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Baryonic properties of broken $U(3)_V$ Skyrmions

D. Masak
Institut für Theoretische Physik I, Universität Giessen, Giessen, West Germany
and Physics Department, Tel Aviv University, Ramat Aviv, Israel
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We calculate the baryonic properties, such as masses and magnetic moments, for the octet and decuplet baryons. The finite mass of the strange quark is taken into account. Results are presented for hidden gauge and massive Yang-Mills-type models.

I. INTRODUCTION

Recently, a large interest in strange Skyrmions has developed. In particular, models which use the Callan-Klebanov bound-state quantization method[1] have been investigated.[2,3] One of these models[3] incorporates vector mesons into the strange Skyrmion, leading to remarkably good agreement between theory and the experimentally known $S=1$ baryon masses. Similar results have been obtained in strange chiral bag models.[4,5]

We present here a perturbative approach to the problem of incorporating nonvanishing strange-quark mass. We use the term perturbatively in the sense that we include the mass of the strange quark after we calculated the underlying Skyrmion. This is somewhat opposite to the Callan-Klebanov method, which assumes that the strange quark is very heavy. The problem of extending the SU(2) pseudoscalar Skyrmion, containing only pions, to an SU(3) pseudoscalar Skyrmion, i.e., describing the baryon octet (spin $\frac{1}{2}$) and decuplet (spin $\frac{3}{2}$), has been addressed earlier.[16–19] However, in the light of the success of models incorporating vector mesons into SU(2) Skyrmions,[6–12] we will attempt the extension of gauged, i.e., including vector mesons, SU(2) Skyrmions to SU(3). Our perturbative approach is presented for the pure pseudoscalar Skyrmion and two gauge variants of the Skyrmion, which include the vector mesons. One method of gauging is the hidden gauge theory[6–9] and the other possible gauge procedure is the massive Yang-Mills model.[10–12]

Besides the octet and decuplet masses we will calculate the magnetic moments and the electromagnetic mass splittings in a perturbative fashion.

The paper is organized as follows. First we shortly review the three different models, including the necessary *Ansätze* for solving the energy and moment of inertia functionals. In the next section we calculate the magnetic moments of the octet baryons, using SU(3)-symmetry arguments and the vector-meson-dominance approximation. The following section treats the SU(3) mass symmetry breaking due to the nonvanishing quark masses. In the last section we present a discussion about the quality of the predictions and the involved approximations.

II. THE MODELS

There exist several models in order to describe the nucleon and the delta in an SU(2)-Skyrme theory. One can use either the pure Skyrmion models with[13] or without pion mass terms[14] or a gauged Skyrme model, i.e., including vector mesons. The second category, namely, the gauged Skyrme models, contains a whole variety of different approaches and approximations.

We are classifying the gauged Skyrme model by their way of gauging. There are two choices. Either the global symmetries of the nonlinear σ model are gauged, the massive Yang-Mills case,[10,12] or a hidden local symmetry is used, the hidden gauge model.[6–9] It can be shown that these two types of gauging can be reproduced by a more general Lagrangian and they are thus the "same" at an effective level,[15] but this is only of academic interest here, since the "equivalence" on an effective level does not imply that both models lead, *a priori*, to the same physics.[15]

We split this section into three parts, the pure Skyrme model, the massive Yang-Mills model, and the hidden gauge model, which we are reviewing briefly.

The main idea is to extend the well-known methods for SU(2) Skyrmions to SU(3) by first assuming that all the quarks have vanishing mass and then calculating the perturbations caused by nonvanishing quark masses. In a second approach we assume that all pseudoscalar mesons have the same mass, the mean SU(3) pseudoscalar-meson mass ≈ 408 MeV. Note that we assume throughout the paper $f_\pi = f_K = f_\eta$.

A. Pure Skyrme model

The first to treat the SU(2) Skyrmion extension to the SU(3) Skyrmion was Guadagnini,[16] followed by several other authors.[17–19] All treat the SU(3) problem the same way, by assuming complete SU(3) symmetry and then treating the symmetry-breaking terms perturbatively. However, somewhat different attempts have been made to tackle the SU(3) Skyrmion problem, by diagonalizing the broken Lagrangian,[20] by assuming that strange baryons consist of a SU(2) Skyrmion and a bound K-meson state,[1] and via algebraic quantization.[21]

Here we will restrict ourselves to the method introduced first by Guadagnini,[16] which we describe below. Consider the pure massless Skyrme model together with the Wess-Zumino-Witten term. The action is then

$$\Gamma_{\mathrm{SkWZ}}=\int \left[\frac{f_\pi^2}{4}\mathrm{Tr}(L_\mu^2)+\frac{1}{32e^2}\mathrm{Tr}[L_\mu,L_\nu]^2\right]d^4x+\frac{iN_C}{240\pi^2}\int_{\mathcal{M}^5}\epsilon^{\mu\nu\alpha\beta\gamma}\mathrm{Tr}(L_\mu L_\nu L_\alpha L_\beta L_\gamma)d^5x\ , \quad (1)$$

where the currents are

$$L_\mu=U^\dagger\partial_\mu U$$

and the chiral field is

$$U=e^{(i/f_\pi)\lambda_a\Phi_a}\ ,$$

where Φ_a ($a=1,\ldots,8$) is the pseudoscalar octet and e is a parameter. Witten[22] showed that the Wess-Zumino-Witten-term ensures the quantization of the Skyrmion as a fermion (spin $\frac{1}{2}$, $\frac{3}{2}$) and that for the Skyrmion soliton solution the baryon number is equivalent to the topologically conserved winding number $[\pi_3(\mathrm{SU}(3))=\mathbb{Z}]$. Note that the Wess-Zumino-Witten term does not contribute to the classical energy nor to the moment of inertia, due to the fact that $\pi_5(\mathrm{SU}(2))=0$.

The stationary Skyrmion soliton solution is achieved via the celebrated hedgehog *Ansatz:* namely,

$$U=\begin{bmatrix}\cos\Theta+i\tau\cdot\hat{\mathbf{r}}\sin\Theta & 0\\ 0 & 1\end{bmatrix} . \quad (2)$$

The choice $\Theta(0)=\pi$ and $\Theta(\infty)=0$ ensures that the winding number, i.e., the topological charge connected with the mapping equation (2), which is equal to the baryon number, is 1.

The quantization of the Lagrangian connected with the action equation (1), is achieved by the collective-coordinate method[14]

$$U(t)=A(t)UA^\dagger(t)\ , \quad (3)$$

where U is given by Eq. (2) and the time-dependent matrix $A\in\mathrm{SU}(3)$.

The baryonic wave functions can now be described by the generalized SU(3) $\mathcal{D}$ functions: i.e.,

$$\psi(A)=\mathcal{D}^{[n]}_{\alpha\beta}(A)=\langle\alpha|\mathcal{D}^{[n]}|\beta\rangle\ , \quad (4)$$

where $[n]$ denotes the representation of SU(3), $\alpha=(Y,I,I_3)$ whereas $\beta=(1,S,-S_3)$.[16] Some wave functions are, e.g.,

$$\psi^{p}_{S_3=-1/2}=2\sqrt{2}\langle 1,\tfrac{1}{2},\tfrac{1}{2}|\mathcal{D}^{[8]}|1,\tfrac{1}{2},\tfrac{1}{2}\rangle\ ,$$

$$\psi^{\Sigma^+}_{S_3=-1/2}=2\sqrt{2}\langle 0,1,1|\mathcal{D}^{[8]}|1,\tfrac{1}{2},\tfrac{1}{2}\rangle\ ,$$

$$\psi^{\Omega^-}_{S_3=3/2}=\sqrt{10}\langle -2,0,0|\mathcal{D}^{[10]}|1,\tfrac{3}{2},-\tfrac{3}{2}\rangle\ .$$

The SU(3) $\mathcal{D}$ functions are normalized according to

$$\int(\mathcal{D}^{[n]}_{\alpha\beta})^*\mathcal{D}^{[m]}_{\gamma\sigma}d\mu(a)=\frac{1}{\dim[n]}\delta^{nm}\delta_{\alpha\gamma}\delta_{\beta\sigma}\ .$$

The adiabatic rotation quantization procedure according to Eq. (3) leads to the Hamiltonian of the form

$$H=M+\frac{1}{2\Lambda_2}(C_2-\tfrac{3}{4})+\frac{1}{2}\left[\frac{1}{\Lambda_2}-\frac{1}{\Lambda_1}\right]\mathbf{J}^2+\cdots\ , \quad (5)$$

where $\mathbf{J}$ is the spin of the baryon and the ellipsis represent a number of additional terms depending only on N_C (Ref. 23). Note, that the hedgehog *Ansatz* equation (2) implies that after quantization spin and isospin are identical $\mathbf{I}=\mathbf{J}$.

The constant C_2 in the Hamiltonian is the quadratic Casimir invariant and depends only on the representation $[n]$: e.g.,

$$C_2[8]=3,\quad C_2[10]=6\ .$$

As a result of the specific values for C_2 the nucleon-delta splitting is the same for SU(2) as for SU(3) Skyrmions:

$$M_{\Delta N}=\frac{3}{2\Lambda_1}+(\text{symmetry-breaking terms})\ .$$

The mass and the moments of inertia for the pure pseudoscalar Skyrme Lagrangian are given by

$$M=4\pi\int_0^\infty\left[\frac{f_\pi^2}{2}(r^2\Theta'^2+2\sin^2\Theta)+\frac{\sin^2\Theta}{2e^2}\left[2\Theta'^2+\frac{\sin^2\Theta}{r^2}\right]\right]dr\ , \quad (6)$$

$$\Lambda_1=4\pi\tfrac{2}{3}\int_0^\infty\left[r^2f_\pi^2\sin^2\Theta+\frac{\sin^2\Theta}{e^2}(r^2\Theta'^2+\sin^2\Theta)\right]dr\ , \quad (7)$$

$$\Lambda_2=4\pi\tfrac{1}{2}\int_0^\infty\left[r^2f_\pi^2(1-\cos\Theta)+\frac{1-\cos\Theta}{4e^2}(r^2\Theta'^2+2\sin^2\Theta)\right]dr\ , \quad (8)$$

$$0=\left[\frac{\tilde{r}^2}{4}+2\sin^2\Theta\right]\Theta''+\Theta'^2\sin2\Theta+\frac{\tilde{r}}{2}\Theta'-\left[\frac{1}{4}+\frac{\sin2\Theta}{\tilde{r}^2}\right]\sin2\Theta\ , \quad (9)$$

where the equation of motion, Eq. (9) is achieved by minimizing the energy functional M Eq. (6). The variable $\tilde{r}$ is defined by $\tilde{r}=ef_\pi r$. The resulting values for the functionals, Eqs. (6)–(8) as functions of f_π and e are[19]

$$M\approx 73\frac{f_\pi}{e},\quad \Lambda_1\approx\frac{56}{f_\pi e^3},\quad \Lambda_2\approx\frac{19}{f_\pi e^3}\ .$$

Note that we are able to express all physical quantities in terms of a geometrical factor times a product of powers of e and f_π in the pure Skyrme model,[14] due to the transformation $r\rightarrow\tilde{r}$.

The Hamiltonian is not unique, we can add terms which vanish in the classical limit. These terms can change the total energy.[24]

The method of quantization and the form of the Hamiltonian are preserved in the gauged Skyrmion models, with some special choice for the vector-meson fields.

B. The massive Yang-Mills model

The massive Yang-Mills model starts with the nonlinear sigma model together with the Wess-Zumino-Witten action. We consider the nonlinear σ model first. The Lagrangian for the nonlinear σ model is

$$\mathcal{L}_\sigma=\frac{f_\pi^2}{4}\text{Tr}(\partial_\mu U\partial_\mu U^\dagger) \tag{10}$$

with f_π as the pion decay constant, $f_\pi\approx 93$ MeV, and U the usual chiral matrix, see Eq. (2).

We can now introduce the vector mesons are gauge particles of a $U(3)_V$ symmetry; i.e.,

$$D_\mu U=\partial_\mu U-i[V_\mu,U]\ , \tag{11}$$

where we absorbed the coupling constant into the vector-meson field. After adding the kinetic energy of the gauge fields and their masses we get

$$\mathcal{L}'=\frac{f_\pi^2}{4}\text{Tr}(D_\mu UD_\mu U^\dagger)-\frac{1}{4g^2}F_{\mu\nu}^2+\frac{m^2}{2}V_\mu^2\ , \tag{12}$$

where we assumed that all vector mesons have the same mass m; i.e., $\mathcal{L}'$ is SU(3) symmetric. From now on we will always use the phenomenologically successful Kawarabayashi-Suzuki-Fayyazuddin-Riazuddin (KSFR) relation[25] for the determination of the coupling strength g:

$$m^2=m_\rho^2=m_\omega^2=2g^2f_\pi^2\ . \tag{13}$$

Note that V_μ consists of a nonet of vector mesons $V_\mu^a\lambda^a$. There are two of them $[\lambda^0=(\frac{2}{3})^{1/2}$ and $\lambda^8\sim Y]$ which play a special role: namely, the ω and the ϕ. We will always assume ideal ω-ϕ mixing, i.e., $\omega\sim(\frac{2}{3})^{1/2}\lambda^0+(1/\sqrt{3})\lambda^8$ and $\phi\sim(\frac{1}{6})^{1/2}\lambda^0-(1/\sqrt{3})\lambda^8$. If we choose the hedgehog *Ansatz* for U Eq. (2) and take the *Ansätze* for the vector mesons consistently, we see that the ϕ field decouples from the Lagrangian even including the Wess-Zumino-Witten term.

The Lagrangian $\mathcal{L}'$ is not complete yet, the gauged Wess-Zumino-Witten action is still missing. Gauging the Wess-Zumino-Witten action leads to[10]

$$\begin{aligned}\mathcal{L}_{\text{YM}}=\mathcal{L}'&+\frac{N_C}{64\pi^2}\omega_{\mu\nu}\epsilon_{\mu\nu\alpha\beta}\text{Tr}[2V_\alpha(U^\dagger\partial_\beta U+\partial_\beta UU^\dagger)\\&+V_\alpha U^\dagger V_\beta U]\\&+\frac{N_C}{48\pi^2}\omega_\mu\epsilon_{\mu\nu\alpha\beta}\text{Tr}(U^\dagger\partial_\nu UU^\dagger\partial_\alpha UU^\dagger\partial_\beta U)\ .\end{aligned} \tag{14}$$

We could have added an SU(3)-symmetric term containing the pseudoscalar-meson masses of the type

$$\begin{aligned}\mathcal{L}_{\text{ps}}&=\frac{3m_\pi^2+4m_K^2+m_\eta^2}{32}f_\pi^2\text{Tr}(U+U^\dagger-2)\\&=\frac{m_{\text{ps}}^2}{4}f_\pi^2\text{Tr}(U+U^\dagger-2)\end{aligned} \tag{15}$$

without spoiling the properties of the Lagrangian equation (14). The necessary *Ansätze* for solving the Lagrangian equation (14) are

$$U=\begin{bmatrix}\cos\Theta+i\tau\cdot\hat{\mathbf{r}}\sin\Theta & 0\\ 0 & 1\end{bmatrix}, \tag{16}$$

$$V_i=A\epsilon_{jki}\tau\cdot\hat{\mathbf{r}}^k\frac{G(r)}{2r}\lambda^jA^\dagger+2A\epsilon_{ijk}\tau\cdot\hat{\mathbf{r}}^jk^k\Omega(r)A^\dagger\ , \tag{17}$$

$$\begin{aligned}V_0=&A\omega(r)\left[\left[\frac{2}{3}\right]^{1/2}\frac{\lambda^0}{2}+\frac{\lambda^8}{2\sqrt{3}}\right]A^\dagger\\&+2A[G_1(r)k^i\lambda^i+G_2(r)\lambda^i\tau\cdot\hat{\mathbf{r}}^i\tau\cdot\hat{\mathbf{r}}^jk^j\\&+S_V(r)\lambda^ak^a]A^\dagger\ ,\end{aligned} \tag{18}$$

where the i,j,k run from $1,\dots,3$ and the a from $4,\dots,7$. The k's are defined as $k^\alpha=-(i/2)\text{Tr}(A^\dagger\dot{A}\lambda_\alpha)$ with $\alpha=1,\dots,8$. Again the Hamiltonian is, similar to the pure Skyrme-model equation (5),

$$H=M+\frac{1}{2\Lambda_2}\left[C_2-\frac{3}{4}\right]+\frac{1}{2}\left[\frac{1}{\Lambda_2}-\frac{1}{\Lambda_1}\right]\mathbf{J}^2 \tag{19}$$

with the functionals now given by (see Ref. 12)

$$\begin{aligned}M=4\pi\int_0^\infty\Bigg[&\frac{f_\pi^2}{2}[r^2\Theta'^2+2\sin^2\Theta+2\sin^2\Theta G(G-2)]+2f_\pi^2G^2-f_\pi^2r^2\omega^2+\frac{1}{g^2}[-r^2\omega'^2+G'^2+\tfrac{1}{2}G^2(G-2)^2]\\&+\frac{N_C}{16\pi^2}[4\omega\Theta'\sin^2\Theta+\omega'\sin2\Theta G(G-2)]\Bigg]dr\ ,\end{aligned} \tag{20}$$

$$\Lambda_1=4\pi\int_0^\infty\left[\tfrac{2}{3}f_\pi^2r^2\sin^2\Theta(G_1-1)^2+2f_\pi^2[r^2(G_1^2+\tfrac{2}{3}G_1G_2+\tfrac{1}{3}G_2^2)-\tfrac{1}{6}\Omega^2]\right.$$
$$+\frac{1}{3g^2}[4G^2(G_1^2+G_1G_2-2G_1-G_2+1)+3r^2G_1'^2+r^2G_2'^2+2r^2G_1'G_2'+2(G^2-2G+2)G_2^2]$$
$$\left.-\frac{1}{6g^2}\left[\Omega'^2+2\frac{\Omega^2}{r^2}+2f^2g^2\Omega^2\right]+\frac{1}{8\pi^2}[2\Omega\Theta'(2\sin^2\Theta-G_1-G_2)+\Omega'\sin2\Theta(GG_1-G-G_2)]\right]dr\ , \tag{21}$$

$$\Lambda_2=4\pi\int_0^\infty\left[\frac{f_\pi^2}{2}r^2[(1-\cos\Theta)(1-S_V)^2+4S_V^2+4S_V^2]+\frac{1}{2g^2}[2r^2S_V'^2+G^2(S_V-1)^2]\right]dr\ . \tag{22}$$

A pseudoscalar-meson mass term $\mathcal{L}_{ps}$ would change the functional M equation (20) to

$$M'=M+m_{ps}^2f_\pi^2(1-\cos\Theta)\ . \tag{23}$$

From the functionals (20)–(22) we get the equations of motion via a variation of the functionals versus the fields

$$\Theta''=-\frac{2}{r}\Theta'+\frac{(1-G)^2}{r^2}\sin2\Theta+\frac{N_C}{8\pi^2f_\pi^2r^2}\omega'[-2\sin^2\Theta+G(G-2)\cos2\Theta]\ , \tag{24}$$

$$G''=\frac{1}{r^2}G(G-1)(G-2)+2f_\pi^2g^2G+g^2f_\pi^2\sin^2\Theta(G-1)-\frac{g^2N_C}{16\pi^2}\omega'\sin2\Theta(G-1)\ , \tag{25}$$

$$\omega''=-\frac{2}{r}\omega'+2f_\pi^2g^2\omega-\frac{N_C}{4\pi^2r^2}g^2\Theta'\sin^2\Theta+\frac{N_C}{8\pi^2r^2}g^2[G(G-2)\Theta'\cos2\Theta+G'(G-1)\sin2\Theta]\ , \tag{26}$$

$$G_1''=-\frac{2}{r}G_1'+g^2f_\pi^2[\sin^2\Theta(G_1-1)+2G_1]+\frac{1}{r^2}[G^2(G_1-1)+2(G-1)G_2']-\frac{g^2N_C}{32\pi^2r^2}[\Omega'(1-G)\sin2\Theta]\ , \tag{27}$$

$$G_2''=-\frac{2}{r}G_2'-g^2f_\pi^2[\sin^2\Theta(G_1-1)-2G_2]+\frac{1}{r^2}[G^2G_1-G^2+2(G^2-3G+3)G_2]$$
$$-\frac{N_Cg^2}{8\pi^2r^2}[\Omega\Theta'-\tfrac{1}{4}\Omega(1-G)\sin2\Theta]\ , \tag{28}$$

$$\Omega''=2g^2f_\pi^2\Omega+\frac{2}{r^2}\Omega+\frac{N_Cg^2}{8\pi^2}[-2\Theta'(2\sin^2\Theta-G_1-G_2)-2\Theta'\cos2\Theta(G-GG_1+G_1)$$
$$+\sin2\Theta(G'G_1-G'+GG_1'-G_1')]\ , \tag{29}$$

$$S_V''=-\frac{2}{r}S_V'+\frac{G^2}{2r}(S_V-1)+f_\pi^2g^2[(1-\cos\Theta)(1-S_V)+2S_V]\ . \tag{30}$$

The incorporation of $\mathcal{L}_{ps}$ would add only $m_{ps}^2f_\pi^2\sin\Theta$ to the right-hand side of Eq. (24). Note that the Λ_2 and the equation of motion for S_V do not contain remnants of the Wess-Zumino-Witten term, due to the special form of the *Ansatz* for Λ_2. The equations of motion, together with the condition that at $r=\infty$ the vacuum should be obtained and that $\Theta(0)=\pi$, which translates into a baryon-number-1 solution, are leading to the following set of boundary conditions:

$$\begin{aligned}
&\Theta(0)=\pi, && \Theta(\infty)=0,\\
&G(0)=0, && G(\infty)=0,\\
&\omega'(0)=0, && \omega(\infty)=0,\\
&G_1'(0)=0, && G_1(\infty)=0,\\
&G_2(0)=0, && G_2(\infty)=0,\\
&\Omega(0)=0, && \Omega(\infty)=0,\\
&S_V'(0)=0, && S_V(\infty)=0.
\end{aligned}$$

The resulting values for Λ_i and M are, if we take $m^2=2f_\pi^2g^2=(770\text{ MeV})^2$ and $f_\pi=93$ MeV,

$$M\approx976(1143)\text{ MeV}\ ,$$
$$\Lambda_1\approx0.36(0.22)\text{ fm}\ ,$$
$$\Lambda_2\approx0.13(0.07)\text{ fm}\ .$$

The numbers in parentheses are obtained if we take $\mathcal{L}_{ps}$ with $m_{ps}\approx408$ MeV into consideration. Surprisingly, the mass of the soliton is close to the experimental nucleon mass, but the moment of inertia Λ_1 is much too small. It should be ≈1 fm in order to agree with the octet-decuplet splitting. If we include the pseudoscalar-meson mass we get a soliton mass which is about 20% larger than in the massless case. The values for the moments of inertia are smaller, which can be easily seen if we recall that a finite pseudoscalar-meson mass changes the convergence for large distances of the chiral field Θ from a polynomial to an exponential behavior.

Note that Λ_2 is not directly accessible via experiment.

C. The hidden gauge model

In the case of hidden gauge theories the method of construction is different from the Yang-Mills case. The stress is in the hidden gauge case more on transformation properties of chiral nonlinear Lagrangians. The construction of chiral nonlinear Lagrangians from required underlying transformation properties dates back to the late 1960s.[27,28] We can postulate a hidden symmetry group $U(3)_V$ in the nonlinear σ model by parametrizing the chiral matrix U. Consider the chiral matrix U in the form

$$U(x)=\xi_L^\dagger(x)\xi_R(x) , \tag{31}$$

where the ξ's are SU(3) matrices. Additionally we introduce a U(3) gauge field $V_\mu(x)$ with

$$V_\mu(x)=V_\mu^a(x)\frac{\lambda^a}{2} . \tag{32}$$

The transformation group now reads

$$[SU(3)_L\times SU(3)_R]_{\text{global}}\times[SU(3)_L\times SU(3)_R]_{\text{local}}$$

with

$$\begin{aligned}\xi_L(x)&\to h(x)\xi_L(x)g_L^\dagger ,\\ \xi_R(x)&\to h(x)\xi_R(x)g_R^\dagger ,\\ V_\mu(x)&\to h(x)V_\mu(x)h^\dagger(x)+\frac{i}{g}h(x)\partial_\mu h^\dagger(x) ,\end{aligned} \tag{33}$$

where $h(x)\in U(3)_V$. Obviously

$$U=\xi_L^\dagger\xi_R\to(g_L\xi_L^\dagger h^\dagger)(h\xi_R g_R^\dagger)=g_L U g_R^\dagger .$$

The covariant derivative is defined by

$$D_\mu\xi_{(L,R)}(x)=(\partial_\mu-iV_\mu)\xi_{(L,R)}(x) ; \tag{34}$$

i.e., we do not simply replace $\partial_\mu U$ by $D_\mu U$. Again we absorbed the coupling constant g into V_μ. We can construct two $[SU(3)_L\times SU(3)_R]_{\text{global}}\times[SU(3)_V]_{\text{local}}\times[\text{parity}]$ invariants from these derivatives:

$$\mathcal{L}_A=-\frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu\xi_L\xi_L^\dagger-D_\mu\xi_R\xi_R^\dagger)^2 , \tag{35}$$

$$\mathcal{L}_V=-\frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu\xi_L\xi_L^\dagger+D_\mu\xi_R\xi_R^\dagger)^2 . \tag{36}$$

Gauge fixing with

$$\xi_L^\dagger=\xi_R=\xi \tag{37}$$

leads to

$$\begin{aligned}\mathcal{L}_A&=f_\pi^2\mathrm{Tr}\left[\frac{1}{2i}(\partial_\mu\xi\xi^\dagger-\partial_\mu\xi^\dagger\xi)\right]^2\\ &=\frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U\partial_\mu U^\dagger)\end{aligned} \tag{38}$$

and

$$\mathcal{L}_V=f_\pi^2\mathrm{Tr}\left[V_\nu-\frac{1}{2i}(\partial_\mu\xi\xi^\dagger+\partial_\mu\xi^\dagger\xi)\right]^2 . \tag{39}$$

We can construct a Lagrangian from these two invariants which, after adding a kinetic energy for the gauge fields, reads

$$\begin{aligned}\mathcal{L}_{\text{HG}}^{(0)}=&-\frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu\xi_L\xi_L^\dagger-D_\mu\xi_R\xi_R^\dagger)^2\\ &-a\frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu\xi_L\xi_L^\dagger+D_\mu\xi_R\xi_R^\dagger)^2-\frac{1}{2g^2}\mathrm{Tr}(F_{\mu\nu}^2) .\end{aligned} \tag{40}$$

The choice $a=2$ ensures the famous KSFR relation.[25] It is interesting to note that ξ is in the $B=1$ sector a singular map from $\mathcal{R}^3\cup\infty$ to SU(2). This singularity is the driving force for the nontrivial ρ field.

The field-strength tensor is given by

$$F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu-i[V_\mu,V_\nu] .$$

The Wess-Zumino-Witten term can be introduced by a procedure which was first used by Fujiwara *et al.*[26] They showed that one can add a number of terms to the Wess-Zumino-Witten term without spoiling its anomalous properties:

$$\Gamma_{\text{WZ}}=\Gamma_{\text{WZ}}^{(0)}+\sum_{i=1}^{6}\int c_i\mathcal{L}_i d^4x . \tag{41}$$

The $\mathcal{L}_i$ are invariants which conserve physical but break intrinsic parity.

Using the following definitions (the A_μ's are global gauge fields, i.e., independent from V_μ)

$$\alpha_\mu^{(L,R)}=D_\mu\xi_{(L,R)}\xi_{(L,R)}^\dagger , \tag{42}$$

$$F_{\mu\nu}^V=\partial_\mu V_\nu-\partial_\nu V_\mu-i[V_\mu,V_\nu] , \tag{43}$$

$$\begin{aligned}A_{\mu\nu}^{(L,R)}=&\xi_{(L,R)}(\partial_\mu A_\nu^{(L,R)}-\partial_\nu A_\mu^{(L,R)}\\ &-i[A_\mu^{(L,R)},A_\nu^{(L,R)}])\xi_{(L,R)}^\dagger ,\end{aligned} \tag{44}$$

the invariants are given as

$$\mathcal{L}_1=\epsilon^{\mu\nu\sigma\beta}\mathrm{Tr}(\alpha_\mu^L\alpha_\nu^L\alpha_\sigma^L\alpha_\beta^R-\alpha_\mu^L\alpha_\nu^R\alpha_\sigma^L\alpha_\beta^R) , \tag{45}$$

$$\mathcal{L}_2=\epsilon^{\mu\nu\sigma\beta}\mathrm{Tr}(\alpha_\mu^L\alpha_\nu^L\alpha_\sigma^L\alpha_\beta^R+\alpha_\mu^L\alpha_\nu^R\alpha_\sigma^L\alpha_\beta^R) , \tag{46}$$

$$\mathcal{L}_3=i\epsilon^{\mu\nu\sigma\beta}\mathrm{Tr}[\tfrac{1}{2}F_{\mu\nu}^V(\alpha_\sigma^L\alpha_\beta^L-\alpha_\sigma^R\alpha_\beta^R)] , \tag{47}$$

$$\mathcal{L}_4=i\epsilon^{\mu\nu\sigma\beta}\mathrm{Tr}[\tfrac{1}{2}F_{\mu\nu}^V(\alpha_\sigma^L\alpha_\beta^R-\alpha_\sigma^R\alpha_\beta^L)] , \tag{48}$$

$$\mathcal{L}_5=i\epsilon^{\mu\nu\sigma\beta}\mathrm{Tr}(\tfrac{1}{2}F_{\mu\nu}^L\alpha_\sigma^R\alpha_\beta^R-\tfrac{1}{2}F_{\mu\nu}^R\alpha_\sigma^L\alpha_\beta^L) , \tag{49}$$

$$\mathcal{L}_6=i\epsilon^{\mu\nu\sigma\beta}\mathrm{Tr}(\tfrac{1}{2}F_{\mu\nu}^L\alpha_\sigma^L\alpha_\beta^R-\tfrac{1}{2}F_{\mu\nu}^R\alpha_\sigma^R\alpha_\beta^L) . \tag{50}$$

The phenomenologically best actions are given by[26]

$$\Gamma_{\text{WZ}}=\Gamma_{\text{WZ}}^{(0)}-15C(c_1\mathcal{L}_4+\mathcal{L}_6+c_2\mathcal{L}_1) , \tag{51}$$

where the choice $c_1=1$ and $c_2=-\frac{1}{2}$ leads to partial vector-meson dominance (PVMD) and the choice $c_1=1$ and $c_2=-\frac{1}{6}$ to complete vector-meson dominance[26,9] (CVMD) with

$$C=-\frac{iN_C}{240\pi^2}\ .$$

Now the Lagrangian reads

$$\mathcal{L}_{\rm HG}=\mathcal{L}_{\rm HG}^{(0)}+\sum_{i=1}^{6}c_i\mathcal{L}_i+\mathcal{L}_\omega\ , \tag{52}$$

where the $\mathcal{L}_{\rm HG}^{(0)}$ is defined in Eq. (40) and the $\mathcal{L}_i$ are given in Eq. (41). $\mathcal{L}_\omega$ are the kinetic and massive parts of the ω meson

$$\mathcal{L}_\omega=-\frac{1}{2g^2}\omega'^2-\frac{m_\omega^2}{2}\omega^2\ .$$

Again we could have added the pseudoscalar-meson mass

$$\mathcal{L}_{\rm ps}=\frac{m_{\rm ps}^2}{4}f_\pi^2{\rm Tr}(U+U^\dagger-2)$$

without spoiling the properties of the Lagrangian equation (52). The Hamiltonian for the hidden gauge theory is

$$H=M+\frac{1}{2\Lambda_2}\left[C_2-\frac{3}{4}\right]+\frac{1}{2}\left[\frac{1}{\Lambda_1}-\frac{1}{\Lambda_2}\right]\mathbf{J}^2\ . \tag{53}$$

Taking now the same *Ansätze* as in the massive Yang-Mills case yields[9]

$$M=\int d^3r\left[\frac{f_\pi^2}{2}\left[\Theta'^2+2\frac{\sin^2\Theta}{r^2}\right]+2\frac{f_\pi^2}{r^2}(G-1+\cos\Theta)^2+\frac{1}{2g^2}\left[2G'^2+\frac{G^2(G-2)^2}{r^2}\right]-\frac{1}{2g^2}(\omega'^2+2g^2f_\pi^2\omega^2)\right.$$
$$\left.-\frac{3N_C}{16\pi^2r^2}\left[-2c_2\omega\Theta'\sin^2\Theta+\frac{c_1}{3g}[-\omega\Theta'G(G+2)+2\sin\Theta(-\omega G'+G\omega')+\omega'(2\sin\Theta-\sin2\Theta)]\right]\right]\ , \tag{54}$$

$$\Lambda_1=4\pi\int\left[\tfrac{2}{3}f_\pi^2r^2[\sin^2\Theta+3G_1^2+2G_1G_2+G_2^2-2\sin^2\Theta+4(G_1-1)(\cos\Theta-1)]\right.$$
$$+\frac{1}{3g^2}[4G^2(G_1^2+G_1G_2-2G_1-G_2+1)+3r^2G_1'^2+r^2G_2'^2+2r^2G_1'G_2'+2(G^2-2G+2)G_2^2]$$
$$-\frac{1}{6g^2}\left[\Omega'^2+2\frac{\Omega^2}{r^2}+2f_\pi^2g^2\Omega^2\right]$$
$$+\frac{N_C}{4\pi^2}\left[2c_2\Omega\Theta'\sin^2\Theta+\frac{c_1}{3g}[\Omega\Theta'(2G_2+G_1-G-GG_1)+\Omega'(2\sin\Theta-\sin2\Theta)-\Omega\sin\Theta(G'+G_1')\right.$$
$$\left.\left.+\Omega'\sin\Theta(G+G_1)]\right]\right]dr\ , \tag{55}$$

$$\Lambda_2=4\pi\int\left\{\frac{f_\pi^2}{2}r^2\left[(1-\cos\Theta)+4\left[S_V-1+\cos\frac{\Theta}{2}\right]^2\right]+\frac{1}{2g^2}[2r^2S_V'^2+G^2(S_V-1)^2]\right\}dr\ . \tag{56}$$

A nonvanishing pseudoscalar-meson mass changes the mass functional to

$$M'=M+m_{\rm ps}^2f_\pi^2(1-\cos\Theta)\ . \tag{57}$$

The equations of motion resulting from minimizing the functionals M, Λ_1, and Λ_2 Eqs. (54)–(56) are given then as

$$\Theta''=-\frac{2}{r}\Theta'+\frac{4}{r^2}\sin\Theta(1-G-\tfrac{1}{2}\cos\Theta)$$
$$+\frac{N_C}{16\pi^2r^2}\{c_1[2\omega(GG'-G'+G'\cos\Theta)+\omega'(G^2-2G-2+4\sin^2\Theta)]+6c_2\omega'\sin^2\Theta\}\ , \tag{58}$$

$$G''=\frac{1}{r^2}G(G-1)(G-2)+2g^2f_\pi^2(G-1+\cos\Theta)-\frac{N_Cg^2}{16\pi^2r^2}c_1[\omega\Theta'(G+\cos\Theta-1)+2\omega'\sin\Theta]\ , \tag{59}$$

$$\omega''=-\frac{2}{r}\omega'+2g^2f_\pi^2\omega+\frac{N_Cg^2}{16\pi^2r^2}\{c_1[2\Theta'(1-G)\cos\Theta+4\sin\Theta(\Theta'\sin\Theta-G')+\Theta'(G^2-2G-2)]+6c_2\Theta'\sin^2\Theta\}\ , \tag{60}$$

$$G_1''=-\frac{2}{r}G_1'+2g^2f_\pi^2G_1-4g^2f_\pi^2\sin^2\tfrac{1}{2}\Theta$$
$$+\frac{1}{r^2}[G^2(G_1-1)+2(G-1)G_2]+\frac{N_Cg^2}{16\pi^2r^2}c_1[\Omega\Theta'(G-1+\cos\Theta)+2\Omega'\sin\Theta]\ , \tag{61}$$

$$G_2''=-\frac{2}{r}G_2'+2g^2f_\pi^2G_2+4g^2f_\pi^2\sin^2\tfrac{1}{2}\Theta$$
$$+\frac{1}{r^2}[G^2(G_1-1)+2(G^2-3G+3)G_2]-\frac{N_Cg^2}{16\pi^2r^2}c_1[\Omega\Theta'(G-5+\cos\Theta)+2\Omega'\sin\Theta]\ , \tag{62}$$

$$\Omega''=\frac{2}{r^2}\Omega+2g^2f_\pi^2\Omega+\frac{N_Cg^2}{4\pi^2}(c_1\{\Theta'[(G_1-G+2)\cos\Theta-2G_2-GG_1-G_1-G-2+4\sin^2\Theta]$$
$$+2(G_2'-G')\sin\Theta\}+6c_2\Theta'\sin^2\Theta)\ , \tag{63}$$

$$S_V''=-\frac{2}{r}S_V'+\frac{G^2}{2r}(S_V-1)+2f_\pi^2g^2\left[S_V-1+\cos\frac{\Theta}{2}\right]. \tag{64}$$

If we take the nonvanishing pseudoscalar-meson mass into account the right-hand side of Eq. (58) gets an additional term $m_{\rm ps}^2f_\pi^2\sin\Theta$. The equations of motion are subject to the boundary conditions, due to the fact that we want to have a $B=1$ solution and vacuum at $r=\infty$:

$$\begin{aligned}
\Theta(0)&=\pi, & \Theta(\infty)&=0,\\
G(0)&=2, & G(\infty)&=0,\\
\omega'(0)&=0, & \omega(\infty)&=0,\\
G_1'(0)&=0, & G_1(\infty)&=0,\\
G_2'(0)&=0, & G_2(\infty)&=0,\\
\Omega(0)&=0, & \Omega(\infty)&=0,\\
S_V'(0)&=0, & S_V(\infty)&=0.
\end{aligned}$$

The resulting values for the moments of inertia Λ_i and the mass M are, if we take $m^2=2f_\pi^2g^2=(770\ \text{MeV})^2$ and $f_\pi=93$ MeV,

$$\begin{aligned}
M&\approx 1547(1817)\ \text{MeV}\ \ (\text{PVMD})\\
&\approx 1293(1412)\ \text{MeV}\ \ (\text{CVMD})\ ,\\
\Lambda_1&\approx 1.84(1.06)\ \text{fm}\ \ (\text{PVMD})\\
&\approx 0.85(0.53)\ \text{fm}\ \ (\text{CVMD})\ ,\\
\Lambda_2&\approx 0.56(0.23)\ \text{fm}\ \ (\text{PVMD})\\
&\approx 0.28(0.14)\ \text{fm}\ \ (\text{CVMD})\ ,
\end{aligned}$$

where the numbers in parentheses correspond to the case including $\mathcal{L}_{\rm ps}$.

The masses are larger than in the massive Yang-Mills model, due to the nontrivial boundary conditions in the ρ field. The moment of inertia Λ_1 is surprisingly close to the experimental value of ≈ 1 fm.

Again, including the pseudoscalar-meson mass yields smaller Λ_i and a larger mass M. This is a result of the change of convergence of Θ from polynomial to exponential.

III. ELECTROMAGNETIC PROPERTIES

The electromagnetic properties of the classical and the gauged Skyrmions can be extracted by constructing the Noether currents corresponding to the global symmetry $U(3)_L\times U(3)_R$ of the Lagrangian.

The electromagnetic current can be easily read off:

$$J_\mu^{\rm em}|_{I=0}\sim L_\mu^{\lambda 8}+R_\mu^{\lambda 8}\ , \tag{65}$$

$$J_\mu^{\rm em}|_{I=1}\sim L_\mu^{\tau 3}+R_\mu^{\tau 3}\ . \tag{66}$$

Alternatively we can use the notion of current field identities in connection with vector-meson dominance (VMD) for the construction of the electromagnetic currents. The VMD current field identities are

$$\mathbf{J}_\mu^\rho=-\frac{m_\rho^2}{g}\rho_\mu\ , \tag{67}$$

$$\mathbf{J}_\mu^\omega=-\frac{m_\omega^2}{3g}\omega_\mu\ . \tag{68}$$

We can identify these currents, Eqs. (67) and (68) as the electromagnetic $I=0$, $I=1$ currents, respectively.

We consider nucleons first. In all models either pure or gauged we are able to read off the form factors $G_{E,M}$ immediately, once the currents are given. Using the Breit frame we get for the spin-$\frac{1}{2}$ Skyrmion[30,7] the following nucleon matrix elements between final state f and initial state i with the momenta $\mathbf{q}/2$ and $-\mathbf{q}/2$, respectively:

$$\left\langle N_f\left(-\frac{\mathbf{q}}{2}\right)\right|J^0(0)\left|N_i\left(-\frac{\mathbf{q}}{2}\right)\right\rangle=G_E(-q^2)\langle s_3^f|s_3^i\rangle\ ,$$

$$\left\langle N_f\left(-\frac{\mathbf{q}}{2}\right)\right|\mathbf{J}(0)\left|N_i\left(-\frac{\mathbf{q}}{2}\right)\right\rangle=\frac{G_M(-q^2)}{M_N}\langle s_3^f|\sigma\times\mathbf{q}|s_3^i\rangle.$$

The s_3 are the initial and final spins of the nucleon along the z axis and σ is the spin operator. For the axial-vector current we take only the transversal part. The adiabatical quantization procedure leads to $\sigma = -2i\Lambda\,\mathrm{Tr}(A^\dagger \dot{A}\tau)$. The magnetic moments are given by $G_M(0)$.

The magnetic moments of the other octet baryons can be calculated by using the transformation property of the charge operator Q (Ref. 29), since our baryon wave functions have good hypercharge and good isospin. The transformations of Q under SU(3) leads to

$$\mu_p = \mu_{\Sigma^+}, \quad \mu_n = \mu_{\Xi^0} ,$$

$$\mu_{\Xi^-} = \tfrac{1}{2}\mu_n, \quad \mu_{\Sigma^0} = -\tfrac{1}{2}\mu_n ,$$

$$\mu_\Lambda = -(\mu_p + \mu_n), \quad \mu_{\Sigma^-} = \mu_{\Xi^-} .$$

In the pure pseudoscalar case without pseudoscalar mass the magnetic form factors are given by

$$G_M^{p,n} = \frac{4\pi M}{q}\int_0^\infty r j_1(qr)\left\{-\frac{1}{4\pi^2\Lambda_1}\Theta'\sin^2\Theta \pm \frac{2}{3}\left[2f_\pi^2\sin^2\Theta + \frac{\sin^2\Theta}{e^2}\left[\Theta'^2 + \frac{\sin^2\Theta}{r^2}\right]\right]\right\}dr \tag{69}$$

leading to the magnetic moments via $\mu_{S,V} = \lim_{q\to 0} G^{S,V}(q^2)$ (Ref. 30)

$$\tfrac{1}{2}(\mu_p + \mu_n) = 0.489 ,$$

$$\tfrac{1}{2}(\mu_p - \mu_n) = \frac{2690}{e^4} \approx 3.05 .$$

In the determination of the magnetic moments for the gauged Skyrmions we will use the approximation of vector-meson dominance leading to the form factors

$$G_M^S = -\frac{M}{\Lambda_1}\frac{m_\rho^2}{3g^2}\frac{2\pi}{q}\int_0^\infty r\Omega j_1(qr)dr , \tag{70}$$

$$G_M^V = \frac{8\pi}{3}\frac{Mm_\rho^2}{g^2 q}\int_0^\infty rGj_1(qr)dr . \tag{71}$$

This vector-meson dominance approximation leads to the magnetic moments (the numbers in parentheses are the results including the $\mathcal{L}_{ps}$)

$$\tfrac{1}{2}(\mu_p + \mu_n) = \begin{cases} 0.26(0.27) & (\text{PVMD}) , \\ 0.20(0.25) & (\text{CVMD}) , \\ 0.47(0.51) & (\text{YM}) , \end{cases}$$

$$\tfrac{1}{2}(\mu_p - \mu_n) = \begin{cases} 3.27(1.71) & (\text{PVMD}) , \\ 1.80(1.11) & (\text{CVMD}) , \\ 0.52(0.29) & (\text{YM}) . \end{cases}$$

If we use the Noether currents together with the approximation of the Wess-Zumino-Witten term by a $\gamma\pi^3$ term we get the hidden gauge model

$$\tfrac{1}{2}(\mu_p + \mu_n) = \begin{cases} 0.20(0.22) & (\text{PVMD}) , \\ 0.20(0.24) & (\text{CVMD}) , \end{cases}$$

$$\tfrac{1}{2}(\mu_p - \mu_n) = \begin{cases} 2.52(1.38) & (\text{PVMD}) , \\ 1.99(0.97) & (\text{CVMD}) . \end{cases}$$

Note that the results differ only slightly for the scalar magnetic moments. The large difference in the isovectorial magnetic moment between the Yang-Mills model and the hidden gauge model is due to the boundary condition $G(0)=2$ in the hidden gauge case. Note that in the massive pseudoscalar-meson case the vectorial moments are much too small.

IV. EXPLICIT SYMMETRY BREAKING

There are two different sources for the explicit symmetry breaking. The first contribution resides in the pseudoscalar-meson sector

$$\mathcal{L}_m^{SU(3)} = \frac{m^2}{4}f_\pi^2\mathrm{Tr}(U + U^\dagger - 2) - \frac{m_K^2 - m_\pi^2}{2\sqrt{3}}f_\pi^2\mathrm{Tr}[\lambda_8(U + U^\dagger)] \tag{72}$$

with $m_{ps}^2 = \frac{1}{8}(3m_\pi^2 + 4m_K^2 + m_\eta^2)$. The first term in Eq. (72) expresses the nonvanishing pseudoscalar-meson mass and the second the mass difference between pions and kaons. The first term is included into the numbers quoted in parentheses.

The two terms lead to the following contributions to the Hamiltonian:

$$\Delta H_m = \int d^3x \frac{m_K^2 - m_\pi^2}{2\sqrt{3}}f_\pi^2\mathrm{Tr}[\lambda_8(U + U^\dagger)] , \tag{73}$$

$$\delta H_\pi = -\frac{m^2}{4}f_\pi^2\int d^3x\, \mathrm{Tr}(U + U^\dagger - 2) . \tag{74}$$

Note that the first term breaks the SU(3) invariance due to the occurrence of λ_8.

The second source resides in the vector-meson sector

$$\Delta H_\rho = \int d^3r \frac{m_{K^*}^2 - m_\rho^2}{4\sqrt{3}}\mathrm{Tr}(\lambda_8 V_\mu^2) . \tag{75}$$

Both Eqs. (73) and (75) show the same property; i.e.,

$$\Delta H = \Delta m\, \mathrm{Tr}(A^\dagger\lambda_8 A) . \tag{76}$$

However Δm is

$$\Delta m_\pi = \frac{8\pi}{3}f_\pi^2(m_K^2 - m_\pi^2)\int(1 - \cos\Theta)r^2 dr \tag{77}$$

for the pseudoscalars (73) and for the vector mesons (75),

$$\Delta m_\rho=\frac{m_{K^*}^2-m_\rho^2}{8g^2}4\pi\int r^2dr\left[-\frac{C_2-\frac{3}{4}-\mathbf{I}^2}{\Lambda_2^2}S_V^2+\frac{\mathbf{I}^2}{6\Lambda_1^2}\left[4G_2^2+8G_2G_1+12G_1^2-3\frac{\Omega^2}{r^2}\right]+\frac{G^2}{r^2}-\frac{\omega^2}{2}\right] \tag{78}$$

for the hidden gauge as well as for the massive Yang-Mills model.

C_2 is the quadratic Casimir invariant and $\mathbf{I}^2$ the expectation value of the square of the isospin.

Following Ref. 18 we get, for the octet (spin-$\frac{1}{2}$ baryons),

$$\langle N|\Delta H|N\rangle=-\tfrac{3}{10}\Delta m\ , \tag{79}$$

$$\langle \Lambda|\Delta H|\Lambda\rangle=-\tfrac{1}{10}\Delta m\ , \tag{80}$$

$$\langle \Sigma|\Delta H|\Sigma\rangle=+\tfrac{1}{10}\Delta m\ , \tag{81}$$

$$\langle \Xi|\Delta H|\Xi\rangle=+\tfrac{2}{10}\Delta m\ , \tag{82}$$

and for the decuplet (spin-$\frac{3}{2}$ baryons)

$$\langle \Delta|\Delta H|\Delta\rangle=-\tfrac{1}{8}\Delta m\ , \tag{83}$$

$$\langle \Sigma^*|\Delta H|\Sigma^*\rangle=0\ , \tag{84}$$

$$\langle \Xi^*|\Delta H|\Xi^*\rangle=+\tfrac{1}{8}\Delta m\ , \tag{85}$$

$$\langle \Omega|\Delta H|\Omega\rangle=+\tfrac{2}{8}\Delta m\ . \tag{86}$$

But now with

$$\Delta m^{(8)}=\Delta m_\pi+\Delta m_\rho^{(8)}\ , \tag{87}$$

$$\Delta m^{(10)}=\Delta m_\pi+\Delta m_\rho^{(10)}\ . \tag{88}$$

All baryons get additionally a contribution δH:

$$\delta H^{(8)}=\delta H_\rho^{(8)}+\delta H_\pi,\quad \delta H^{(10)}=\delta H_\rho^{(10)}+\delta H_\pi\ ,$$

with

$$\delta H_\pi=6\pi f_\pi^2 m_{\rm ps}^2\int(1-\cos\theta)r^2dr\ .$$

Note that $\delta H_\pi=0$ if we include $\mathcal{L}_{\rm ps}$ into our calculations. The vector mesons give the contribution

$$\delta H_\rho=\frac{3m_\rho^2+4m_{K^*}^2+m_\omega^2-8m_\rho^2}{32}\int \mathrm{Tr}(V_\mu^2)d^3r \tag{89}$$

$$=\frac{M_{K^*}^2-m_\rho^2}{8g^2}4\pi\int r^2dr\left[\frac{G^2}{r^2}-\frac{\omega^2}{2}+2\frac{(C_2-\frac{3}{4}-\mathbf{I}^2)S_V^2}{\Lambda_2^2}+\frac{\mathbf{I}^2}{6\Lambda_1^2}\left[-3\frac{\Omega^2}{r^2}+4G_2^2+8G_2G_1^2+12G_1^2\right]\right]\ . \tag{90}$$

For the different models we get

$$\Delta M^{(8)}=\Delta m_\pi+\Delta m_\rho^{(8)}$$

=388 MeV Skyrme

=692(180) MeV hidden gauge (PVMD)

=300(92) MeV hidden gauge (CVMD)

=209(29) MeV Yang-Mills ,

$$\Delta m^{(10)}=\Delta m_\pi+\Delta m_\rho^{(10)}$$

=388 MeV Skyrme

=695(182) MeV hidden gauge (PVMD)

=306(98) MeV hidden gauge (CVMD)

=204(78) MeV Yang-Mills ,

$$\delta H^{(8)}=\delta H_\rho^{(8)}+\delta H_\pi\quad(\delta H_\rho)$$

=2148 MeV Skyrme

=1925(95) MeV hidden gauge (PVMD)

=1199(89) MeV hidden gauge (CVMD)

=362(4) MeV Yang-Mills ,

$$\delta H^{(10)}=\delta H_\rho^{(10)}+\delta H_\pi\quad(\delta H_\rho)$$

=2148 MeV Skyrme

=1928(98) MeV hidden gauge (PVMD)

=1205(98) MeV hidden gauge (CVMD)

=358(16) MeV Yang-Mills .

The numbers in parentheses refer to the massive pseudoscalar-meson case. Alternatively we can calculate the singlet, the 27-plet, and the F and D masses leading to

$$m_{[1]}=\tfrac{1}{8}(2m_N+2m_\Xi+m_\Lambda+3m_\Sigma)$$

$$=m_N+\tfrac{3}{10}\Delta H^8\ ,$$

$$m_{[27]}=\tfrac{1}{8}(\tfrac{3}{5})^{1/2}(2m_N+2m_\Xi-3m_\Lambda-m_\Xi)$$

$$=0\ ,$$

$$m_F=\tfrac{1}{8}\sqrt{2}(2m_N-2m_\Xi)$$

$$=-\tfrac{1}{8}\sqrt{2}\,\Delta H^8\ ,$$

$$m_D=-\tfrac{1}{8}(\tfrac{2}{5})^{1/2}(2m_N+2m_\Xi+2m_\Lambda-6m_\Sigma)$$

$$=-\tfrac{1}{8}(\tfrac{2}{5})^{1/2}\Delta H^8\ .$$

TABLE I. The mass splittings in MeV for physical input parameters. The experiment is taken from Ref. 31. The numbers in parentheses correspond to the massive pseudoscalar-meson case.

	The mass splittings				
	PVMD	CVMD	YM	Sk	Expt
f_π	93	93	93	65	93
$g(e)$	5.85	5.85	5.85	5.45	5.85
M_N	3568(2923)	3014(3156)	2617(4803)	3402	939
$M_{N\Lambda}$	138(36)	60(19)	42(19)	78	176
$M_{N\Sigma}$	275(72)	120(37)	84(37)	155	254
$M_{N\Xi}$	346(90)	150(46)	105(46)	194	379
$M_{[1]}$	3776(2977)	3104(3185)	2779(4830)	3528	1151
M_{8-10}	164(278)	354(562)	811(1351)	293	231
$M_{N\Delta}$	285(309)	406(578)	848(1370)	370	293
$M_{\Delta\Sigma^*}$	88(23)	38(13)	26(10)	41	155
$M_{\Delta\Xi^*}$	172(46)	76(25)	52(20)	82	307
$M_{\Delta\Omega}$	264(68)	114(37)	78(29)	123	442

The numerical value of the singlet mass comes out much too large about 200%, see Table I. Additionally we can calculate the electromagnetic $\Delta I = 1$ splitting for $\Delta_{Y,i} = M_{YII_3} - M_{YII_3+1}$ as

$$
\begin{aligned}
5\Delta_n &= 2\Delta_\Sigma \\
&= \tfrac{1}{4}\Delta_\Xi \\
&= \frac{8\pi}{3}(m_{K^0}^2 - m_{K^+}^2) f_\pi^2 \int_0^\infty (1-\cos\Theta) r^2 dr \qquad (91) \\
&= \begin{cases} 3.3 \ (\text{Skyrme}) , \\ 1.6(0.4) \ (\text{PVMD}) , \\ 0.7(0.1) \ (\text{CVMD}) , \\ 0.6(0.1) \ (\text{YM}) . \end{cases}
\end{aligned}
$$

The numbers in parentheses correspond to the massive pseudoscalar-meson case. Note that the values for the splittings are much too small, see Table II.

V. DISCUSSION

In Table I we have listed the mass and mass splittings for the octet and decuplet baryons. The very large nucleon masses are due to the small values of Λ_2, in all models. If we do not include the effects of the quantization Λ's, we get better nucleon masses, see the individual listings in Sec. II. Note that the 27-plet mass $M_{[27]}$ vanishes which is in good agreement with the experiment (-2.7 MeV) (Ref. 31). However, the singlet mass is too large and the F/D ratio turns out to be $\sqrt{5}$ as compared to 4.06 in the experiment.[31] One of course rediscovers the Gell-Mann–Okubo relations:

TABLE II. The magnetic moments for physical input parameters. The primed quantities for the hidden gauge-model calculate the magnetic moments with the help of the vector-meson-dominance approximation. The experiment is taken from Ref. 31. Note that the values in the massive pseudoscalar-meson case are much worse then the ones quoted in this table.

	The electromagnetic properties						
	PVMD′	PVMD	CVMD′	CVMD	YM	Sk	Expt
f_π	93	93	93	93	93	65	93
$g(e)$	5.85	5.85	5.85	5.85	5.85	5.45	5.85
μ_p	3.53	2.72	2.00	2.19	1.09	3.53	2.79
μ_n	-3.01	-2.32	-1.60	-1.79	-0.05	-2.56	-1.91
μ_{Σ^+}	3.53	2.72	2.00	2.19	1.09	3.53	2.38
μ_{Ξ^0}	-3.01	-2.32	-1.60	-1.79	-0.05	-2.56	-1.25
μ_{Σ^-}	-1.5	-1.16	-0.80	-0.90	-0.02	-1.28	-1.14
μ_{Σ^-}	-0.52	-0.40	-0.40	-0.40	-0.94	-0.98	-0.69
μ_Λ	-0.52	-0.40	-0.40	-0.40	-0.94	-0.98	-0.61
Δ_N	0.32		0.14		0.12	0.66	1.29
Δ_Σ	0.80		0.35		0.30	1.6	3.3
Δ_Ξ	6.4		2.8		2.4	1.3	2.4

$$2(M_N+M_\Xi)=3M_\Lambda+M_\Sigma\ ,$$

$$M_\Delta-M_{\Sigma^*}=M_{\Sigma^*}-M_{\Xi^*}=M_{\Xi^*}-M_{\Omega^-}\ .$$

The deviations of the F/D ratio and the much too large singlet mass suggest that the model does not have the correct baryon wave functions or in the case of the singlet mass that the absolute scale of the vacuum energy of the theory is not well chosen.

The same applies to the magnetic moments given in Table II. Note that the predictions from the Noether currents are superior to the vector-meson-dominance approximation.

The electromagnetic $\Delta I=1$ splitting is much too small in all models, which suggest that the splitting mechanism needs additional contributions. The large contributions of Λ_2 to the total energy are casting doubts on the validity of the quantization procedure, although the octet splittings are well reproduced for the vanishing pseudoscalar-mass case. We can, therefore, conclude that the perturbative approach to the SU(3) Skyrmion problem is not successful.

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