the eikonal integral samples the gauge field only at points inside the annihilation atom. The gauge field within an atom is due largely to the electrons and protons associated with that atom. Fields coming from other atoms contribute little (since atoms are neutral), leading to only a weak A dependence that saturates quickly as A increases.

So initial-state interactions *do* occur, and in particular there are interactions between the beam particle and spectator partons in the target. These active-spectator interactions give the positron small kicks in momentum. However, the longitudinal momentum transferred is negligible compared with the positron's initial momentum; and the cross section $d\sigma/dQ^2$ is insensitive to the positron's transverse momentum once the integration over $\mathbf{Q}_\perp$ is performed. Even in the integrated cross section $d\sigma/dQ^2$, the soft initial-state interactions produce phases, but these phases can be absorbed into the definition of the target's distribution function. In fact, the phases that arise in the Drell-Yan process as a consequence of soft initial-state interactions are very similar to the phases that arise in deeply inelastic scattering as a consequence of soft final-state interactions. One can show that the differences in the corresponding distribution amplitudes amount to hard, perturbatively calculable corrections.[2,8]

Thus, any physical effects which would provide a distinctive signature for active-spectator initial-state interactions are suppressed in $d\sigma/dQ^2$ by a power of $1/Q^2$. However, active-spectator initial-state interactions are important if the target is very long or if the positron's energy is too low. The almost complete cancellation of initial-state interactions in $d\sigma/dQ^2$ relies upon the validity of the eikonal approximation for the positron's Hamiltonian (2.6). The part of the Hamiltonian that is dropped in this approximation, $(\mathbf{p}_\perp^2+m^2)/(2p)$, is small compared with the term p that is retained. However, it does contribute a phase as a part of the time evolution operator $\exp(-i\hat{H}t)$, and that phase will be important if the time available to resolve a change in the projectile's energy is large enough. The resolving time of the target is just $t\approx L$, the length of the target, and so the piece of the Hamiltonian that is dropped in the eikonal approximation is unimportant only if

$$\frac{\langle \mathbf{p}_\perp^2+m^2\rangle}{2P}L \ll 1 . \tag{2.11}$$

Thus if the beam energy P is too low, a new random phase appears, destroying the coherence of the beam and invalidating the parton-model (factored) form of the cross section (1.1). Since $\langle \mathbf{p}_\perp^2\rangle$ grows like L, the minimum beam energy required for factorization grows like $L^2 \propto A^{2/3}$.

If the target-length condition of (2.11) is satisfied, then one must look beyond $d\sigma/dQ^2$ in order to see the effects of active-spectator initial-state interactions. A sensitive quantity is the distribution in $\mathbf{Q}_\perp$ of the lepton pair. Initial-state interactions broaden this distribution: the positron receives small kicks in $\mathbf{p}_\perp$, executing a random walk in $\mathbf{p}_\perp$ space, and then passes its accumulated transverse momentum on to the lepton pair. Thus one expects an increase in $\langle \mathbf{Q}_\perp^2\rangle$ for molecular targets as compared to atomic targets, the increment growing with the length L of the target:

$$\Delta\langle \mathbf{Q}_\perp^2\rangle \propto L \propto A^{1/3} . \tag{2.12}$$

One can be more systematic in computing this effect. With $\mathbf{Q}_\perp$ fixed, the interfering annihilations need no longer be at the same impact parameter, and so the eikonal phases no longer cancel. As a result one is left with phase factors such as

$$U_I=\exp\left[ie\int_{-\infty}^{z_1}[\mathcal{A}^0(z,\mathbf{z}_{1,\perp})-\mathcal{A}^0(z,\mathbf{z}_{2\perp})]dz\right] . \tag{2.13}$$

To first approximation this is

$$U_I\approx\exp\left[ie\int_{-\infty}^{z_1}(\mathbf{z}_{1\perp}-\mathbf{z}_{2\perp})\cdot\nabla_\perp\mathcal{A}^0(z,\mathbf{z}_{1\perp})dz\right]=e^{i\mathbf{z}_\perp\cdot\delta\mathbf{p}_\perp}, \tag{2.14}$$

where

$$\delta\mathbf{p}_\perp=e\int_{-\infty}^{z_1}\nabla_\perp\mathcal{A}^0dz \tag{2.15}$$

is roughly the transverse momentum accumulated by the positron up to position z_1. In general $\delta\mathbf{p}_\perp$ receives a sum of contributions, one from each of the atoms through which the positron passes. The signs of these contributions fluctuate in a fairly random fashion, so we estimate that

$$|\delta\mathbf{p}_\perp|\sim\sqrt{N}\,\left|e\int_{\text{atom}}\nabla_\perp\mathcal{A}^0dz\right|\sim\sqrt{N}R\,|e\nabla_\perp\mathcal{A}^0| \sim\sqrt{N}e^2/R , \tag{2.16}$$

where $N\approx A^{1/3}$ is the number of atoms traversed by the positron, and R is a typical atomic radius. Since this momentum is ultimately transferred to the lepton pair, (2.16) leads immediately to (2.12), at the same time providing an estimate of the proportionality constant in that equation.

Note that these effects are gauge independent. The phase U_I (2.13) can be made manifestly gauge invariant by redefining it as

$$\exp\left[ie\int_{C_I}\mathcal{A}\cdot dz\right] , \tag{2.17}$$

where the contour C_I ranges from $(z,\mathbf{z}_\perp)=(-\infty,\mathbf{z}_{1\perp})$ to $(z_1,\mathbf{z}_{1\perp})$ to $(z_1,\mathbf{z}_{2\perp})$ and finally to $(-\infty,\mathbf{z}_{2\perp})$. The extra piece that must be added to (2.13) to give (2.17) changes none of our conclusions. In fact this piece is negligible for our target, since it is proportional to the three-vector potential, which, in the Coulomb gauge, is suppressed by a factor of $\langle v/c\rangle \ll 1$ associated with either the nucleus or the electron.

We should note that it is only because the photon is a vector particle that active-spectator initial-state interactions can have any effect to leading order in $1/s$. From the positron's point of view, the time available for an interaction is very short—i.e., of order Lm^2/s once the Lorentz contraction of the target is taken into account. It is only because the amplitude for scattering via vector exchange grows linearly with s that there is a finite probability of an interaction as $s\to\infty$. Were the photon a scalar particle, all active-spectator initial-state interactions would be suppressed by m^2/s.

B. Inelastic interactions

Elastic active-spectator initial-state interactions have a negligible effect on the positron's longitudinal momentum. However, it is well known that bremsstrahlung induced by low-$p_\perp$ scattering can greatly deplete the longitudinal momentum of a high-energy positron. In order to illustrate some of the issues involved, let us consider the forward radiation induced by a soft scattering from a Coulomb potential. The lowest-order amplitude, which corresponds to the two diagrams of Fig. 3(a), has the form

$$e(1-z)\left[-\frac{\boldsymbol{\epsilon}_\perp\cdot\mathbf{q}_\perp}{q_\perp^2}+\frac{\boldsymbol{\epsilon}_\perp\cdot(\mathbf{q}_\perp-z\boldsymbol{l}_\perp)}{(\mathbf{q}_\perp-z\boldsymbol{l}_\perp)^2}\right]V_c(\boldsymbol{l}) , \tag{2.18}$$

where $\mathbf{q}=(zP,\mathbf{q}_\perp)$ is the momentum carried off by the radiated photon, $\boldsymbol{\epsilon}$ is its polarization vector, $V_c(\boldsymbol{l})$ is the Fourier transform of the Coulomb potential, and $\boldsymbol{l}$ is the momentum transfer. In arriving at (2.18), we have assumed that the momentum transfer is soft ($|\boldsymbol{l}| \ll P$) and

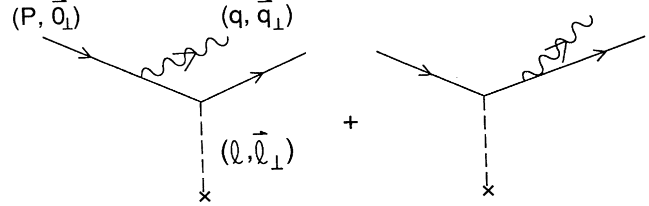

FIG. 3. The two lowest-order amplitudes for radiation induced by interaction with a potential. Wavy lines denote transverse photons and crosses denote the sources of external potentials.

that the radiation is in the forward direction ($q \gg q_\perp$), and for convenience we have taken the Coulomb potential to be independent of the longitudinal coordinate (that is, we have taken $l \equiv l_z = 0$). The amplitude of (2.18) allows photon momenta through the entire range $0 \le z \le 1$. Thus the photon can carry off any fraction of the positron's momentum. If this were to occur as the result of an initial-state interaction in a Drell-Yan reaction, it would greatly reduce the cross section at fixed Q^2.

To see what really happens in the Drell-Yan case we have to examine the physics of radiation a bit more closely. Note the strong cancellation between the two amplitudes in (2.18) for $q_\perp > z l_\perp$. The first term by itself allows $\mathbf{q}_\perp^2$ to be as large as the kinematic limit $2(1-z)P|l|$, and yet all of the hard radiation cancels when the second term is included. Why does this happen? The Feynman diagrams not only describe the collision, but also the evolution of the physical positron state $|\bar{e}\rangle_{\rm phys}$ from the bare state $|\bar{e}\rangle_0$ as QED is adiabatically switched on. The incoming physical state is a superposition,

$$|\bar{e}\rangle_{\rm phys} = \sqrt{Z_2}\,|\bar{e}_0\rangle_0 + \int dx\, d^2\mathbf{q}_\perp |e\gamma\rangle_0 \psi_{\bar{e}\gamma}(x,\mathbf{q}_\perp) + \cdots \ , \tag{2.19}$$

that contains $\bar{e}\gamma$ states with arbitrarily large $\mathbf{q}_\perp$, since $\psi_{\bar{e}\gamma} \sim e\epsilon_\perp \cdot \mathbf{q}_\perp (1-z)/\mathbf{q}_\perp^2$. These components of the positron state correspond to the large-$q_\perp$ contribution from the first term in (2.18). When the positron scatters, its large-$q_\perp$ components are essentially unaffected. Only those components having $q_\perp$ of order $l_\perp$ or less are substantially modified. The large-$q_\perp$ part of the initial positron evolves directly into the large-$q_\perp$ part of the final-state positron. The low-$q_\perp$ part is ultimately resolved into two components: the low-$q_\perp$ part of the final positron, and a physical photon. This is why photons are radiated with only low transverse momentum.

If the positron annihilates, as in the Drell-Yan process, there is no positron in the final state and, photons of all $q_\perp$ are emitted. In processes that involve only the active partons, one can associate this radiation with the Q-dependent distribution functions for the positron and the atom or, depending on the particular choice of factorization scheme, with radiative corrections to the basic hard process. But what happens if we have an active-spectator initial-state interaction followed by the annihilation [see Fig. 6(b) in Sec. III C]? The components of the incoming state $|\bar{e}\rangle_{\rm phys}$ for which $q_\perp$ is large compared with $l_\perp$ are undisturbed by the collision, and all of the radiation can be associated with the positron's distribution function or with radiative corrections to the basic process. However, the components of the incoming state for which $q_\perp$ is of order $l_\perp$ are disturbed by the collision, and, given enough time, additional radiation would develop in the resulting outgoing state. By the uncertainty principle, the light-cone time $\Delta\tau$ ($\tau \equiv t + z$) required to resolve a change in the state grows with the beam energy:

$$\Delta\tau \sim 1/\Delta p^- \ , \tag{2.20}$$

where $p^- \equiv p^0 - p^3$ and

$$\Delta \mathbf{p}^- \sim \frac{\mathbf{q}_\perp^2}{2z(1-z)P} \sim \frac{\langle \mathbf{p}_\perp^2 \rangle}{2P} \ . \tag{2.21}$$

On the other hand, the light-cone resolving time of the target is just L, the length of the target. Thus, there is no induced radiation provided that[9]

$$\frac{\langle \mathbf{p}_\perp^2 \rangle}{2P} L \ll 1 \ . \tag{2.22}$$

This is the same condition as the one (2.11) that guarantees that initial-state interactions are unimportant in $d\sigma/dQ^2$. If the condition (2.22) is not satisfied, as in Drell-Yan annihilation on a lead brick, there is copious radiation induced by active-spectator initial-state interactions. Then corrections for the energy lost by the projectile to radiation must be made before applying the standard Drell-Yan formalism.

III. DETAILED ANALYSIS

A. Introduction

To illustrate the issues raised in Sec. II, we now examine in detail the positron-hydrogen Drell-Yan process in QED, using Coulomb gauge in the atomic rest frame. The cross section is largest when Q^2 and x are such that only nonrelativistic momenta need flow through the wave function. We restrict our discussion to this case. We also treat the electron, positron, and proton as pointlike scalar particles. This choice of model is advantageous for several reasons.

(i) There are four small parameters in the model: m^2/Q^2, $\langle v/c \rangle$ for the electron in the atom, $\langle v/c \rangle$ for the proton in the atom, and α the QED fine-structure constant. Usually we compute only to leading order in each of these small parameters, although the Coulomb potential is treated to all orders. This greatly simplifies the analysis.

(ii) The coupling of transverse photons to the constituents of the atom is always suppressed by at least a factor $\langle v/c \rangle$. Thus all radiation is emitted by the projectile to leading order in $\langle v/c \rangle$.

(iii) The kinematics are greatly simplified since $|\mathbf{k}| \ll m$ and effectively $|k^0| \ll |\mathbf{k}|$ for most loop momenta relevant to the initial-state interactions.

(iv) Ordinary time-ordered perturbation theory (TOPTh) can be used, in place of covariant perturbation theory. The projectile is essentially in the infinite-momentum frame, while the target is highly nonrelativistic. Consequently, the plethora of additional diagrams that normally plagues TOPTh is suppressed by factors of $1/P$ or $1/m$ [i.e., $\langle v/c \rangle$]. What remains is relatively simple, and quite intuitive. Also the use of TOPTh allows us to employ ordinary nonrelativistic quantum mechanics in describing the target.

(v) Our primary interest is in initial-state interactions involving spectator constituents in the target; only these can lead to A-dependent cross sections. In QED, interactions involving the spectator partons are in a different gauge class from those involving the active constituent. Thus, it is usually possible to consider one without worrying about the other.

(vi) Diagrams containing fermion loops are not crucial to the qualitative analysis of initial-state interactions. Since, in QED, these diagrams are in a different gauge class from diagrams without fermion loops, we can omit them from our model calculations.[10]

Within the context of this model we can examine the salient features of initial-state interactions while avoiding many of the complications of a general treatment.

B. Elastic interactions

The lowest-order (in α) contribution to our Drell-Yan process was analyzed in Sec. II [see (2.4)]. The simplest diagram with an interaction between the positron and a spectator (i.e., the proton) is shown in Fig. 4(a). This contribution is identical to that in lowest order [(2.4)] but with $\psi(\mathbf{k})$ replaced by

$$e^2 \int \frac{d^3 l}{(2\pi)^3} \frac{\psi(\mathbf{k}-\boldsymbol{l})}{|\boldsymbol{l}|^2} \left[-l+\epsilon-\frac{(\mathbf{k}-\boldsymbol{l})^2}{2m}-\frac{\mathbf{k}^2}{2M} - \frac{l_\perp^2}{2(P+l)}+i\epsilon \right]^{-1} . \tag{3.1}$$

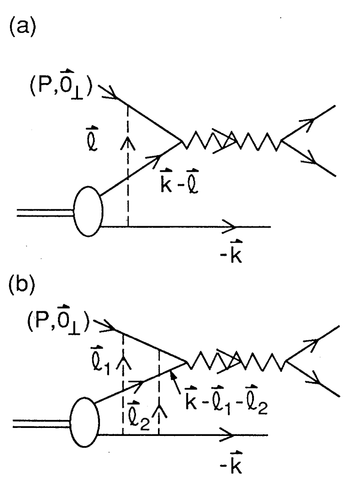

FIG. 4. Active-spectator initial-state interactions for the process $\bar{e}+\mathrm{H}\to l\bar{l}+p$, (a) the relative order-α amplitude, (b) the relative order-α^2 amplitude.

Here $\boldsymbol{l}=(l,\boldsymbol{l}_\perp)$, and the last factor is just the propagator $1/(E_{\text{initial}}-E_{\text{intermediate}}+i\epsilon)$ in time-ordered perturbation theory. We neglect $\boldsymbol{l}$ relative to P since the magnitude of $\boldsymbol{l}$ is limited by the wave function and Coulomb propagator. To leading order in $\langle v/c \rangle$ and $1/P$ the energy denominator has the eikonal form $-l+i\epsilon$, and the Fourier transform of (3.1) becomes

$$\left[ie \int_{-\infty}^{z_1} \mathcal{A}_p^0(z,\mathbf{z}_\perp)dz \right] \psi(z_1,\mathbf{z}_\perp) , \tag{3.2}$$

where $\mathcal{A}_p^\mu$ is the vector potential for the proton's Coulomb field. This, together with a similar contribution due to electron's Coulomb field, is just the first-order term from the expansion of the eikonal phase in (2.9).

The second-order term comes from diagrams such as that shown in Fig. 4(b). The contribution of Fig. 4(b) is identical to that in (2.4) but now with $\psi(\mathbf{k})$ replaced by

$$e^4 \int \frac{d^3 l_1 d^3 l_2}{(2\pi)^6} \frac{\psi(\mathbf{k}-\boldsymbol{l}_1-\boldsymbol{l}_2)}{|\boldsymbol{l}_1|^2 |\boldsymbol{l}_2|^2} \frac{1}{-l_1+i\epsilon} \frac{1}{-l_1-l_2+i\epsilon} . \tag{3.3}$$

Fourier transforming this expression, one obtains the proton's contribution to the second-order term in the expansion of (2.9). Higher-order terms follow in an obvious fashion.

There is a variety of other diagrams at the two-loop level and beyond (Fig. 5), but for the most part these are negligible in our model. Coulombic interactions involving seagull vertices or Z graphs [Fig. 5(a)] are always down by $1/P$ or $\langle v/c \rangle$. Active-spectator initial-state interactions involving the exchange of a transverse photon [Fig. 5(b)] are suppressed by a factor of $\langle v/c \rangle$, this coming from the photon-atom coupling. Diagrams with an active-spectator initial-state interaction followed by in-

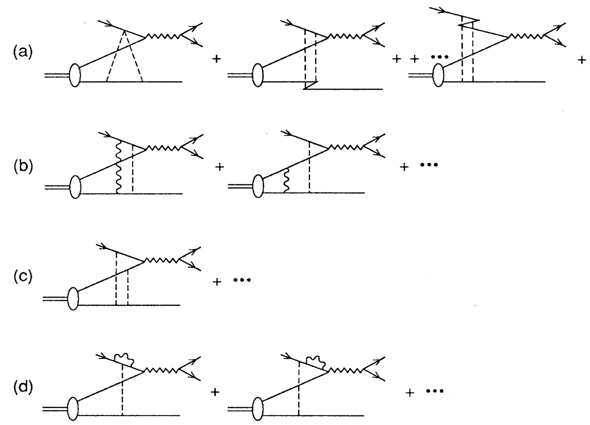

FIG. 5. Examples of elastic initial-state interactions involving the spectator for the process $\bar{e}+\mathrm{H}\to l\bar{l}+p$, (a) examples of Coulombic interactions involving seagulls or Z graphs, (b) examples of interactions involving exchange of a transverse photon, (c) an example of an active-spectator interaction followed by an interaction between the target's constituents, (d) examples of active-spectator interactions accompanied by radiative corrections.

teractions within the target [Fig. 5(c)] are suppressed by $\langle v/c \rangle$, since the time required for atomic interactions is typically $O(mL^2)$, where L is the atom's length, while the time available in this case is only $O(L) \ll mL^2$. Virtual-photon exchanges involving only the active partons, along with similar processes involving real photons, are, of course, a part of the usual QCD radiative corrections to the parton model. Such radiative corrections have no dependence on the target structure and are in a different gauge class from the active-spectator initial-state interactions, so we do not discuss them further. Diagrams mixing active-spectator initial-state interactions with radiative corrections [Fig. 5(c)] do depend on the target's structure and contribute in leading order in $1/Q^2$ and v/c. These diagrams are generally combined with diagrams involving radiation (Sec. III C). At large Q^2 they can put into the factored form of (1.1) (Refs. 2 and 3). Hence, at large Q^2 all of the target-structure effects in such interactions are contained in the distribution functions, which we have already analyzed. [At small Q^2, the target-length condition comes into play, just as in the case of the pure active-spectator interactions. See (2.22) and also the discussion in the remainder of this subsection.]

The eikonal approximation used in going from (3.1) to (3.2) breaks down for long targets. This breakdown is most easily analyzed in the limit of infinite proton mass M. Then the set of diagrams in Fig. 5(c) combines naturally with the one-loop diagram in Fig. 4(a). The net effect is to replace the energy denominator in (3.1) by

$$-l+\epsilon-\hat{H}_{\rm atom}-O(l_\perp^2/P)+i\epsilon \ , \tag{3.4}$$

where $\hat{H}_{\rm atom}$ is the Hamiltonian operator for the atom. [The kinetic energy part of $\hat{H}_{\rm atom}$ is already present in (3.1); the remaining diagrams introduce the potential energy term.] Since $(\epsilon-\hat{H}_{\rm atom})\psi=0$ (Ref. 11), the energy denominator may be further simplified to read

$$-l-O(l_\perp^2/P)+i\epsilon \ . \tag{3.5}$$

Thus, the deviation from the eikonal form $-l+i\epsilon$ is suppressed by $1/P$ rather than $1/m$. This deviation is negligible provided that the wave function in (3.1) is insensitive to shifts in l of $O(l_\perp^2/P)$. By the uncertainty principle this is true for an atom of length L only if

$$\frac{l_\perp^2}{P}L \ll 1 \ . \tag{3.6}$$

This is precisely the condition (2.11) derived in Sec. II.

C. Inelastic interactions

All radiation comes from the positron to leading order in $\langle v/c \rangle$ in our model. Consequently, there is only one diagram to leading order in α [Fig. 6(a)]. It gives an amplitude identical to that without radiation, but with the wave function replaced by

$$e\psi(\mathbf{k})\frac{2\epsilon\cdot\mathbf{P}}{P-q}\left[q-|\mathbf{q}|-\frac{\mathbf{q}_\perp^2}{2(P-q)}+i\epsilon\right]^{-1} . \tag{3.7}$$

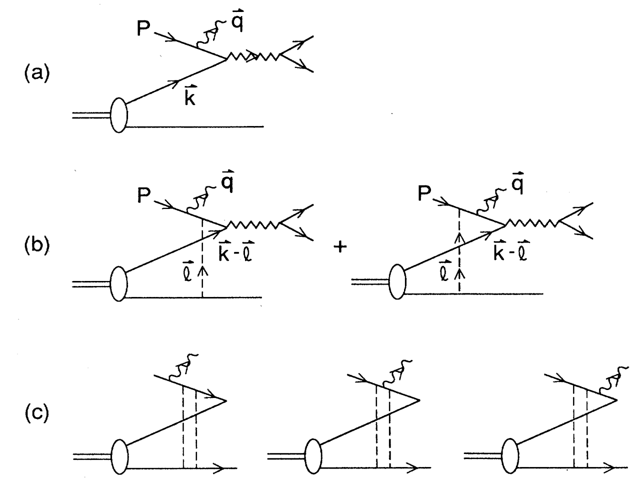

FIG. 6. Examples of radiation in the process $\bar{e}+\mathrm{H}\to l\bar{l}+X$, (a) the lowest-order amplitude for radiation, (b) the lowest-order amplitudes for radiation induced by active-spectator initial-state interactions, (c) examples of higher-order amplitudes for radiation induced by initial-state interactions.

Here $\mathbf{q}=(q,\mathbf{q}_\perp)$ is the radiated photon's momentum and ϵ is its polarization vector. This amplitude contributes to each of the three factors in the parton-model cross section given in (1.1). The contribution from $q_\perp \approx Q$ is absorbed into the cross section for the hard subprocess. The contribution from smaller $q_\perp$ is absorbed into the positron's distribution function when the photon is approximately collinear with the positron: i.e., when $q^+=|\mathbf{q}|+q$ is $O(P)$. Similarly low-$q_\perp$ photons contribute to the atoms' distribution function when $q^-=|\mathbf{q}|-q$ is of $O(m)$ (Ref. 12). The precise range in $q_\perp$ associated with each of the distribution functions and with the subprocess cross section is a matter of convention. The main point here is that the entire contribution to the cross section from this diagram can be put into the factored form (1.1); and therefore the radiation associated with the beam particle is unaffected by the target.

We now consider the lowest-order diagrams that involve both active-spectator initial-state interactions and radiation. There are two amplitudes that contribute [Fig. 6(b)]. In the first, the wave function is replaced by

$$e^3\int\frac{d^3l}{(2\pi)^3}\frac{\psi(\mathbf{k}-l)}{|l|^2}\frac{\epsilon\cdot\mathbf{P}}{P-q}\left[q-|\mathbf{q}|-\frac{\mathbf{q}_\perp^2}{2(P-q)}+i\epsilon\right]^{-1} \times\left[q-|\mathbf{q}|-\tilde{l}-\frac{(\mathbf{q}_\perp-l_\perp)^2}{2(P-q)}+i\epsilon\right]^{-1} , \tag{3.8a}$$

where

$$\tilde{l}\equiv l-\epsilon+\frac{|\mathbf{k}-l|^2}{2m}+\frac{\mathbf{k}^2}{2M} \ . \tag{3.8b}$$

In the second, the wave function is replaced by

$$e^3\int\frac{d^3l}{(2\pi)^3}\frac{\psi(\mathbf{k}-\mathbf{l})}{|\mathbf{l}|^2}\frac{\epsilon\cdot\mathbf{P}}{P-q}\left[-\tilde{l}-\frac{l_\perp^2}{2P}+i\epsilon\right]^{-1}\left[q-|\mathbf{q}|-\tilde{l}-\frac{(\mathbf{q}_\perp-\mathbf{l}_\perp)^2}{2(P-q)}+i\epsilon\right]^{-1}. \quad (3.9)$$

We again neglect l relative to P in these amplitudes. When $q_\perp$ is large compared with $l_\perp$, the expressions (3.8) and (3.9) simplify and combine to give a factored amplitude:

$$\left[e^2\int\frac{d^3l}{(2\pi)^3}\frac{\psi(\mathbf{k}-\mathbf{l})}{|\mathbf{l}|^2}\left[-\tilde{l}-\frac{l_\perp^2}{2P}+i\epsilon\right]^{-1}\right]\times\left[\frac{e\epsilon\cdot\mathbf{P}}{P-q}\left[\tilde{q}-\frac{\mathbf{q}_\perp^2}{2(P-q)}+i\epsilon\right]^{-1}\right]. \quad (3.10)$$

This is just the expected factored form: the expression (3.10) contains the amplitude for an elastic active-spectator initial-state interaction (3.1) multiplied by the amplitude for a one-photon correction to the basic Drell-Yan process (3.7). The factorization of the amplitude (3.10) can be generalized to all orders in α. For example, in second order in the active-spectator interaction, one would need to combine the diagrams shown in Fig. 6(c) in order to obtain the factored form.

Usually, the amplitudes (3.8) and (3.9) combine into the factored form (3.10) when $q_\perp$ is small as well. Provided that the target-length condition (3.6) is satisfied, all terms of order $\mathbf{q}_\perp^2/P \sim l_\perp^2/P$ can be neglected in the denominators, and the factored form results. However, factorization fails when the target is very long. In that case one expects radiation beyond that which is accounted for in the factored cross section (1.1). This additional radiation can carry off an arbitrarily large fraction of the positron's (longitudinal) momentum, greatly reducing the Drell-Yan cross section at fixed Q^2.

IV. CONCLUSIONS AND APPLICATIONS

A rather simple picture emerges from our study of initial-state interactions in Drell-Yan processes in QED. There are important active-spectator initial-state interactions in QED because the interactions proceed via the exchange of a vector particle, the photon. The main observable effect of these is to broaden the transverse-momentum distribution of the beam particles. The broadening of the projectile transverse-momentum distribution has no effect on the cross section $d\sigma/dQ^2$, since that cross section contains an integration over the transverse momentum of the lepton pair. However, in $d\sigma/dQ^2 d\mathbf{Q}_\perp^2$ the $\mathbf{Q}_\perp$ distribution of the lepton pairs is affected by the active-spectator interactions, with the change in $\langle \mathbf{Q}_\perp^2\rangle$ growing linearly with the length of the target. When the target is very long ($L > P/\langle \mathbf{p}_\perp^2\rangle$) it can resolve the destructively interfering amplitudes for the emission of collinear radiation before and after an active-spectator initial-state interaction. Then spectator-induced high-energy collinear radiation occurs, destroying the parton-model factorization of (1.1). Such radiation seriously degrades the effective energy of the beam particles and, hence, significantly reduces the Drell-Yan cross section at fixed Q^2. This type of phenomenon is familiar to experimentalists who routinely deal with the consequences of thick-target radiation.

Although the discussion we have given in this paper is couched in terms of a weak-coupling nonrelativistic model, our principal conclusions are based on the following results: the factored form of the cross section at large Q^2, the eikonal expression for the parton distributions, and the target-length condition [(2.11)]. All of these results can be derived in QCD as well.[2,3] Hence, we expect the phenomena that we have uncovered to play a role in hadronic collisions as well.

As we have emphasized, an important consequence of active-spectator initial-state interactions is that one expects A-dependent broadening of the $\mathbf{Q}_\perp$ distribution for Drell-Yan reactions with nuclei. The strength of this effect provides a direct measure of the transparency of nuclear matter to quarks. The increase in $\langle \mathbf{Q}_\perp^2\rangle$ is just the average number of collisions the quark undergoes multiplied by the average momentum transferred in each collision. The average number of collisions can be expressed in terms of the nuclear radius R_A and the mean free path ξ_q for quarks in nuclear matter:

$$N_{\text{collisions}}=\frac{\frac{4}{3}R_A}{\xi_q} \quad (4.1)$$

for a spherical nucleus. Thus the increase in $\langle \mathbf{Q}_\perp^2\rangle$ is

$$\Delta\langle \mathbf{Q}_\perp^2\rangle=\frac{4R_A l_\perp^2}{3\xi_q}=\frac{4R_N l_\perp^2 A^{1/3}}{3\xi_q}, \quad (4.2)$$

where R_N is the radius of a nucleon, and where $l_\perp \approx 250$ MeV is the typical transverse momentum transferred to the quark in a single collision. (Michael and Wilk[13] have discussed the smearing of the lepton-pair transverse momentum in terms of a particular model for multiple scattering and have reached similar conclusions.)

In fact there is evidence for this $\mathbf{Q}_\perp$ broadening in the π^--nucleus Drell-Yan data produced by the NA10 Collaboration.[14] The $\mathbf{Q}_\perp$ distribution in a tungsten target is enhanced at large $\mathbf{Q}_\perp$ relative to the $\mathbf{Q}_\perp$ distribution in a deuterium target and depleted at small $\mathbf{Q}_\perp$; this is consistent with the expected A dependence. The difference in $\langle \mathbf{Q}_\perp^2\rangle$ for tungsten and deuterium is roughly 0.15 GeV^2/c^2. This result together with (4.2) implies a mean free path ξ_q for the quark of about 2 fm. A phenomenological analysis of the NA10 data along these lines has been given by Chiappetta and Pirner.[15] Recent evidence for initial-state active-spectator interactions in gluon-induced reactions has been discussed by Hufner, Kurihara, and Pirner.[16]

The interpretation of such data as evidence for active-spectator initial-state interactions is complicated by several issues. Nuclear Fermi motion can also broaden the $\mathbf{Q}_\perp$ spectrum. Given that the maximum momentum of a nucleon in a nucleus is only of order 250 MeV, Fermi motion can shift $\langle \mathbf{Q}_\perp^2\rangle$ in tungsten relative to hydrogen by only about 0.06 GeV^2/c^2; the effect should be much smaller when one compares tungsten and deuterium. Potentially more important are dynamical phenomena that are related to the European Muon Collaboration (EMC) effect.[17] Several experiments[18,19] have demonstrated that the quark structure functions for nucleons are modified in nuclei. This could imply a nuclear modification of the transverse-momentum distribution of the struck quark in the target. That is, there could be a "transverse EMC effect." However, such a contribution to the $\mathbf{Q}_\perp$ distribution would saturate as A increases, in contrast with the effect due to active-spectator initial-state interactions. The transverse EMC effect is itself worthy of study. Experimental measurements of this effect might well allow us to discriminate between different models of the EMC effect. Also, the sign and magnitude of the transverse EMC effect may depend upon whether the active quark in the target is a valence quark or sea quark. Then the extent of the $\mathbf{Q}_\perp$ smearing would depend upon whether pion beams or proton beams were used in the experiment.

A related phenomenon, which is due to final-state interactions, should occur in jet production in deeply inelastic scattering on nuclear targets. In this process, formation of the leading particles in the quark jet occurs well outside the nuclear volume at high energies. The final-state interactions between the struck quark and the rest of the nucleus broaden the $p_\perp$ distribution of the quark jet, with $\Delta p_\perp^2 \propto A^{1/3}$.

Similarly, the smearing of $p_\perp$ distributions that results from initial- and final-state interactions in high-$p_\perp$ inclusive hadronic reactions can have a very significant effect upon the cross sections, since those sections decrease rapidly with $p_\perp$.

Another way that one can look for active-spectator initial-state interactions is by studying the failure of the Drell-Yan formula (1.1) for large targets. There is a minimum laboratory beam momentum $P_{\min}$ below which the Drell-Yan formula fails. Our analysis shows that this minimum momentum is dependent upon the size of the target. For large enough nuclei we expect [see (2.11)]

$$x_b P_{\min} \approx l_\perp A^{2/3} , \tag{4.3}$$

where x_b is the momentum fraction carried by the beam quark, $l_\perp$ is a typical hadronic scale, and we have neglected the quark mass. The Drell-Yan cross section is significantly reduced if P is below this value. Of course, there are many competing mechanisms with the Drell-Yan process at low Q^2, so this effect may not show up except in the very largest nuclei. From (4.3) we see that the Drell-Yan formula could well fail for a ^{238}U target even when Q^2 is as large as 10 or 20 GeV$^2/c^2$. On the other hand, if one were to collide nuclei at the Superconducting Super Collider, with a typical center-of-mass energy 20 TeV, then for $x_b \approx 1$ the target-length condition of (2.11) would be satisfied for nuclei up to about 10^{-5} cm in length—almost a macroscopic size.

As we have seen, the study of initial- and final-state interactions in hadronic collisions leads us to confront many of the implications of the full QCD gauge-field theory of the strong interactions. In so doing, we explore our partonic pictures of high-energy processes, both by discovering new phenomena that are not contained in the naive parton model and by revealing the range of validity of the partonic framework itself.

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