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The reaction $\gamma\gamma \to \pi\pi$ at low energy

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The results of a previous calculation by two of the present authors are presented in a form more suitable for comparison with recent experiments on $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$. The relation of the process $K_S \to \gamma\gamma$ to these calculations is recalled briefly.

I. INTRODUCTION

The experimental study of $\gamma\gamma \to \pi\pi$ near threshold has reached a new level of maturity with the presentation of data from the Crystal Ball Collaboration on $\gamma\gamma \to \pi^0\pi^0$ (Ref. 1). New data from the JADE Collaboration[2] on $\gamma\gamma \to \pi^0\pi^0$ appear to have similar energy dependence, but are preliminary. Together with data on $\gamma\gamma \to \pi^+\pi^-$ (Refs. 3–7), these results have been analyzed recently,[8–11] and have been shown to provide effective tests of various models[8–17] proposed in the past few years for the low-energy $\pi\pi$ and $K\bar{K}$ systems.

The low-energy dynamics of the $\pi\pi$ system is almost completely specified by the constraints of crossing, unitarity, and current algebra.[18–20] Thus, one would expect a description based on these features and on the Born term for $\gamma\gamma \to \pi^+\pi^-$ to provide a suitable framework for analyzing the $\gamma\gamma \to \pi\pi$ reaction. Such an analysis was in fact performed by several authors,[21–23] shortly after the work[24] which called attention to the accessibility of the two-photon process in e^+e^- collisions. The adequacy of the above assumptions for describing $\gamma\gamma \to \pi\pi$ near threshold was noted some time ago in Ref. 25.

In view of the current interest in low-energy $\pi\pi$ interactions, with new data available on $\gamma\gamma \to \pi\pi$ and related processes such as $K_S \to \gamma\gamma$, we felt it timely to present the predictions of one of the early models[22] in a manner more suitable for comparison with recent experiments. A preliminary account of this work has been presented in Ref. 26. The model is of interest not only with regard to its implications for the reaction $\gamma\gamma \to \pi\pi$, but as a possible example of how strong-interaction dynamics could be applicable to physics at the TeV scale.[27]

Our results are, in brief, as follows.

The old predictions for $\gamma\gamma \to \pi^+\pi^-$ led one to expect relatively modest modifications of the Born term at low energies. The effect of the $I=0$ S-wave scattering enhancement in $\pi\pi$ scattering (the σ) was found to be substantially shifted downward in energy for this process. These predictions are rather similar to those presented in Fig. 10 of the second paper of Ref. 9. The results from experiment are ambiguous. The study of $\gamma\gamma \to \pi^+\pi^-$ requires careful separation of pion pairs from $\mu^+\mu^-$ and e^+e^- backgrounds, a challenging task at low energies. Measured cross sections disagree with one another and with most theoretical predictions below $E_{\mathrm{c.m.}} \simeq 500$ MeV. A careful compilation of the data (with various angular selections) and comparison with predictions has been performed in Refs. 8–10.

The process $\gamma\gamma \to \pi^0\pi^0$, on the other hand, has been measured recently with little background all the way down to threshold.[1,2] The old prediction for the total $\gamma\gamma \to \pi^0\pi^0$ cross section is not far from experiment, but the data appear somewhat flatter as a function of $m_{\pi\pi} \equiv E_{\mathrm{c.m.}}/c^2$ than the prediction.

The decay $K_S \to \gamma\gamma$ offers an independent test of the model description of $\gamma\gamma \to \pi\pi$. Recent data on this decay confirm the assumption we have made of smooth variation in the amplitudes. The data are not sufficiently precise, as yet, to resolve ambiguities in model parameters.

Because the original publication of the framework was relatively long ago, we have organized the paper as a self-contained presentation. In Sec. II we give a brief discussion of elastic pion-pion scattering following the approach of Refs. 18 and 19. This formalism is then applied to the case of two-pion production by two photons in Sec. III, and we recapitulate some key results of Ref. 22. In that work some predictions were also made for the process $\gamma\gamma \to 2\pi^+2\pi^-$. The situation with regard to experimental information on that reaction is brought up to date in Sec. IV. Some remarks on the process $K_S \to \gamma\gamma$ are contained in Sec. V. We summarize in Sec. VI.

II. ELASTIC PION-PION SCATTERING

The assumptions of current algebra, unitarity, and crossing symmetry can be combined to give a simple model of elastic $\pi\pi$ scattering, valid up to almost 1 GeV in the center of mass.[18,19]

In the zero-pion-mass limit, current-algebra amplitudes for S- and P-wave pion-pion scattering are of the form $t_I^{(\mathrm{CA})}(s)\sim p_i\cdot p_j/f_\pi^2$, where I labels the isospin and p_i are four-momenta. Here f_π is the pion decay constant, whose value is 93 MeV. When pion masses are taken into account, one finds[28]

$$t_0^{(\mathrm{CA})}(s)=(16\pi f_\pi^2)^{-1}(2s-m_\pi^2)\ , \tag{2.1a}$$

$$t_2^{(\mathrm{CA})}(s)=-(16\pi f_\pi^2)^{-1}(s-2m_\pi^2)\ , \tag{2.1b}$$

$$t_1^{(\mathrm{CA})}(s)=(16\pi f_\pi^2)^{-1}(s-4m_\pi^2)/3\ , \tag{2.1c}$$

where $s=E_{\mathrm{c.m.}}^2$ is the squared center-of-mass energy. The $I=0,2$ amplitudes are S waves, while $I=1$ is P wave. Here the amplitudes t_I are normalized in such a way that

$$t_I(s)=(\sqrt{s}/k)e^{i\delta_I}\sin\delta_I\ , \tag{2.2}$$

with $k=(s/4-m_\pi^2)^{1/2}$ equal to the magnitude of the center-of-mass three-momentum of either pion.

Next, one imposes elastic unitarity on the $\pi\pi$ scattering amplitude. In effect, one is resumming $\pi\pi$ scattering "bubbles." One finds an amplitude

$$t_I(s)=t_I^{(\mathrm{CA})}(s)D_{lI}^{-1}(s)\ , \tag{2.3}$$

where the function D, labeled by orbital angular momentum l and isospin I, parametrizes the effect of rescattering. A convenient function for this purpose[18] is

$$D_{lI}(s)=\frac{M_I^2-s}{M_I^2-s_{0I}}+[h_1(s)-h_1(|M_I^2|)-ik/\sqrt{s}\,]t_I^{(\mathrm{CA})}(s)\ , \tag{2.4}$$

with

$$h_1(s)\equiv(\beta/2\pi)\ln[(1+\beta)/(1-\beta)] \tag{2.5}$$

and $\beta\equiv 2k/\sqrt{s}=(1-4m_\pi^2/s)^{1/2}$. In (2.4) the parameters s_{0I} are chosen to be the zeros of the current-algebra amplitudes (2.1), so that the amplitudes will have the correct slopes in s at those points. The parameters M_I, when M_I is real, describe resonance masses. The real part of D_{lI} vanishes at $s=M_I^2$ for $I=0,1$. If M_I is imaginary, the effect of a repulsive $\pi\pi$ interaction in the $I=2$ channel can be imitated by having $\mathrm{Re}D_{02}=0$ at $s=-|M_2^2|$.

The above unitarization scheme involves a single free parameter M_I for each isospin channel. If one uses a specific value of the ρ mass, one obtains a relation between the ρ mass and the width $\Gamma(\rho\rightarrow\pi\pi)$ familiar from other approaches.[29]

The unitarization scheme of Ref. 18 destroys the crossing symmetry of the original current-algebra amplitudes (2.1), since it is only applied in one channel (the s channel) (Ref. 30). The restoration of this symmetry was discussed, for example, in Refs. 19 and 20. It was found in Ref. 19 that for a restricted range of masses M_I, the crossing symmetry (as checked with the help of dispersion relations) was obeyed in an approximate fashion. This provided absolute predictions of resonance masses. For zero pion mass, the energy scale was set entirely by f_π. One obtained in that limit $m_\rho\sim 2\pi f_\pi$, $m_\sigma\sim m_\rho$, and

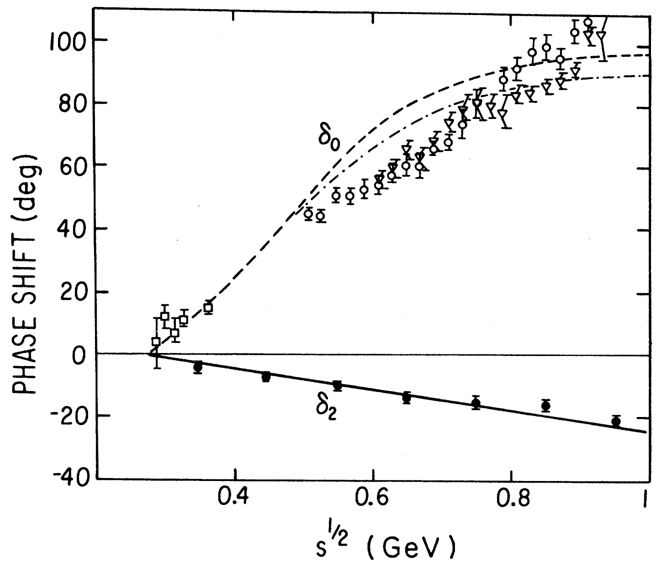

FIG. 1. S-wave phase shifts δ_I predicted by the model of Ref. 19, compared with results of analyses of experimental data. Solid line: predicted δ_2 for $M_2^2=-(685\ \mathrm{MeV}/c^2)^2$; dashed line: predicted δ_0 for $M_0=755\ \mathrm{MeV}/c^2$; dashed-dotted line: predicted δ_0 for $M_0=900\ \mathrm{MeV}/c^2$. Data points: open squares, Ref. 31 ($I=0$); open circles, Ref. 32 ($I=0$); triangles, Ref. 33 ($I=0$); closed circles, Ref. 34 ($I=2$).

an imaginary $I=2$ parameter $M_2\simeq im_\rho$. The inclusion of a small additional D-wave contribution from the $f_2(1270)$ resonance led to the predictions $M_0=730\ \mathrm{MeV}/c^2$, $M_1=690\ \mathrm{MeV}/c^2$, $M_2^2=-(710\ \mathrm{MeV}/c^2)^2$. The inclusion of pion-mass effects changed the $I=0$ mass predictions to $M_0=755\ \mathrm{MeV}/c^2$, $M_2^2=-(685\ \mathrm{MeV}/c^2)^2$. The predicted phase shifts (from Fig. 2 of Ref. 19) are compared with data[31–34] for $I=0$ and $I=2$ channels in Fig. 1. Also shown is a prediction with $M_0=900\ \mathrm{MeV}/c^2$. The model slightly overestimates δ_0 above 500 MeV/c^2, but less drastically for the higher value of M_0. Further increases in M_0 appear to have little effect. The $I=2$ phase shifts appear to have been correctly predicted in Ref. 19.

To summarize, low-energy $\pi\pi$ scattering is an exceptional physical system. The low mass of the pion means that crossing symmetry, unitarity, and current algebra represent very strong constraints on dynamical behavior and, indeed, the qualitative features of $\pi\pi$ dynamics are determined by these constraints. These features are $I=0$ S-wave and $I=1$ P-wave resonances, a relationship between the widths and masses of these resonances, and a repulsive S-wave $I=2$ interaction. The scale for the masses is set by $2\pi f_\pi$. Actual values of the masses are sensitive to high-energy effects (at the 25–30% level) which can be represented by polynomial ambiguities in the scattering amplitudes.

III. THE REACTION $\gamma\gamma\rightarrow\pi\pi$

The reaction $\gamma\gamma\rightarrow\pi\pi$ is governed by S- and D-wave $I=0$ and 2 $\pi\pi$ amplitudes up to center-of-mass energies well above 1 GeV. We shall take into account the effects of final-state interactions in the S waves, using the model for the $\pi\pi$ interaction described in the previous section. A small additional contribution to the D waves below 1 GeV in the center of mass comes from the tail of the $f_2(1270)$ resonance, which we shall add in a manner described in the first of Refs. 7.

The S-wave Born-term contribution to the charged-pion pair-production cross section is

$$\sigma_{SB}=\sigma_1\beta(1-\beta^2)|f_0|^2 , \tag{3.1}$$

where

$$\sigma_1\equiv\pi\alpha^2/8m_\pi^2=4.17\times10^{-31}\ \mathrm{cm}^2 , \tag{3.2}$$

$$\beta\equiv(1-4m_\pi^2/s)^{1/2}=2k/\sqrt{s} , \tag{3.3}$$

and

$$f_0\equiv f_{\mathrm{Born}}^{(I=0)}\equiv f_{\mathrm{Born}}^{(I=2)}\equiv\frac{1-\beta^2}{\beta}\ln\frac{1+\beta}{1-\beta} . \tag{3.4}$$

The *total* Born term contribution is

$$\sigma_B=\sigma_1\beta(1-\beta^2)\left[8-4\beta^2-2\frac{1-\beta^4}{\beta}\ln\frac{1+\beta}{1-\beta}\right] . \tag{3.5}$$

The S-wave contributions to the charged- and neutral-pion production cross sections are defined, respectively, as σ_{SC} and σ_{SN}. They may be expressed in terms of amplitudes $f^{(I=0)}$ and $f^{(I=2)}$ which reduce to the amplitude f_0 of Eq. (3.4) in the absence of final-state interactions. With the normalizations adopted above, we have

$$\sigma_{SC}=\sigma_1\beta(1-\beta^2)|f_C|^2,\quad \sigma_{SN}=\sigma_1\beta(1-\beta^2)|f_N|^2/2 , \tag{3.6}$$

with

$$f_C\equiv(2f^{(I=0)}+f^{(I=2)})/3,\quad f_N\equiv2(f^{(I=0)}-f^{(I=2)})/3 . \tag{3.7}$$

The factor of 2 in the denominator of the expression (3.6) for σ_{SN} accounts for the identity of the two pions in the final state. In the absence of rescattering, the terms $f^{(I=0)}$ and $f^{(I=2)}$ in f_N would cancel one another.

The form for the amplitudes $f^{(I)}$ taken in Ref. 22 is

$$f^{(I)}=[P_I(s)f_0^2+Q_I(s)f_0+R_I(s)]/D_{0I}(s) , \tag{3.8}$$

where

$$P_I(s)=\frac{k^2}{4\pi m_\pi^2}t_I^{(\mathrm{CA})}(s) , \tag{3.9}$$

$$Q_I(s)=\frac{M_I^2-s}{M_I^2-s_{0I}}-t_I^{(\mathrm{CA})}h_1(|M_I|^2) , \tag{3.10}$$

$$R_I(s)=\frac{\pi m_\pi^2}{s}t_I^{(\mathrm{CA})}(s)-\frac{1}{\pi}t_I^{(\mathrm{CA})}(0)+O(s/m_\rho^2) . \tag{3.11}$$

Note that P_I, Q_I, and R_I are completely specified in terms of the current-algebra amplitudes and resonance masses, except for the term $O(s/m_\rho^2)$ in $R_I(s)$. The motivation for the form (3.8) is given in detail in Ref. 22. Here we merely remark that it is a simple ansatz which takes account of the rescattering of pions in the direct channel; it avoids introducing spurious singularities in the left-half s plane, but, as described in the previous section, accounts in an approximate fashion for crossing symmetry of the elastic amplitude through the choice of the parameters M_I^2.

The last term in Eq. (3.11) expresses the uncertainty in our calculation, which is motivated purely by the low-energy theorem for $\gamma\gamma\to\pi\pi$ and by current algebra. A polynomial uncertainty of this order should, in fact, be a feature of *any* attempt to relate elastic $\pi\pi$ scattering to inelastic processes involving the production of two pions without the use of further theoretical assumptions.

The total charged-pion production cross section in the low-energy region may be expressed as

$$\sigma_C\equiv\sigma(\gamma\gamma\to\pi^+\pi^-)=\sigma_{SC}+\sigma_B-\sigma_{SB} , \tag{3.12}$$

where the last two terms take account of the Born-term contribution to higher partial waves. [Here we do not include the effects of the $f_2(1270)$.] In Fig. 2 we quote the Born term cross section σ_B and the prediction for σ_C. The shaded band expresses the polynomial ambiguity mentioned earlier.

The effects of S-wave final-state interactions on the Born term in $\gamma\gamma\to\pi^+\pi^-$ are relatively modest, and are similar to "solution A" of Ref. 9. A consensus[8–10,22] appears to be that for $m_{\pi\pi}$ above 350 MeV/c^2, the predicted cross section can actually fall *below* the Born term, depending on the polynomial ambiguity mentioned in Ref. 22. The experimental cross sections[3–7] appear to lie higher than the Born term, but generally within the limits entailed by the predictions of Fig. 2. As in Ref. 9, we are unable to account for the data of the DM1+DM2 Collaborations[4] below 500 MeV/c^2 or of PLUTO (Ref. 3) below 400 MeV/c^2. At such low dipion masses, the process $\gamma\gamma\to\mu^+\mu^-$ is a potential contaminant.

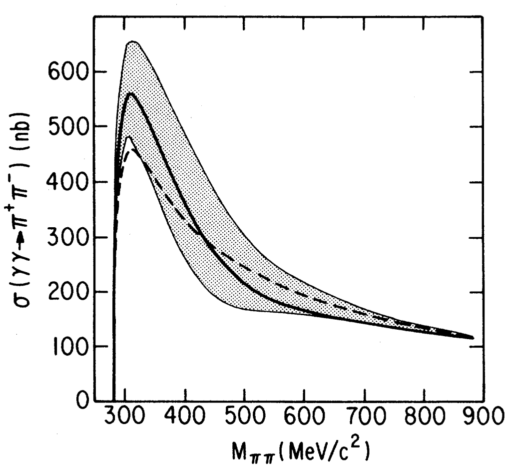

FIG. 2. Cross section for $\gamma\gamma\to\pi^+\pi^-$ as a function of $\pi\pi$ effective mass. Dashed line: Born term σ_B [Eq. (3.5)]; solid line: predicted cross section σ_C [Eq. (3.12)]. Shaded band: effect of the uncertainty of $\pm s/m_\rho^2$ in the term $R_0(s)$ in Eq. (3.11).

The cross section σ_N for neutral-pion production may be calculated by adding a contribution σ_{DN} for the $f_2(1270)$ to the S-wave cross section σ_{SN} of Eq. (3.6):

$$\sigma_N = \sigma_{SN} + \sigma_{DN} \ , \tag{3.13}$$

$$\sigma_{DN} = \frac{40\pi m_f}{\sqrt{s}} \frac{\Gamma_{\pi^0\pi^0}\Gamma_{\gamma\gamma}}{(m_f^2 - s)^2 + m_f^2 \Gamma_{\rm tot}^2} \ , \tag{3.14}$$

where m_f is the f_2 mass, $\Gamma_{\pi^0\pi^0} = B_{\pi^0\pi^0}\Gamma_{\rm tot}$ and $\Gamma_{\gamma\gamma}$ are the partial widths of f_2 into $\pi^0\pi^0$ and $\gamma\gamma$, and the total width is given by the energy-dependent expression[35]

$$\Gamma_{\rm tot} = \Gamma_0 \left[\frac{k}{k_0} \right]^5 \frac{(k_0 r_0)^4 + 3(k_0 r_0)^2 + 9}{(k r_0)^4 + 3(k r_0)^2 + 9} \ , \tag{3.15}$$

where Γ_0 is the physical width of the f_2, k is the magnitude of the three-momentum in the $\pi\pi$ system (as in Sec. II), k_0 is the corresponding value at the f_2 resonance mass, and r_0 is a hadronic radius. For the parameters in Eqs. (3.14) and (3.15) we have taken the values $m_f = 1264$ MeV/c^2, $\Gamma_0 = 190$ MeV, $B_{\pi^0\pi^0} = 0.281$, $r_0 = 1$ fm, and $\Gamma_{\gamma\gamma} = 3.26$ keV, which provide an approximate fit[26] to the $\pi^0\pi^0$ spectrum presented in the second of Ref. 1 (Ref. 36).

The prediction for σ_N is compared with experimental values[1] in Fig. 3. The increase in the predicted cross section above 700 MeV/c^2 is due to the contribution of the f_2 tail. Only the resonant contribution is included; the Born term does not contribute directly to $2\pi^0$ production.

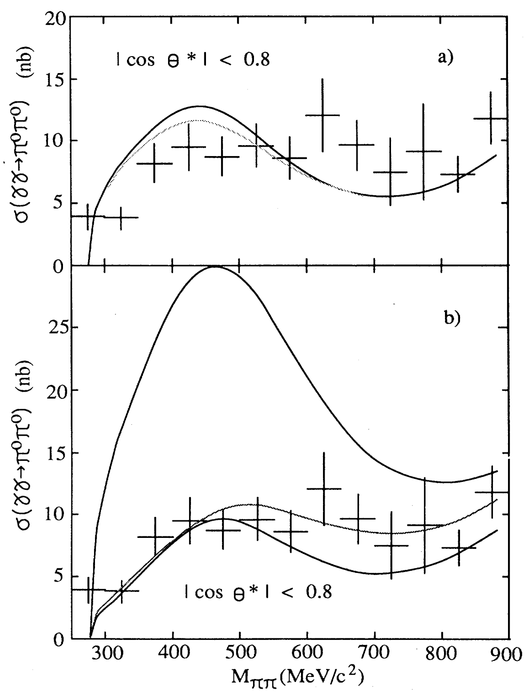

FIG. 3. Cross section $\sigma_N \equiv \sigma(\gamma\gamma \to \pi^0\pi^0)$ [Eq. (3.13)] as a function of effective $\pi\pi$ mass for $|\cos\theta^*| \le 0.8$. Data points are from the second of Refs. 1. The $f_2(1270)$ tail has been added as described in the text. (a) Predictions based on central values for $f^{(I=0)}$. Upper curve: $M_0 = 755$ MeV/c^2; lower curve: $M_0 = 900$ MeV/c^2. (b) Upper and lower curves: range of predictions corresponding to term $\pm s/m_\rho^2$ in Eq. (3.11). Central curve: effect of adding a term $-0.85\ s/m_\rho^2$ to $R_0(s)$ in Eq. (3.11). Here $M_0 = 755$ MeV/c^2 for all curves.

The predicted cross section shown in Fig. 3(a) [based on the *central* values for $f^{(I=0)}$, without the additional polynomial term in Eq. (3.11)] appears more rapidly varying than the data as a function of $m_{\pi\pi}$. Nonetheless, the average magnitude appears correct. The low-energy "bulge" predicted for $M_0 = 755$ MeV/c^2 (the value in Ref. 22) is slightly reduced if M_0 is raised to 900 MeV/c^2, but, just as for elastic scattering, further increases in M_0 seem to have little effect. One ingredient which has been left out of the present calculation is the activity in the S-wave $\pi\pi$ amplitude around $m_{\pi\pi} = 1$ GeV/c^2, correlated with the presence of the S^* [now known as $f_0(975)$] and perhaps other[37] resonances and with the opening of the $K\bar{K}$ threshold.

The effect of the polynomial ambiguity in $R_0(s)$, mentioned earlier, is shown in Fig. 3(b). To be precise, we shall define the additional term $O(s/m_\rho^2)$ in Eq. (3.11) for $I=0$ to be $\Delta R_0(s)$. The upper and lower solid curves are the maximum and minimum predicted values of $\sigma(\gamma\gamma \to \pi^0\pi^0)$ as $\Delta R_0(s)$ ranges over the values between $-s/m_\rho^2$ and s/m_ρ^2. An acceptable fit to the low-energy data is obtained for $\Delta R_0(s) \simeq -0.85 s/m_\rho^2$, a polynomial term within these limits.

A recent prediction[10] for the low-energy behavior of $\sigma(\gamma\gamma \to \pi^0\pi^0)$ makes direct use of *experimental* values of $\pi\pi$ phase shifts. This is very much in the spirit advocated in Ref. 21. Impressive agreement is obtained with the recent data.[1] However, as we have seen, the existence of a polynomial ambiguity means that there will not be a unique connection between elastic phase shifts and the reaction $\gamma\gamma \to \pi\pi$, without additional assumptions.

We wish to compare our approach with the widely discussed model of Mennessier.[12] Both use low-energy theorems, chiral pion-pion scattering and unitarity. Our approach is to treat these as constraints, and to represent any remaining uncertainty with polynomial ambiguities. The Mennessier model satisfies the constraints by means of a specific chiral Lagrangian. In Mennessier's parametrization further attention is paid to the possibility of S-wave resonances in the 1-GeV region and to the interference of the $f_2(1270)$ resonant amplitude with the Born term in $\gamma\gamma \to \pi^+\pi^-$. However, the underlying physics is very similar in the two models.

Recently Bijnens and Cornet and Donoghue, Holstein, and Lin[17] have applied chiral perturbation theory to the reaction $\gamma\gamma \to \pi\pi$. Bijnens and Cornet find modest modifications of the Born term for charged-pion production, in accord with results of other approaches. For neutral-pion pairs, they find a cross section which rises approximately linearly from threshold up to $\sqrt{s} = 700$ MeV, to a value at that energy of 21.5 nb. A similar result is found by Donoghue *et al.* This behavior is in rough accord with the data (shown in Fig. 3) below about 500 MeV. Above that energy, Donoghue *et al.* have shown that unitarization of the chiral-perturbation-theory result should play an important role in keeping the cross section from rising further. Our approach contains such unitarization from the start.

We have compared the threshold behavior of our

$\gamma\gamma \to \pi^0\pi^0$ cross section with the chiral-perturbation-theory results of Ref. 17. Our approach predicts

$$\sigma(\gamma\gamma \to \pi^0\pi^0) = \frac{\pi\alpha^2\beta m_\pi^2}{2^{14} f_\pi^4}\left[3+\frac{4}{\pi^2}\right]^2 ,$$

whereas Ref. 17 predicts the quantity in parentheses to be $(-1+4/\pi^2)^2$. The difference appears to stem from the presence in the chiral Lagrangian approach of additional pion-loop contributions involving four-pion–photon and four-pion–two-photon point interactions, not contained in our treatment. Such terms lie within the ambiguity due to the remainder term $\Delta R_0(s) = O(s/m_\rho^2)$ in our Eq. (3.11). Present data are not of sufficient accuracy near threshold to distinguish between the two predictions.

The question is sometimes raised of whether one can add resonances in $\gamma\gamma \to \pi\pi$ to a "continuum" in describing the data. We would eschew such an approach except for narrow structures. Our construction shows that the "continuum" and the "σ" are different aspects of the same underlying physical structure. Great care must be taken, for example, in interpreting the residue of a pole at M_0 in S-wave $\gamma\gamma \to \pi\pi$ scattering in terms of a "$\sigma\gamma\gamma$" coupling. Under no circumstances would we advocate simply adding a "σ" to a Born term in $\gamma\gamma \to \pi^+\pi^-$ to reproduce the observed charged and neutral pion pair cross section up to $\sim$900 MeV. In the region of 1 GeV it may be permissible to add a narrow "$f_0(975)$" to an otherwise smoothly varying S-wave amplitude provided that the constraint of unitarity is carefully respected.

IV. THE PROCESS $\gamma\gamma \to 2\pi^+ 2\pi^-$

A further set of predictions in Ref. 22 concerned the reactions $e^+e^- \to 2\pi^+2\pi^-$ and $e^+e^- \to e^+e^-2\pi^+2\pi^-$ very near threshold. Present data[38–40] are not sufficiently accurate at low energies to test the threshold predictions. At a c.m. energy of 893 MeV, for example, the authors of Ref. 38 quote

$$\sigma(e^+e^- \to 2\pi^+2\pi^-) < 1.6 \text{ nb} , \tag{4.1}$$

to be compared with the predicted value[22] of 0.15 nb. The result of Ref. 39 is

$$\sigma(\gamma\gamma \to 2\pi^+2\pi^-) < 6 \text{ nb} \tag{4.2}$$

in the range $0.75 \le E_{\text{c.m.}} \le 1.15$ GeV, while Terazawa (Ref. 23) and the authors of Ref. 22 find a value of about 2 nb for this process at $E_{\text{c.m.}}/c^2 = 6m_\pi$ (Ref. 41). The very large values found for $\sigma(\gamma\gamma \to 2\pi^+2\pi^-)$ at higher energies[40,42] are not explained, of course, within this approach.

V. THE DECAY $K_S \to \gamma\gamma$

The assumption of a small or vanishing polynomial ambiguity in $R_0(s)$ can be tested in the recently measured[43] decay $K_S \to \gamma\gamma$. It was pointed out in Ref. 44 that a change in $R_0(s)$ affects the relation between the amplitude $A_{\gamma\gamma}$ for this process and that $(A_{\pi\pi})$ for the decay $K_S \to \pi^+\pi^-$:

$$A_{\gamma\gamma} = \alpha A_{\pi\pi}[F(m_K^2) + \Delta R_0(m_K^2)]/2 + s\bar{A}_{\gamma\gamma}(s) , \tag{5.1}$$

where

$$F(s) = \tfrac{1}{2}h_1 f_0 - h f_0 + \frac{\pi m_\pi^2}{s} - \frac{1}{\pi} , \tag{5.2}$$

$$h(s) = h_1(s) - i\beta/2 , \tag{5.3}$$

and the functions $f_0(s)$ and $h_1(s)$ were defined in Eqs. (3.4) and (2.5). The remaining term in Eq. (5.1), $s\bar{A}_{\gamma\gamma}(s)$, represents that portion of the $\gamma\gamma$ decay amplitude which is not dominated by the two-pion intermediate state. In Eq. (5.1) it is postulated that the $\gamma\gamma$ decay amplitude is dominated by the two-pion intermediate state. For $m_{K^0} = 497.7$ MeV/c^2 and $m_{\pi^\pm} = 139.6$ MeV/c^2 (we use the charged pion mass here), we find

$$F(m_K^2) = -0.21 + 0.37i . \tag{5.4}$$

If $\bar{A}_{\gamma\gamma}(s)$ is very small so that two-pion states dominated, then[44]

$$\frac{\Gamma(K_S \to \gamma\gamma)}{\Gamma(K_S \to \pi^+\pi^-)} = \frac{\alpha^2}{4\beta(m_K^2)}|F(m_K^2) + \Delta R_0(m_K^2)|^2 \tag{5.5a}$$

$$= 2.94 \times 10^{-6} \quad \text{for } \Delta R_0(m_K^2) = 0 . \tag{5.5b}$$

Experimentally,[43]

$$\frac{\Gamma(K_S \to \gamma\gamma)}{\Gamma(K_S \to \pi^+\pi^-)} = \frac{(2.4 \pm 1.2) \times 10^{-6}}{0.686} = (3.5 \pm 1.8) \times 10^{-6} . \tag{5.6}$$

The close agreement between Eqs. (5.5b) and (5.6) supports the idea[44–46] that $K_S \to \gamma\gamma$ is indeed dominated by the two-pion intermediate state, and that $\Delta R_0(m_K^2)$ is not too large.[47] At the 90% confidence level (1.64σ) we find the predicted $K_S \to \gamma\gamma$ rate (5.5a) is compatible with Eq. (5.6) only if

$$-0.7\frac{m_K^2}{m_\rho^2} \le \Delta R_0(m_K^2) \le 1.7\frac{m_K^2}{m_\rho^2} . \tag{5.7}$$

More precise data would provide a low-energy value for $R_0(s)$ which would be compared with the $\gamma\gamma \to \pi\pi$ data.

VI. CONCLUSIONS

We have compared some old predictions for two- and four-pion production by two photons with recent data. The qualitative agreement of the data on $\gamma\gamma \to \pi^0\pi^0$ at low energies with a calculation based on low-energy theorems, supplemented by unitarity and a rudimentary approximation to crossing symmetry, is an encouraging confirmation of these general principles. Further confirmation is provided by recent experimental measurements of the decay $K_S \to \gamma\gamma$.

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