Dray–'t Hooft geometries and the death of white holes

Steven K. Blau
Theoretical Division T8, Mail Stop B-285, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 28 November 1988)

Astrophysical white holes may be modeled as special cases of the Dray–'t Hooft geometries. With this observation one may better understand the physics underlying the death of these white holes.

In a discerning and provocative Letter,[1] Eardley considered astrophysical white holes and reasoned thus: a white hole is gravitationally attractive and so any ambient matter will be accelerated toward it. (By "astrophysical white hole," I mean the time-reversed picture of the gravitational collapse of matter to form a black hole. The paradigm of such collapse is Ref. 2. I distinguish between astrophysical white holes and the white-hole region of the extended Schwarzschild spacetime. In particular, the former has a nonsingular $r=0$ line and does not sample the full Schwarzschild geometry.) But, as no matter can cross the white-hole particle horizon, given sufficient time, a macroscopic mass will be accreted in an arbitrarily thin shell at the horizon. Now consider the fate of a spherical bit of matter ejected from the white hole. Just after this matter crosses the horizon its radius is less than the Schwarzschild radius of the enclosed material, which includes the original white-hole mass along with the accreted ambient matter. Thus, what was ejected from the white hole is refocused onto a future singularity and is unobservable from afar. This is the death of white holes. Since ambient material is condensed onto an arbitrarily thin shell, the amount of matter accreted may be arbitrarily small. (Astrophysical black holes which are the time-reversed picture of Oppenheimer-Snyder collapse have been studied in detail in Ref. 3.)

It may not be evident that the matter ejected from the white hole is trapped within a future horizon, and not within a perturbed past horizon. In any event, it seems remarkable that the future singularity formed by an arbitrarily small mass is powerful enough to suck in all the matter ejected by a large white hole. My interpretation as to how this comes about is somewhat unconventional. Eardley's original argument does not consider the question as to what happens if a white hole emits a significant portion of its mass in a single instant. Nor does it provide a quantitative framework for determining how long is "sufficiently long" for matter to be suitably accumulated, and for ascertaining the effect of not waiting "sufficiently long." All these concerns may be addressed by modeling an astrophysical white hole as a special case of the Dray–'t Hooft geometries.[4–6]

Dray–'t Hooft geometries are spacetimes containing several regions of Schwarzschild space sewn across null junctions of singular energy density. The simplest such geometry is easy to visualize. Consider the Schwarzschild line element written in null Kruskal-Szekeres coordinates:

$$ds^2=\frac{-32M^3}{R}e^{-R/2M}du\,dv+R^2(d\theta^2+\sin^2\theta\,d\phi^2) \tag{1}$$

with R a function of u and v given implicitly by

$$\left[\frac{R}{2M}-1\right]e^{R/2M}=-uv\ . \tag{2}$$

The extended Schwarzschild spacetime is illustrated in Fig. 1(a). The horizons are located at $u=0$ and $v=0$. At the horizon $v=0$, say, the operation $u\mapsto u+\Delta$ is a symmetry of the metric. Thus the sliding operation resulting in Fig. 1(b) yields a well-behaved spacetime manifold. The energy-momentum tensor vanishes everywhere in this spacetime except on the horizon where it is singular and given by

$$T=\frac{e}{256\pi M^4}\Delta\delta(v)\partial_u\otimes\partial_u\ .$$

The simplest Dray–'t Hooft geometry is two Schwarzschild spacetimes of the same mass separated by a singular energy density at the horizon. Two Schwarzschild spacetimes may also be connected across a null sheet at some $v=v_0$ which I take positive for definiteness. In this case the mass in the region $v>v_0$ must be greater than the mass in the region $v<v_0$. This is because an asymptotic observer in the region of large v will be passed by the incoming null sheet and will interpret the positive mass difference as the energy being carried by the sheet. Generic Dray–'t Hooft geometries in-

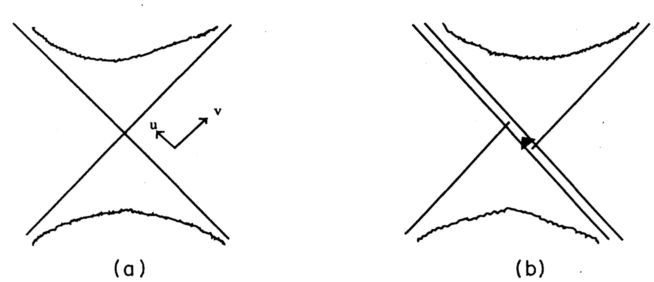

FIG. 1. The extended Schwarzschild spacetime (a) and the simplest of the Dray–'t Hooft geometries (b). The doubled line on the horizon of (b) represents the null surface of singular energy density.

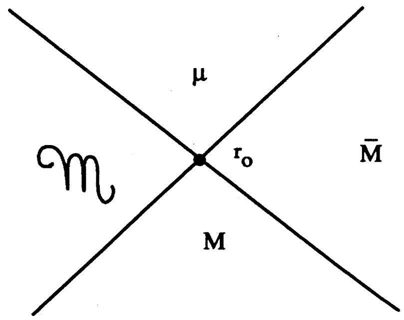

FIG. 2. The spacetimes obtained when two null sheets cross are specified by the masses of four distinct regions and the crossing radius r_0 of the two sheets.

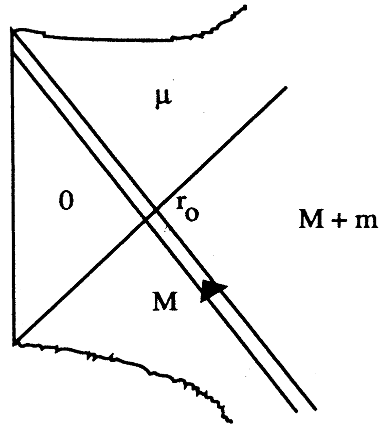

FIG. 3. The spacetime containing an astrophysical white hole of mass M ejecting all its material at once, along with an accreted shell of mass m.

volve a complicated pattern of crisscrossing null sheets, but for my purposes it suffices to consider just two crossing sheets. The spacetimes obtained may be enumerated with illustrations like Fig. 2 which display the masses of the four distinct spacetime regions, along with the radius r_0 at which the sheets cross. The causal and singularity structures of the resulting spacetimes are determined by these five parameters but there is a constraint:[6]

$$(r_0-2\bar{M})(r_0-2\mathcal{M})=(r_0-2\mu)(r_0-2M) \ . \tag{3}$$

This constraint allows one to put Eardley's argument on a quantitative footing. The realization of an astrophysical white hole as a Dray–'t Hooft geometry provides a framework for understanding the physics behind the argument.

If an astrophysical white hole is at all able to escape the death predicted it by Eardley, then the most efficient mechanism would be for the white hole to eject all its matter at a single instant along an outgoing null trajectory. Suppose that the white hole has lived sufficiently long before emitting its matter that ambient material has condensed into a thin sheet traveling inward along an (approximately) null trajectory. Let r_0 denote the radius at which the two trajectories cross. The spacetime obtained is illustrated in Fig. 3. In the figure, M is the mass of the white hole. An observer at asymptopia measures mass $M+m$ and so attributes a mass m to the incoming sheet of accreted matter. The leftmost wedge is flat space (i.e., has zero mass) because it is interior to both incoming accreting matter and the matter ejected from the white hole. The parameter μ represents the strength of the future singularity which forms when the incoming matter focuses on the origin and 2μ is the radius of the future horizon. If $2\mu > r_0$, all matter emitted from the white hole will be trapped. After defining ϵ by $r_0 \equiv 2(M+\epsilon)$, the constraint equation (3) gives

$$\mu=(M+\epsilon)\left[\frac{m}{\epsilon}\right] . \tag{4}$$

The condition that all matter ejected from the white hole be trapped is thus $\epsilon < m$. For larger ϵ matter ejected from the white hole can in principle escape, but a particular model of white-hole emission is needed in order to make a precise statement.

Determining the length of time a white hole must sit around before ejecting its material, in order that its matter be trapped, is no easy job; it depends strongly on one's picture of the ambient matter surrounding the white hole. (Indeed the use of the word "time" is problematic in general relativity, all the more in this case in which there are four different Schwarzschild times. For purposes of my estimate a crude usage of the word "time" should suffice. Precise statements can be made after studying in detail how the four Schwarzschild regions are sewn together. Note, however, that statements phrased in terms of the crossing radius r_0 do have invariant meaning.) For example, if the white hole sits in a medium of constant density all the way down to the past horizon, then all ejected matter is necessarily trapped, no matter when it is emitted. The reason is that a thin shell of radius ϵ at the horizon contains a mass of order $\sqrt{\epsilon}$. If, on the other hand, the white hole is immersed in a medium which is moving away from it, then sufficient accretion may never occur. In order to estimate the time scale involved in the death of white holes, I shall consider a white hole of mass M surrounded by a thin shell of mass m initially at rest. The trajectory of the matter in the shell is given by

$$E \equiv \left[1-\frac{2M}{r}\right]\frac{dt}{d\tau}=\text{const} , \tag{5}$$

with t the Schwarzschild time of the white-hole region and τ the proper time.[7] The time at which the shell approaches within a distance $2m$ of the white-hole horizon is a very weak function of E, provided E is neither very large nor very small. For small m, this time is well approximated by

$$M \ln\left[\frac{M}{m}\right]=5\times 10^{-6}\left[\frac{M}{M_\odot}\ln\left[\frac{M}{m}\right]\right] \text{sec} , \tag{6}$$

in accord with Eardley's estimate.[1]

With the help of Fig. 3 one can see how it is possible that the future singularity created by the collapse of a shell of arbitrarily small mass m can be powerful enough to converge all the matter ejected from an astrophysical white hole. This seems to me to be the essential physics behind the death of white holes. As the accreting matter accelerates toward r_0 its kinetic energy increases, yet the mass of the shell is always perceived to be m; evidently the increased kinetic energy is compensated by the increased (negative) binding energy between the shell and the white hole. Once r_0 is crossed, however, the in-going shell is interior to the white-hole matter. There is no more gravitational binding and effectively the mass of the shell suddenly increases to μ. The quantity $\mu - m$, then, is an invariant measure of this sudden loss of binding energy. Note that in the limit $\epsilon \mapsto \infty$ there is never any binding and $\mu \mapsto m$.

While I have focused on the gravitational binding of accreting matter, other authors have identified the increasing kinetic energy of the infalling matter as the "cause" of the death of white holes. The usual argument[1] begins with the observation that a translation of Schwarzschild time, $t \mapsto t + \Delta t$, becomes, in null Kruskal-Szekeres coordinates, $u \mapsto \exp(\Delta t/4M)$, $v \mapsto \exp(-\Delta t/4M)$. Given sufficient time, the accreting matter may be approximated as a null sheet at the horizon; the energy-momentum tensor then has only one nonvanishing component $T_{vv} = a\delta(v)$. Time translation corresponds to $a \mapsto a\exp(\Delta t/4M)$ and one observes an exponential "blue-shift" with time. In fact the T_{vv} of accreting matter is not changing with the (affine) time of the matter's geodesic. What is being compared are components of the energy-momentum tensor in different coordinate systems related by time translation. It is not clear how physics is to be gleaned from such a comparison.[1]

Because of the exponential blue-shift just described, the white-hole horizon is sometimes said to be unstable. I believe, at least in the context of Dray–'t Hooft geometries, that this characterization is unwarranted. (The stability of white-hole horizons in other contexts has been studied by several authors. See, for example, Refs. 8–11.) The reason is that exponential blue-shifts are found in Dray–'t Hooft geometries containing a region of flat space connected across a junction to Schwarzschild space.[12] Since such spacetimes need not have a white-hole horizon at all, it seems improper to interpret the blue-shift as indicating an instability of the past horizon. In my view, the instability that leads to the death of white holes is the mundane instability associated with the fact that gravitational interactions are attractive; given sufficient time, and a gravitating source, matter initially at rest tends to condense to arbitrarily large densities. The peculiar geometry of white holes gives two nonintuitive features to this condensation process. The matter accretes in a thin shell at the horizon instead of concentrating at the origin. And it was Eardley's insight that, since a white hole is enclosed within its horizon, this concentration could reconverge emergent matter and lead to the death of white holes.

Many of the ideas in this paper were crystalized in the course of conversations with Eduardo Guendelman and Alan Guth. I would also like to thank Emil Mottola and Stuart Raby for useful discussions.

[1] Douglas M. Eardley, Phys. Rev. Lett. **33**, 442 (1974).
[2] J. R. Oppenheimer and H. Snyder, Phys. Rev. **55**, 455 (1939).
[3] H. Tuohimaa, Lett. Nuovo Cimento **20**, 361 (1977).
[4] Ian H. Redmount, Prog. Theor. Phys. **73**, 1401 (1985).
[5] Tevian Dray and Gerard 't Hooft, Nucl. Phys. **B253**, 173 (1985).
[6] Tevian Dray and Gerard 't Hooft, Commun. Math. Phys. **99**, 613 (1985).
[7] Robert M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).
[8] Y. Gürsel *et al.*, Phys. Rev. D **19**, 413 (1979).
[9] Y. Gürsel *et al.*, Phys. Rev. D **20**, 1260 (1979).
[10] J. M. McNamara, Proc. R. Soc. London **A358**, 499 (1978).
[11] F. Tipler, Phys. Rev. D **16**, 3359 (1977).
[12] E. I. Guendelman, Alan H. Guth, and Steven K. Blau (in preparation).