Chiral-symmetry constraints on the critical temperature in QCD

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We study the restoration of chiral symmetry at finite temperatures in QCD. We focus on the role light particles play in disordering the system and compute the critical temperature in the linear σ model, the Nambu–Jona-Lasinio model, and in the nonperturbative one-gluon-exchange approximation in QCD. In all three cases we obtain $T_c = 2f_\pi \approx 180$ MeV, which might be a model-independent result.

I. INTRODUCTION

It has been known for some time that field theories with spontaneously broken symmetries exhibit symmetry restoration at high temperatures.[1,2] The reason for this is very simple. At finite temperatures, vacuum expectation values are replaced by thermal averages. Thermal fluctuations tend to disorder the system and, at the point where order-parameter fluctuations become comparable to the order parameter itself, its value averages to zero and the symmetry is restored.[3] At zero temperature the energy (density) of the vacuum is lowered by spontaneous symmetry breakdown. Increase of the temperature gives rise to an increasing tunneling rate between two vacua (symmetric and broken). When this tunneling rate is such that the system is in either vacuum with the same probability, the two vacua become degenerate and there is no advantage breaking the symmetry.

Considerable effort has been made in understanding the behavior of a variety of field-theoretical models, as well as that of the global symmetries at finite temperatures.[4] In recent years there has been an increasing activity to determine the phase diagram of QCD because of its relevence for the high-energy heavy-ion collisions and the physics of the early Universe.[5] Recent numerical studies suggests the existence of two phase transitions: deconfinement and chiral-symmetry restoration.[6]

It is believed that deconfinement comes from the gluon sector of the theory involving heavy quarks. On the other hand, chiral symmetry is the symmetry of the light-quark (u,d) sector where the action does not possess a scale; instead the scale is generated dynamically. At zero temperature, chiral symmetry is broken spontaneously: quarks acquire dynamical mass and $\bar{\psi}\psi$ develops a nonvanishing vacuum expectation value. The mechanism of chiral-symmetry breakdown is modeled[7,8] after BCS theory of superconductivity,[9] where the three scales of the theory (ω_D, Δ, T_c) are related by two BCS equations. Analogy between the two phenomena suggests that a similar connection between the fundamental scales of QCD might exist as well.[10]

Recently we have seen a major breakthrough in numerical simulations of QCD on the lattice.[11] Computer calculations of various physical quantities such as quark masses and the bound-state spectrum[12] have led to (numerical) results that are in rather good agreement with experiment. On the other hand, our analytical tools have been rendered inadequate by the complicated nonlinear structure of low-energy QCD. Therefore, understanding of these numerical results is still incomplete. Most of our analytical knowledge about the finite-temperature phase transitions in QCD comes from studying simple models and from making various approximations of QCD (Refs. 10, 13, and 14).

In this paper we study the constraints that chiral symmetry places on possible values of the critical temperature. In Sec. II we discuss the low-temperature expansion and the role of the degeneracy factor for light particles. We shall apply these ideas to particular models starting with the finite-temperature treatment of the linear σ model in Sec. III. Then the restoration temperature in the Nambu–Jona-Lasinio (NJL) model is calculated in Sec. IV. Finally, the Schwinger-Dyson equation calculation for QCD in the operator-product-expansion-(OPE-) improved one-gluon-exchange approximation is treated in Sec. V. For all three models, we obtain the result for the critical temperature $T_c = 2f_\pi \approx 180$ MeV (for $N_c = 3, N_f = 2$) as first found in Ref. 10. In Sec. VI we draw the relevant conclusions and suggest that such a relation between the chiral-symmetry-restoration temperature and the pion-decay constant has model-independent significance.

II. ROLE OF LIGHT PARTICLES FOR CHIRAL-SYMMETRY RESTORATION

In a recent paper,[15] it was shown that chiral symmetry constrains the low-temperature structure of QCD. At low energies the pion-decay constant sets the scale, and the low-temperature expansion of the (chiral-symmetry-breaking) quark condensate is

$$\langle \bar{\psi}\psi \rangle_T = \langle \bar{\psi}\psi \rangle \left[1 - \frac{N_f^2 - 1}{N_f} \frac{T^2}{12 f_\pi^2} + \cdots \right], \qquad (1)$$

where the pion-decay constant is $f_\pi \approx 90$ MeV in the chiral limit. The degeneracy factor $N_f^2 - 1$ counts the number of (massless) Nambu-Goldstone bosons in the theory which give the leading contribution to the thermal average in (1). The contribution of massive excitations (e.g., baryons and massive mesons) in (1) is exponentially suppressed. As long as the number of light particles does not change, the expansion (1) remains unaltered even at higher temperatures.

It is then reasonable to invoke such an expansion as the first estimate of the critical temperature T_c. Setting $\langle \bar{\psi}\psi \rangle_{T_c} = 0$ due only to the first two terms in (1) gives

$$T_c \approx 2f_\pi \left[\frac{3N_f}{N_f^2-1} \right]^{1/2} . \tag{2}$$

For $N_f=2$, Eq. (2) leads to $T_c \approx 2\sqrt{2} f_\pi \approx 255$ MeV. This estimate should be thought of as an upper bound to the critical temperature; as the number of light degrees of freedom increases, the degeneracy factor N_f^2-1 will be replaced by a larger number, thus lowering T_c in (2). This is the case for a second-order phase transition. Also, the flavor dependence of T_c in (2) gives the correct large-N_f limit;[2,16] as the number of light particles increases, the system becomes disordered and the critical temperature is lowered.

Regardless of the accuracy of the low-temperature expansion estimates of T_c, Eq. (1) also suggests the role that the degeneracy factor for light particles plays in obtaining the precise value of the critical temperature. However, counting the contribution of these light degrees of freedom in an interacting theory is model dependent. Nevertheless, in the sections to follow, we shall demonstrate that T_c is model independent and its value is determined by chiral symmetry.

It is interesting that one can obtain an approximate critical temperature of the same magnitude as found from (2) along with a similar qualitative dependence on N_f by assuming that the transition occurs at a point where the light particles (pions) "touch" each other. The number density of pions as a function of temperature is

$$\begin{aligned} n_\pi &= (N_f^2-1)\int \frac{d^3q}{(2\pi)^3}\frac{1}{e^{q/T}-1} \\ &= (N_f^2-1)\frac{T^3\zeta(3)}{\pi^2} . \end{aligned} \tag{3a}$$

When T reaches the critical value, space becomes filled with pions, and at that close-packing point, the pion-number density can be alternatively expressed as

$$n_\pi \approx \frac{1}{\frac{4}{3}\pi r_\pi^3} , \tag{3b}$$

where $r_\pi \approx 0.67$ fm is the experimental pion charge radius. Equating the two results for n_π, we arrive at

$$T_c \approx \left[\frac{3\pi}{4\zeta(3)(N_f^2-1)} \right]^{1/3} \frac{1}{r_\pi} . \tag{4}$$

For $N_f=2$, the above equation gives $T_c \approx 255$ MeV. Again, it is clear that an increase of the degeneracy factor reduces T_c below 255 MeV. In particular, contributions to the degeneracy factor will come from other bosonic hadrons (predominantly σ) and nucleons. Nevertheless, this geometrical model of the transition gives the right scale and approximate degeneracy factor dependence for T_c; it even predicts that $T_c \sim f_\pi$ because[17] $r_\pi \sim 1/f_\pi$.

III. LINEAR σ MODEL

In this section we shall apply the general ideas of Sec. II to the concrete example of an interacting theory—the linear σ model.[18] At $T=0$ the theory breaks chiral $SU(N_f)\times SU(N_f)$ symmetry spontaneously: quarks acquire dynamical mass, pions are massless Goldstone bosons, and the σ mass is proportional to the order parameter. As the system is heated, the splitting between the σ and π's narrows. At T_c the theory undergoes a second-order phase transition: the quarks and σ meson, in addition to the pions, become massless. The thermal averages are then no longer dominated by pions; the σ meson and quarks contribute with equal weight to the degeneracy factor.

The σ-model Lagrangian density is[18]

$$\begin{aligned} \mathcal{L} &= \bar{\psi}(i\partial\!\!\!/ + g\sigma + ig\gamma_5 \boldsymbol{\pi}\cdot\boldsymbol{\tau})\psi \\ &\quad + \tfrac{1}{2}(\partial_\mu\sigma)^2 + \tfrac{1}{2}(\partial_\mu\boldsymbol{\pi})^2 - \tfrac{1}{2}\mu^2(\sigma^2+\boldsymbol{\pi}^2) - \frac{\lambda}{4}(\sigma^2+\boldsymbol{\pi}^2)^2 . \end{aligned} \tag{5}$$

When $\mu^2<0$, the σ field develops a vacuum expectation value and the chiral symmetry is spontaneously broken. In the tree approximation, the particles have masses

$$m_\pi^2 = 0, \quad m_\sigma^2 = -2\mu^2, \quad m_{\rm dyn} = f_\pi g , \tag{6}$$

where $f_\pi^2 = -\mu^2/\lambda$ and $m_{\rm dyn}$ is the dynamically generated quark mass.

As the temperature is increased, the typical contribution to the scalar mass will be positive and proportional to the temperature. On the grounds of dimensional analysis, this contribution from the scalar sector gives

$$\mu^2(T) = \mu^2 + C^2\lambda T^2 , \tag{7}$$

where C is a real numerical constant. At the point where $\mu^2(T)$ vanishes, symmetry is restored and we have

$$T_c^2 = -\frac{\mu^2}{\lambda C^2} \equiv \frac{f_\pi^2}{C^2} . \tag{8}$$

The only problem, therefore, is to establish the value of C which is essentially a degeneracy factor. For that purpose we calculate $m_\sigma(T)$ and look for the temperature at which it vanishes. At the critical point, there are only three graphs that contribute to the σ self-energy (Fig. 1). Diagrams that are induced by spontaneous symmetry breaking (not explicitly displayed in Fig. 1), such as trilinear σ and $\sigma\pi\pi$ terms, are all proportional to the order parameter and, thus, vanish at the critical point.

Detailed calculations of the first diagram of Fig. 1 were given in Ref. 10 and we will not repeat them here. The other two terms are easy to calculate (the first and second

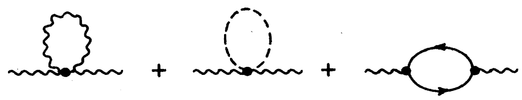

FIG. 1. Self-energy of σ meson in linear σ model. Wavy lines represent σ, dashed lines π's, and solid lines quarks.

graphs differ only by a combinatorial factor). In the limit $m_\sigma(T)\to 0$, the equation for T_c becomes

$$m_\sigma^2=\frac{T_c^2}{6}[\lambda(3+N_f^2-1)-2g^2N_c] , \tag{9a}$$

where $T_c^2/6$ is the thermal distribution factor, the first and second terms are due to the σ and π loops, respectively, and the third term $(-2g^2N_c)$ comes from the fermion loop (thus the "minus" sign). In order to display the combinatorial factor and compare T_c with f_π, as we did using the dimensional arguments in (8), we use $\lambda=m_\sigma^2/2f_\pi^2$ and $g^2=m_\sigma^2/4f_\pi^2$. The former relation is merely a definition of f_π from (6) and the latter is a modified version of the Goldberger-Trieman relation $(m_{\rm dyn}=f_\pi g)$ supplemented with the QCD result[19] $m_\sigma=2m_{\rm dyn}$. Combining the above equations with (9a), we arrive at

$$m_\sigma^2=\frac{T_c^2}{12f_\pi^2}m_\sigma^2(3+N_f^2-1-N_c) . \tag{9b}$$

Factorization of $m_\sigma^2\neq 0$ in (9b) then results in

$$T_c^2=f_\pi^2\frac{12}{3+(N_f^2-1)-N_c} \tag{10a}$$

and correspondingly, the combinatorial factor of (8) is $C^2=(3+N_f^2-1-N_c)/12$. When $N_f=2$ and $N_c=3$, the chiral-symmetry-restoration temperature becomes

$$T_c=2f_\pi . \tag{10b}$$

Here, an interesting cancellation has occurred in (10a): N_c from the quark loop has been balanced by the degeneracy factor for the σ loop, leaving the (light) pion degeneracy factor (N_f^2-1) to control the critical temperature as we anticipated from the low-temperature expansion in Sec. II. Also, for large N_f we find from (10a) that $T_c\sim f_\pi/N_f$ which is the correct behavior of the critical temperature in this limit.[2,3]

IV. NJL FOUR-FERMION MODEL

Like the linear σ model, the Nambu–Jona-Lasinio (NJL) model satisfies basic relations of current algebra that are determined by chiral symmetry. Perhaps the biggest difference between the two models is the way they describe scalar mesons. In the NJL model, σ and π mesons are realized as quark-antiquark composites, whereas in the σ model they are elementary. The two models are thought to correspond to low-energy (σ model) and intermediate-energy[20] (NJL) effective theories of QCD. The NJL model is a four-fermion (nonrenormalizable) theory defined by the Lagrangian density

$$\mathcal{L}=\bar\psi(i\not\partial+g\sigma+ig\gamma_5\boldsymbol{\pi}\cdot\boldsymbol{\tau})\psi-\tfrac{1}{2}(\sigma^2+\boldsymbol{\pi}^2) , \tag{11}$$

where σ and $\boldsymbol{\pi}$ are auxiliary fields that need to be integrated out in the Hamiltonian. Fermion fields transform according to the fundamental representation of the color-SU(N_c) and flavor-SU(N_f) groups; $\boldsymbol{\tau}$ are the generators of the flavor group and color indices have been suppressed. The NJL coupling constant g has dimension of inverse mass. An ultraviolet cutoff Λ will be used in order to regulate short-distance singularities.

A finite-temperature treatment of the NJL model is expected to be an adequate alternative to the much more complex low-energy QCD. The model has been treated from different points of view by several authors.[13,14] Almost uniformly, the general strategy was to fit the values of the cutoff Λ and coupling constant g so that the model reproduces phenomenological values of $\langle\bar\psi\psi\rangle$ and f_π or some other choice of measurable parameters, and thereafter T_c was determined. However, as we shall demonstrate in this section, an important identity[14] between T_c, f_π, and the gap mass Δ has not been emphasized. This relation fixes the value of T_c in terms of f_π (or Δ), regardless of the choice of Λ and g:

$$\left[\frac{\pi T_c}{\sqrt{3}}\right]^2=\frac{2\pi^2}{N_c}f_\pi^2+\frac{\Delta^2}{2} . \tag{12}$$

In order to derive (12), we start with the Schwinger-Dyson (SD) equation for fermion dynamical mass Δ at finite temperature:

$$\Delta(T)=G\int\frac{d^3q}{(2\pi)^3}\frac{\Delta(T)}{\omega_q}\left[1-\frac{2}{e^{\omega_q/T}+1}\right] , \tag{13}$$

where $G\equiv g^2[2N_fN_c+(N_f^2-2)/2]$ and $\omega_q=[q^2+\Delta^2(T)]^{1/2}$ is the energy of massive quarks. The first term $(2N_fN_c)$ in G comes from the Hartree (σ tadpole) contribution to the self-energy, while the second term (N_f^2-2) is composed of two parts: π exchange (N_f^2-1) and σ exchange (1 with opposite sign).

At zero temperature the critical coupling, below which quarks remain massless, is $G_c=4\pi^2/\Lambda^2$. On the other hand, the restoration temperature, which is obtained from the gap equation (13) in the limit $\Delta(T_c)\to 0^+$, is[14]

$$\left[\frac{\pi T_c}{\sqrt{3}}\right]^2=4\pi^2\frac{G-G_c}{GG_c} . \tag{14}$$

In Ref. 14 it was argued that for $(G-G_c)G_c^{-1}\ll 1$, chiral-symmetry breakdown occurs in the infrared regime of the theory, and the way one regularizes the ultraviolet singularities becomes unimportant. It is therefore possible to establish the relationship between the physical quantities relevant for chiral symmetry without reference to the cutoff. The expression for the pion decay constant at $T=0$ is[14]

$$f_\pi^2=N_c\int\frac{d^3q}{(2\pi)^3}\frac{\Delta^2}{\omega_q^3} . \tag{15}$$

Using the gap equation (13), this relation (15) leads directly to the important identity (12).

Connection between f_π and Δ can be obtained from the definition of quark-pion coupling ($g_{\pi qq}$):

$$if_\pi k_\mu = g_{\pi qq} N_c \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}[\gamma_\mu \gamma_5 S(q) \gamma_5 S(q+k)] \ . \tag{16}$$

Solving the integral in (16) is straightforward, but not necessary. At an intermediate stage, Eq. (16) can be written as

$$f_\pi = g_{\pi qq} N_c \int \frac{d^3q}{(2\pi)^3} \frac{\Delta}{\omega_q^3} \ . \tag{17}$$

Comparing (15) and (17), we arrive at the Goldberger-Treiman relation

$$\Delta = g_{\pi qq} f_\pi \ . \tag{18}$$

Thus, in terms of f_π, the relation (12) for T_c becomes

$$N_c \frac{T_c^2}{12 f_\pi^2} = \frac{1}{2}\left[1 + \frac{N_c g_{\pi qq}^2}{4\pi^2}\right] \ . \tag{19}$$

To simplify this expression further, we adopt a recent QCD result[21] $g_{\pi qq} \approx 2\pi/\sqrt{N_c}$. Substituting this in (19) leads to $T_c = 2\sqrt{3/N_c} f_\pi$ and specializing to $N_c = 3$, we obtain the desired relation $T_c = 2f_\pi$, the same value of T_c that we have derived in the σ model Eq. (10).

V. QCD CALCULATION OF QUARK SELF-ENERGY

Finally, we turn to the calculation of the Schwinger-Dyson (SD) equation for the fermion self-energy in QCD at finite temperatures. Such an approach has been attempted with the simplified assumption of a constant quark mass and coupling as a function of momenta.[10] This crude approximation required an (artificial) ultraviolet (Debye) cutoff in the spirit of the nonrelativistic BCS theory of superconductivity.[9] Demanding $\alpha_s \approx \pi/4$, where one expects the running QCD coupling to freeze-out,[22] the implied k_D cutoff in turn led to a BCS-type equation for T_c, which was numerically determined to be[10] $T_c \approx 176$ MeV.

Here we improve this approach by introducing a momentum-dependent dynamical quark mass in the spirit of the operator-product expansion (OPE) in QCD (Ref. 23), which then provides a natural cutoff in the gap equation. Such a dynamical quark mass $m_{\mathrm{dyn}}(p^2)$ is thought to run like $1/p^2$ for[24] $p^2 > m_{\mathrm{dyn}}^2$ (neglecting logarithms) but freezes out when[25] $p^2 < m_{\mathrm{dyn}}^2$. Translating this timelike result to the spacelike region, we introduce a damping factor $-m_{\mathrm{dyn}}^3/p^2$ for the self-energy.

At finite temperatures, the gap equation has the form

$$i\Sigma(p) = (-ig)^2 C_F \int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(p-q)\gamma^\mu S(q)\gamma^\nu \ , \tag{20}$$

where $D_{\mu\nu}$ and $S(q)$ are the finite-temperature gluon and quark propagators and $C_F = \frac{4}{3}$. We do calculations in the real-time formalism[16] in Landau gauge, where wave-function renormalization is absent [therefore, we have $\Sigma(p^2 = m_{\mathrm{dyn}}^2) = m_{\mathrm{dyn}}$].

In this case, we will use the following generalization of the asymptotic form of the dynamical mass at finite temperatures:

$$\Sigma(p^2) = +m_{\mathrm{dyn}}(T)\frac{m_{\mathrm{dyn}}^2}{p^2} \tag{21}$$

for $p^2 > m_{\mathrm{dyn}}^2$. Constant integration in the q_0 plane[10] for the SD equation (20), incorporating the natural cutoff (21), gives the gap equation for the critical temperature in terms of the zero-temperature dynamical mass and coupling constant:

$$m_{\mathrm{dyn}}(T) = \frac{2\alpha_s}{\pi} m_{\mathrm{dyn}}(T)\left[\int_0^{m_{\mathrm{dyn}}} \frac{dk}{E_k(T)} \tanh\frac{E_k(T)}{2T} + \int_{m_{\mathrm{dyn}}}^\infty \frac{dk}{E_k(T)} \frac{m_{\mathrm{dyn}}^2}{\mathbf{k}^2} \tanh\frac{E_k(T)}{2T} + \int_{m_{\mathrm{dyn}}}^\infty \frac{dk}{E_k(T)}\left[\frac{m_{\mathrm{dyn}}}{2|\mathbf{k}| - m_{\mathrm{dyn}}} - \frac{m_{\mathrm{dyn}}^2}{\mathbf{k}^2}\right]\right] , \tag{22}$$

where we have denoted $E_k(T) = [\mathbf{k}^2 + m_{\mathrm{dyn}}^2(T)]^{1/2}$. The value of the coupling constant α_s in (22) is fixed by the zero-temperature gap equation, which numerically turns out to be $\alpha_s \approx \pi/3$. Substituting the latter value back into (22), and factorizing $m_{\mathrm{dyn}}(T)$, the value of T_c at which $m_{\mathrm{dyn}}(T_c) = 0$ is found numerically to be

$$T_c \approx 0.52 m_{\mathrm{dyn}} \ . \tag{23}$$

For $m_{\mathrm{dyn}} = 320$ MeV, Eq. (23) gives $T_c \approx 170$ MeV.

Possible improvements of the above result could be achieved by implementing the renormalization-group corrections into the running coupling constant. We have repeated the calculation of the critical temperature with such a (T independent) QCD running coupling constant and found that these improvements lead to an insignificant correction to the previous result. As for the T dependence of[5] α_s, the fact that our quark model predicts $T_c \approx 170$ MeV implies that variation of α_s with T can be neglected in the broken phase. (For $N_f = 2$ and $N_c = 3$, $\Lambda_{\mathrm{QCD}} \approx 270$ MeV $> T_c$.) It is also possible to calculate f_π in this OPE-improved quark-model scheme,[21] with the result $f_\pi \approx 87$ MeV. Therefore, the conclusion that $T_c \sim 170$ MeV in this model reinforces, with a high degree of accuracy, our belief once again that $T_c = 2f_\pi$.

In these approximate calculations of the gap equation, we have neglected some very important aspects of finite-temperature QCD. Whereas the relevance of confinement on chiral-symmetry breaking at zero temperature is arguable, it is clear that at finite temperatures, the two issues have to be addressed together. There are clear indications that both deconfinement and chiral transitions occur at the same point or, at least, very close to each other.[5,6] Also, it has been known for some time that deconfinement is driven by Debye screening[6] and, therefore, one expects that this effect should have some bearing on the chiral-symmetry-restoration transition. Calculations of the lower-dimensional (confining) gauge theories indicate that both transitions are driven by the screening.[26]

In light of the above remarks, it is rather surprising that our simple approximation leads to a consistent prediction for T_c. We believe that one of the reasons for this is due to the phenomenological inequality: $T_D \le T_c < \Lambda_{QCD} < m_{dyn}$, where T_D is the deconfinement temperature. Thus, the crude approximations of this section, neglecting the confinement, Debye screening, and T dependence of the coupling constant, are justified.

Similar calculations of the SD equation in Coulomb-gauge QCD (in the instantaneous approximation) were performed in Ref. 27. It is interesting to note that despite the different nature of the approximation the authors used, their results led to the conclusion that $T_c \approx 2f_\pi$ (numerically), regardless of the choice of the parameters they used to fit the experimental data.

VI. CONCLUSIONS

We have calculated the chiral-symmetry-restoration temperature in three different models related to QCD. Regardless of the model, and the differences between the degrees of freedom each of them describes, the result $T_c \approx 2f_\pi$ seems to prevail. We demonstrate how chiral symmetry constrains the value of T_c in a similar manner as it constrains the low-temperature expansion. Contributions from the light excitations with masses $M \ll T_c$ dominate the thermodynamic averages and drive the restoration transition. A simple explanation of the relationship between T_c and f_π is given in terms of the counting factors for light particles that are determined by the chiral symmetry and the mechanism of its breaking. The value of $T_c \approx 180$ MeV is consistent with the latest lattice predictions.[28] This increases our hope that $T_c \approx 2f_\pi$ might be a result of some importance beyond the simple models we used in this paper.

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