Thermodynamics of (2+1)-dimensional four-fermion models

B. Rosenstein and B. J. Warr
Theory Group, Department of Physics, University of Texas, Austin, Texas 78712

S. H. Park
Center for Relativity, Department of Physics, University of Texas, Austin, Texas 78712

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The thermodynamics of the (2+1)-dimensional Gross-Neveu model is analyzed. The model is not renormalizable in the weak-coupling expansion but becomes renormalizable in the $1/N_f$ expansion. We calculate the critical temperature and find the phase structure. The critical exponents α, α', β, γ, γ', δ, η, ν, and ν' are calculated to leading order in $1/N_f$, and do not satisfy the "scaling relations."

I. INTRODUCTION

It was found recently[1] that a wide class of fermionic quantum field theories is renormalizable in $d=2+1$ dimensions. (For the simple four-fermion model presented in this paper the renormalizability was first noted by Gross[2] and Parisi,[3] and a proof was sketched by Shizuya.[4]) These theories are nonrenormalizable in a "weak-coupling expansion," but become renormalizable in the $1/N_f$ expansion[5] (or so-called "auxiliary field method").[6] Moreover, the $1/N_f$ expansion allows us to study in a very explicit way many interesting "nonperturbative" phenomena, such as dynamical symmetry breaking and bound states.[7,8]

In this paper we study the thermodynamics of the simplest fermionic theory in $d=2+1$ dimensions: namely, the scalar-scalar four-fermion model. We are motivated by its close analogies to a Bardeen-Cooper-Schrieffer-(BCS-) type theory of superconductivity in two spatial dimensions which recently attracted attention due to the planar character of high-T_c superconducting ferromagnets.[9]

The Lagrangian is

$$\mathcal{L}=\bar{\psi}i\partial\!\!\!/\psi+\frac{g^2}{2N_f}(\bar{\psi}\psi)^2\ , \tag{1}$$

which has a discrete chiral symmetry, $\psi\rightarrow\gamma_5\psi$, precluding a bare fermion mass. This model was introduced into particle physics by Nambu and Jona-Lasinio[10] to describe chiral-symmetry breaking in strong interactions in $d=3+1$ dimensions. The model was then generalized to lower dimensions by Gross and Neveu,[11] who discovered the property of "asymptotic freedom" in $d=1+1$.

The thermodynamics of (1) was studied extensively for $d=1+1$, where the model is renormalized in *both* the $1/N_f$ expansion and weak coupling. At *exactly* zero temperature the chiral symmetry is dynamically broken,[11] generating a mass M for the fermions. At nonzero temperatures a naive application of the $1/N_f$ expansion then yields a critical temperature for symmetry restoration, given in leading order by[12]

$$T_c=(0.57)M\ . \tag{2}$$

However, this result is *wrong*, as was illuminated by Ref. 13. In fact for any finite N_f the critical temperature is rigorously *zero*, and the situation is analogous to the Ising model in one space dimension. In both cases "kink" configurations are unsuppressed, so at low temperatures the system is segmented into regions of *alternating signs* of the order parameter. The net average value of the order parameter is then zero. The $1/N_f$ expansion in $d=1+1$ misses this effect because the energy per kink goes to infinity as $N_f\rightarrow\infty$. Thus in leading order the system really is spatially homogeneous, and the expansion in $1/N_f$ just measures the effects of small fluctuations about this state. The contribution from the kinks has the form e^{-N_f}, which is nonanalytic in $1/N_f$. To sum up, in one space dimension there is no superconductivity, but this fact is obscured in the $1/N_f$ expansion.

The situation changes, however, when we turn to the case of two space dimensions. A simple energy-entropy argument now shows that "domains" *will* be suppressed at low enough temperatures, so that the critical temperature will now be finite. This phenomenon was first calculated in the analogous case of the Ising model by Onsager.[14] Thus we expect a large-N_f calculation in the four-fermion model to be reliable in $d=2+1$, and our result for the critical temperature is

$$T_c=\frac{M}{2\ln 2}\ . \tag{3}$$

(This is in agreement with the calculations of Ref. 15. In $d=3+1$ the model is not renormalizable, but leading-order calculations can be carried out.[16])

In the next section we review the finite-temperature formalism and compute the Landau free energy function. This yields the critical temperature (3), and also the discontinuity in specific-heat capacity across the transition. We compute the critical exponents α, α', and β, and then from the thermal Green's functions we obtain ν and η. Next, we introduce a chemical potential μ to probe the effects of a finite fermion density.

Finally, we subject the system to an external source coupled to the order parameter $\bar{\psi}\psi$ and derive the exponents γ, γ', and δ. We observe that the "scaling relations" for the exponents are *not* obeyed, and discuss this in the conclusion.

II. TEMPERATURE FORMALISM

The Hamiltonian of the model is

$$H=\int d^2x\left[-i\bar{\psi}\left[\gamma^1\frac{\partial}{\partial x^1}+\gamma^2\frac{\partial}{\partial x^2}\right]\psi-\frac{g^2}{2N_f}(\bar{\psi}\psi)^2\right]. \tag{4}$$

The quantum statistical partition function is then

$$Z(\beta)=\mathrm{Tr}\exp[-(\beta H)]. \tag{5}$$

To calculate $Z(\beta)$ it is simpler to use the equivalent form[17]

$$Z(\beta)=\int D\psi(x)D\bar{\psi}(x)\times\exp\left[-\int_\beta d^3x\left[\bar{\psi}\partial\!\!\!/\psi+\frac{g^2}{2N_f}(\bar{\psi}\psi)^2\right]\right], \tag{6}$$

where the fermion fields [in Eq. (6) the notation is such that the γ matrices are 4×4 and Hermitian] are now antiperiodic functions on $R^2\times[0,\beta]$. The $1/N_f$ expansion of (6) is most easily described using an auxiliary field $\sigma(x)$ (Ref. 5). We rewrite (6) as

$$Z(\beta)=\int D\psi D\bar{\psi}D\sigma\times\exp\left[-\int_\beta d^3x\left[\bar{\psi}\partial\!\!\!/\psi+\sigma\bar{\psi}\psi-\frac{N_f}{2g^2}\sigma^2\right]\right], \tag{7}$$

where σ is periodic on $[0,\beta]$ since it is bosonic. The functional integral is now quadratic in the fermion fields so we integrate to obtain an "effective action" for $\sigma(x)$:

$$S_{\rm eff}=-N_f\left[\int_\beta d^3x\frac{\sigma^2}{2g^2}+\mathrm{tr}\ln(\partial\!\!\!/+\sigma)_\beta\right]. \tag{8}$$

The procedure now is to find a minimum point of $S_{\rm eff}$ for some constant configuration, and then expand in fluctuations. This expansion is regularized using a momentum cutoff Λ, and is guaranteed to be renormalizable for any finite β by the renormalizability proof for $\beta=\infty$ (Ref. 1). Note that the "bare coupling" g_Λ^2 must be taken equal to its $\beta=\infty$ value.

It is convenient to insert the cutoff Λ on the "spatial momenta," $\mathbf{p}=(p_1,p_2)$ only. At $\beta=\infty$ the stationary equation (gap equation) reads

$$0=\sigma\left[\frac{-1}{g_\Lambda^2}-4\int\frac{dE\,d^2\mathbf{p}}{(2\pi)^3}\frac{1}{E^2+\mathbf{p}_\Lambda^2+\sigma^2}\right]. \tag{9}$$

Provided g_Λ^{-2} lies in the range

$$0\ge g_\Lambda^{-2}\ge g_{\rm crit}^{-2}=-4\int\frac{dE\,d^2\mathbf{p}}{(2\pi)^3}\frac{1}{E^2+\mathbf{p}_\Lambda^2}, \tag{10a}$$

we can define a finite mass M through

$$-\frac{1}{g_\Lambda^2}=4\int\frac{dE\,d^2\mathbf{p}}{(2\pi)^3}\frac{1}{E^2+\mathbf{p}_\Lambda^2+M^2}, \tag{10b}$$

and the minimum configuration at $\beta=\infty$ occurs at $|\sigma|=M$. From the $1/N_f$ Feynman rules (Ref. 1) this quantity M is the zero-temperature mass of the fermions.

At finite temperature we substitute (10b) into (8), and for constant configurations we have

$$S_{\rm eff}=N_f\beta(\text{area})F_{\rm Lan}(\sigma,\beta), \tag{11a}$$

where $F_{\rm Lan}$ is the "Landau free energy per unit area," and is given by

$$\frac{\partial}{\partial\sigma}F_{\rm Lan}(\sigma,\beta)=\sigma\left[-\frac{1}{g_\Lambda^2}-\frac{4}{\beta}\sum_{n=-\infty}^{\infty}{}'\int\frac{d^2\mathbf{p}}{(2\pi)^2}\frac{1}{(2n+1)^2\frac{\pi^2}{\beta^2}+\mathbf{p}_\Lambda^2+\sigma^2}\right]. \tag{11b}$$

Using a Poisson summation formula and a contour rotation we obtain (see Ref. 13 for details)

$$\frac{\partial}{\partial\sigma}F_{\rm Lan}(\sigma,\beta)=4\sigma\int\frac{d^2\mathbf{p}}{2(2\pi)^2}\left[\frac{1}{E_M}-\frac{1}{E\sigma}\left[1-\frac{2}{e^{\beta E_\sigma}+1}\right]\right], \tag{12}$$

where $E_\sigma^2=\mathbf{p}_\Lambda^2+\sigma^2$ and similarly for E_M. Equation (12) can be explicitly evaluated, and then integrated to yield $F_{\rm Lan}(\sigma,\beta)$ using $F_{\rm Lan}(0,\beta)=0$. The result is finite as $\Lambda\to\infty$ and we have

$$\frac{\partial}{\partial\sigma}F_{\rm Lan}(\sigma,\beta)=\frac{1}{\pi}\sigma\left[|\sigma|-M+\frac{2}{\beta}\ln(1+e^{-\beta|\sigma|})\right], \tag{13a}$$

$$F_{\rm Lan}(\sigma\ge0,\beta)=\frac{1}{\pi\beta^3}\int_0^{\beta\sigma}dx[x-\beta M+2\ln(1+e^{-x})]. \tag{13b}$$

We see from (13) that the transition from the "superconducting phase" ($\langle\sigma\rangle\neq0$) to "normal phase"

($\langle\sigma\rangle=0$) is *second order,* since $\langle\sigma\rangle_\beta$ is continuous. The order parameter drops to zero at $\beta=\beta_c$, where

$$\beta_c M = 2\ln 2 \tag{14}$$

and (14) is just the advertised result (3). For temperatures just below T_c the effective mass of the fermion (which is just $\langle\sigma\rangle$), is very small, and from (13a) we get

$$M_f^2(\beta)=8\ln 2\frac{\beta-\beta_c}{\beta_c^3} . \tag{15}$$

Thus in the terminology of Ref. 18 the critical exponent β is $\frac{1}{2}$.

With regard to the free energy, above the critical temperature F vanishes, and below T_c we have

$$F(T<T_c)=-\frac{4(\ln 2)^2}{\pi}N_f(T_c-T)^2T_c . \tag{16a}$$

Thus the specific heat (per unit area) is

$$C_A(T<T_c)=\frac{8(\ln 2)^2}{\pi}N_fT_c^2 , \tag{16b}$$

and we observe that at leading order the specific heat is finite but discontinuous at $T=T_c$. The critical exponents α and α' are therefore zero.

III. THE DYNAMICAL CRITICAL EXPONENTS

We would like to know the thermal average $\langle\sigma(\mathbf{x})\sigma(\mathbf{y})\rangle_\beta$ near the critical temperature, to see the nature of the fluctuations. The σ propagator at leading order in $1/N_f$ is given by functionally differentiating (8) twice with respect to $\sigma(\mathbf{x})$, and at $\beta=\infty$ is[1]

$$D_\sigma(k^2,\beta=\infty)=\frac{2\pi}{N_f}\frac{\sqrt{k^2}}{(k^2+4M^2)\arctan(\sqrt{k^2}/2M)} . \tag{17}$$

Here $k^2=\epsilon^2+\mathbf{p}^2$, and we have full three-dimensional Euclidean invariance. As β is reduced (17) modifies in an important way, and is in particular no longer Euclidean invariant. It is *not* true that to get $D_\sigma(k^2,\beta)$ you just replace $M\to M_f(\beta)$. A slightly lengthy calculation reveals, below the critical temperature,

$$D_\sigma^{-1}(\epsilon=0,\mathbf{p}^2,\beta\ge\beta_c)=\frac{N_f(\mathbf{p}^2+4\sigma^2)}{2\pi\sqrt{\mathbf{p}^2}}\times\int_0^1\frac{x\,dx}{\sqrt{1-x^2}\sqrt{a^2+x^2}}\times\left[1-\frac{2}{e^{\tilde\beta\sqrt{x^2+a^2}}+1}\right] , \tag{18}$$

where $a=2\langle\sigma\rangle_\beta/\sqrt{\mathbf{p}^2}$ and $\tilde\beta=\frac{1}{2}\beta\sqrt{\mathbf{p}^2}$. For temperatures above the critical temperature we get

$$D_\sigma^{-1}(\epsilon=0,\mathbf{p}^2,\beta\le\beta_c)=\frac{2N_f\ln 2}{\pi}\left[\frac{1}{\beta}-\frac{1}{\beta_c}\right]+\frac{N_f\sqrt{\mathbf{p}^2}}{2\pi}\int_0^1\frac{dx}{\sqrt{1-x^2}}\times\left[1-\frac{2}{e^{\tilde\beta x}+1}\right] . \tag{19}$$

At exactly the critical temperature $\langle\sigma\rangle_\beta$ vanishes and we have, for $\beta_c\sqrt{\mathbf{p}^2}\ll 1$,

$$D_\sigma(\mathbf{p}^2,\beta_c)=\frac{8\pi}{N_f}\frac{1}{\beta_c\mathbf{p}^2} . \tag{20}$$

Thus the critical exponent η is zero. For small $\mathbf{p}^2$ and $\beta-\beta_c>0$ we have

$$D_\sigma(\mathbf{p}^2,\beta>\beta_c)=\frac{8\pi}{N_f}\frac{1}{\beta_c(\mathbf{p}^2+4\langle\sigma\rangle_\beta^2)} \tag{21a}$$

so the correlation length ξ is

$$\xi(\beta>\beta_c)=\frac{1}{2\langle\sigma\rangle_\beta}=\left[\frac{\beta_c^3}{32\ln 2}\right]^{1/2}\frac{1}{\sqrt{\beta-\beta_c}} , \tag{21b}$$

and $\nu'=\frac{1}{2}$. Finally for small $\mathbf{p}$ and $\beta-\beta_c<0$ we have

$$D_\sigma(\mathbf{p}^2,\beta<\beta_c)=\frac{8\pi}{N_f}\frac{1}{\beta_c\left[\mathbf{p}^2+\frac{16\ln 2}{\beta_c^3}(\beta_c-\beta)\right]} \tag{22a}$$

so

$$\xi(\beta<\beta_c)=\left[\frac{\beta_c^3}{64\ln 2}\right]^{1/2}\frac{1}{\sqrt{\beta_c-\beta}} , \tag{22b}$$

and $\nu=\frac{1}{2}$. The correlation length behaves similarly above and below the critical temperature (as expected) but the overall magnitude changes.

IV. FINITE FERMION DENSITY

The effects of a chemical potential μ are given by shifting the energy levels in (11b) by μ (Ref. 13). The identical manipulations as before now yield, in correspondence to (13a),

$$\frac{\partial F_{\rm Lan}}{\partial\sigma}(\sigma,\beta,\mu)=\frac{\sigma}{\pi}\left[|\sigma|-M+\frac{1}{\beta}\int_{\beta|\sigma|}^{\infty}du\left[\frac{1}{e^{u-\beta\mu}+1}+\frac{1}{e^{u+\beta\mu}+1}\right]\right]$$
$$=\frac{\sigma}{\pi}\left[|\sigma|-M+\frac{1}{\beta}\ln(1+2e^{-\beta|\sigma|}\cosh\mu\beta+e^{-2\beta|\sigma|})\right] . \tag{23}$$

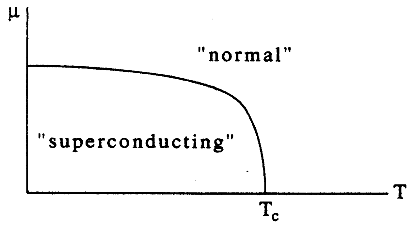

FIG. 1. Phase diagram of plane superconductor.

At zero temperature we see that if $\mu < M$ the absolute minimum still occurs at $|\sigma| = M$. If $\mu > M$ then there is just a unique minimum at $\sigma = 0$, so $\mu = M$ is a critical value. For higher temperatures the critical value of μ drops and reaches zero at $T = T_c$. Thus we have the phase diagram of Fig. 1.

We have a *line* of second-order phase transitions given analytically by

$$\beta M = \ln(2 + 2\cosh\mu\beta) \ . \tag{24}$$

V. EFFECT OF EXTERNAL SOURCE

For simplicity let us first set $\beta = \infty$, $\mu = 0$ and consider an external source coupled to the order parameter $\bar{\psi}\psi$ in (7). This breaks the chiral symmetry and induces under renormalization a term *linear* in the auxiliary field $\sigma(x)$. The bare "Lagrangian" at leading order becomes (without loss of generality we take M and m to be positive)

$$\mathcal{L} = \bar{\psi}(\partial\!\!\!/ - m)\psi + \sigma\bar{\psi}\psi - \frac{N_f}{2g_\Lambda^2}\sigma^2 - N_f m B_\Lambda \sigma \ , \tag{25a}$$

where g_Λ^2 is still given by (10b), and the renormalization constant B_Λ is given by

$$B_\Lambda = 4\int \frac{dE\, d^2\mathbf{p}}{(2\pi)^3} \frac{1}{E^2 + \mathbf{p}_\Lambda^2} \ . \tag{25b}$$

It may appear that there should be three parameters: namely, the couplings of $\bar{\psi}\psi$, σ^2, and σ, but one of these can be removed by a field redefinition $\sigma \to \sigma + c$ (Ref. 2).

The model (25a) is closed under renormalization, but there is a generalization which is also allowed: namely, to add a term $|\sigma|^3$. Provided $\langle \sigma \rangle \neq 0$ there is no harmful effect from the nonanalyticity, and the $1/N_f$ expansion is well defined. However this term radically changes the nature of the field theory, which in terms of the fermion fields alone is no longer a polynomial interaction. We shall consider this interesting case in a future publication.[7]

The Landau energy of the model Eq. (25) is now given by

$$\frac{\partial F_{\rm Lan}}{\partial\sigma}(\sigma,\infty) = \frac{1}{\pi}(\sigma - m)(|\sigma - m| - M) - \frac{mM}{\pi} \ . \tag{26a}$$

Comparing to (13a) we see that for $\beta = \infty$, $\mu = 0$ the effect of the source is to shift $\sigma \to \sigma - m$ and to add an extra inhomogeneous term $-mM/\pi$. It is easy to see that this mnemonic remains true even when β,μ are finite, so the general expression is, cf. (23),

$$\frac{\partial F_{\rm Lan}}{\partial\sigma}(\sigma,\beta,\mu) = \frac{1}{\pi}(\sigma - m)\left[|\sigma - m| - M + \frac{1}{\beta}\ln(1 + 2e^{-\beta|\sigma - m|}\cosh\mu\beta + e^{-2\beta|\sigma - m|})\right] - \frac{mM}{\pi} \ . \tag{26b}$$

The effective mass of the fermion $M_f(\beta)$ is $\langle(\sigma - m)\rangle_\beta$. Setting $\mu = 0$, we find that at the critical temperature M_f vanishes as the source is removed, and obeys

$$M_f^3(\beta_c) = \frac{8\ln 2}{\beta_c^3} m \ . \tag{27}$$

Thus the critical exponent δ is 3. Defining a "susceptibility" by $\partial M_f(\beta,m)/\partial m$ then as $m \to 0$ there is the behavior

$$\frac{\partial M_f}{\partial m}(\beta > \beta_c) = \frac{\beta_c}{2(\beta - \beta_c)} \ , \tag{28a}$$

$$\frac{\partial M_f}{\partial m}(\beta < \beta_c) = \frac{\beta_c}{\beta_c - \beta} \ , \tag{28b}$$

and so the critical exponents γ' and γ are equal to 1. To summarize, the critical exponents have been collected in Table I.

TABLE I. Critical exponents of the four-fermion theory.

α	α'	β	γ	γ'	δ	η	ν	ν'
0	0	$\frac{1}{2}$	1	1	3	0	$\frac{1}{2}$	$\frac{1}{2}$

VI. DISCUSSION AND CONCLUSIONS

In this paper we have studied the thermodynamics of the renormalizable Gross-Neveu model in $d = 2 + 1$. The model represents a relativistic superconductor in two spatial dimensions, and has a very simple "gap equation" given by (9). The phase structure for the effects of temperature and chemical potential is given by Eq. (24) and Fig. 1. The reason why the gap equation is *algebraic,* as opposed to the *integral* equation one usually obtains in BCS theory (Ref. 9), is that the underlying interaction of the fermions is just a delta function. The BCS theory involves a nonlocal potential, which has a physical cutoff in it related to the "Debye screening length." The major difference is that, in the language of particle physics, the BCS model is not renormalizable.

Let us now discuss the values of the critical exponents given in the table. Ignoring η, ν, ν' for the moment, the exponents take on the "mean-field values," first predicted by Landau.[18] This is essentially because fluctuations are suppressed by powers of $1/N_f$. However, we were able to look at fluctuations to compute η, ν, ν', and of course all these values will be modified in next to leading orders. This will be computed elsewhere.[19] An interesting feature is that the critical exponents are *not* those of the Gaussian fixed point, and *nor* do they obey the "scaling relations" (Ref. 18).

This is the first explicit example of this phenomenon know to us. We do not get Gaussian behavior, since after all the four-point function of the fermions is not zero at *any* temperature. [The four-point function is related to the thermal σ propagator given in Eqs. (18)–(22b), see Ref. 1.] Moreover the model does not satisfy "scaling" either. This last is the property that near the critical temperature there are *no* length scales in the problem apart from the correlation length ξ, which implies that in fact the critical exponents are not independent. Scaling relates the exponents by

$$\alpha=\alpha'=2-\nu d \ , \tag{29a}$$

$$\beta=\tfrac{1}{2}\nu(d+2+\eta) \ , \tag{29b}$$

$$\gamma=\gamma'=\nu(2-\eta) \ , \tag{29c}$$

$$\delta=(d+2-\eta)/(d-2+\eta) \ , \tag{29d}$$

$$\nu'=\nu \ , \tag{29e}$$

where d is the spatial dimension (in our case $d=2$.) The values in Table I do not agree with (29), and nor will their corrections order by order in $1/N_f$. To illuminate this we consider the thermal propagator at $\beta=\beta_c$. From (18),

$$D_\sigma^{-1}(0,\mathbf{p}^2,\beta_c) = \frac{N_f}{2\pi}\sqrt{\mathbf{p}^2}\int_0^1 \frac{dx}{\sqrt{1-x^2}}\left[1-\frac{2}{\exp(\frac{1}{2}\beta_c\sqrt{\mathbf{p}^2})+1}\right] . \tag{30}$$

For low momenta this looks like an ordinary massless boson, but for large momenta we have the noncanonical behavior $D_\sigma(\mathbf{p}^2)\sim 1/\sqrt{\mathbf{p}^2}$. Thus the critical Green's functions are *not* given by power laws (modulo logarithms), and scaling breaks down. This effect is due to the composite nature of the order parameter, and we expect similar nonscaling behavior in finite-temperature QCD.

As a final remark it would be very interesting to verify the method of the $1/N_f$ expansion for the model on the lattice. Our calculations of finite temperature can then be used to simulate "finite-size effects." Similar calculations have been done using various methods in the (1+1)-dimensional case.[20]

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